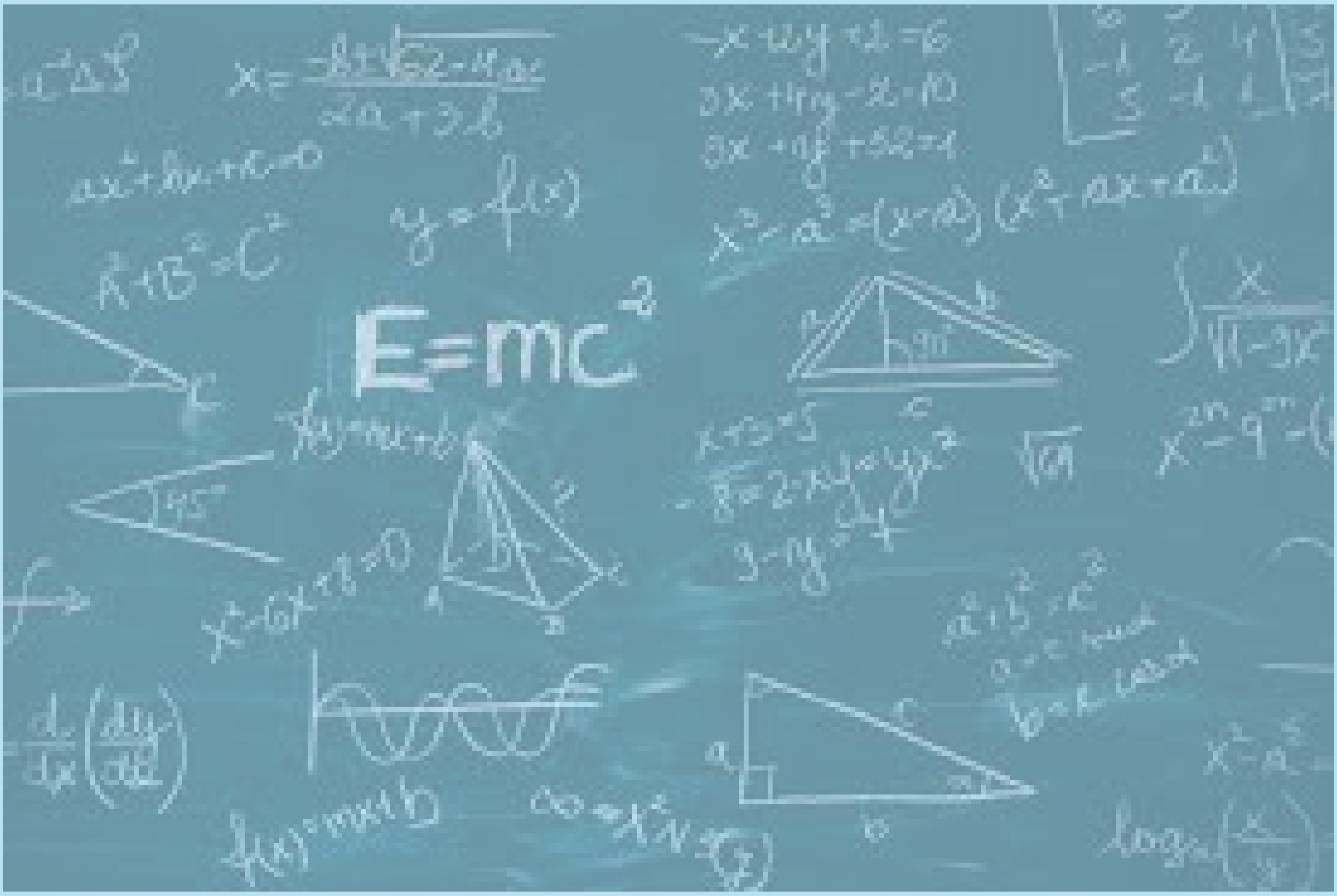
CLASS XII

CRPF PUBLIC SCHOOL, ROHINI

MATHEMATICS

PRACTICE PAPERS (WITH SOLUTIONS)

SESSION 2024-25



CRPF PUBLIC SCHOOL, ROHINI, DELHI-110085

PRACTICE PAPER-01 (2024-25)

CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has **18 MCQ's and 02 Assertion Reasoning based questions** of 1 mark each.

3. Section B has **5 Very Short Answer (VSA)-type questions** of 2 marks each.

4. Section C has **6 Short Answer (SA)-type questions** of 3 marks each.

5. Section D has **4 Long Answer (LA)-type questions** of 5 marks each.

6. Section E has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A (MCQ) 1 Mark Questions	
Q1	If $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to (a) 0 (b) 1 (c) 2 (d) -1
Q2	If $A=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -3 & 4 & 1 \end{bmatrix}$, the value of $\lvert adjA \rvert$ is: (a) 2^0 (b) 2^1 (c) 2^2 (d) 2^3
Q3	If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$, then the magnitude of $\vec{x}$ is: $(a)\ \sqrt{12}$ (b) 12 (c) 13 $(d)\ \sqrt{13}$
Q4	If $f(x)=\begin{cases} \dfrac{3\sin \pi x}{5x} & ,x\neq 0 \\ 2k & ,x=0 \end{cases}$ is continuous at $x=0$, then the value of k is: (a) $\dfrac{\pi}{10}$ (b) $\dfrac{3\pi}{10}$ (c) $\dfrac{3\pi}{2}$ (d) $\dfrac{3\pi}{5}$
Q5	The value of $\int \dfrac{1}{x\cos^2(1+\log x)}dx$ is: (a) $\tan(1+\log x)+c$ (b) $\cot(1+\log x)+c$ (c) $\sec(1+\log x)+c$ (d) $\cos(1+\log x)+c$

Q6

The solution of the differential equation $\dfrac{dy}{dx}+\sqrt{\dfrac{1-y^2}{1-x^2}}=0$ is:

(a) $y+\sin^{-1}y=\sin^{-1}x+c$ (b) $\sin^{-1}y-\sin^{-1}x=c$

(c) $\sin^{-1}y+\sin^{-1}x=c$ (d) $\sin^{-1}y-\sin^{-1}x=cxy$

Q7

Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).

Let F = 4x + 6y be the objective function. The minimum value of F occurs at:

(a) Only (0, 2)

(b) Only (3, 0)

(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)

(d) any point on the line segment joining the points (0, 2) and (3, 0)

Q8

If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}.\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|$ is: equal to:

(a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) 7 (d) 3

Q9

$\displaystyle\int_0^{\frac{2}{3}}\frac{dx}{4+9x^2}$ is equal to:

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{9}$ (c) $\dfrac{\pi}{24}$ (d) $\dfrac{\pi}{36}$

Q10

If A and B are 2 × 2 square matrices and A + B = $\begin{bmatrix}4 & -3\\ 1 & 6\end{bmatrix}$ and A – B = $\begin{bmatrix}-2 & -1\\ 5 & 2\end{bmatrix}$,

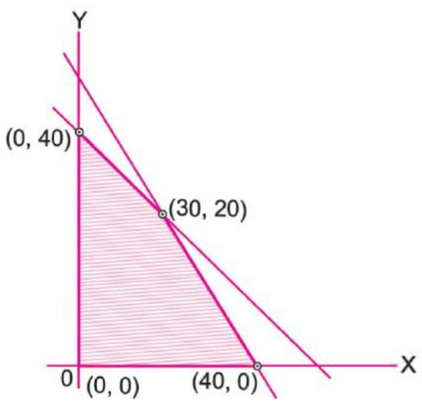
then the value of AB is:

(a) $\begin{bmatrix}-7 & 5\\ 1 & -5\end{bmatrix}$ (b) $\begin{bmatrix}7 & -5\\ 1 & 5\end{bmatrix}$ c) $\begin{bmatrix}7 & -1\\ 5 & -5\end{bmatrix}$ (d) $\begin{bmatrix}7 & -1\\ -5 & 5\end{bmatrix}$

Q11

Feasible region (shaded) for a LPP is shown in the given figure.

The maximum value of the Z = 0.4x + y is:



(a) 45 (b) 40 (c) 50 (d) 41

Q12

If $\begin{vmatrix}2x+5 & 3\\ 5x+2 & 9\end{vmatrix}$ = 0, then the value of *x* is:

(a) 13 (b) 3 (c) -13 (d) √3

Q13	If $A=\begin{bmatrix}4 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} exist if (a) $\lambda=2$ (b) $\lambda\neq 2$ (c) $\lambda\neq -2$ (d) $\lambda=-2$
Q14	If A and B are two events such that P(A) = $\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P\left(\frac{A}{B}\right)=\frac{1}{4}$, then $P(A'\cap B')$ equals (*a*) $\frac{1}{12}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{16}$
Q15	The integrating factor for solving the differential equation $x\frac{dy}{dx}-y=2x^2$ is : (a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$
Q16	If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$, then $\frac{dy}{dx}$ is : (*a*) $\cot\theta$ (*b*) $\tan\theta$ (*c*) $a\cot\theta$ (*d*) $a\tan\theta$
Q17	If three points A, B and C have position vectors $\hat{i}+x\hat{j}+3\hat{k}$, $3\hat{i}+4\hat{j}+7\hat{k}$ and $y\hat{i}-2\hat{j}-5\hat{k}$ respectively are collinear, then (*x*, *y*) is: (*a*) (2, –3) (*b*) (–2, 3) (*c*) (–2, –3) (*d*) (2, 3)
Q18	A line is such that it is inclined with *y*-axis and *z*-axis at 60º, then the angle this line is inclined with *x*-axis, is: (*a*) 45º (*b*) 30 º (*c*) 75 º (*d*) 60 º
	Assertion Reasoning Based Questions
Q19	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R** **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **Reason (R) :** Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
Q20	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

	Assertion A: If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is $\vec{r}=5\hat{i}-4j+6k+\lambda(3\hat{i}+7j+2k)$ **Reason R:** The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x-3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{-2}=\frac{y-4}{4}=\frac{z+8}{-5}$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
	SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each
Q21	Let $f: R-\{-1\}\rightarrow R$ be defined by, $f\left(x\right)=\frac{1+x^2}{1+x}$. Show that f is not 1-1. **OR** If y = $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$
Q22	A sphere increases its volume at the rate of π cm^3/s . Find the rate at which its surface area increases, when the radius is 1 cm.
Q23	Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that angle between them is 60^0 and their scalar product is $\frac{9}{2}$. **OR** Find the vector equation of the line passing through the point $A(1,2,-1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.
Q24	Differentiate $y=\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.
Q25	Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).
	SECTION – C (Short Answer (SA)-type questions) 3 Marks Each
Q26	Evaluate : $\int\frac{8}{(x+2)(x^2+4)}dx$

