AN EDUCATIONAL INSTITUTE

CBSE Class XI Mathematics
Sample Paper 05
As per pattern issued by CBSE for (2023-24)

Maximum Marks : 80 **Time : 3 hrs.**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
6. Section E has 4 questions carrying 05 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A
(Question 1 to 20 carry 1 mark)

Q1. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for both physics and chemistry. How many opt for none of the subjects?

a)17	b) 67
c)80	d)53

Q2. R={ (A, B) : $A^2 + B^2 = 25$, A,B ∈ N} on the sets has the following relation

a)(A,A)∈ $R\ for\ all\ A$	b) (A,B) and (B,A) ∈ $R\ for\ all\ A, B$
c)(A,B) ,(B ,C) and (A,C)∈ $R\ for\ all\ A, B, C$	d) none of these

Q.3.The range of the function f(x) = 3x – 2, is

a) (- ∞, ∞)	b) R – {3}
c)(- ∞, 0)	d)(0, – ∞)

Q4.If A, B, C be three sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C, then,

a)B=C	b) A=C
c)A=B=C	d)A=B

Q5.Four dice are rolled. The number of possible outcomes in which at least one dice show 2 is

a)1296	b) 671
c) 625	d)585

Q6. The least value of n for which $\{(1 + i)/(1 - i)\}^n$ is real, is

a)1	b) 2
c)3	d) 4

Q7. Let U={1,2,3............40} ; A={x : x is divisible by 5 and 10} and B={x : x=5n ,n∈ N } then n(A∩B) is

a)0	b) 1
c)2	d) 3

Q8. At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate

a) (1, 1)	b) (3, 1)
c) (-3, 1)	d) (-3, -3)

Q9. The locus of a point, whose abscissa and ordinate are always equal is

a)x + y + 1 = 0	b) x – y = 0
c)x + y = 1	d)none of these.

Q10. If y = $\sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at x = 1 is :

a) 1	b) 1/2
c) $1/\sqrt{2}$	d) 0

Q11.The solution of the inequality $|x - 1| < 2$ is

a) (1, ∞)	b) (-1, 3)
c)(1, -3)	d) (∞, 1)

Q.12. The value of cos 5π is

a)0	b) 1
c)-1	d)none of these

Q.13 If repetition of the digits is allowed, then the number of even natural numbers having three digits is

a)250	b) 350
c)450	d)550

Q14.The mean deviation about mean for the data : 7 , 6 ,4 , 3 , 8 , 2 is:

a) 0	b) 1
c) 2	d) 3

Q15.The tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2m}{n}$ where m and n respectively are

a) $a^2 - b^2, ab$	b) $ab, a^2 - b^2$
c) $a^2 + b^2, ab$	d) $ab, a^2 + b^2$

Q16. The point $(-2,-3,-4)$ lies in the

a) First Octant	b) seventh Octant
c) second octant	d) Eighth Ocatant

Q17. If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$,then x is

a)90	b) 100
c)80	d)95

Q18. If A and B are two given sets, then A ∩ (A ∩ B)′ is equal to

a)A	b) B′
c)ϕ	d)A-B

Q19. Assertion (A) The domain of the real function f defined by $f(x)=\sqrt{x-1}\ \ is\ \ R-\{1\}$
Reason (R) The range of the function defined by $f(x)=\sqrt{x-1}$ is [0,∞)

a)Both assertion and reason are true and reason is the correct explanation of assertion	b)Both assertion and reason are true but reason is not the correct explanation of assertion
c)Assertion is true but reason is false.	d) Assertion is false but reason is true.

Q.20 Assertion (A) : The value of $i^2 + i^4 + i^6 + \ldots\ldots\ldots + i^{20}$ is 1 .
Reason (R) : $i^{4n} = 1$, n ∈ Z .

a)Both assertion and reason are true and reason is the correct explanation of assertion	b)Both assertion and reason are true but reason is not the correct explanation of assertion
c)Assertion is true but reason is false.	d) Assertion is false but reason is true.

SECTION B
(Question 21 to 25 carry 2 mark)

Q21. If (1+ i)(1+ 2i) (1+ 3i) ………… (1 +ni)= a+ib .Prove that 2.5.10….. $(1+ n^2) = a^2 + b^2$.

