BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade XII

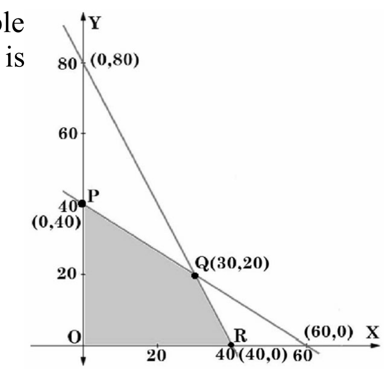
Class: - XII | **Grade 12 Preparatory EXAMINATION-02** | Date: -

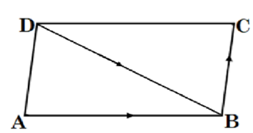
Time: - 3 hours | **SUBJECT: MATHEMATICS, SET A** | Marks: - 80

General Instructions:

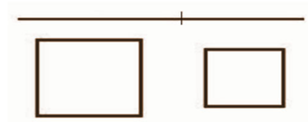
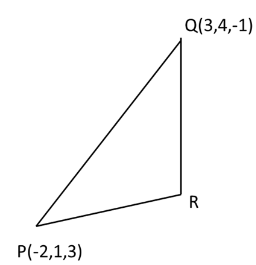
- This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

	Section A	
1	For the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible the value of λ is (a) 0 (b) 10 (c) R - {10} (d) R - {-10}	1
2	P is a square matrix such that P. (adj P) = $\begin{bmatrix} -2025 & 0 & 0 \\ 0 & -2025 & 0 \\ 0 & 0 & -2025 \end{bmatrix}$, then |P| + |adj P| = (a) $2025^2 \times 2024$ (b) -2024 (c) 2025×2024 (d) $(-2025)^2 + 2025$	1
3	The corner points of the bounded feasible region determined by a system of linear constraints are (0,3), (1,1) and (3,0). Let Z = $px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is (a) p = q (b) p = 2q (c) p = 3q (d) 2p = q	1
4	If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, the order of matrix B is (a) m × n (b) n × n (c) n × m (d) m ×m	1
5	If $AA^T = I$, then matrix A is called an orthogonal Matrix. Given that A is an orthogonal matrix, the value of | (adj (adj(A)) | is (a) 1 (b) ±1 (c) -1 (d) 0	1
6	If m and n, respectively, are the order and degree of DE $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^4\right) = 0$, then m + n = (a) 1 (b) 2 (c) 3 (d) 4	1
7	The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ (a) $e^{2\sqrt{x}}$ (b) $2\sqrt{x}$ (c) $e^{-2\sqrt{x}}$ (d) $-2\sqrt{x}$	1

8	$\int \frac{e^x}{x+1}\{1+(x+1)\log(x+1)\}\ dx$ (a) $e^x \log(x+1) + e^x + C$ (b) $e^x \log(x+1) + C$ (c) $\frac{e^x}{x+1} + C$ (d) $\frac{e^x x}{x+1} + C$	1
9	A problem in Mathematics is given to three students whose chances of solving it are ½, ⅓ and ¼ respectively. If the events of their solving the problem are independent, then the probability that the problem will be solved, is (a) ¼ (b) ⅓ (c) ½ (d) ¾	1
10	The value of k for which the function $\begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ is (a) 3 (b) -3 (c) 6 (d) -6	1
11	If A = $\begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of 2a – (b + c) is (a) 10 (b) -10 (c) 0 (d) 1	1
12	If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots\ldots\ldots \infty}}}$ then $\frac{dy}{dx}$ is (a) $\frac{\sin x}{2y-1}$ (b) $\frac{\cos x}{2y-1}$ (c) $\frac{\cos x}{y-1}$ (d)none of these	1
13	For an LPP, the objective function is Z= 4x + 3y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.  Which one of the following statements is true? (a) Maximum value of Z is at R (b) Maximum value of Z is at Q (c) Value of Z at R is less than the value at P (d) Value of Z at Q is less than the value of R	1
14	f(x) = x^x has a stationary point at (a) x = e (b) x = $\frac{1}{e}$ (c) x = – e (d) x = $\sqrt{e}$	1
15	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases (in cm^2/sec) when the side is 10 cm is (a) 10 (b) $\sqrt{3}$ (c) $10\sqrt{3}$ (d) $\frac{10}{3}$	1
16	Area bounded by the curve $y = x^3$, the x-axis and the ordinates x = -2 and x = 1 is (a) – 9 (b) $\frac{-15}{4}$ (c) $\frac{15}{4}$ (d) $\frac{17}{4}$	1