Q27	One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. **OR** Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Let X denote the number of red cards drawn. Find the probability distribution of X. Also, find the mean of this distribution.
Q28	Evaluate : $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$ **OR** Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x . \operatorname{cosec} x} dx$
Q29	Solve the following differential equation: $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x=y\sin\left(\frac{y}{x}\right)$; $y(1)=\frac{\pi}{2}$ **OR** Solve the following differential equation: $\frac{dy}{dx}=-\frac{x+y\cos x}{1+\sin x}$
Q30	Solve the following Linear Programming Problem graphically: Maximise Z = x + 2y Subject to the constraints: $x + 2y \geq 100$; $2x - y \leq 0$; $2x + y \leq 200$; $x, y \geq 0$
Q31	Evaluate : $\int \frac{x^3+x+1}{x^2-1} dx$
SECTION – D (Long Answer (LA)-type questions) 5 Marks Each	
Q32	Consider $f: R-\left\{-\frac{4}{3}\right\} \to R-\left\{\frac{4}{3}\right\}$ given by $f(x)=\frac{4x+3}{3x+4}$. Show that f is one-one and onto. **OR** Show that the relation S in the set $A=\{x \in Z : 0 \leq x \leq 10\}$ given by $S=\{(a,b): a, b \in Z, \|a-b\|$ is divisible by 4$\}$. Show that S an equivalence relation. Find the set of all elements related to 1.
Q33	Find the sub-intervals in which $f(x) = \log(2+x) - \frac{x}{2+x}$, $x > -2$ is increasing or decreasing.
Q34	Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B. **OR**

	Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.
Q35	Using Matrix Method, solve the following system of linear equations. $x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$
	SECTION – E (Case Study Based Questions) 4 Marks Each
Q36	Read the following passage and answer the questions given below. In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$  (i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. (ii) Find the critical point of the function. (iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area. **OR** (iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

Q37	Rohit cuts a cake with a knife on his eighteenth birthday. The circular cake is represented by $x^2 + y^2 = 4$. The sharp edge of the knife represents a straight line given by $x = \sqrt{3}y$.   Based on the above information, answer the following questions. (a) Draw the suitable geometrical figure representing the above situation. Also shade the smaller region formed by the line and the circle in the first quadrant and above x-axis. (b) Using integration, find the area of the smaller region shaded above.
Q38	   A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time. B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random. (a) What is the probability of getting a defective item? (b) If the item so chosen is found to be defective. What is the probability that it was produced by B?

CRPF PUBLIC SCHOOL, ROHINI, DELHI-110085

PRACTICE PAPER-02 (2024-25)

CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E.** Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

Q1. If A is a square matrix of order 3 such that $|adj\ A|=144$, the value of $|A^T|$ is:

(a) 0 (b) 144 (c) $\pm$ 12 (d) 12

Q2. If $0<x<\pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, the value(s) of x is:

(a) $\pi/3$ (b) $\pi/6$ (c) $5\pi/6$ (d) $2\pi/3,\ \pi/3$

Q3 If the points $A(3,-2), B(k,2)$ and $C(8,8)$ are collinear, then the value of k is:

(a) 2 (b) -3 (c) 4 (d) -4

Q4. If $A=\begin{bmatrix} 0 & x+2 \\ 2x-3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then x is equal to:

(a) $\frac{1}{3}$ (b) 5 (c) 3 (d) 1

Q5. If $y=\tan^{-1}(\sec x+\tan x)$, then $\frac{dy}{dx}$ is:

(a) $1/2$ (b) $-1/2$ (c) 1 (d) none of these

Q6. If the function $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2} & ,x\neq 0\\ k & ,x=0\end{cases}$ is continuous at $x=0$, then value of k is:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Q7. $\int \frac{x+3}{(x+4)^2}e^x dx = ?$

(a) $\frac{e^x}{x+4}+c$ (b) $\frac{e^x}{x+3}+c$ (c) $\frac{1}{(x+4)^2}+c$ (d) $\frac{e^x}{(x+4)^2}+c$

Q8. $\int \frac{dx}{e^x+e^{-x}}$ is equal to

(a) $\tan^{-1}(e^x)+C$ (b) $\tan^{-1}(e^{-x})+C$

(c) $\log(e^x-e^{-x})+C$ (d) $\log(e^x+e^{-x})+C$

Q9. The sum of the cofactors of element 13 and 1 in $\begin{vmatrix}10 & 19 & 2\\ 0 & 13 & 1\\ 9 & 24 & 2\end{vmatrix}$ is:

(a) 71 (b) -69 (c) -67 (d) -71

Q10. The integrating factor of the differential equation $(x\log x)\frac{dy}{dx}+y=2\log x$ is:

(a) $\log(\log x)$ (b) e^x (c) $\log x$ (d) x

Q11. Sum of order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^5+4x=0$ is

(a) 6 (b) 3 (c) 4 (d) 5

Q12. The p.v.'s of the points A,B,C are $\hat{i}+x\hat{j}+3\hat{k}$, $3\hat{i}+4\hat{j}+7\hat{k}$ and $y\hat{i}-2\hat{j}-5\hat{k}$ respectively, if A,B,C are collinear, then (x,y) = ?

(a) $(2,-3)$ (b) $(-2,3)$ (c) $(0,3)$ (d) $(2,3)$

Q13. If $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=|\vec{b}|=\sqrt{2}$ and $\vec{a}.\vec{b}=-1$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\pi/3$ (b) $\pi/4$ (c) $2\pi/3$ (d) $\frac{\pi}{2}$

Q14. The diagonals of a parallelogram are represented by the vectors $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$. The area of the parallelogram is:

(a) $7\sqrt{3}$ sq.units (b) $5\sqrt{3}$ sq.units (c) $3\sqrt{5}$ sq.units (d) $\frac{3\sqrt{2}}{2}$ sq. units

Q15. If a line makes angle $\pi/3$ and $\pi/4$ with x-axis and y-axis respectively, then the acute angle made by the line with z-axis is:

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $5\pi/12$

Q16. A bag contains 5 red, 7 green and 4 white balls, three balls are drawn one after the other without replacement. Then the probability that the balls drawn are white, green and green respectively, is:

(a) $\frac{1}{20}$ (b) $\frac{3}{20}$ (c) $\frac{7}{20}$ (d) $\frac{3}{7}$

Q17.

The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

Q18.

The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2 (b) 3

(c) 4 (d) 5

Assertion Reasoning Based Questions

Q19. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A)

The equation of the line passing through $(1,1,2)$ *and* $(2,3,-1)$ is $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{-3}$

Reason (R)

Equation of the line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

(a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(c) **A** is correct but **R** is not correct
(d) **A** is not correct but **R** is correct

Q20. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

(a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(c) **A** is correct but **R** is not correct
(d) **A** is not correct but **R** is correct

SECTION B

Q21. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represents two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

If $\vec{a}$, $\vec{b}$ are unit vectors such that the vector $\vec{a} + 3\vec{b}$ is perpendicular to the vector $7\vec{a} - 5\vec{b}$ and $\vec{a} - 4\vec{b}$ is perpendicular to $7\vec{a} - 2\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

Q22. If A, B, C and D are the points with position vectors

$\hat{i} - \hat{j} + \hat{k}, 2\hat{i} - \hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ respectively, then find the projection of $\overline{AB}$ along $\overline{CD}$.

Q23. Write the following in the simplest form: $sin\left(2tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

OR

A relation R is defined on a set of real numbers $\mathbb{R}$ as

R = {(x, y) : x . y is an irrational number}.

Check whether R is reflexive, symmetric and transitive or not.

Q24. If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

Q25. Sand is pouring from a pipe at the rate of 12 cm^3/sec The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

SECTION C

Q26.

Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1+5^x}\,dx$$

OR

Evaluate: $\int_1^4 |x-1| + |x-2| + |x-3| dx$

Q27. Evaluate the following integral x: $\int \frac{3x+1}{(x-2)^2(x+2)} dx$

Q28.

Find : $\int \frac{x^3+x}{x^4-9}\,dx.$

Q29. Solve the following differential equation:

$$(tan^{-1}y - x)dy = (1+y^2)dx$$

OR

Solve the following differential equation:

$$x\frac{dy}{dx} - y + xsin\left(\frac{y}{x}\right) = 0, \text{given that when } x = 2, y = \pi$$

Q30.

Let X denote the number of colleges where you will apply after your results and $P(X=x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x) = \begin{cases} kx & \text{, if } x = 0 \text{ or } 1 \\ 2kx & \text{, if } x = 2 \\ k(5-x) & \text{, if } x = 3 \text{ or } 4 \\ 0 & \text{, if } x > 4 \end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in

(i) exactly one college

(ii) atmost 2 colleges

(iii) atleast 2 colleges.

OR

There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Find the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution.

Q31. Solve the following Linear Programming Problem graphically:

Maximize $z=2x+5y$ subject to the following constraints:

$2x+4y\le 8$

$3x+y\le 6$

$x+y\le 4$

$x\ge 0, y\ge 0$

SECTION D

Q32. If $A=\begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations:

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4$.

OR

Find the product of matrices $A=\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$.

Q33.

Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

Q34. Consider $f: R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Check it is bijective or not. Justify your answer.

OR

Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

Q35. Dr. Dewan residing in Delhi went to see an apartment of 3BHK in Noida. The window of the house was in the form of rectangle surmounted by a semicircular opening having a perimeter of the window 10m as shown in the figure.

If x and y represents the length and breadth of the rectangular region, what is the relation between the variables?

(i) What is the area of the window in terms of x?

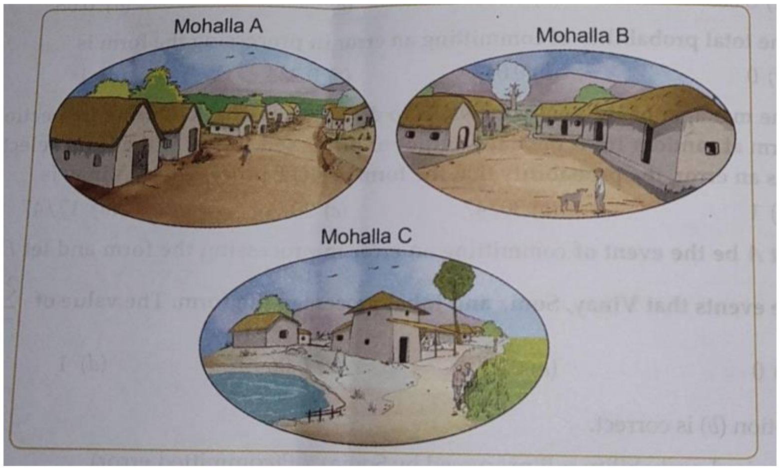
(ii) What should be the value of x for the area to be maximum?

SECTION E

(Question numbers 36 to 38 carry 4 mark each.)

This section contains three Case-study/Passage based questions.
First two questions have three sub-parts (i), (ii) and (iii) of marks 1, 1, and 2 respectively.
Third question has two sub-parts of 2 marks each.