Q22. Prove that

$$\frac{1}{2}cos\,x-\frac{\sqrt{3}}{2}sin\,x=\cos\left(\frac{\pi}{3}+x\right)$$

OR

Find the value of $\sqrt{3}$ cosec 20° – sec 20°

Q23. Find the equation of circle passing through (2,3) and centre lies on (3,-1)

Q24. The letters of the word “MUMMY” are placed at random in a row.What is the chance that letters at the extreme are both M?

OR

Events E and F are such that P (not E or not F) = 0.25 state whether E and F are mutually exclusive.

Q25. Solve the inequality $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \left(\frac{2-x}{5}\right)$

SECTION C
(Question 26 to 31 carry 3 mark)

Q26. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): a ∈ A, b ∈ B, a is a factor of b and a < b}, then find in Roster form. Also find its domain and range.

Q27.The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line y = 2x + c. Find c and remaining two vertices.

OR

If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

Q28. If z is a complex number such that $|z|$=1, prove that $\left(\frac{z-1}{z+1}\right)$ is purely imaginary.

Q29.Using properties of sets and their complements prove that (A ∪ B) ∩ (A ∪ B′) = A.

Q30. Find the length of major, minor axis and latus rectum of the following ellipse, $16x^2 + 25y^2 = 400$

OR

Find equation of an ellipse having vertices (0, ±5) and foci (0, ±4).

Q31. Prove that

$cos^2\alpha + cos^2(\alpha + 120^0) + cos^2(\alpha - 120^0) = \frac{3}{2}$.

SECTION D
(Question 32 to 35 carry 4 mark)

Q32. Let A, B be any two (non-empty) sets and R be a relation from A to B, then the inverse of relation R denoted by R^{-1} is a relation from B to A i.e. $R^{-1} \subset B \times A$. Also R^{-1}= { (b , a) : (a , b) R },Clearly (a , b) ∈ R⟺(b , a) ∈R^{-1}.
If A = { 2 , 3, 4 , 5 } , B = {3 ,6 ,7 , 10 } and a relation R from A to B is defined as R = { (x , y) : x divides y, x ∈ A , y ∈ B }
Based above information, answer the following questions : -
(i) Draw the arrow diagram of above relation.
(ii) Write R^{-1} as a set of ordered pairs.
(iii) Write domain and range of R^{-1}
or
Write domain and range of R.

Q33. Rana visited a dentist for his tooth problem .The probability that he will have his tooth extracted is 0.06 ,the probability that he will have a cavity filled is 0.2 and the probability that he will have a tooth extracted or a cavity filed is 0.23 .Answer the below given questions :

(i) For any event E ,What can be the minimum and maximum value of probability .

(ii)If E_1 and E_2 be two events such that $E_1 \subseteq E_2$ then what is the relationship between $P(E_1)$ & $P(E_2)$.
(iii)What is the probability that he will have his tooth extracted as well as cavity filled ?

Or

Find the probability that he neither have tooth extracted nor filled cavity?

Q34. Rahul and Ravi planned to play Business (board game) in which they were supposed to use dice. They decide a rule themselves to start or open the coin(token) until "2" appears.

(i)How many elements of sample space correspond to the event if 2 appear on second throw of a dice?
(ii) How many elements of sample space correspond to the event if 2 appear on third throw of a dice?

(iii)How many elements of the sample space correspond to the event that the 2 appears on the kth roll of the die?

Or

How many elements of the sample space correspond to the event that the 2 appears not later than the kth roll of the die?

SECTION E
(Question 35 to 32 carry 5 mark)

Q35.Solve for x ,$\left|\frac{2x-1}{x-1}\right| > 2, x \in R$

Or

Solve for x,$|x+1| + |x| > 3, x \in R$

Q36. If a and b are the roots of $x^2 - 3x + p = 0$ and c and d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP. Then prove that $\frac{q+p}{q-p} = \frac{17}{15}$.

Q37.If p_1 and p_2 are the lengths of perpendiculars from the origin to the line $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\vartheta - y \sin\theta = a\cos 2\theta$respectively then prove that $4P_1^2 + p_2^2 = a^2$

OR

Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2-b^2}, 0)$ and ($-\sqrt{a^2 - b^2}$, 0) to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

Q38. Find the mean deviation about the mean for the following :

X_i	5	10	15	20	25
F_i	7	4	6	3	5

SECTION – A

(Question number **1 to 20** carry **1 marks** each)

Q1.	**B**
Q2.	**B**
Q3.	**A**
Q4.	**A**
Q5.	**A**
Q6.	**B**
Q7.	**C**
Q8.	**B**
Q9.	**B**
Q10.	**D**
Q11.	**B**
Q12.	**C**
Q13.	**C**
Q14.	**B**
Q15.	**B**
Q16.	**B**
Q17.	**B**
Q18.	**D**
Q19.	**D**
Q20.	**B**

SECTION – B

(Question number **21 to 25** carry **2 marks** each)

Q21.

