BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)

CLASS: XII **SUBJECT: MATHEMATICS** **DATE: 20.12.2024**

TIME: - 3 HRS **CODE (041)(SET-2)** **MARKS: - 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:
(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion - Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) - type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) - type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) - type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study - based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

Section A $[\mathbf{1 \times 20 = 20}]$

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

1. The principal value of $\operatorname{cosec}^{-1}(2)$is

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{6}$

2 The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

a) $\cos x - \sin\left(\frac{y}{x}\right)$ b) $\frac{y}{x}$ c) $\frac{x^2+y^2}{xy}$ d) $\cos^2\left(\frac{x}{y}\right)$

3 Which of the following is true for the function f(x) = 9x - 5?

a) f(x) is strictly increasing on R
b) f(x) is decreasing on R
c) Both f(x) increasing on R and f(x) decreasing on R are false
d) f(x) is strictly decreasing on R

4 The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is

a) $-\frac{\pi}{3}$ b) 0 c) $\frac{2\pi}{3}$ d) $\frac{\pi}{3}$

5 Which of the following functions is decreasing on $(0, \frac{\pi}{2})$

a) sin2x b) tan x c) cos x d) cos3x

6 For what values of m are the points with the position vector $10i + 3j$, $12i - 5j$ and $mi + 11j$ collinear

a)-8 b) 8 c) 4 d) -4

7 The system of linear equations: $x + y + z = 0, 2x + y - z = 0$, $3x + 2y = 0$ has

a) No solution b) a unique solution

c) an infinitely many solutions d) none of these

8 If two events A and B are such that $P(A') = 0.3$ $P(B) = 0.4$ and $P(A \cap B')$ = 0. Then $P(B/A \cup B'))$ =

$a)\frac{1}{4}$ b) $\frac{1}{5}$ c) $\frac{3}{5}$ d) $\frac{2}{5}$

9 If A is a square matrix such that $(A - 2I)(A + I) = 0$ then $A^{-1} =$

a) $\frac{A-I}{2}$ b) $\frac{A+I}{2}$ c) 2(A-I) d) 2A+ I

10 If $\vec{a}$ and $\vec{b}$ are two unit vectors then which of the following values of $(\vec{a}.\vec{b})$ is not possible:

a) $\sqrt{3}$ b) $\sqrt{3}$ / 2 c) 1/$\sqrt{2}$ d) -1/2

11 The solution set of the inequality 3x + 5y < 4 is

a) an open half - plane not containing the origin.

b) an open half - plane containing the origin.

c) a closed half plane containing the origin.

d) the whole XY - plane not containing the line 3x + 5y = 4.

12 $\int \frac{(10x^9+10^x \log_e 10)\, dx}{x^{10}+ 10^x}$ is :

a) $10^x - 10^x$ + C b) $10^x + 10^x$ +c c) ($x^{10} + 10^x$)$^{-1}$ +c d)log ($x^{10} + 10^x$)+c

13 If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$ then find the value of K

a) 1 b) 2 c) -2 d) 4

14 If A = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then A^{16} is equal to

a)$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

15 If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying $AA^T = 9 I_3$ then the values of a & b respectively are

a)1,2 b) -2,-1 c) -1,2 d) -2,1

16 If a point (h, k) satisfies an in equation $ax + by \geq 4$ then the half plane represented by the inequation is

a) The half plane containing the point (h , k) but excluding the points on $ax + by = 4$

b) The half plane containing the point (h , k) and the points on $ax + by = 4$

c) Whole xy plane

d) none

17 The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

a) $20\pi^2$ sq. units b) 25π sq. units

c) 20π sq. units d) $16\pi^2$ sq. units

18 If y = x^x find $\frac{d^2y}{dx^2}$

a) 0 b) x^x (1 - log x) ^{2} + $\frac{1}{x}$

c) x^x (1 + log x) ^{2} + $\frac{1}{x}$ d) x^x (1 + log x) ^{2} - $\frac{1}{x}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two

statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

a) Both (A) and (R) are true and (R) is the correct explanation of (A).

b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

c) (A) is true but (R) is false.

d) (A) is false but (R) is true

19 **Assertion (A):** The function f: R→ R given by f(x) = x^3 is injective

Reason (R): The function f: X → Y is injective if f(x) = f(y) implies x = y , x,y ∈ X

Assertion (A):

20

Assertion (A): A function f(x) = $\begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is discontinuous at x = 0.