Q36. In a village there are three mohallas A, Band C. In A,60% farmers believe in new technology of agriculture, while in B, 70% and in C, 80%. A farmer is selected at random from village.



(i) What is the conditional probability that a farmer believe in new technology if he belongs to mohalla A?

(ii) What is the total probability that a farmer believe in new technology of agriculture?

(iii) District agriculture officer selects a farmer at random in a village and he found that selected farmer believe in new technology of agriculture, what is the probability that the farmer belongs to mohalla B ?

Q37.

Ravindra started to run a small factory of manufacturing LED bulbs. He can sell x bulbs at a price of ₹ (300 – x) each. The cost price of x bulbs is ₹ $(2x^2 - 60x + 18)$.

Based on the above information, answer the following questions :

(i) Find the profit function P(x) for selling x bulbs.

(ii) What is $\frac{d}{dx}[P(x)]$?

(iii) (a) How many bulbs should he sell to earn maximum profit ?

OR

(iii) (b) How many bulbs is he selling if he is incurring a loss of ₹ 18 ?

Q38. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ respectively.

Based on the above information, answer the following questions:

(i) Find the shortest distance between the given lines.

(ii) Find the point at which the motorcycles may collide.

CRPF PUBLIC SCHOOL, ROHINI, DELHI-110085

PRACTICE PAPER-3 (2024-25)

CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

GENERAL INSTRUCTIONS

(i) This Question paper contains- **five sections A, B, C, D** and **E.** Each section is compulsory. However, there are internal choices in some questions.

(ii) **Section A has 18 MCQ's and 2 Assertion Reasoning based Questions** of 1 mark each.

*(iii)***Section B has 5 Very short Answer (VSA) – type questions** of 2 mark each.

(iv) **Section C has 6 Short Answer (SA) – type questions** of 3 mark each.

(v) **Section D has 4 Long Answer (LA) – type questions** of 5 mark each.

(vi) **Section E has 3 source based / case based /integrated units of assessment** (4 mark each) with sub parts.

SECTION A

Q1. Let $A = \{3,5\}$ then the number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

Q2. If the points $A(3,-2), B(k,2)$ and $C(8,8)$ are collinear, then the value of k is:

(a) 2 (b) –3 (c) 5 (d) –4

Q3. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is:

(a) 6 (b) −6 (c) 0 (d) −7

Q4. If $|A| = |KA|$, where A is a square matrix of order 2, then sum of all possible values of K is

(a) 1 (b) −1 (c) 2 (d) 0

Q5. If $A.(adjA) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |adjA|$ is equal to:

(a) 12 (b) 9 (c) 3 (d) 27

Q6. If $f(x) = \begin{cases} \frac{sinx}{x} + cosx & ,x \neq 0 \\ 2k & ,x = 0 \end{cases}$ is continuous at $x = 0$, the value of k is:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

Q7. If $x = a\cos^2\theta, y = b\sin^2\theta$,then $\frac{dy}{dx}$ is:

(a) $-\frac{a}{b}$ (b) $\frac{a}{b}\cot\theta$ (c) $-\frac{b}{a}$ (d) none of these

Q8. The feasible region for an LPP is always a ________________ polygon.

(a) Convex (b) Concave (c) the feasible region depends on LPP (d) none of these

Q9. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}\, dx = ?$

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) none of these

Q10. What is the product of order and the degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 siny + \left(\frac{dy}{dx}\right)^3 cosy = \sqrt{y}$$

(a) 4 (b) 6 (c) 3 (d) not defined

Q11. The integrating factor of the differential equation $xlogx\frac{dy}{dx} + y = \frac{2logx}{x}$ is

(a) x (b) $\log(logx)$ (c) $logx$ (d) $\frac{1}{x}$

Q12. The projection of the vector $\vec{a} = 3\hat{i} - \hat{j} - 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ is:

(a) $\sqrt{14}/2$ (b) $14/\sqrt{2}$ (c) $\sqrt{14}$ (d) 7

Q13. If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\pi/6$ (b) $2\pi/3$ (c) $5\pi/3$ (d) $\pi/3$

Q14. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ is:

(a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$

Q15. The value of λ for which the lines $\frac{x-5}{7}=\frac{2-y}{5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{2y-1}{\lambda}=\frac{z}{3}$ are at right angles, is

(a) 2 (b) 4 (c) -2 (d) -4

Q16. Direction cosines of a line perpendicular to both $x-axis\ and\ z-axis$ are

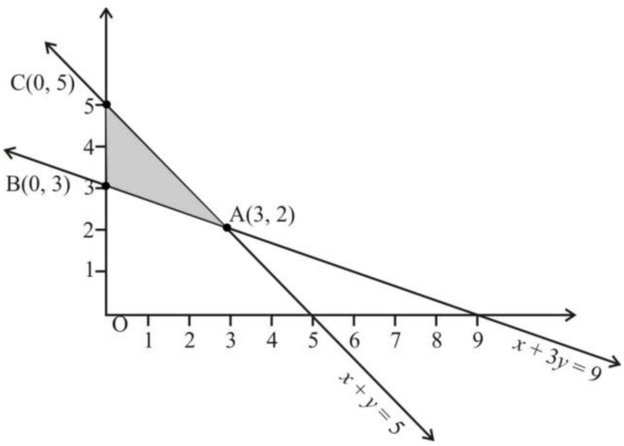
(a) 1,0,1 (b) 1,1,1 (c) 0,0,1 (d) 0,1,0

Q17. $\int e^{-x}\left(\frac{x+1}{x^2}\right)dx$ is equal to:

(a) $\frac{e^{-x}}{x}+C$ (b) $\frac{e^{x}}{x}+C$ (c) $\frac{e^{x}}{x^2}+C$ (d) $-\frac{e^{-x}}{x}+C$

Q18. The feasible region for an LPP is shown in the following figure. Then the minimum value

of Z = 11x + 7y is



(a) 21 (b) 47 (c) 20 (*d*) 31

Assertion Reasoning Based Question

Q19. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : The value of $cot\left(cos^{-1}\frac{7}{25}\right) is\ \frac{7}{24}$

Reason (R): $cot^{-1}(cot\theta)=\theta\ for\ all\ \theta\in(0,\pi)$

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c) **A** is correct but **R** is not correct
d) **A** is not correct but **R** is correct

Q20. Given below are two statements: one is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion (A) : If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, then $P(A \cap B) = \frac{4}{5}$, if A ,B are independent events.

Reason (R): If A and B are independent events, then $P(A \cap B) = P(A).P(B)$

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

a) Both **A** and **R** are correct and **R** is the correct explanation of **A**
b) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c) **A** is correct but **R** is not correct
d) **A** is not correct but **R** is correct

SECTION B

Q21. Evaluate the following: $\sin\left(\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

OR

Write the following in the simplest form: $y = \sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$

Q22. Find the intervals in which the following function is increasing or decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Q23. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1: 4, where

$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

OR

If the sum of two unit vector is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Q24. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=3\hat{i}-4\hat{j}-\hat{k}$.

Q25. Evaluate the following : $\int \frac{\sin 3x}{\sin x}\,dx$

SECTION C

Q26. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

Find $\frac{dy}{dx}$ if: $y=e^{\sin x}+(\tan x)^x$

Q27. Evaluate: $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$

Q28. Find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$.

Q29. Solve the following differential equation:

$$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy=\left[2x\sin\left(\frac{y}{x}\right)-y\cos\left(\frac{y}{x}\right)\right]dx$$

OR

Find the particular solution of the differential equation

$(x^2+1)\frac{dy}{dx}-2xy=(x^4+2x^2+1)\cos x$, given that $y(0)=0$

Q30. Two persons A and B throw a coin alternately till one of them gets a ‘head’ and win the game. Find their respective probabilities of winning if A starts first.

Q31. Solve the following linear programming problem (L.P.P) graphically.

Minimize $Z=5x+10y$

Subject to constraints:

$$x+2y\le 120;\, x+y\ge 60;\, x-2y\ge 0;\, x,y\ge 0$$

SECTION D

Q32. Find the inverse of matrix $A=\begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 2 \\ 4 & 2 & -3 \end{bmatrix}$

Hence solve the given system of equations:

$2x+3y+4z=17, 3x-2y+2z=11, 4x+2y-3z=8$.

Q33. Let $A = \{1,2,3,\ldots,9\}$ and R is the relation in $A \times A$ defined by $(a,b)R(c,d) \Leftrightarrow a+d = b+c$ for all $(a,b),(c,d)\ in\ A \times A$ .Prove that R is an equivalence relation. Also obtain the equivalence class $[(2,5)]$

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \dfrac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

Q34. A perpendicular is drawn from the point $(0,2,7)$ to the line $\dfrac{x+2}{-1} = \dfrac{y-1}{3} = \dfrac{z-3}{-2}$.

Find (i) foot of perpendicular (ii) length of perpendicular (iii) image of point in the line.

OR

Find the shortest distance between the following pair of parallel lines:

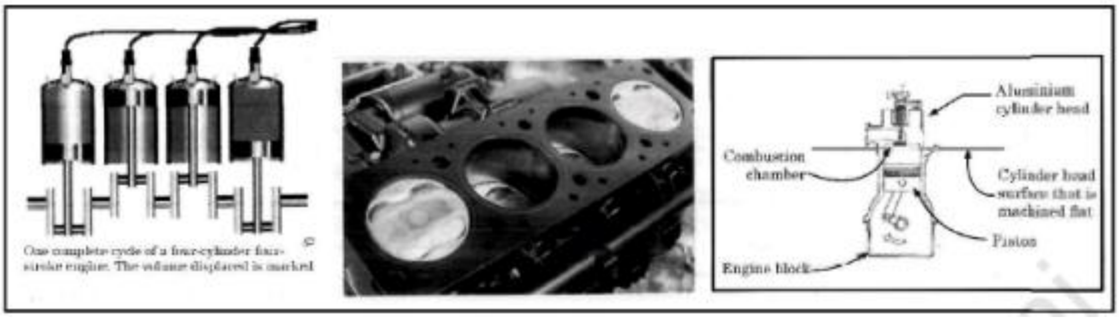
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}\ ;\ \frac{x-3}{4} = \frac{3-y}{-6} = \frac{z+5}{12}$$

Q35. Evaluate : $\displaystyle\int_{-5}^{0} \left(|x| + |x+2| + |x+5|\right) dx$

SECTION E

Q36. Read the following passage and answer the questions given below:

Engine displacement is the measure of the cylinder volume swept by all the pistons of the piston engine. The piston moves inside the cylinder bore. The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$.



(i) If the radius of the cylinder is $r\ cm$ and height is $h\ cm$, then write the volume V of the cylinder in terms of radius r

(ii) Find $\frac{dV}{dr}$

(iii) Find the radius of the cylinder when its volume is maximum

OR

For maximum volume, $h > r$. State true or false and justify.