Ans.)

$(1 + i)(1 + 2i)(1 + 3i) \ldots\ldots(1 + ni) = a + ib$

Taking modulus on both the sides, we get:

$$|(1+i)(1+2i)(1+3i)\ldots\ldots\ldots\ldots(1+ni)| = |a+ib|$$

$|(1+i)(1+2i)(1+3i)\ldots\ldots\ldots\ldots(1+ni)|$ can be written as

$$|(1+i)|\,|(1+2i)|\,|(1+3i)|\ldots\ldots\,|(1+ni)|$$

$$\sqrt{1^2+1^2}\times\sqrt{1^2+2^2}\times\sqrt{1^2+3^2}\times\ldots\times\sqrt{1+n^2}=\sqrt{a^2+b^2}$$

$$\Rightarrow \sqrt{2}\times\sqrt{5}\times\sqrt{10}\times\ldots\times\sqrt{1+n^2}=\sqrt{a^2+b^2}$$

Squaring on both the sides, we get:

$$2\times5\times10\times\ldots\times(1+n^2)=a^2+b^2$$

Q22. Prove that $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos\left(\frac{\pi}{3}+x\right)$

Ans.) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

Substituting ,

$A = \frac{\pi}{3}$ and $B = x$,we get

$$\cos\left(\frac{\pi}{3}+x\right) = \cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x$$

$$\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$

$$\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}(\cos x - \sqrt{3}\sin x)$$

Q23. Find the equation of circle passing through (2,3) and centre lies on (3,-1)

Ans.) Let eqn of circle be $(x-3)^2+(y+1)^2=r^2$ —— ①

as (2,3) lies on it

So, satisfy eqn —— ①

$\therefore r^2 = 17$

hence, eqn is $(x-3)^2+(y+1)^2 = 17$.

Q24. The letters of the word "MUMMY" are placed at random in a row.What is the chance that letters at the extreme are both M?

Ans.) The total number of ways of arranging the letters = 5!/3!=20 (because M is present 3 times, so we divide by 3!)

Now, to get the favourable cases, we proceed as follows :

Put M at the first position and M at the last position

Now, we have to arrange M,U,Y only

The number of ways of doing this is 3!=6

So, total favourable ways =6

Total possible ways =20

Probability = 6/20=0.3

OR

Events E and F are such that P (not E or not F) = 0.25 state whether E and F are mutually exclusive.

Ans.) We have P (not E or not F) =0.25

⇒P(E′∪F′)=0.25

$\Rightarrow P(E\cap F)'=0.25$
$\Rightarrow P(E\cap F)=1-P(E\cap F)'$
$\Rightarrow P(E\cap F)=1-0.25$
$\Rightarrow P(E\cap F)=0.75 \neq 0$
$\Rightarrow E\cap F\neq\phi\Rightarrow$ Thus E and F are not mutually exclusive.

Q25. Solve the inequality $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \left(\frac{2-x}{5}\right)$

Ans.) $(2x-1)/3\geq(3x-2)/4-(2-x)/5$
$\Rightarrow(2x-1)/3\geq (5(3x-2)-4(2-x))/20$
$\Rightarrow(2x-1)/3\geq (15x-10-8+4x)/20$
$\Rightarrow(2x-1)/3\geq(19x-18)/20$
$\Rightarrow 20(2x-1)\geq 3(19x-18)$
$\Rightarrow 40x-20\geq 57x-54$
$\Rightarrow -20+54\geq 57x-40$
$\Rightarrow 34\geq 17x$
$\Rightarrow 2\geq x$
Thus, all real numbers x, which are less than or equal to 2, are the solution, of of the given inequality.
Hence, the solution set of the given inequality is $(-\infty,2]$

SECTION – C

(Question number **26 to 31** carry **3 marks** each)

Q26. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): a∈ A, b∈ B, a is a factor of b and a < b}, then find in Roster form. Also find its domain and range.

