INFINITY THINK BEYOND............
AN EDUCATIONAL INSTITUTE

CBSE Class XI Mathematics
Sample Paper 02
As per pattern issued by CBSE for (2023-24)

Maximum Marks : 80 **Time : 3 hrs.**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
6. Section E has 4 questions carrying 05 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION A
(Question 1 to 20 carry 1 mark)

Q1. Total number of elements in the power set of A containing 15 elements is

a)2^{15}	b) 15^2
c) $2^{15}-1$	d)$2^{15}-1$

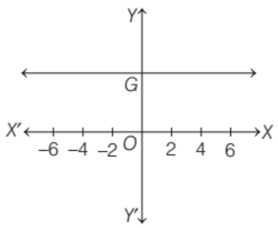
Q2. If $((1-\mathrm{i})^4 = a + \mathrm{i}b$ then the value of a and b are respectively

a)-4,0	b) 0,-4
c)4,0	d) 0,4

Q3. If a relation R is defined on the set Z of integers as follows $(a,b) \in R \Leftrightarrow a^2 + b^2 = 25$,domain is

a){3,4,5}	b) {0,3,4,5}
c)$\{0, \pm 3, \pm 4, \pm 5\}$	d)None of these

Q4. If G represents the name of the function in given graph, then G is a/an



a)identity function	b) constant function
c)modulus function	d)none of these

Q5.Range of tan x

a)R	b)R- (-1,1)
c)R-{0}	d)R-{1,-1}

Q6. The sum of first three terms of a GP is 13 /12 and their product is −1then the common ratio of the GP is :

a)-4/3 or -3/4	b) ¾ or 4/3
c)1/4 or -1/4	d)5/3 or -3/5

Q7. Let A = {x: x is a square of a natural number and x is less than 100} and B is a set of even natural numbers. The cardinality of A∩B is

a)4	b) 5
c)9	d)None of these

Q8. If (x+y)+i(x-y)=4+6i,then xy is

a)5	b) -5
c)4	d)-4

Q9. Slope of a line if angle of inclination with positive x axis is 135 degree.

a)0	b) not defined
c)1	d)-1

Q10. Gemoteric means between 3 and 96 are

a)6,12,24,28	b) 6,10,24,48
c)6,10,40,48	d)48,24,10,5

Q11. Find x if $\frac{x-2}{x+5} > 2$

a)[-12,-5)	b) [-12,-5]
c)(-12,-5)	d)none of these

Q12. In a GP, the 3rd term is 24 and the 6th term is 192. Then, the 10th term is

a)1084	b) 3290
c)3072	d)2340

Q13. The total number of terms in expansion of $(n + a)^{100} + (n - a)^{100}$ after simplification is

a)202	b) 51
c)50	d)none of these

Q14. The value of $\lim_{x\to 2} \cdot \frac{x^n-2^n}{x-2} = 80$,n is equal to

a)1	b) 3
c)5	d)7

Q15. The points A(x,4) ,B(3,-2),C (4,-5)are collinear in the value of x

a)1	b) -1
c)2	d)0

Q16. Approx value $(1 \cdot 1)^{10000}$ is

a)greater than 1000	b) less than 1000
c)equal to 1000	d)none of these

Q17. The number of permutation of word 'MESMERISE' is

a)$\frac{9!}{(2!)^2(3!)}$	b)$\frac{9!}{(2!)^3(3!)}$
c)$\frac{9!}{(3!)^2(2!)}$	d)$\frac{5!}{(2!)^2(3!)}$

Q18.Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits

a)1/16	b) 16/25
c)1/65	d)1/25

Q19. Assertion (A) The power set of the set {1, 2} is the set {ϕ, {1}, {1,2}, {2}}
Reason (R) The power set is set of all subsets of the set.

a)Both assertion and reason are true and reason is the correct explanation of assertion	b)Both assertion and reason are true but reason is not the correct explanation of assertion
c)Assertion is true but reason is false.	d) Assertion is false but reason is true.

