BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade XII

Class: - XII **SUBJECT: Applied Mathematics** **Date: - 20.12.2024**
Time: - 3 hours **Code - (241) SET-2** **Marks: - 80**

MARKING SCHEME

SECTION – A

1.	A)	0	1
2.	B)	360 litres	1
3.	B)	38	1
4.	A)	A is a symmetric matrix	1
5.	D)	to each element of a row (or column) is added equi-multiples of the corresponding elements of another row or column	1
6.	C)	289	1
7.	A)	-6/25	1
8.		$\frac{1}{4}\log(x^4+1)+C$	1
9.	D)	2.4	1
10.	A)	0.6826	1
11.	B)	(14.1775, 15.8225)	1
12.	A)	Null hypothesis: H0: μ=150, Alternative hypothesis: H1: μ≠150	1
13.	A)	Trend	1
14.	C)	irregular variations	1
15.	C)	the value of the perpetuity will increase	1
16.	B)	30%	1
17.	A)	Rs. 8,100	1
18.	B)	(5, 1)	1
19.	A)	Both Assertion and Reason are true, and the Reason is the correct explanation of the Assertion.	1
20.	D)	The Assertion is false, but the Reason is true	1

SECTION – B

21.	$x = e^{\frac{x}{y}} \Rightarrow \log x = \log e^{\frac{x}{y}} \Longrightarrow \log x = \frac{x}{y} \Longrightarrow y.\log x = x$ Differentiate with respect to x, $y.\frac{1}{x} + \log x.\frac{dy}{dx} = 1$ $\Longrightarrow \frac{dy}{dx} = \frac{x-y}{x.\log x}$	$\left(\frac{1}{2}\right)$ (1) $\left(\frac{1}{2}\right)$
22.	Take $t = e + \log x \Longrightarrow dt = \frac{dx}{x}$. Also (x: 1, e; t: e, 1 + e) $I = \frac{1}{3}\int_e^{1+e} t\,dt = \frac{1}{3}\left[\frac{t^2}{2}\right]_e^{1+e} = \frac{1}{6}(1+2e).$	$\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$ (1)
	Take $u = \log x$ and $dv = x^2\,dx,$	(1/2)

x	1	e
t	e	$1+e$

	we get $du = \frac{1}{x}dx$ and $v = \frac{x^3}{3}$ $\int x^2.\log x\,dx = \frac{x^3}{3}.\text{Log}\,x - \int \frac{x^3}{3}.\frac{1}{x}dx = \frac{x^3}{3}\left(\log x - \frac{1}{3}\right) + c$	(1/2) (1)
23.	$\mu = \bar{x} \pm M.E = 2000 \pm 1.669 \times \frac{100}{\sqrt{64}}$ $= 2000 \pm 20.8625$ $= (1979.1375, 2020.8625)$	(1) (1)
24.	$Annual\ depreciation = \frac{original\ cost - scrap\ value}{useful\ life}$ $useful\ life = \frac{1{,}20{,}00{,}000 - 24{,}00{,}000}{8{,}00{,}000} = 12\ years$	(1/2) $\left(1 + \frac{1}{2}\right)$
	$effective\ rate\ of\ return(per\ rupee) = \left(1 + \frac{r}{100p}\right)^p - 1$ $= (1.0225)^4 - 1 = 0.093$ $\therefore effective\ rate = 9.3\%$	(1/2) (1) (1/2)
25.	Maximise $Z = 30x + 40y$ Subject to the constraints $2x + 3y \le 120;$ $3x + 2y \le 100;$ $x \ge 0, y \ge 0$	(1/2) (1/2) (1/2) (1/2)

SECTION – C

26.	$30\%\ of\ (x + 1725) < 50\%\ of\ 1725 < 40\%\ of\ (x + 1725)$ $\Longrightarrow 3x + 5175 < 8625 < 4x + 6900$ $\Longrightarrow 431.25 < x < 1150$	(1) (1) (1)
27.	$5A + 3B + 2C = 0 \Longrightarrow \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + 2C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Longrightarrow C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$	(2) (1)
	$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}; 4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}; 5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\left(1\frac{1}{2}\right)$ $\left(1\frac{1}{2}\right)$
28.	$np + npq = 25 \longrightarrow (1)$ $(np)(npq) = 150 \longrightarrow (2)$ $solving\ (1)\ \&\ (2), we\ get: q = \frac{2}{3}, p = \frac{1}{3}$ $sub\ p\ \&\ q\ in\ (1) \Rightarrow n = 45$ Number of trails = 45, probability of success = $\frac{1}{3}$	(1/2) (1) (1/2) (1)
	Given $\mu = 75, \sigma = 4, Z = \frac{X-75}{4}$ $(i)\ P(X < 65) = P(Z < -2.5) = 0.0062$ $(ii) P(X > 80) = P(Z > 1.25) = 1 - P(Z < 1.25) = 0.1056$ $(iii)\ P(70 < X < 85) = P(-1.25 < Z < 2.5)$ $= P(Z < 2.5) - P(Z < -1.25)$ $= 0.998 - 0.1056 = 0.8924$	(1) (1) (1)