Reason (R): The function f(x) = $\begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ is continuous for all values of x.

Section B $[\mathbf{2 \times 5 = 10}]$

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21 Write the interval for the principal value of function and draw its graph: $sin^{-1}x$.

22 If $y = sin^{-1}(\sqrt{x})$, then find $\frac{dy}{dx}$.

OR

If $siny = xsin(a + y)$, then prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$.

23 The total revenue in rupees received from the sale of x units of the product is given by$R(x) = 13x^2 + 26x + 15$. Find Marginal Revenue when 17 units are produced.

24 Find a unit vector in the direction of $\overrightarrow{AB}$,where $A(1,2,3)$ and B (4, 5, 6)are the given points.

25 If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$,$\vec{b} = \hat{4\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude 6 units which is parallel to a vector$2\vec{a} - \vec{b} + 3\,\vec{c}$.

OR

If A(2, 3, 4), B(5, 4, - 1), C(3, 6, 2) and D(1, 2, 0) be four points, show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$

Section C $[\mathbf{3 \times 6 = 18}]$

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26 A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

27 Show that the semi - vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

28 Evaluate:$\int \frac{2x}{x^2+7x+10}dx$

OR

Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{sin2x}}\, dx$

29 Solve the following LPP graphically:

Maximize Z = 5x + 3y

Subject to

3x + 5y ≤ 15

5x + 2y ≤ 10

and, x, y ≥ 0

30 Find the angle between the vectors (a + b) and (a – b) if a = (2i – j + 3k) and b = (3i + j – 2k) and hence find a vector perpendicular to both (a + b) and (a – b).

OR

Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ *and* $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\,\vec{a} = 21$.

31 An insurance company insured 3000 scooters, 4000 cars and 5000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0.02, 0.03 and 0.04 respectively. One of the insured vehicles meet with an accident. Find the probability that it is a scooter.

OR

Three cards are drawn one by one with replacement, from a well shuffled deck of 52 cards. Find the probability distribution of number of spades. Also find the mean of the distribution.

SECTION D **[5 × 4= 20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32 Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}$ y = x in the first quadrant, using integration.

33 Using matrices, solve the following system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$,

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$,

$\frac{6}{x} + \frac{9}{y} + \frac{-20}{z} = 2$

34 Find the particular solution of the differential equation: $tanx.\frac{dy}{dx} = 2xtanx + x^2 - y$;

$(tanx \neq 0)\ given\ that\ y = 0\ when\ x = \frac{\pi}{2}$

OR

Find the general solution of the differential equation $(x - y)\frac{dy}{dx} = x + 2y$.

35 Find the shortest distance between the lines l_1 and l_2 whose vector equationsare

$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ ...(1)

and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$ ...(2)

OR

Find the perpendicular distance of the point (1, 0, 0) from the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

Section E $[\mathbf{4 \times 3 = 12}]$

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36 **Case Study-1**

Sherlin and Danju are playing Ludo at home during Covid - 19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set 1, 2, 3, 4, 5, 6. Let A be the set of players while B be the set of all possible outcomes.

A = S, D, B = 1, 2, 3, 4, 5, 6

i. Let R : B→ B be defined by R = (x, y): y is divisible by x. Determine whether R is Reflexive, symmetric or transitive. (1)

ii. Raji wants to know the number of functions from A to B. How many number of functions are possible? (1)

iii. Let R be a relation on B defined by R = (1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5). Then describe R . (2)

OR

Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible? (2)

37 **Case Study-2:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

i. Find the volume of the open box formed by folding up the cutting each corner with x cm. (1)

ii. Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (1)

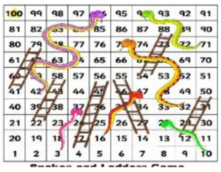
iii. Verify that volume of the box is maximum at x = 3 cm by second derivative test? (2)

OR

Find the maximum volume of the box. (2)

38 **Case Study-3:**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.

Aditya rolled down both black and red die together.

First die is black and second is red.

i. Find the conditional probability of obtaining the sum 10, given that the black die resulted in even number. (2)

ii. Find the conditional probability of obtaining the doublet, given that the red die resulted in a number more than 4. (2)