Q20. Assertion (A) Slope of line 3x-4y+10 =0 is 3 /4 .
Reason (R) x-intercept and y-intercept of 3x-4y+10 = 0 respectively are − 10 /3 and 5 /2 .

a)Both assertion and reason are true and reason is the correct explanation of assertion	b)Both assertion and reason are true but reason is not the correct explanation of assertion
c)Assertion is true but reason is false.	d) Assertion is false but reason is true.

SECTION B
(Question 21 to 25 carry 2 mark)

Q21. Simplify $(x^2 - y)^4 - (x^2 + y)^4$ using binomial expansion.

Q22. Prove that :

$$\frac{1+\sin x-\cos x}{1+\sin x+\cos x}=\tan\frac{x}{2}$$

Q23. Find the derivative of f (x) = $\dfrac{x}{1+\tan x}$

Or

Find the value of $\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$

Q24. Prove that the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) are collinear. Find the ratio in which the first point divides the join of the other two.

Or

Verify that : (-1, 2, 1), (1,-2, 5), (4, -7 8) and (2, -3, 4) are the vertices of a parallelogram.

Q25. Find the mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean ?

SECTION C
(Question 26 to 31 carry 3 mark)

Q26. Find the domain and range of signum function .Also draw its graph

Q27. Find the distance of the point of intersection of the lines 2x-3y+5=0 and 3x+4y=0 from the line 5x-2y=0.

Q.28. If $\left(\frac{1+\mathrm{i}}{1-\mathrm{i}}\right)^3-\left(\frac{1-\mathrm{i}}{1+\mathrm{i}}\right)^3=x+\mathrm{i}y$,then find x and y

Q.29 Four cards are drawn at random from a pack of 52 cards. Find the probability of getting a) all the four cards of the same suit e) all cards of the same colour.

Q.30. Find the equation of the circle passing through the point (–1, 3) and having its centre at the point of intersection of the lines x −2y = 4 and 2x+5y=1

Q.31. Prove that $\cos 20^0\cos 40^0\cos 80^0\cos 60^0=\frac{1}{16}$

OR

Prove that $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)$

SECTION D
(Question 32 to 35 carry 4 mark)

Q.32 During the Mathematics class, A teacher clears the concept of permutations and combinations to the 11th class students. After the class was over he asks the students some more questions :

On the basis of the information given above answer the following:-

(a) Find the number of arrangements of the letters of the word INDEPENDENCE.

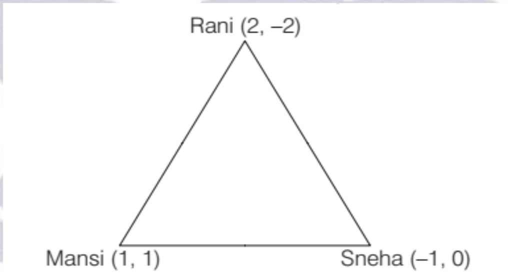
(b) In How many of these do the words begin with I and end in P.

(c) In How many of these do all the vowels never occur together.

OR

In How many of these do all the four E's do not occur together

Q.33. Three girls, Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.



(a)Find the equation of line formed by Rani and Mansi.

(b) Find slope of Equation formed by Rani and Sneha.

(c) Find equation of median of line through Rani

or

Find equation of altitude through Mansi.

Q34. Four candidates A, B, C and D are applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given same chance of being selected, while c is twice as likely to be selected as D, what are the probabilities that

(a)Find the probability that A is selected

(b)Find the probability that C is selected

(c) Find the Probability that A is not selected

or

Find the Probability that C is not selected

SECTION E
(Question 35 to 38 carry 5 mark)

Q35. Find the value of the expression

$$cos^4\frac{\pi}{8} + cos^4\frac{3\pi}{8} + cos^4\frac{5\pi}{8} + cos^4\frac{7\pi}{8}$$

Or

Prove that $sin\, 20^0\, sin\, 40^0\, sin\, 60^0\, sin\, 80^0 = \frac{3}{16}$

Q36.A line is such that its segment between the straight lines 5x-y+4=0 and 3x+4y-4=0 is bisected at the point (1,5). Obtain its equation

Or

Assuming that straight lines work as the plane mirror for a point , find the image of the point (1,2) in the line x-3y+4=0.