29.		1+1+1 (total + Average + Centered average)

Month	4-monthly total	4-monthly average	4-monthly centered average
Jan			
Feb	-2	-0.5	
Mar	13	3.25	1.375
Apr	30	7.5	5.375
May	50	12.5	10
Jun	63	15.75	14.125
Jul	66	16.5	16.125
Aug	59	14.75	15.625
Sep	41	10.25	12.5
Oct	20	5	7.625
Nov			
Dec			

30.	A = 3,30,000-20,000 = 3,10,000; $i = \frac{5}{100}$; $n = 7$ $A = R\left[\frac{(1+i)^n - 1}{i}\right]$ $\Rightarrow 3{,}10{,}000 = R\left(\frac{0.407}{0.05}\right)$ $\Rightarrow R = 38{,}083.53$	(1/2) (1/2) (1) (1)
31.	$CAGR = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1 \Rightarrow 8\% = \left(\frac{2{,}50{,}000}{1{,}00{,}000}\right)^{\frac{1}{n}} - 1$ $\Rightarrow n = \frac{\log(2.5)}{\log(1.08)} = 11.73 = 12\ years(app)$	(1/2 + 1/2) (2)
	(i) $P(1-i)i = 7{,}20{,}000 \longrightarrow (1)$; $P(1-i)^2 i = 6{,}48{,}000 \longrightarrow (2)$ Dividing (2) by (1), $i = \frac{1}{10} \Rightarrow r = 10\%$ (ii) $sub\ i = \frac{1}{10}\ in\ (1) \Rightarrow P = 80{,}00{,}000$	(1) (1) (1)

SECTION – D

32.	Pipe A, B fills the tank in 12, 18 hours, and Pipe C empties the tank in 24 hours. Rate of Pipe A, B and C are $\frac{1}{12}, \frac{1}{18}$ and $-\frac{1}{24}$ tank per hour. **Work Done in the First 3 Hours (Pipe A and Pipe B Open)** Combined amount of A and B filled in 3 hours = $3 \times \frac{5}{36} = \frac{5}{12}$ **Work Done in the Next 2 Hours (Pipe A, Pipe B, and Pipe C Open)** Combined Amount of A, B and C filled in 2 hours = $2 \times \frac{7}{72} = \frac{7}{36}$ **Work Done in the Next 1 Hour (Pipe B and Pipe C Open)** Combined Amount of B and C filled in 1 hour = $1 \times \frac{1}{72} = \frac{1}{72}$ **Work Done in the last 1 Hour (Only Pipe C Open)** Pipe C empties the tank in 1 hour = $-\frac{1}{24}$ **Work done at the end of 7 hours** = $\frac{5}{12} + \frac{7}{36} + \frac{1}{72} - \frac{1}{24} = \frac{7}{12}$ To determine how long it will take to fill the remaining $\frac{5}{12}$ of the tank. Since Pipe C is emptying the tank at $-\frac{1}{24}$ of the tank per hour, and no other pipes are filling the tank, the remaining of the tank will never be	(1/2) (1/2) (1/2) (1/2) (1/2) (1) (1/2) (1)