Q37. Read the following passage and answer the questions given below:

To reduce global warming environmentalists and scientists came up with an innovative idea of developing a spherical bulb that would absorb harmful gases and thereby reduce global warming. But during the process of absorption the bulb would get inflated and it's radius would be increasing at 1cm/sec.

(i) Find the rate at which the volume increases when radius is 6 cm.

(ii) At an instant when volume was increasing at the rate of $400\pi cm^3$/sec find the rate at which it's surface area is increasing?

Q38. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4: 4: 2 respectively. The germination rates of three types of seeds are 45%, 60%, and 35%respectively.

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

CRPF PUBLIC SCHOOL, ROHINI, DELHI-110085

PRACTICE PAPER-4 (2024-25)

CLASS XII

Time: 3 hours **MATHEMATICS** **Max Marks 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has **18 MCQ's and 02 Assertion Reasoning based questions** of 1 mark each.

3. Section B has **5 Very Short Answer (VSA)-type questions** of 2 marks each.

4. Section C has **6 Short Answer (SA)-type questions** of 3 marks each.

5. Section D has **4 Long Answer (LA)-type questions** of 5 marks each.

6. Section E has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A (MCQ) 1 Mark Questions	
Q1	If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to : (a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$
Q2	Let A be a 3×3 matrix such that $\|\text{adj } A\| = 64$. Then $\|A\|$ is equal to : (a) 8 only (b) -8 only (c) 64 (d) 8 or -8
Q3	If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is : (a) 10 (b) 6 (c) 8 (d) 0
Q4	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is (a) $(-1, \infty)$ (b) $(-2, -1)$ (c) $(-\infty, -2)$ (d) $[-1, 1]$

Q5	If the set A contains 5 elements and the set B contains 6 elements, then the number of both one-one and onto mapping from A to B is (a) 720 (b) 120 (c) 30 (d) 0
Q6	The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$, is : (a) 5 (b) 6 (c) 7 (d) 4
Q7	The number of feasible solutions of the linear programming problem given as Maximize z = 15x + 30y subject to constraints : $3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is (a) 1 (b) 2 (c) 3 (d) infinite
Q8	In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to : (a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$ (c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$
Q9	$\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ is equal to : (a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$
Q10	If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of x + y is : (a) -2 (b) 2 (c) 3 (d) -3
Q11	The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is : (a) 2 (b) 0 (c) 1 (d) -1

Q12	The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then : (a) z is maximum at (2, 72), minimum at (15, 20) (b) z is maximum at (15, 20), minimum at (40, 15) (c) z is maximum at (40, 15), minimum at (15, 20) (d) z is maximum at (40, 15), minimum at (2, 72)
Q13	If $\|A\| = 2$, where A is a 2×2 matrix, then $\|4A^{-1}\|$ equals : (a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$
Q14	If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to : (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$
Q15	If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is : (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0
Q16	The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is : (a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$ (c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$
Q17	If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is : (a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log \left\| \sec\left(\frac{\pi}{4} - x\right) \right\|$ (d) $-\log \left\| \sec\left(\frac{\pi}{4} - x\right) \right\|$

Q18	Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are : (a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$
Assertion Reasoning Based Questions	
Q19	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R** *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$. *Reason (R) :* Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
Q20	Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R** *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5). *Reason (R):* Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$. In the light of the above statements, choose the ***most appropriate*** answer from the options given below a. Both **A** and **R** are correct and **R** is the correct explanation of **A** b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A** c. **A** is correct but **R** is not correct d. **A** is not correct but **R** is correct
SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each	
Q21	Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **OR** Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

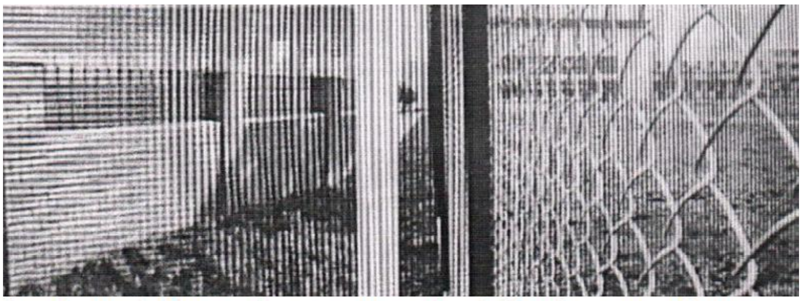
Q22	If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.
Q23	A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a. **OR** Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
Q24	Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.
Q25	Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.
SECTION – C (Short Answer (SA)-type questions) 3 Marks Each	
Q26	Determine graphically the minimum value of the following objective function : $z = 500x + 400y$ subject to constraints $x + y \leq 200$, $x \geq 20$, $y \geq 4x$, $y \geq 0$.
Q27	Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **OR** A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).
Q28	Find : $\int \frac{x^4}{(x-1)(x^2+1)} dx$ **OR** Find : $\int \frac{\cos \theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}} d\theta$

Q29	Find the general solution of the differential equation : $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$ **OR** Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.
Q30	Find the area of the following region using integration : $\{(x, y) : y^2 \le 2x \text{ and } y \ge x-4\}$
Q31	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.
SECTION – D (Long Answer (LA)-type questions) 5 Marks Each	
Q32	If matrix $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $3x+2y+z=2000$ $4x+y+3z=2500$ $x+y+z=900$
Q33	Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines. **OR** A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.
Q34	Evaluate : $\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx$ **OR** Evaluate : $\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)\,dx$

Q35 A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

SECTION – E (Case Study Based Questions) 4 Marks Each

Q36 Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*

Q37 A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Q38 The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**

CLASS XII

CRPF PUBLIC SCHOOL, ROHINI

MATHEMATICS

MARKING SCHEME

SESSION 2024-25

MARKING SCHEME: PRACTICE PAPER-01 (2024-25)

(i)

SECTION-A

(1) $A^2 = I \Rightarrow \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow x = 0$ $\therefore$ option (a)

(2) $|adj A| = |A|^{n-1} = |A|^2$; $|A| = 2$ $\therefore$ Ans $= 2^2$ $\therefore$ option (c)

(3) $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 12 \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \Rightarrow |\vec{x}|^2 = 13 \Rightarrow |\vec{x}| = \sqrt{13}$ $\therefore$ option (d)

(4) $\lim_{x\to 0} f(x) = f(0) \Rightarrow 2k = \lim_{x\to 0} \frac{3\sin \pi x}{5x} \Rightarrow k = \frac{3}{10}\cdot\left(\lim_{x\to 0}\frac{\sin\pi x}{\pi x}\right)\cdot\pi = \frac{3\pi}{10}$ $\therefore$ option (b)

(5) $I = \int \frac{\sec^2(1+\log x)}{x}\,dx$; Put $1+\log x = t \Rightarrow \frac{1}{x}dx = dt$ $\therefore I = \int \sec^2 t\, dt = \tan t + C$

$\therefore I = \tan(1+\log x) + C$ $\therefore$ option (a)

(6) $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \Rightarrow \frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = C$ $\therefore$ option (c)

(7) clearly option (d)

(8) $|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$; $\vec{a}\cdot\vec{b} = |\vec{b}|^2$

$|\vec{a}-\vec{b}|^2 = 7 \Rightarrow |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 7 \Rightarrow 14 - 2|\vec{b}|^2 + |\vec{b}|^2 = 7$

$\Rightarrow |\vec{b}|^2 = 7 \Rightarrow |\vec{b}| = \sqrt{7}$ $\therefore$ option (a)

(9) $I = \int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{2}\times\frac{1}{3}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right] + C$

$\therefore \int_0^{2/3} \frac{dx}{4+9x^2} = \frac{1}{6}\cdot\left[\tan^{-1}1 - \tan^{-1}0\right] = \frac{\pi}{24}$ $\therefore$ option (c)

(10) adding, $2A = \begin{bmatrix} 2 & -4 \\ 6 & 8 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$; subtracting, $2B = \begin{bmatrix} 6 & -2 \\ -4 & 4 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 3+4 & -1-4 \\ 9-8 & -3+8 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ $\therefore$ option (b)

(11) $Z_{(0,0)} = 0$; $Z_{(0,40)} = \underline{40}$; $Z_{(40,0)} = 40(0.4) = 16$; $Z_{(30,20)} = 30(0.4) + 20 = 32$

clearly, $Z_{max} = 40$ $\therefore$ option (b)

(12) $9(2x+5) - 3(5x+2) = 0 \Rightarrow 18x - 15x + 45 - 6 = 0 \Rightarrow 3x = -39 \Rightarrow x = -13$ $\therefore$ option (c)

(13) $|A| \neq 0 \Rightarrow 4(6-5) - \lambda(-5) - 3(-2) \neq 0 \Rightarrow 4 + 5\lambda + 6 \neq 0 \Rightarrow \lambda \neq -2$ $\therefore$ option (c)

(14) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$; $P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(A\cap B) = P(B)\times\frac{1}{4} = \frac{1}{12}$

Now, $P(A'\cap B') = P[(A\cup B)'] = 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right] = 1 - \left(\frac{6+4-1}{12}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore$ option (c)

(15) $m+n = 2+2 = 4$ $\therefore$ option (d)

(16) $x = a(\cos\theta + \theta\sin\theta) \Rightarrow \frac{dx}{d\theta} = a(-\sin\theta + \theta.\cos\theta + \sin\theta.1) = a\theta\cos\theta$

$y = a(\sin\theta - \theta\cos\theta) \Rightarrow \frac{dy}{d\theta} = a\left[\cos\theta - (\theta.(-\sin\theta)) + \cos\theta.1)\right] = a\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$ $\therefore$ option (b)

(17) $\vec{AB} = 2\hat{i} + (4-x)\hat{j} + 4\hat{k}$; $\vec{BC} = (y-3)\hat{i} + 6\hat{j} + 12\hat{k}$

A, B, C are collinear, $\frac{2}{y-3} = \frac{4-x}{6} = \frac{4}{12}$ $\Rightarrow x = 2, y = 3$ $\therefore$ option (a)

(18) $\cos^2\alpha + \frac{1}{4} + \frac{1}{4} = 1 \Rightarrow \cos^2\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$ $\therefore$ option (a)

(19) option (a)

(20) option (c)

Section - B

(21) here, $f(0) = \frac{1+0^2}{1+0} = 1$; $f(1) = \frac{1+1^2}{1+1} = 1$ ie. $f(0) = f(1)$ but $0 \neq 1$

$\therefore f$ is not one-one.