Q37.Find the derivative of F(x)= xsinx by first principal.

Q38.The diameters of circles (in mm) drawn in a design are given below :

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

CBSE Class XI Mathematics Sample Paper 02 Answer key

SECTION – A
(Question number 1 to 20 carry 1 marks each)

Q1.	A
Q2.	A
Q3.	C
Q4.	B
Q5.	A
Q6.	A
Q7.	A
Q8.	B
Q9.	D
Q10.	A
Q11.	C
Q12.	C
Q13.	B
Q14.	C
Q15.	A
Q16.	A
Q17.	A
Q18.	B
Q19.	A
Q20.	B

SECTION – B
(Question number 21 to 25 carry 2 marks each)

Q21.Simplify $(x^2-y)^4-(x^2+y)^4$ using binomial expansion.

$$(x^2-y)^4-(x^2+y)^4$$

$$(x^8-8x^6y+24x^4y^2-32x^2y^3+16y^4)-(x^8+8x^6y+24x^4y^2+32x^2y^3+16y^4)$$

$$-16x^6y-64x^4y^2$$

Q22.Prove that : $\dfrac{1+\sin x-\cos x}{1+\sin x+\cos x}=\tan\dfrac{x}{2}$

Ans

$$LHS = \frac{1+\sin x-\cos x}{1+\cos x+\sin x}$$

$$= \frac{\sin x(1+\sin x-\cos x)}{\sin x(1+\cos x)+\sin^2 x}$$

$$= \frac{\sin x(1+\sin x-\cos x)}{\sin x(1+\cos x)+(1-\cos^2 x)}$$

$$= \frac{\sin x(1+\sin x-\cos x)}{(1+\cos x)(\sin x+1-\cos x)}$$

$$= \frac{\sin x}{1+\cos x}$$

$$= \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}$$

$$= \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$= \tan\left(\frac{x}{2}\right)$$

Q23.Find the derivative of f (x) = $\frac{x}{1+\tan x}$

F'(x) $= \frac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

Or

Find the value of $\lim_{x\to 1}\frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$

$$\lim_{x\to 1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}\right]$$

$$= \lim_{x\to 1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)(x-1)}\right]$$

$$= \lim_{x\to 1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\right]$$

$$= \lim_{x\to 1}\left[\frac{2x-3}{(2x+3)\left(\sqrt{x}+1\right)}\right]$$

= -1/(5)(2)

= -1/10

Q24. Prove that the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) are collinear. Find the ratio in which the first point divides the join of the other two.

AB = $\sqrt{(0-2)^2+(-1-1)^2+(-7+9)^2}$

$\sqrt{4+4+4}=2\sqrt{3}$

BC = $\sqrt{(2-6)^2+(1-5)^2+(-9+13)^2}$

$\sqrt{16+16+16}=4\sqrt{3}$

AC = $\sqrt{(0-6)^2+(-1-5)^2+(-7+13)^2}$

$\sqrt{36+36+36}=6\sqrt{3}$

Now, AB + BC = $2\sqrt{3}+4\sqrt{3}$

= $6\sqrt{3}$

= AC

Thus, A,B & C are collinear.

AB:AC = $2\sqrt{3}:6\sqrt{3}$

= 1:3

Thus, A divides BC externally in the ratio 1:3.

Or

Verify that : (-1, 2, 1), (1,-2, 5), (4, -7 8) and (2, -3, 4) are the vertices of a parallelogram.