Ans.)

Given, A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54} and

R = {(a, b): a ∈ A, b ∈ B, a is a

factor of b and a<b}

Roster form

R = {(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)}

Domain of R = {2, 6, 9}

Range of R = {4, 6, 18, 27, 54}

Q27.The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line y = 2x + c. Find c and remaining two vertices.

Ans.) Since, diagonals of rectangle bisect each other, so mid point of (1,3) and (5,1) must satisfy y=2x+c i.e (3,2) lies on it

$\Rightarrow 2=6+c \Rightarrow c=-4$

Therefore other two vertices lies on y=2x−4

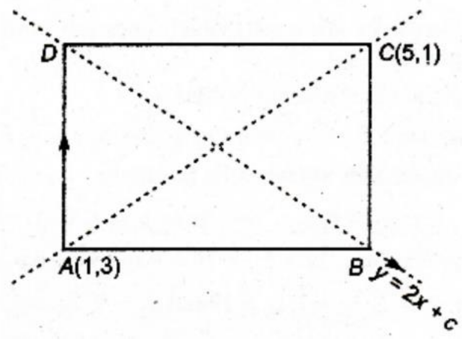
Let the coordinate of B be (x,2x−4)

Therefore slope of AB . slope of BC=−1

$\Rightarrow (2x-4-3)/(x-1) \cdot (2x-4-1)/(x-5)=-1$

$\Rightarrow (x^2-6x+8)=0 \Rightarrow x=4,2 \Rightarrow y=4,0$

Hence, required points are (4,4),(2,0)



OR

If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

Ans.) Let 'P' be the point (2,1) and Q be the image (4,3) of the point P in the line LM/

Mid point of PQ lies on LM.

Mid point of PQ is (3,2)

Slope of line PQ=1

∵ slope of LH x slope of PQ=−1

∴ slope of LH=−1

Equation of LM :x+y=5

Q28. If z is a complex number such that $|z|$=1, prove that $\left(\frac{z-1}{z+1}\right)$ is purely imaginary

Ans.)

Let $z = x + iy$. Then $|z|^2 = x^2 + y^2$.

Therefore the condition $|z| = 1$ is equivalent to

$x^2 + y^2 = 1$.

Now $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$

$= \frac{(x-1+iy)(x+1-iy)}{(x+1+iy)(x+1-iy)}$

$= \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2} = \frac{2iy}{(x+1)^2+y^2}$ by (1)

Hence $\frac{z-1}{z+1}$ is purely imaginary when $|z| = 1$

provided $z \neq -1$.

When $z = 1$, we have $\frac{z-1}{z+1} = 0$.

Now recall that according to the definition 2 given in §2, 0 is a pure imaginary number, since the point 0 which corresponds to z = 0 lies on both real and imaginary axes.

So in this case also, $\frac{z-1}{z+1}$ is a pure imaginary number.

Q29.Using properties of sets and their complements prove that (A ∪ B) ∩ (A ∪ B′) = A.

Ans.) L.H.S = (A ∪ B) ∩ (A ∩ B′) = A ∪ (B ∩ B′) (By distributive law)

= A ∪ ∅ (∴ B ∩ B′ = ∅)

= A R.H.S

Q30. Find the length of major, minor axis and latus rectum of the following ellipse, $16x^2 + 25y^2 = 400$

Ans.) $16x^2+25y^2=400 \Rightarrow x^2/25 + y^2/16 = 1$

Therefore, a=5,b=4

$c = \sqrt{a^2 - b^2} = 3$ Length of major axis: 2a=10

Length of minor axis: 2b=8

Eccentricity (e): 5/3,Latus rectum $= \frac{2b^2}{a} = \frac{2\times16}{5} = \frac{32}{5}$

OR

Find equation of an ellipse having vertices (0, ±5) and foci (0, ±4).