OR

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$\therefore y = \sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2}\right) = \sin^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)$

ie. $y = \sin^{-1}\left(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$

$\therefore \frac{dy}{dx} = \frac{1}{2} \times \frac{-1}{\sqrt{1-x^2}}$ Hence shown

(22) 

$\frac{dV}{dt} = \pi\ cm^3/s$; To find $\left.\frac{dS}{dt}\right]_{r=1cm}$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi.3r^2\frac{dr}{dt} \Rightarrow \cancel{\pi} = \frac{4}{3}\cancel{\pi}.3r^2.\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4r^2}$

Now, $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r.\frac{dr}{dt} \Rightarrow \frac{dS}{dt} = 8\pi r \times \frac{1}{4r^2} = \frac{2\pi}{r}$

$\therefore \left.\frac{dS}{dt}\right]_{r=1} = \frac{2\pi}{1} = \underline{2\pi\ cm^2/s}$

(23) $|\vec{a}| = |\vec{b}| = k$ (say) ; $\theta = 60^\circ$, $\vec{a}.\vec{b} = \frac{9}{2}$ $\Rightarrow |\vec{a}|.|\vec{b}|\cos\theta = \frac{9}{2}$

$\Rightarrow k.k.\cos 60^\circ = \frac{9}{2} \Rightarrow k^2 \times \frac{1}{2} = \frac{9}{2} \Rightarrow \underline{k = 3}$

OR

Passing point is $(1,2,-1)$; line: $5x - 25 = 14 - 7y = 35z$

$\Leftrightarrow 5(x-5) = -7(y-2) = 35z \Leftrightarrow \frac{x-5}{\left(\frac{1}{5}\right)} = \frac{y-2}{\left(-\frac{1}{7}\right)} = \frac{z}{\left(\frac{1}{35}\right)}$

$\Leftrightarrow \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$

$\therefore$ Req. vector eqn:

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

(24) $y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right) = \tan^{-1}\left(\cot\frac{x}{2}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right)$

$\therefore y = \frac{\pi}{2} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

(25) $A(1,1,2),\ B(2,3,5),\ C(1,5,5)$

$\vec{BA} = -\hat{i}-2\hat{j}-3\hat{j}, \quad \vec{BC} = -\hat{i}+2\hat{j}+0\hat{k}$

$ar(\Delta ABC) = \frac{1}{2}|\vec{BA}\times\vec{BC}| = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -2 & -3\\ -1 & 2 & 0\end{vmatrix} = \frac{1}{2}|\hat{i}(6)-\hat{j}(-3)+\hat{k}(-4)|$

$= \frac{1}{2}\sqrt{36+9+16} = \frac{\sqrt{61}}{2}$ sq. units

Section-C

(26) $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \Rightarrow 8 = A(x^2+4) + (Bx+C)(x+2)$

$A+B=0,\ 2B+C=0,\ 4A+2C=8$; $A=1,\ B=-1,\ C=2$

$I = \int\frac{dx}{x+2} + \int\frac{-x+2}{x^2+4}dx = \int\frac{dx}{x+2} - \frac{1}{2}\int\frac{2x}{x^2+4}dx + 2\cdot\int\frac{dx}{x^2+4}$

$= \log|x+2| - \frac{1}{2}\log|x^2+4| + \cancel{2}\times\frac{1}{\cancel{2}}\cdot\tan^{-1}\left(\frac{x}{2}\right) + C$

(27) $P(A) = \frac{4}{5},\ P(B) = \frac{3}{4},\ P(C) = \frac{2}{3}$

(i) $P(B\cap C\cap\bar{A}) = P(B)\cdot P(C)\cdot P(\bar{A}) = \frac{3}{4}\times\frac{2}{3}\times\frac{1}{5} = \frac{1}{10}$

(ii) $P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC) = \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3} + \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} + \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{13}{30}$

OR

X = no. of red cards drawn ; $X = 0,1,2$

$P(X=0) = \frac{26}{52}\times\frac{25}{51} = \frac{650}{2652}$; $P(X=1) = \frac{26}{52}\times\frac{26}{51}\times 2 = \frac{1352}{2652}$; $P(X=2) = \frac{26}{52}\times\frac{25}{51} = \frac{650}{2652}$

Probability Dist. is

X	0	1	2
P(X)	$\frac{650}{2652}$	$\frac{1352}{2652}$	$\frac{650}{2652}$

ie.

X	0	1	2
P(X)	$\frac{25}{102}$	$\frac{52}{102}$	$\frac{25}{102}$

mean = 1

(28) $x^2-2x=0 \Rightarrow x = 0,2 \quad \therefore I = -\int_1^2(x^2-2x)dx + \int_2^3(x^2-2x)dx$

$= -\left[\frac{x^3}{3}-\frac{2x^2}{2}\right]_1^2 + \left[\frac{x^3}{3}-\frac{2x^2}{2}\right]_2^3 = 2$

OR

$I = \int_0^\pi \frac{x\tan x}{\sec x\cdot\operatorname{cosec} x}dx = \int_0^\pi x\cdot\sin^2 x\,dx = \int_0^\pi x\cdot\left(\frac{1-\cos 2x}{2}\right)dx$

$\Rightarrow I = \frac{1}{2}\int_0^\pi x\,dx - \frac{1}{2}\int_0^\pi \underset{I}{x}\cdot\underset{II}{\cos 2x}\,dx$

$= \frac{1}{4}\cdot[x^2]_0^\pi - \frac{1}{2}\left[x\cdot\frac{\sin 2x}{2} - \int_0^\pi 1\cdot\frac{\sin 2x}{2}dx\right]_0^\pi$

$= \frac{\pi^2}{4}$

(29) $\frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)}$

Put $\frac{y}{x} = v$ ie. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{\sin v} \Rightarrow x\frac{dv}{dx} = -\operatorname{cosec} v \Rightarrow \frac{dv}{\operatorname{cosec} v} = -\frac{dx}{x} \Rightarrow \int \sin v\, dv = -\int \frac{dx}{x}$

$\Rightarrow -\cos v = -\log|x| + C \Rightarrow \cos v = \log|x| - C \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + C'$

when $x = 1$, $y = \frac{\pi}{2} \Rightarrow C = 0$ $\therefore$ solution is $\cos\left(\frac{y}{x}\right) = \log|x|$

OR

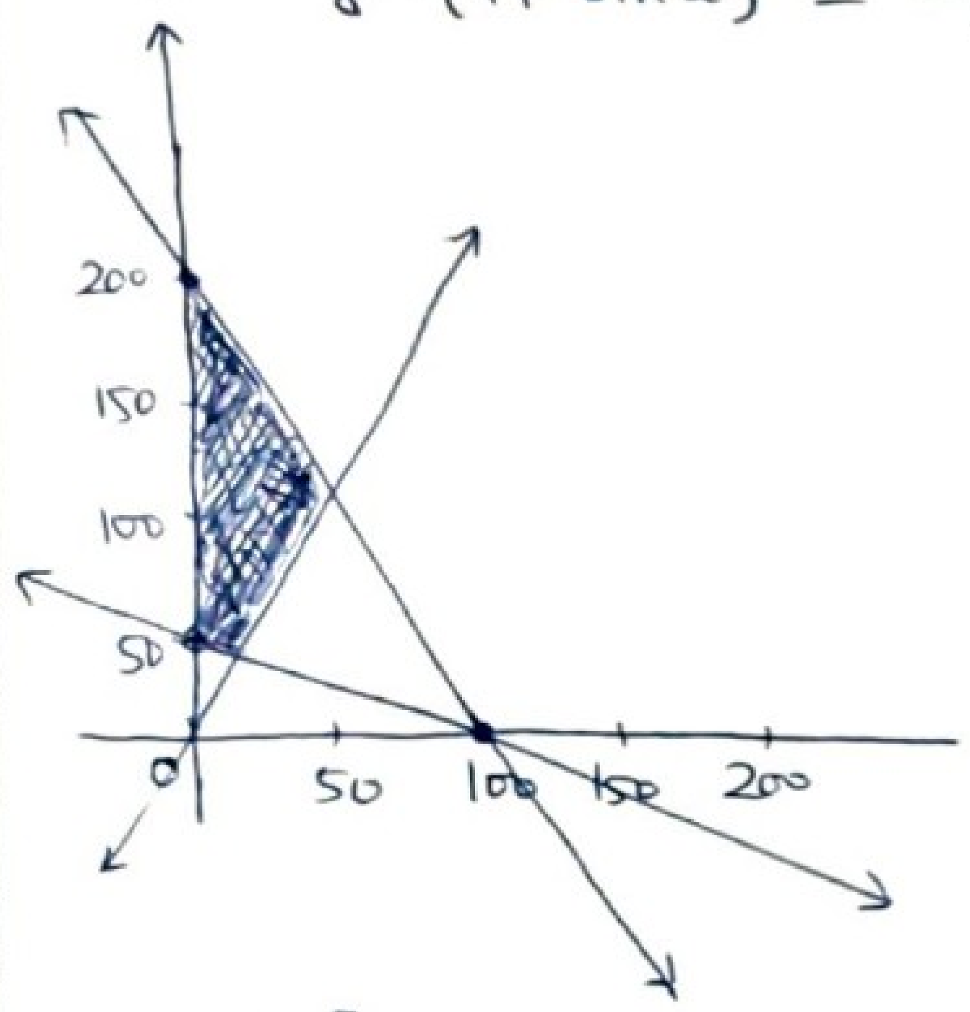
$\frac{dy}{dx} + \underbrace{\left(\frac{\cos x}{1+\sin x}\right)}_{P} \cdot y = \underbrace{-\frac{x}{1+\sin x}}_{Q}$

$I.F. = e^{\int P dx} = 1 + \sin x$

Soln : $y \cdot (1+\sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\, dx = \int -x\, dx = -\frac{x^2}{2} + C$

ie. $y \cdot (1+\sin x) = -\frac{x^2}{2} + C$

(30)



Corner Pt	$Z = x + 2y$
(0,50)	100
(50,100)	250
(0,200)	400
(20,40)	100

$\rightarrow$ max. at $x = 0$, $y = 200$

(31) $I = \int \frac{x^3+x+1}{x^2-1}\, dx = \int x\, dx + \int \frac{2x}{x^2-1}\, dx + \int \frac{dx}{x^2-1}$

$= \frac{x^2}{2} + \log|x^2-1| + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

Section-D

(32) for one-one : let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$

let $f(x_1) = f(x_2)$

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one

for onto let $y = f(x) \Rightarrow y = \frac{4x+3}{3x+4} \Rightarrow x = \frac{3-4y}{3y-4} \in$ co-domain

$\Rightarrow f$ is onto

OR

$|a-a| = 0$, which is divisible by 4 $\therefore$ S is reflexive.

$(a,b) \in S \Rightarrow |a-b|$ is div. by 4 $\Rightarrow |b-a|$ is div. by 4 $\Rightarrow (b,a) \in S \Rightarrow S$ is symm.

$(a,b) \in S$ and $(b,c) \in S \Rightarrow |a-b| = 4\lambda$, $|b-c| = 4\mu \Rightarrow |a-c| = 4\lambda' \Rightarrow S$ is tran.

$\therefore$ S is equivalence.