Let A(–1, 2, 1), B(1, –2, 5), C(4, –7, 8) & D(2, –3, 4)
ABCD can be vertices of parallelogram only if opposite sides are equal.
i.e. AB = CD & BC = AD
Calculating AB
A ≡ (– 1, 2, 1) and B ≡ (1, – 2, 5)

Distance AB $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Distance AB $=\sqrt{(1-(-1))^2+(-2-2)^2+(5-1)^2}$

$=\sqrt{(2)^2+(-4)^2+(4)^2}=\sqrt{4+16+16}=\sqrt{36}=6$

Calculating BC ,B ≡ (1, – 2, 5) and C ≡ (4, – 7, 8)

Distance BC $=\sqrt{(4-1)^2+(-7-(-2))^2+(8-5)^2}$

$=\sqrt{(3)^2+(-5)^2+(3)^2}=\sqrt{9+25+9}=\sqrt{43}$

Calculating CD,C ≡ (4, – 7, 8) and D ≡ (2, – 3, 4)

Distance CD $= \sqrt{(2-4)^2 + (-3-(-7))^2 + (4-8)^2} = \sqrt{(-2)^2 + (4)^2 + (-4)^2}$

$= \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

Calculating DA.D ≡ (2, – 3, 4) and A ≡ (– 1, 2, 1)

Distance DA $= \sqrt{(-1-2)^2 + (2-(-3))^2 + (1-4)^2} = \sqrt{(-3)^2 + (5)^2 + (-3)^2}$

$= \sqrt{9 + 25 + 9} = \sqrt{43}$

Since AB = CD & BC = DA

So, In ABCD both pairs of opposite sides are equal.
Thus, ABCD is a parallelogram.

Q25. Find the mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean ?

Given data is, 3,10,10,4,7,10,5
Here, n=7, $\overline{x}$=(3+10+10+4+7+10+5)/7=7
∴ Mean deviation from mean is =(∑|x− $\overline{x}$|)/n=2.57

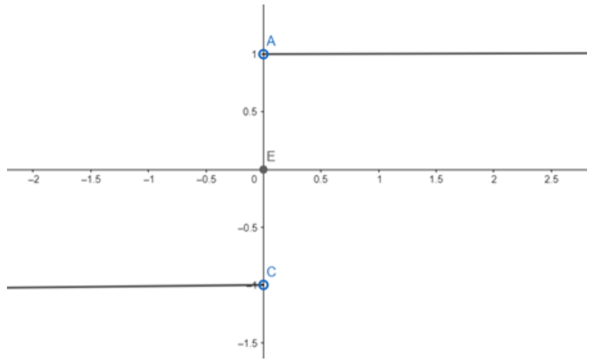
SECTION – C
(Question number 26 to 31 carry 3 marks each)

Q26.Find the and range of signum function .Also draw its graph
Ans)Signum function is often defined simply as 1 for x > 0 and -1 for x < 0. And for x = 0 it is 0.

$$f(x) = \begin{cases} \dfrac{|x|}{x}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases}$$

$$f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$$

,the domain is $(-\infty,\infty)$ and range is {1,-1,0}



Q27.Find the distance of the point of domain intersection of the lines 2x-3y+5=0 and 3x+4y=0 from the line 5x-2y=0.

Given equations are: 2x – 3y + 5 = 0 .....(i)

3x + 4y = 0 ......(ii)

From equation (ii) we get,

4y = – 3x

⇒ y = $\frac{-3}{4}x$ .....(iii)

Putting the value of y in eq. (i) we have

$2x-3\left(\frac{-3}{4}x\right)+5$ = 0

⇒ 8x + 9x + 20 = 0

⇒ 17x + 20 = 0

⇒ x = $\frac{-20}{17}$

Putting the value of x in equation (iii) we get

y = $\frac{-3}{4}\left(\frac{-20}{17}\right)$

⇒ y = $\frac{15}{17}$

∴ Point of intersection is $\left(-\frac{20}{17},\frac{15}{17}\right)$.