Ans.) Since the foci are on y axis, the equations of the ellipse is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Given: vertices are (0, ± 5), a = 5

Also, since foci are (0, ± 4), c = 4 and $b^2 = a^2 - c^2 = 25 - 16 = 9$

Therefore, the equation of the hyperbola is $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Q31.Prove that $\boldsymbol{cos^2 x + cos^2(x + 120^0) + cos^2(x - 120^0) = \frac{3}{2}}$

Ans.) L.H.S. = $\cos^2 x + \cos^2(x + 120°) + \cos^2(x - 120°)$

$$= \frac{1+\cos 2x}{2} + \frac{1+\cos 2(x+120°)}{2} + \frac{1+\cos 2(x-120°)}{2} \text{......} \left[\because \cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$$

$$= \frac{3}{2} + \frac{1}{2}[\cos 2x + \cos(2x+240°) + \cos(2x-240°)]$$

$$= \frac{3}{2} + \frac{1}{2}(\cos 2x + \cos 2x \cos 240° - \sin 2x \sin 240° + \cos 2x \cos 240° + \sin 2x \sin 240°)$$

$$= \frac{3}{2} + \frac{1}{2}(\cos 2x + 2\cos 2x \cos 240°)$$

$$= \frac{3}{2} + \frac{1}{2}[\cos 2x + 2\cos 2x \cos(180° + 60°)]$$

$$= \frac{3}{2} + \frac{1}{2}[\cos 2x + 2\cos 2x(-\cos 60°)]$$

$= \frac{3}{2} + \frac{1}{2}\left[\cos 2x - 2\cos 2x\left(\frac{1}{2}\right)\right]$

$= \frac{3}{2} + \frac{1}{2}(\cos 2x - \cos 2x)$

$= \frac{3}{2} + \frac{1}{2}(0)$

$= \frac{3}{2}$

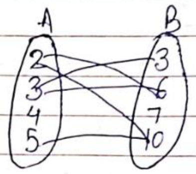
= R.H.S.

SECTION – D

(Question number **32 to 34** carry **4 marks** each)

Q32.

(i)



(ii) $R^{-1} = \{(6,2)\ (6,3)\ (3,3)\ (10,2)(10,5)\}$

(iii) Domain of R^{-1} = $\{6, 3, 10\}$

Range of R^{-1} = $\{2, 3, 5\}$

OR

Domain of R = $\{2, 3, 5\}$

Range of R = $\{3, 6, 10\}$

Q33.

(i) Minimum probability '0'

Maximum probability '1'

(ii) $P(E_1) \leq P(E_2)$

(iii) A: Tooth extract

B: Cavity filled

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.06 + 0.2 - 0.23$

$= 0.03$

OR

$P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.23$

$= 0.77$

Q34.

(i) 5 elements (1,2), (3,2) (4,2) (5,2) (6,2)

(ii) 25 elements (1,3,2) (1,1,2) (4,3,2) (5,3,2) . . . . . (6,6,2)

(iii) first (k-1) rolls have no. other than 2.

$$\frac{5}{1^{st}} \times \frac{5}{2^{nd}} \times \frac{5}{3^{rd}} \times \ldots \frac{5 \text{ways}}{(k-1)^{th}} \times \frac{2 \text{ (1 way)}}{k^{th} \text{ roll}}$$

Total elements $5 \times 5 \times \ldots 5$ (k-1) times

$= 5^{k-1}$ elements

OR

If 2 appear on first roll = 1 way
If 2 appear on 2nd roll = 5 way
⋮
If 2 appear on $(k-1)^{th}$ roll $= 5^{k-2}$
If 2 appear on k^{th} roll $= 5^{k-1}$

Total ways $1 + 5 + 5^2 + \ldots 5^{k-1}$ (k terms)

$$= \frac{5^k - 1}{5 - 1} = \frac{5^k - 1}{4}$$

SECTION – E

(Question number **35 to 38** carry **5 marks** each)

Q35. Solve for x , $\left|\frac{2x-1}{x-1}\right| > 2, x \in R$

Ans.) $\frac{2x-1}{x-1} < -2$ and $\frac{2x-1}{x-1} > 2$

$\frac{2x-1}{x-1} + 2 < 0$ and $\frac{2x-1}{x-1} - 2 > 0$

$\frac{2x-1+2x-2}{x-1} < 0$ and $\frac{2x-1-2x+2}{x-1} > 0$

$\frac{4x-3}{x-1} < 0$ and $\frac{1}{x-1} > 0$

$+ \quad \frac{3}{4} \quad - \quad 1 \quad +$

$x \in \left(\frac{3}{4}, 1\right)$ and $x - 1 > 0$

$x > 1$

$x \in (1, \infty)$

Thus, $x \in \left(\frac{3}{4}, 1\right) \cup (1, \infty)$

Or

Solve for x, $|x+1| + |x| > 3, x \in R$

$-(x+1) - x > 3$, $-2x - 1 > 3$, $-2x > 4$, $x < -2$ | $-(x+1) + x > 3$, $-1 > 3$ false | $(x+1) + x > 3$, $2x + 1 > 3$, $x > \frac{2}{2}$, $x > 1$ | $(x+1) - x > 3$, $1 > 3$ false

$\therefore x \in (-\infty, -2) \cup (1, \infty)$

Q36.. If a and b are the roots of $x^2 - 3x + p = 0$ and c and d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP. Then prove that $\frac{q+p}{q-p} = \frac{17}{15}$.