$[1] = \{1, 5, 9\}$

(33) $f'(x) = x^3 - 3x^2 - 10x + 24 = (x-2)(x-4)(x+3)$

$f'(x) = 0 \Rightarrow x = -3, 2, 4$

$$\begin{array}{ccccccccc} & - & & + & & - & & + & \\ -\infty & & -3 & & 2 & & 4 & & \infty \end{array}$$

f is inc. when $(-3, 2) \cup (4, \infty)$

f is dec. when $(-\infty, -3) \cup (2, 4)$

(34) Let eq^n of req. line is $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$

here, $3a - 16b + 7c = 0$ and $3a + 8b - 5c = 0$

solving: $\frac{a}{24} = \frac{b}{36} = \frac{c}{72} = \lambda'$ or $\frac{a}{2} = \frac{b}{3} = \frac{z}{6}$

Req. Ans: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

OR

here, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$, $\vec{a}_2 = \hat{i} + \hat{j} + 2\hat{k}$; $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$

given lines are parallel.

here, $\vec{a}_2 - \vec{a}_1 = 3\hat{k}$; $|\vec{b}| = \sqrt{6}$

Req. Dist. $= \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right| = \frac{|3\hat{i} - 6\hat{j}|}{\sqrt{6}} = \frac{\sqrt{45}}{\sqrt{6}} = \sqrt{\frac{15}{2}}$

(35) $|A| = 6 \neq 0 \Rightarrow A^{-1}$ exists.

Cofactors: $A_{11} = -6$; $A_{12} = 14$; $A_{13} = -15$

$A_{21} = 17$; $A_{22} = 5$; $A_{22} = 9$

$A_{31} = 13$; $A_{32} = -8$; $A_{33} = -1$

$$\text{adj}\, A = \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}\, A = \frac{1}{6} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$$

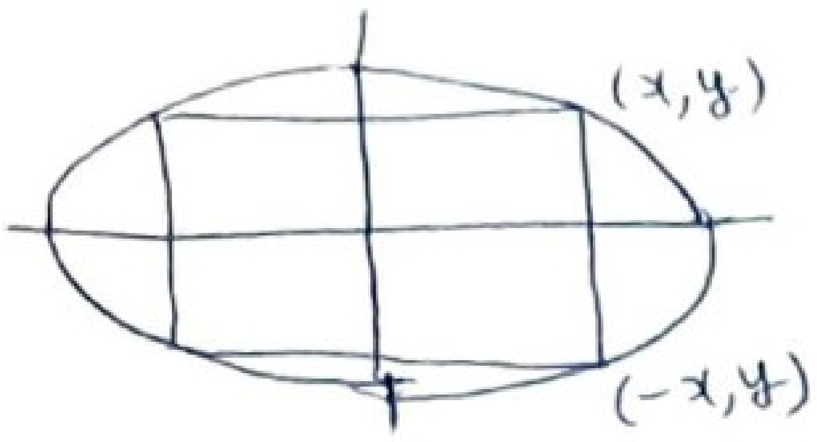
$$X = A^{-1} \cdot B = \frac{1}{6} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 201 \\ -134 \\ 67 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

$\Rightarrow x = 3$, $y = 2$, $z = -1$ Ans

Section D

(36)



$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

(i) $A = (2x)(2y) = 4x \cdot \frac{b}{a}\sqrt{a^2-x^2} \Rightarrow A = \frac{4b}{a}\, x \sqrt{a^2-x^2}$

(ii) $A^2 = A' = \frac{16b^2}{a^2} \cdot x^2(a^2-x^2)$

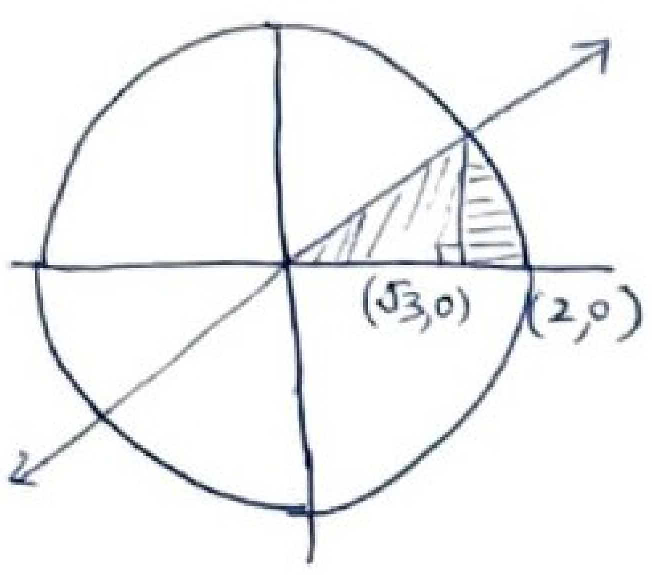
$\frac{dA'}{dx} = x(2a^2 - 4x^3) = 0 \Rightarrow x = \pm\frac{a}{\sqrt{2}}, 0 \Rightarrow x = \frac{a}{\sqrt{2}}$; when $x = \frac{a}{\sqrt{2}}$,

(iii) $\left.\frac{d^2A'}{dx^2}\right]_{x<\frac{a}{\sqrt{2}}} = +ve$; $\left.\frac{d^2A'}{dx^2}\right]_{x>\frac{a}{\sqrt{2}}} = -ve$ $\therefore$ A' or A is max.

also, $\frac{d^2A'}{dx^2} = \frac{16b^2}{a^2}\cdot\left(2x^2 - \frac{12a^2}{2}\right) < 0$ $\therefore$ A' or A is max. at $x = \frac{a}{\sqrt{2}}$

Now, $l = 2x = \sqrt{2}\,a$ and $b = 2y = \sqrt{2}\,b$

(37)



$$A = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

$$= \frac{1}{2\sqrt{3}}\left[x^2\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2} = \frac{\pi}{3} \text{ sq. units}$$

(38) E_1: item manufactured by A

E_2: " " " B

E_3: " " " C

A : defective item is found.

(a) $P(A) = \frac{50}{100}\times\frac{1}{100} + \frac{30}{100}\times\frac{5}{100} + \frac{20}{100}\times\frac{7}{100} = \frac{340}{10000} = \frac{17}{500}$

(b) $P(E_2|A) = \dfrac{\frac{30}{100}\times\frac{5}{100}}{P(A)} = \frac{15}{34}$

CRPF PUBLIC SCHOOL, ROHINI, DELHI-110085

MARKING SCHEME: PRACTICE PAPER-02 (2024-25)

Section - A

(1) $|adj A| = |A|^2 \Rightarrow 144 = |A|^2 \Rightarrow |A| = \pm 12$ $\therefore$ option (c)

(2) $\begin{vmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{vmatrix} = 0 \Rightarrow 4\sin^2 x - 3 = 0 \Rightarrow \sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore$ option (d)

(3) $ar(\Delta ABC) = 0 \Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0 \Rightarrow k = 5$ $\therefore$ option (b)

(4) here, $x+2 = -(2x-3) \Rightarrow x = \frac{1}{3}$ $\therefore$ option (a)

(5) $y = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)$

$= \tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$

$= \tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$= \frac{\pi}{4}+\frac{x}{2}$ $\therefore \frac{dy}{dx} = \frac{1}{2}$

$\therefore$ option (a)

(6) $k = \lim_{x\to 0} \frac{1-\cos 4x}{8x^2} = \lim_{x\to 0} \frac{2\sin^2 2x}{8x^2}$

$= \lim_{2x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 = 1^2 = 1$ $\therefore$ option (a)

(7) $I = \int \frac{x+3}{(x+4)^2} \cdot e^x dx = \int \frac{x+4-1}{(x+4)^2} \cdot e^x dx = \int \left[\frac{1}{x+4} - \frac{1}{(x+4)^2}\right] \cdot e^x dx$

$= \frac{e^x}{x+4} + C$ $\therefore$ option (a)

(8) $I = \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x}+1} dx$ Put $e^x = t$ $\therefore I = \int \frac{dt}{1+t^2} = \tan^{-1} t + C$

$= \tan^{-1} e^x + C$ $\therefore$ option (a)

(9) $A_{22} = \begin{vmatrix} 10 & 2 \\ 9 & 2 \end{vmatrix} = 2$; $A_{23} = -\begin{vmatrix} 10 & 19 \\ 9 & 24 \end{vmatrix} = -69$

$\therefore A_{22} + A_{23} = 2 + (-69) = -67$ $\therefore$ option (c)

(10) $\frac{dy}{dx} + \frac{1}{x\log x} \cdot y = \frac{2}{x}$ $\therefore$ I.F. $= e^{\int \frac{1}{x\log x} dx} = e^{\log(\log x)} = \log x$

$\therefore$ option (c)

(11) clearly $2+3=5$ $\therefore$ option (d)

(12) $\vec{AB} = 2\hat{i} + (4-x)\hat{j} + 4\hat{k}$; $\vec{BC} = (y-3)\hat{i} - 6\hat{j} - 12\hat{k}$

here, $\frac{2}{y-3} = \frac{4-x}{-6} = \frac{4}{-12}$ $\Rightarrow x=2, y=-3$ $\therefore$ option (a)

(13) $\vec{a}\cdot\vec{b} = -1 \Rightarrow |\vec{a}|.|\vec{b}|.\cos\theta = -1 \Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

$\therefore$ option (c)

(14) $A = \frac{1}{2}|\vec{d_1}\times\vec{d_2}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \frac{1}{2}|-2\hat{i} - 14\hat{j} - 10\hat{k}|$

$= \frac{1}{2}\sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \frac{\sqrt{300}}{2} = 5\sqrt{3}$

$\therefore$ option (b)

(15) $\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1 \Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1 \Rightarrow \cos\gamma = \frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3}$

$\Rightarrow$ option (b)

(16) $P(W,G,G) = \frac{4}{16}\times\frac{7}{15}\times\frac{6}{14} = \frac{1}{20}$ $\therefore$ option (a)

(17) $Z = 11x + 7y$

$Z_{(0,3)} = 21$; $Z_{(0,5)} = 35$; $Z_{(3,2)} = 47$ $\therefore$ option (a)

(18) option (d)

(19) option (a)

(20) option (d)

Section-B

(21) $\vec{d_1} = \vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ $\therefore |\vec{d_1}| = \sqrt{49} = 7$

$\vec{d_2} = \vec{b} - \vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$ $\therefore |\vec{d_2}| = \sqrt{69}$

$\therefore$ unit vectors along $\vec{d_1}$ & $\vec{d_2}$ are $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$ & $\frac{1}{\sqrt{69}}(\hat{i} + 2\hat{j} - 8\hat{k})$

OR

here, $|\vec{a}| = |\vec{b}| = 1$

also, $(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0$

$\Rightarrow 7|\vec{a}|^2 + 16(\vec{a}.\vec{b}) - 15|\vec{b}|^2 = 0$

$\Rightarrow 16.|\vec{a}|.|\vec{b}|.\cos\theta = 15 - 7 \Rightarrow \cos\theta = \frac{8}{16} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