Now perpendicular distance from the point $\left(-\frac{20}{17},\frac{15}{17}\right)$ to the given line 5x – 2y = 0 is

$$\left|\frac{5\left(-\frac{20}{17}\right)-2\left(\frac{15}{17}\right)}{\sqrt{25+4}}\right|=\left|\frac{(-100)17-\frac{30}{17}}{\sqrt{29}}\right|$$

= $\frac{130}{17\sqrt{29}}$

Q28.If $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=x+iy$,then find x and y

Ans)x = 0, and y = -2

Q29.(a)

Probability= $\frac{\text{conditionalcase}}{\text{totalcase}}$

For this conditional case of selecting all four same suit, it can be either heart or spades or clubs or diamonds.

So, total number of conditional case=No. of case of selecting all 4 cards of hearts suit + no. of case of selecting all 4 cards of spades suit + no. of case of selecting all 4 cards of clubs suit + no. of case of selecting all 4 cards of diamonds suit

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4 = \frac{4 \times 13!}{9!4!}$

$= \frac{4 \times 13 \times 12 \times 11 \times 10 \times 9!}{9! \times 4 \times 3 \times 2 \times 1} = 2860$

Total number of cases = ${}^{52}C_4 = \frac{52!}{48!4!} = 270725$

So, probability of getting all 4 cards of the same suit = $\frac{2860}{270725} = 0.0106.$

Sol^n ⇒ Since 4 Cards are drawn from a well suffled pack of cards.

$n(S) = {}^{52}C_4$

Let E be the event of getting 4 cards of the same colour

Since there are 2 colours of cards

$n(E) = 2\ {}^{26}C_4$

$P(E) = 2\frac{{}^{26}C_4}{{}^{52}C_4} = 2 \times \frac{26 \times 25 \times 24 \times 23}{52 \times 51 \times 50 \times 49}$

$P(E) = \frac{92}{833}$ Ans

Q30.

Find the equation of the circle passing through the point (−1, 3) and having its centre at the point of intersection of the lines x −2y = 4 and 2x+5y=1

Let, the required equation of circle be —

$(x-h)^2 + (y-k)^2 = r^2$ —— (i)

where (h,k) is the centre and r = radius.

∴ The point of intersection of the given equation of lines,

$x - 2y = 4$

$2x + 5y + 1 = 0$ [Solving by cross multiplication method]

Point of intersection = $(2,-1)$

∴ Radius of circle $= \sqrt{(2+1)^2 + (-1-3)^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

From (i),

$(x+1)^2 + (y-3)^2 = 5^2$

$\Rightarrow x^2+1+2x+y^2+9-6y = 25 \Rightarrow x^2+y^2+2x-6y-15=0$

Q31.

L.H.S. = cos 20°. cos 40°. cos 60°. cos 80°

$= \cos 20^\circ . \cos 40^\circ . \frac{1}{2} . \cos 80^\circ$

$= \frac{1}{2 \times 2}(2\cos 40^\circ . \cos 20^\circ). \cos 80^\circ$

$= \frac{1}{4}[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)]. \cos 80^\circ$

$= \frac{1}{4}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ$

$= \frac{1}{4}\cos 60^\circ . \cos 80^\circ + \frac{1}{4}\cos 20^\circ . \cos 80^\circ$

$= \frac{1}{4}\left(\frac{1}{2}\right)\cos 80^\circ + \frac{1}{2 \times 4}(2\cos 80^\circ \cos 20^\circ)$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}[\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)]$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}(\cos 100^\circ + \cos 60^\circ)$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}\cos 100^\circ + \frac{1}{8}\cos 60^\circ$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}\cos(180^\circ - 80^\circ) + \frac{1}{8} \times \frac{1}{2}$

$= \frac{1}{8}\cos 80^\circ - \frac{1}{8}\cos 80^\circ + \frac{1}{16}$ ...[∵ cos(180° – θ) = – cos θ]

$= \frac{1}{16}$

= R.H.S.