	filled, while only Pipe C is open. Therefore, the tank cannot be completely filled under these conditions.	
33.	Let T be the temperature of the body of the victim and S be the room temperature. $$\frac{dT}{dt} = -k(T - S) \Rightarrow T = S + Ce^{-kt} \rightarrow (1)$$ At t = 0, S = 68, T = 85 from (1) ⇒ $C = 17$. $\therefore T = 68 + 17e^{-kt} \rightarrow (2)$ At t = 2, T = 78 from (2) ⇒ $k = -0.227$ $\therefore T = 68 + 17e^{0.227t} \rightarrow (3)$ At T = 98.6 from (3) ⇒ $t = 2.59$ *hours* i.e., 2 hours 36 minutes (app) Time of death of the victim is 2 hrs 36 mins before 8:00 AM = 5:24 AM	$(1\frac{1}{2})$ (1) (1) (1) (1/2)
	$$\frac{dy}{dt} - \frac{t}{1+t^2}y = \frac{2t}{1+t^2}$$ $$\Rightarrow P = -\frac{t}{1+t^2} \ \& \ Q = \frac{2t}{1+t^2},$$ $$I.F = \frac{1}{\sqrt{1+t^2}}$$ General solution is $\frac{y}{\sqrt{1+t^2}} = -\frac{2}{\sqrt{1+t^2}} + c \Rightarrow y = -2 + c\sqrt{1+t^2}$ When $y(0) = -1 \Rightarrow c = 1 \therefore y = -2 + \sqrt{1+t^2}$ Now $y(1) = -2 + \sqrt{1+1^2} = -2 + \sqrt{2}$	(1/2) (1/2) (1) $(1\frac{1}{2})$ (1) (1/2)
34.	(i) $P(X = 5) = \frac{3^5e^{-3}}{5!} = 0.101$ (ii) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ $= \frac{3^0e^{-3}}{0!} + \frac{3^1e^{-3}}{1!} + \frac{3^2e^{-3}}{2!} = 0.4233$ (iii) $P(X > 4) = 1 - P(X \leq 4)$ $= 1 - \big(P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)\big)$ $= 1 - 0.9836 = 0.0164$	(1) (2) (2)
35.	Corner points: A (10, 0) → $Z = 24x + 36y$ = 240; B (2, 4) → 192; C (1, 5) → 204; D (0, 8) → 288 The feasible region is unbounded region. The minimum value is 192. We draw the line $24x + 36y = 192$ $(i.e., 2x + 3y = 16)$ and note that the half plane $2x + 3y < 16$ has no common point with the feasible region. So, Z has minimum value of 192 at the point (2, 4).	Graph $(1\frac{1}{2})$ Table $(1\frac{1}{2})$ (1) (1)
	Let x and y be the number of units of product 1 and product 2 respectively. *objective function is maximise the profit,* $Z = 120x + 100y$ Constraints are $4x + 3y \leq 200;$ $3x + 4y \leq 180;$ $2x + 3y \leq 150;$ $x \leq 30;$ $y \leq 40;$	Graph $(1\frac{1}{2})$ $(1\frac{1}{2})$

Table for Q35 (corner points):

Corner points	$Z = 24x + 36y$
A (10, 0)	240
B (2, 4)	192
C (1, 5)	204
D (0, 8)	288

	$x \geq 0, y \geq 0$ <table><tr><td>Corner points</td><td>$Z = 120x + 100y$</td></tr><tr><td>(30, 40)</td><td>7600</td></tr><tr><td>(30, 30)</td><td>6600</td></tr><tr><td>(20, 35)</td><td>5900</td></tr><tr><td>(25, 30)</td><td>6000</td></tr></table>Maximum profit is Rs. 7600 attained at the point (30, 40).	Table ($1\frac{1}{2}$) (1/2)

SECTION – E

36.	(i) Circumference = $2\pi r \Rightarrow x = 2\pi r$ (ii) Area of the circle = $\frac{1}{4\pi}x^2$ sq.units (OR) Area of the square = $\frac{1}{16}(28-x)^2$ sq.units (iii) Combined area = $\frac{1}{4\pi}x^2 + \frac{1}{16}(28-x)^2$ $\frac{dA}{dx} = \frac{x}{2\pi} - \frac{1}{8}(28-x) = 0 \Rightarrow x = \frac{28\pi}{4+\pi}$ is the critical point. $\frac{d^2A}{dx^2} = \frac{1}{2\pi} + \frac{1}{8} > 0$. So, area is minimum when $x = \frac{28\pi}{4+\pi}$	(1) (1) (1) (1) (1)
37.	(i) Let x and y denotes the length and breadth of the plot. $(x-45)(y+45) = xy \Rightarrow x - y = 45 \longrightarrow (1)$ $(x-20)(y-10) = xy - 4600 \Rightarrow x + 2y = 480 \longrightarrow (2)$ (ii) $\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45 \\ 480 \end{bmatrix}$ (iii) Area of the plot = $xy = (190)(145) = 27550\ m^2$	 (1) (1) (1) (1)
38.	(i) $EMI = \frac{P \times r \times (1+r)^n}{(1+r)^n - 1} = \frac{40,000 \times 0.01 \times 1.26824}{1.26824 - 1} =$ $Rs. 1,888\ (app)$ (ii) Total payment at the end of 2 years = $24 \times 1,888 =$ $Rs. 45,312$(app) (iii) Total interest paid at the end of 2 years $= Rs. 45,312 - Rs. 40,000$ $= Rs. 5,312 (app)$	(2) (1) (1)