(22) $\vec{AB} = \vec{OB} - \vec{OA} = (2\hat{i} - \hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + \hat{k}) = \hat{i} + 2\hat{k}$

$\vec{CD} = \vec{OD} - \vec{OC} = (3\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} - 3\hat{k}) = \hat{i} - 2\hat{j} + 4\hat{k}$

Req. Projection $= \dfrac{\vec{AB} \cdot \vec{CD}}{|\vec{CD}|} = \dfrac{(1)(1) + 2(4)}{\sqrt{1+4+16}} = \dfrac{9}{\sqrt{21}}$ Ans

(23) let $y = \sin\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$\therefore y = \sin\left(2\tan^{-1}\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}\right) = \sin\left(2\tan^{-1}(\tan\theta)\right) = \sin(2\theta)$

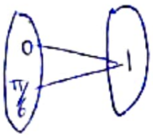
$= \sin(\cos^{-1}x)$

$= \sin(\sin^{-1}\sqrt{1-x^2})$

OR

$= \sqrt{1-x^2}$ Ans

$f: R \to R$; $f(x) = \sin^2 x + \cos^2 x$



$\therefore$ not one-one ; Not onto as $R_f = \{1\} \neq$ Co-domain

$\Rightarrow f$ is not bijective

(24) $y = [\log(x + \sqrt{x^2+1})]^2$

$\Rightarrow \dfrac{dy}{dx} = 2 \cdot \log(x + \sqrt{x^2+1}) \cdot x\left[1 + \dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}\right] \times \dfrac{1}{x + \sqrt{x^2+1}}$

$= 2\log(x + \sqrt{x^2+1}) \cdot \left(\dfrac{\cancel{\sqrt{x^2+1} + x}}{\sqrt{x^2+1}} \times \dfrac{1}{\cancel{x + \sqrt{x^2+1}}}\right)$

$\Rightarrow \sqrt{1+x^2} \cdot \dfrac{dy}{dx} = 2\log(x + \sqrt{x^2+1})$

diff. again,

$\sqrt{1+x^2} \cdot \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} \cdot \dfrac{1}{2\sqrt{1+x^2}} \cdot (2x) = 2 \times \dfrac{1}{x + \sqrt{x^2+1}} \times \left(1 + \dfrac{2x}{2\sqrt{x^2+1}}\right)$

$\Rightarrow (1+x^2) \cdot \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 2$. Hence proved

(25)

$\dfrac{dV}{dt} = 12\ cm^3/s$; $h = \frac{1}{6}r \Rightarrow r = 6h$

Now, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6h)^2 \times h \Rightarrow V = 12\pi h^3$

$\Rightarrow \dfrac{dV}{dt} = 36\pi h^2 \dfrac{dh}{dt} \Rightarrow 12 = 36 \times \pi h^2 \dfrac{dh}{dt}$

$\Rightarrow \left.\dfrac{dh}{dt}\right]_{h=4cm} = \dfrac{12}{36\pi(4)^2} = \dfrac{1}{48\pi}$ cm/s Ans

Section-C

(26) $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ —— ①

$= \int_0^{\pi} \frac{(\pi - x) . \tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} dx$

$I = \int_0^{\pi} \frac{+(\pi - x) \tan x}{+(\sec x + \tan x)} dx$ —— ②

adding ① + ②

$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} dx$

$\Rightarrow I = \frac{\pi}{2} \cdot \int_0^{\pi} \frac{\sec x \tan x - \tan^2 x}{1} dx$

$= \frac{\pi}{2} \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1) dx$

$= \frac{\pi}{2} \left[\sec x - \tan x + x\right]_0^{\pi} = \frac{\pi}{2} (\pi - 2)$ Ans

OR

$I = \int_1^4 |x-1| + |x-2| + |x-3| \, dx$

$= \int_1^2 (-x+4) dx + \int_2^3 x \, dx + \int_3^4 (3x-6) dx$

$= \left[-\frac{x^2}{2} + 4x\right]_1^2 + \left[\frac{x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2} - 6x\right]_3^4 = \frac{19}{2}$ Ans

(27) $I = \int \frac{3x+1}{(x-2)^2 (x+2)} dx$

Let $\frac{3x+1}{(x-2)^2 (x+2)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$\Rightarrow 3x+1 = A(x-2)^2 + B(x-2)(x+2) + C(x+2)$

$\Rightarrow 3x+1 = A(x^2-4x+4) + B(x^2-4) + C(x+2)$

Equating, $A + B = 0$, $-4A + 2C = 6$; $4A + 2C = 1$

Solving, $A = -\frac{5}{16}$, $B = \frac{5}{16}$, $C = \frac{7}{4}$

$I = \frac{-5}{16} \int \frac{dx}{x+2} + \frac{5}{16} \int \frac{dx}{x-2} + \frac{7}{4} \int \frac{dx}{(x-2)^2}$

$= \frac{5}{16} \log \left|\frac{x-2}{x+2}\right| - \frac{7}{4(x-2)} + C$

(28) $I = \int \cos(\log x)\,.dx$

Put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$I = \int \underset{I}{\cos t}\,.\,\underset{II}{e^t}\;dt$

$= \cos t\,.\,e^t - \int(-\sin t).e^t\,dt$

$= \cos t\,.\,e^t + \int \underset{I}{\sin t}\,.\,\underset{II}{e^t}\;dt$

$= \cos t\,.\,e^t + \sin t\,.\,e^t - \int \cos t\,.e^t\,dt$

$= e^t(\cos t + \sin t) - I$

$\Rightarrow 2I = e^t(\cos t + \sin t) \quad \Rightarrow I = \frac{1}{2}e^t(\cos t + \sin t) + C$

$= \frac{1}{2}x.[\cos(\log x) + \sin(\log x)] + C$ Ans

(29) $(\tan^{-1}y - x).dy = (1+y^2).dx$

$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}.x = \frac{\tan^{-1}y}{1+y^2}$

$I.F. = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Solution is: $y.e^{\tan^{-1}y} = \underbrace{\int \frac{\tan^{-1}y}{1+y^2}.e^{\tan^{-1}y}\,dy}_{I}$

for I, $\tan^{-1}y = t \quad \therefore I = \int t.e^t\,dt = (t-1).e^t + C$

$\Rightarrow y = \tan t \qquad = (\tan^{-1}y - 1).e^{\tan^{-1}y} + C$

$\therefore$ soln is. $y.e^{\tan^{-1}y} = (\tan^{-1}y - 1).e^{\tan^{-1}y} + C$

OR

clearly given DE is homogeneous.

$\frac{dy}{dx} - \frac{y}{x} + \sin\left(\frac{y}{x}\right) = 0$

Put $\frac{y}{x} = v$ ie $y = vx \quad \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Given DE becomes, $v + x\frac{dv}{dx} = v - \sin v$

$\Rightarrow \int \operatorname{cosec} v\,dv = -\int\frac{dx}{x}$

$\Rightarrow \log\left|\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x}\right| = -\log x + \log c$

$\Rightarrow \operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x} = \frac{c}{x}$

when $x = 2,\ y = \pi \Rightarrow c = 2$

$\therefore$ Req. solution is $\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x} = \frac{2}{x}$ Ans

(30) $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{2}$; $P(\bar{A})=\frac{1}{2}$, $P(\bar{B})=\frac{1}{2}$

Prob. of winning by A $= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + \dots \infty$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^2\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^4\cdot\frac{1}{2} + \dots$

$= \dfrac{\frac{1}{2}}{1-\frac{1}{4}} = \underline{\frac{2}{3}}$ $\left(\because a+ar+\dots\infty = \frac{a}{1-r}\right)$

$P(B) = 1-\frac{2}{3} = \underline{\frac{1}{3}}$

OR

Total people = 50

E : People believe in non-violence ; F : People believe in violence

$P(E)=\frac{20}{50}$, $P(\bar{E})=\frac{30}{50}$

X : No. of non-violent person ; $x=0,1,2$

$P(X=0) = \frac{30}{50}\times\frac{29}{49}$

$P(X=1) = 2\times\frac{30}{50}\times\frac{20}{49}$

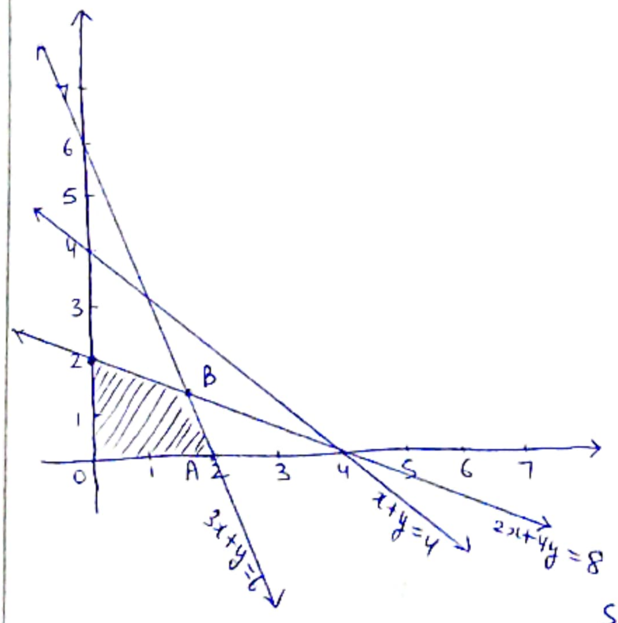
$P(X=2) = \frac{20}{50}\times\frac{19}{49}$

P.D. table

X	0	1	2
P(X)	$\frac{30}{50}\times\frac{29}{49}$	$2\times\frac{30}{50}\times\frac{20}{49}$	$\frac{20}{50}\times\frac{19}{49}$

$E(X) = \frac{196}{245}$

(31)



Point	$Z=2x+5y$
(0,0)	0
(2,0)	4
(0,2)	10
$\left(\frac{8}{5},\frac{6}{5}\right)$	$\frac{46}{5}$

$\longrightarrow$ max

$\therefore$ Z is max. at (0,2).

Max. value $= \underline{10}$

Section - D

(32) $|A| = 1200 \neq 0 \Rightarrow A^{-1}$ exists

$A^{-1} = \frac{1}{|A|}\cdot adj A = \frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$

$AX = B \Rightarrow X = A^{-1}.B = \frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{3} \\ \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix}$

$\therefore x=2, y=-3, z=5$ Ans

OR

$AB = 4I \Rightarrow B^{-1} = \frac{1}{4} \cdot A$

Given system $\quad BX = C \Rightarrow X = B^{-1}C = \frac{1}{4} A \cdot C$

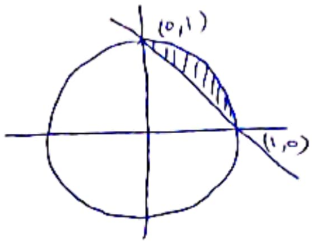
$$= \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore \underline{x = 2,\ y = 1,\ z = -1}$ Ans.