OR

Prove that $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)$

LHS = Cosα +Cosβ + Cosγ + Cos(α + β+ γ)

= 2Cos((α + β)/2)Cos((α - β)/2) + 2 Cos((α + β+ 2γ)/2)Cos((-α - β)/2)

Using Cos(-A) = CosA

= 2Cos((α + β)/2)Cos((α - β)/2) + 2 Cos((α + β+ 2γ)/2)Cos((α + β)/2)

= 2Cos((α + β)/2) (Cos((α - β)/2) + Cos((α + β+ 2γ)/2))

= 2Cos((α + β)/2) (2 Cos((α + γ)/2)Cos((-β - γ)/2)

= 2Cos((α + β)/2) (2 Cos((α + γ)/2) (Cos((β + γ)/2))

= 4 Cos((α + β)/2)Cos((β + γ)/2)Cos((α + γ)/2)

= RHS

SECTION – D

(Question number 32 to 34 carry 4 marks each)

Q32. a)

Total arrangements $= \frac{12!}{3!4!2!}$

$= \mathbf{1663200}$

b)

Number of arrangements $= \frac{n!}{p_1!p_2!p_3!}$

$= \frac{11!}{4!\,3!\,2!}$

$= \mathbf{138600}$

c)

Required number of arrangements

$= \frac{8!}{3!\,2!} \times \frac{5!}{4!}$

$= \frac{(8\times7\times6\times5\times4\times3!)\times(5\times4!)}{3!\times4!\times2}$

$= \mathbf{16800}$

Q33. a)Find the equation of line formed by Rani and Mansi.(Ans: 3x+y-4=0)

b) Find slope of Equation formed by Rani and Sneha.(Ans: -2/3)
c) Find equation of median of line through Rani (Ans: 4y+5x-2=0)
or

Find equation of altitude through Mansi(Ans: 3x-2y-1 =0)

34.

Q34.
A is twice likely to be selected as B, P(A) = 2 P(B)
& C is twice likely to be selected as D, P(C) = 2 P(D)
It is given that B & C have about the same chance
Thus, P(B) = P(C)
Now, sum of all probabilities is 1,
Thus,
P(A) + P(B) + P(C) + P(D) = 1
P(A) + P(B) + P(B) + P(D) = 1
Thus,
P(A) + P(A)/2 + P(A)/2 + P(C)/2 = 1
[2 P(A) + P(A) + P(A) + P(B)] /2 = 1
4 P(A) + P(A) / 2 = 2
[8 P(A) + P(A)] / 2 = 2
9 P(A) = 4
P(A) = 4/9

a)Find the probability that A is selected=4/9
b)Find the probability that C is selected =2/9
c) Find the Probability that A is not selected=5/9
or
Find the Probability that C is not selected=7/9

SECTION – E
(Question number 35 to 38 carry 5 marks each)

Q35.Find the value of the expression

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

$\cos^4\pi/8 + \cos^4 3\pi/8 + \cos^4 5\pi/8 + \cos^4 7\pi/8$

$\Rightarrow \cos^4\pi/8 + \cos^4 3\pi/8 + \cos^4(\pi - 3\pi/8) + \cos^4(\pi - \pi/8)$

$\Rightarrow \cos^4\pi/8 + \cos^4 3\pi/8 + \cos^4\pi/8 + \cos^4 3\pi/8$

$\Rightarrow 2(\cos^4\pi/8 + \cos^4 3\pi/8)$

$\Rightarrow 2[\cos^4\pi/8 + \cos^4(\pi/2 - \pi/8)]$

$\Rightarrow 2[\cos^4\pi/8 + \sin^4\pi/8]$

$\Rightarrow 2[(\cos^2\pi/8 + \sin^2\pi/8)^2 - 2.\cos^2\pi/8.\sin^2\pi/8]$

$\Rightarrow 2[1 - 2.\cos^2\pi/8.\sin^2\pi/8]$

$\Rightarrow 2[1 - 1/2.4.\cos^2\pi/8.\sin^2\pi/8] = 2[1 - 1/2.(\sin^2\pi/4)^2]$

$\Rightarrow 2[1 - 1/2 \times 1/2] = 3/2$

Or

Prove that $\boldsymbol{\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}}$