Ans

We have,

a +b = 3, ab = p, c + d =12 and cd = q

a, b, c and d form a G.P.

∴ First term = a, b = ar, c = ar^2 and d = ar^3

Then, we have

a + b = 3 and c + d = 12

$\Rightarrow a + ar = 3$

$\Rightarrow a(1 + r) = 3 \ldots (i)$

Similarly, $ar^2(1 + r) = 12 \ldots (ii)$

$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$

$\Rightarrow r^2 = 4$

$\Rightarrow r = 2$

$\therefore a(1+r) = 3$

$\Rightarrow a = 1$

Now , $p = ab$

$\Rightarrow p = a \times ar = 2$

And, $q = cd$

$\Rightarrow q = ar^2 \times ar^3 = 2^5 = 32$

$\therefore \frac{q+p}{q-p} = \frac{32+2}{32-2} = \frac{34}{30} = \frac{17}{15}$

Q37.If p_1 and p_2 are the lengths of perpendiculars from the origin to the line $x\, sec\, \theta + y\, cosec\, \theta = a$ and $x \cos \vartheta - y \sin \theta = a \cos 2\theta$respectively then prove that $4P_1^2 + p_2^2 = a^2$

Ans.) The given lines are x sec θ + y cosec θ = a … (1)

x cos θ − y sin θ = a cos 2 θ … (2)

let p1 and p 2 are the perpendiculars from the origin upon the lines (1) and (2), respectively

$$p_1 = \left| -\frac{a}{\sqrt{\sec^2\theta + \csc^2\theta}} \right| \text{ and } p_2 = \left| -\frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right|$$

$$\Rightarrow p_1 = \frac{1}{2}\left| -a \times 2\sin\theta\cos\theta \right| \text{ and } p_2 = \left| -a\cos 2\theta \right|$$

$$\Rightarrow p_1 = \frac{1}{2}\left| -a\sin 2\theta \right| \text{ and } p_2 = \left| -a\cos 2\theta \right|$$

$\Rightarrow 4p_1^2 + p_2^2 = a^2 (\sin^2 2\theta + \cos^2 2\theta) = a^2$

Q38. Find the mean deviation about the mean for the following :

X_i	5	10	15	20	25
F_i	7	4	6	3	5

First we will calculate mean

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
5	7	$7 \times 5 = 35$	$\lvert 5 - 14\rvert = \lvert -9\rvert = 9$	$7 \times 9 = 63$
10	4	$4 \times 10 = 40$	$\lvert 10 - 14\rvert = \lvert -4\rvert = 4$	$4 \times 4 = 16$
15	6	$6 \times 15 = 90$	$\lvert 15 - 14\rvert = \lvert -1\rvert = 1$	$6 \times 1 = 6$
20	3	$3 \times 20 = 60$	$\lvert 20 - 14\rvert = \lvert 6\rvert = 6$	$3 \times 6 = 18$
25	5	$5 \times 25 = 125$	$\lvert 25 - 14\rvert = \lvert 11\rvert = 11$	$5 \times 11 = 55$
$\sum f_i = 25$		$\sum f_ix_i = 350$		$\sum f_i\lvert x_i - \bar{x}\rvert = 158$

$$\text{Mean}(\bar{x}) = \frac{\sum f_ix_i}{\sum f_i}$$

$$\bar{x} = \frac{350}{25}$$

$$\bar{x} = 14$$

$$\text{Mean deviation about mean} = \frac{\sum f_i|x_i - \bar{x}|}{\sum f_i}$$

Putting $\sum f_i|x_i - \bar{x}| = 158$, $\sum f_i = 25$

$$\text{M.D.}\ (\bar{x}) = \frac{1}{25} \times 158$$

$$= \mathbf{6.32}$$