17	$\int \frac{1}{x\,(x^5+1)}\,dx$ (a) $\frac{1}{5}\log\left\|\frac{x^5}{x^5+1}\right\| + c$ (b) $\frac{1}{5}\log\left\|x^5\,(x^5+1)\right\| + c$ (c) $\frac{1}{5}\log\left\|\frac{x^5+1}{x^5}\right\| + c$ (d) None of these	1
18	If α, β and γ are the angles which a line makes with the positive directions of x, y, z axes respectively, then which of the following is **not** true? a) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ c) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ d) $\cos\alpha + \cos\beta + \cos\gamma = 1$	1
	Assertion Reason Type Questions Directions: In the following questions, a statement of assertion (A) is followed by a statement of the reason (R). Mark the correct choice as: (a) Both assertion and reason are true and reason is the correct explanation of assertion. (b) Both assertion and reason are true but reason is not the correct explanation of assertion. (c) Assertion is true but reason is false. (d) Assertion is false but reason is true.	
19	Assertion: The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is $\frac{\pi}{3}$. Reason: If $\tan^{-1}(x) = y$, then the principal value of y is $\frac{-\pi}{2} < y < \frac{\pi}{2}$	1
20	Assertion (A): The function $f(x) = (\|x-6\| \cos x)$ is differentiable in $R - \{6\}$. Reason (R): If a function f is continuous at a point c, then it is also differentiable at that point.	1
	Section B	
21	Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$ in the simplest form. Justify. **OR** Draw the graph of $\cos^{-1}x$ where $x \in [-1, 0]$. Also write the range of $\cos^{-1}x$, $\tan^{-1}x$.	2
22	Check the differentiability of the function $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at x =1.	2
23	Find the interval(s) in which the function $f(x) = x^2e^{-x}$ is strictly increasing. **OR** Show that the function $\left(\frac{4\sin x}{2+\cos x} - x\right)$ is strictly increasing in $\left[0, \frac{\pi}{2}\right]$.	2
24	The diagram given represents a parallelogram ABCD. If $\overrightarrow{AB} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\overrightarrow{DB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the area of the parallelogram ABCD.  	2
25	Find the coordinates of the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at the distance of 5 units from the point (1,3,3).	2

	Section C	
26	If x= a cos θ + b sin θ and y = a sin θ − b cos θ then show that $\frac{dy}{dx} = \frac{-x}{y}$ and hence prove that $y^2 \frac{d^2y}{dx^2}$ - $x\frac{dy}{dx} + y = 0$.	3
27	A and B throw a die alternatively till one of them gets a 'six' and wins the game. Find their respective probabilities of winning, if A starts the game. **OR** There are four cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn. Find the probability distribution of X and the mean of X.	3
28	Find $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx$	3
29	Find the general solution of the DE: $\left(x^2y + yx\sqrt{y^2-x^2}\right)dx - x^3\,dy = 0$. **OR** Solve the DE: $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$	3
30	Solve the LPP graphically: Maximise Z = 50x + 30y, subject to constraints: $x + 2y \le 12$, $2x + y \le 12$, $4x + 5y \ge 20$, $x \ge 0$, $y \ge 0$.	3
31	Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ **OR** Evaluate $\int_{-1}^{2} \lvert x^3 - x \rvert\, dx$	3
	Section D	
32	Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) ⇔ ad (b + c) = bc (a + d) for all (a, b), (c, d) ϵ N × N. Show that R is an equivalence relation on N × N. **OR** Show that the function f: R → R defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor Onto. Further find Set A so that the given function f: R → A becomes an onto function.	5
33	If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: 3x + 4y + 7z = 14, 2x – y + 3z = 4, x + 2y – 3z = 0.	5

34	Using Integration, find the area of the triangle whose vertices are (-1,1), (0,5) and (3,2). Draw the rough sketch and show necessary steps. **OR** Draw the rough sketch of the curve y = 10 cos 2x ; where $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$. Using integration, find the area of region bounded by y = 10 cos 2x from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$ and the x-axis.	5
35	Find the Cartesian equation of a line L_2 which is the mirror image of the line L_1 with respect to line L: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line L_1 passes through the point P (1, 6, 3) and is parallel to line L.	5
	Section E	
	Case Study 1	
36	Simran cuts a metallic wire of length 'a' metre into two pieces. She uses both pieces to create two squares of different side lengths. Assume that the wire of length '*x*' metres be used to make the first square. 	
(i)	Express the side lengths of both the squares in terms of 'a' and '*x*'. Find an expression for the combined area (A) of both the squares as a function of *x*.	2
(ii)	Using derivatives, determine the side lengths of both the squares (in terms of '*a*') for which the Combined area A is minimum. Justify. **OR** Using derivatives, find the minimum value of the combined area of both squares in terms of '*a*'. Justify.	2
	Case Study 2	
37	The flight path of two aeroplanes in a flight simulator game are shown. The coordinates of the airports P (-2, 1, 3) and Q (3, 4, -1) are given. Aeroplane 1 flies directly from P to Q. Aeroplane 2 that starts from P has a layover at R and then flies to Q.  The Path of Aeroplane 2 from P to R can be represented by the vector $5\hat{i} + \hat{j} - 2\hat{k}$. (Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)	
(i)	What is the angle between the flight paths of Aeroplane 1 and Aeroplane 2 just after take-off?	2
(ii)	Write the vector representing the path of Aeroplane 2 from R to Q. Consider that Aeroplane 1 started the flight with a full fuel tank. Find the position vector of the point where a third of the fuel runs out if the entire fuel is required for the flight.	2

	Case Study 3	
38	A company conducts a mandatory health check-up for all newly hired employees, to check for infections that could affect other regular employees. A blood infection affects roughly 5% of the population. The probability of a false positive on the test for this infection is 4%, while the probability of a false negative on the test is 3%.  (Note: A false positive on a test refers to a case when a person is not infected, but tests positive for the infection. A false negative on a test refers to a case when a person is infected, but tests negative for the infection.)	
(i)	What is the probability that a person tests positive given that he is actually infected? What is the probability that the employee tests positive for the infection?	2
(ii)	If a person tests positive for the infection, (a) what is the probability that the employee is infected? (b) What is the probability that employee is not infected? **OR** What is the probability that a person tests negative for the infection? If a person tests negative for the infection, what is the probability that the employee is actually infected?	2

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