L.H.S. = sin 20°· sin 40°· sin 60°· sin 80°

$= \frac{\sqrt{3}}{2} \cdot \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ \ldots.. \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{4}(2 \sin 40^\circ \cdot \sin 20^\circ) \cdot \sin 80^\circ$

$= \frac{\sqrt{3}}{4}[\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)] \times \sin 80^\circ$

$= \frac{\sqrt{3}}{4}[\cos 20^\circ - \cos 60^\circ] \cdot \sin 80^\circ$

$= \frac{\sqrt{3}}{8}[2 \sin 80^\circ \cdot \cos 20^\circ - 2 \cos 60^\circ \cdot \sin 80^\circ]$

$= \frac{\sqrt{3}}{8}\left[\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - 2 \times \frac{1}{2} \cdot \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8}[\sin 100^\circ + \sin 60^\circ - \sin 80^\circ]$

$= \frac{\sqrt{3}}{8}\left[\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ\right]$

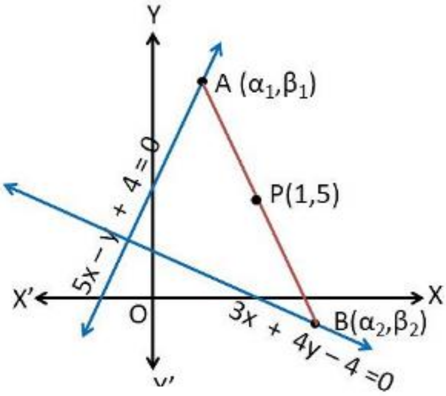
$= \frac{\sqrt{3}}{8}\left(\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ\right)$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{16}$

= R.H.S.

Q36.A line is such that its segment between the straight lines 5x-y+4=0 and 3x+4y-4=0 is bisected at the point (1,5). Obtain its equation



$5x-y+4=0$ ... (1)

$3x+4y-4=0$ ... (2)

Let the required line intersects the lines (1) and (2) at the points, (α_1, β_1) and (α_2, β_2), respectively (Fig10.24). Therefore

$5\alpha_1 - \beta_1 + 4 = 0$ and

$3\alpha_2 + 4\beta_2 - 4 = 0$

or $\beta_1 = 5\alpha_1 + 4$ and $\beta_2 = \dfrac{4-3\alpha_2}{4}$.

We are given that the mid point of the segment of the required line between (α_1, β_1) and (α_2, β_2) is (1, 5). Therefore

$$\frac{\alpha_1+\alpha_2}{2} = 1 \text{ and } \frac{\beta_1+\beta_2}{2} = 5,$$

or
$$\alpha_1+\alpha_2 = 2 \text{ and } \frac{5\alpha_1+4+\dfrac{4-3\alpha_2}{4}}{2} = 5,$$

or $\alpha_1+\alpha_2 = 2$ and $20\alpha_1 - 3\alpha_2 = 20$ ... (3)

Solving equations in (3) for α_1 and α_2, we get

$$\alpha_1 = \frac{26}{23} \text{ and } \alpha_2 = \frac{20}{23} \text{ and hence, } \beta_1 = 5.\frac{26}{23}+4 = \frac{222}{23}.$$

Equation of the required line passing through (1, 5) and (α_1, β_1) is

$$y-5 = \frac{\beta_1-5}{\alpha_1-1}(x-1) \text{ or } y-5 = \frac{\frac{222}{23}-5}{\frac{26}{23}-1}(x-1)$$

Equation of the required line passing through (1, 5) and (α_1, β_1) is

$$y-5 = \frac{\beta_1-5}{\alpha_1-1}(x-1) \text{ or } y-5 = \frac{\frac{222}{23}-5}{\frac{26}{23}-1}(x-1)$$

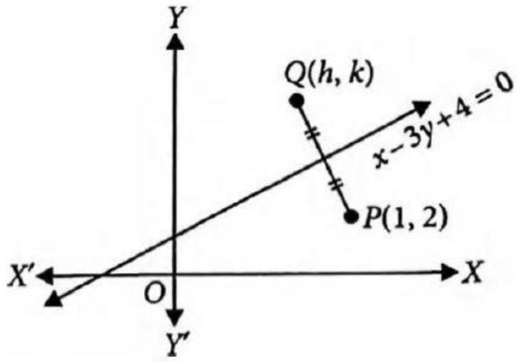
or $107x - 3y - 92 = 0$,

which is the equation of required line.

Or

Assuming that straight lines work as the plane mirror for a point , find the image of the point (1,2) in the line x-3y+4=0.

Let say point Q(h , k) is the image of point P(1, 2) in the line x-3y+4=0.



Then mid point of QP will lie on x-3y+4=0.
=> (h+ 1)/2 , (k + 2)/2 will lie on x-3y+4=0.
=> (h + 1)/2 - 3 (k + 2)/2 + 4 = 0
=> h + 1 - 3k - 6 + 8 = 0
=> h – 3k = -3
Also line between (h, k) and (1, 2) will be perpendicular to x-3y+4=0
Slope between (h, k) and (1, 2) = (k - 2)/(h-1)
Slope of x-3y+4=0 => 3y = x + 4 => y = (1/3)x + 4/3 slope = 1/3
(k - 2)/(h-1) * (1/3) = - 1
=> k - 2 = -3(h - 1)
=> k - 2 = -3h + 3
=> 3h + k = 5
h – 3k = -3 Eq1
3h + k = 5 Eq2
Eq2 - 3 *Eq1
10k = 14 => k = 1.4,h = 1.2
Point is (1.2 , 1.4)
Q37.Find the derivative of F(x)= xsinx by first principal

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x\{\sin(x+h) - \sin x\} + h\sin(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{x\,.\,2\operatorname{Cos}\left(\frac{x+h+x}{2}\right)\operatorname{Sin}\left(\frac{x+h-x}{2}\right) + h\operatorname{Sin}(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{2x\operatorname{Cos}\left(\frac{2x+h}{2}\right)\operatorname{Sin}\left(\frac{h}{2}\right)}{h} + \lim_{h\to 0} \frac{h\sin(x+h)}{h}$$

$$= 2x\cos\left(\frac{2x+0}{2}\right).\frac{1}{2} + \sin(x+0)$$

= x cos x + sin x ∴ f'(x) = x cos x + sin

Q38.

Ans

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 42.5}{4}$	${y_i}^2$	$f_i y_i$	$f_i {y_i}^2$
$32.5 - 36.5$	15	34.5	-2	4	-30	60
$36.5 - 40.5$	17	38.5	-1	1	-17	17
$40.5 - 44.5$	21	42.5	0	0	0	0
$44.5 - 48.5$	22	46.5	1	1	22	22
$48.5 - 52.5$	25	50.5	2	4	50	100
	100					199

mean = $A + [\sum f_i y_i]/N \times h$

$= 42.5 + 25/100 \times 4$

$= 42.5 + 1$

$= 43.5$

Variance$(\sigma^2) = h^2/N^2\ [N \sum(f_i y_i)^2 - (\sum f_i y_i)^2]$

$= (16)/(10000)\ [100 \times 199 - (25)^2]$

$= (16)/(10000)\ [19900 - 625]$

$= (16)/(10000) \times 19275$

$= 30.84$

Standard deviation,

$(\sigma) = \sqrt{30.84}$

$= 5.55$

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