(33) $A = \{(x,y) : x^2 + y^2 \le 1 \le x + y\}$

Consider, $x^2 + y^2 = 1$ and $x + y = 1$



$A = \int_0^1 (y \text{ of circle})\,dx - \int_0^1 (y \text{ of line})\,dx$

$= \int_0^1 \sqrt{1-x^2}\,dx - \int_0^1 (1-x)\,dx$

$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1 = \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units.

(34) $f(x) = 5x^2 + 6x - 9$

for one-one $\quad$ Let $x_1, x_2 \in R_+$

let $f(x_1) = f(x_2)$

$\Rightarrow 5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9$

$\Rightarrow (x_1 - x_2)(5x_1 + 5x_2 + 6) = 0$

$\Rightarrow x_1 = x_2 \quad \Rightarrow f$ is one-one

for onto : $\quad y = 5x^2 + 6x - 9$

$\Rightarrow 5x^2 + 6x - 9 - y = 0 \quad \Rightarrow x = \frac{-6 \pm \sqrt{36 + 4 \times 5(9+y)}}{10} > 0$

$\Rightarrow -6 \pm \sqrt{216 + 20y} > 0$

$\Rightarrow 216 + 20y > 36 \quad \Rightarrow y \in (-9, \infty) =$ Co-domain.

$\therefore f$ is onto

$\Rightarrow f$ is bijective fn.

OR

$R = \{(a,b) : a, b \in A\ ;\ |a-b|$ is div. by $4\}$

for reflexive: $|a-a|$ i.e. 0, is div. by $4 \Rightarrow (a,a) \in R \Rightarrow R$ is reflexive.

for symmetric: Let $(a,b) \in R \Rightarrow |a-b|$ is div. by 4

$\Rightarrow |b-a|$ is div. by 4 $\Rightarrow (b,a) \in R \Rightarrow R$ is symm.

for transitive: Let $(a,b), (b,c) \in R$

$\Rightarrow |a-b|$ is div. by 4 and $|b-c|$ is div. by 4

$\Rightarrow a - b = \pm 4\lambda$ and $b - c = \pm 4\mu$

$\Rightarrow a - c = \pm 4(\lambda + \mu) = \pm 4\lambda' \Rightarrow |a-c|$ is div. by 4 $\Rightarrow (a,c) \in R \Rightarrow R$ is transitive

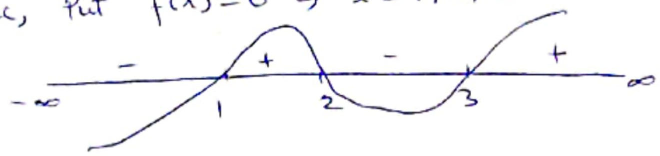
$\therefore R$ is equivalence Relation.

Now; $[1] = \{1, 5, 9\}$ and $[2] = \{2, 10\}$

(35) $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$f'(x) = 4x^3 - 24x^2 + 44x - 24 = 4(x^3 - 6x^2 + 11x - 6) = 4(x-1)(x-2)(x-3)$

for inc/dec, Put $f'(x) = 0 \Rightarrow x = 1, 2, 3$



(a) f is inc when $x \in [1,2] \cup [3,\infty)$

(b) f is dec. when $x \in (-\infty,1] \cup [2,3]$

Section-E

(36) E_1: farmer is selected from village A

E_2: " " " " " B

E_3: " " " " " C

A: farmer selected believes in technology

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A|E_1) = \frac{60}{100}$, $P(A|E_2) = \frac{70}{100}$, $P(A|E_3) = \frac{80}{100}$

(i) $P(A|E_1) = \frac{60}{100}$

(ii) $P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + P(E_3).P(A|E_3)$

$= \frac{1}{3} \times \frac{60}{100} + \frac{1}{3} \times \frac{70}{100} + \frac{1}{3} \times \frac{80}{100} = \frac{210}{300} = \frac{7}{10}$

(iii) $P(E_2|A) = \dfrac{P(E_2).P(A|E_2)}{P(A)}$

$= \dfrac{\frac{1}{3} \times \frac{70}{100}}{\frac{210}{300}} = \frac{1}{3}$

(37) (i) $2y + x + \pi\left(\frac{x}{2}\right) = 10 \Rightarrow y = \frac{1}{4}(20 - \pi x - 2x)$

(ii) $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{4}.x(20 - \pi x - 2x) + \frac{1}{8}\pi x^2$

$\Rightarrow \frac{dA}{dx} = \dfrac{20 - 2\pi x - 4x}{4} + \frac{1}{4}\pi x = 0 \Rightarrow x = \dfrac{20}{4+\pi}$; $\dfrac{d^2A}{dx^2} < 0$ here.

OR

(iii) $A = \frac{x}{4}(20 - \pi x - 2x) + \dfrac{\pi x^2}{8} = \dfrac{50}{\pi + 4}$ Sq. units

(38) (i) $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$; $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$ $\therefore |\vec{b}_1 \times \vec{b}_2| = 3\sqrt{3}$

$\therefore SD = \dfrac{3(3) + 3(-3) + 0}{3\sqrt{3}} = 0$

(ii) line 1: $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = \lambda \Rightarrow$ Any point $(\lambda, 2\lambda, -\lambda)$

line 2: $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = \mu \Rightarrow$ Any point $(2\mu+3, \mu+3, \mu)$

Pt. of collision, $\lambda = 2\mu + 3$; $2\lambda = \mu + 3$; $-\lambda = \mu$ $\Rightarrow \lambda = 1, \mu = -1$

Req. Point is $(1, 2, -1)$

SECTION-A

A1. $n(A)=2$ ∴ No. of reflexive relations $= 2^{n(n-1)} = 2^{2\times1} = 4$ ∴ option (b)

A2. A, B, C are collinear ⇒ ar(ΔABC) = 0

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0 \Rightarrow 3(2-8)+2(k-8)+1(8k-16)=0 \Rightarrow k=5$$

∴ option (c)

A3. $\begin{vmatrix} x & 2 \\ 3 & x-2 \end{vmatrix} = 0 \Rightarrow x^2-2x-6=0$

∴ Product of all possible values of x is $\frac{c}{a}$ ie. $-\frac{6}{1}=-6$

∴ option (b)

A4. $|A| = |kA| \Rightarrow |A| = k^n.|A| \Rightarrow k^2=1 \Rightarrow k=\pm1$

∴ sum $= -1+1 = 0$ ∴ option (d)

A5. $A.(\text{adj}A) = 3I \Rightarrow |A|.I = 3I \Rightarrow |A|=3$

also, $|\text{adj}A| = |A|^{n-1} = 3^2 = 9$

∴ $|A| + |\text{adj}A| = 3+9 = 12$ ∴ option (a)

A6. As f is cts. at $x=0$

$\Rightarrow \lim_{x\to0} f(x) = f(0) \Rightarrow 2k = \lim_{x\to0}\left(\frac{\sin x}{x}+\cos x\right) \Rightarrow 2k = 1+1 \Rightarrow k=1$

∴ option (a)

A7. $x = a\cos^2\theta \Rightarrow \frac{dx}{d\theta} = 2a\cos\theta(-\sin\theta)$

$y = b\sin^2\theta \Rightarrow \frac{dy}{d\theta} = 2b\sin\theta(\cos\theta)$

∴ $\frac{dy}{dx} = \frac{2b\sin\theta\cos\theta}{-2a\sin\theta\cos\theta} = -\frac{b}{a}$ ∴ option (c)

A8. option (a)

A9. $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$ —(1)

$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$ —(2)

add, $2I = \int_0^{\pi/2} 1.dx \Rightarrow 2I = [x]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$ ∴ option (c)

A10. order = 2, degree = 2 ∴ Product = 2×2 = 4 ∴ option (a)

A11. $\frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{2\log x}{x^2.\log x}$; $P = \frac{1}{x\log x}$ ∴ $IF = e^{\int\frac{1}{x\log x}dx} = e^{\int\frac{dt}{t}} = e^{\log t}$

$(\because \log x = t \Rightarrow \frac{1}{x}dx = dt)$

$= e^{\log(\log x)}$

$= \log x$

∴ option (c)

A12. Req. Projection $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \dfrac{3(1)+(-1)(2)+(-2)(-3)}{\sqrt{1+4+9}} = \dfrac{\sqrt{14}}{2}$ $\therefore$ option (a)

A13. $\vec{a}+\vec{b} = -\vec{c} \Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = (-\vec{c})\cdot(-\vec{c}) \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|\,|\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow (3)^2 + (5)^2 + 2(3)(5)\cos\theta = (7)^2$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\therefore$ option (d)

A14. $\vec{b} = \hat{i} - \vec{a} = \hat{i} - 2\hat{i} + 2\hat{j} - 2\hat{k} \Rightarrow \vec{b} = -\hat{i} + 2\hat{j} - 2\hat{k} \Rightarrow |\vec{b}| = \sqrt{1+4+4}$

$\Rightarrow |\vec{b}| = 3$ $\therefore$ option (b)

A15 d.r. of I line $= \langle 7, -5, 1 \rangle$

d.r. of II line $= \langle 1, \frac{\lambda}{2}, 3 \rangle$

here, $7(1) + (-5)\left(\frac{\lambda}{2}\right) + 1(3) = 0 \Rightarrow -\frac{5\lambda}{2} = -10 \Rightarrow \lambda = 4$ $\therefore$ option (b)

A16. req. line refers to y-axis, whose d.c. are 0, 1, 0 $\therefore$ option (d)

A17. $I = \int e^{-x}\left(\frac{x+1}{x^2}\right)dx$; Put $-x = t \Rightarrow dx = -dt$ also, $x = -t$

$\therefore I = \int e^{t}\left(\frac{-t+1}{t^2}\right)dt = \int\left(\frac{1}{t} - \frac{1}{t^2}\right)\cdot e^{t}\,dt = \frac{e^{t}}{t} + C$

i.e. $I = \frac{e^{-x}}{-x} + C = -\frac{e^{-x}}{x} + C$ $\therefore$ option (d)

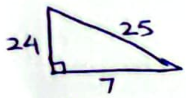
A18. $Z = 11x + 7y$

$\therefore Z_{A(3,2)} = 11(3) + 7(2) = 47$

$Z_{B(0,3)} = 11(0) + 7(3) = 21$

$Z_{C(0,5)} = 11(0) + 7(5) = 35$ $\therefore Z_{min} = 21$ $\therefore$ option (a)

A19. $\cot\left(\cos^{-1}\frac{7}{25}\right) = \cot\left(\cot^{-1}\frac{7}{24}\right) = \frac{7}{24}$



$\therefore$ A is true.

R is also true but not used in A. $\therefore$ option (b)

A20. $P(A) \times P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25} \neq P(A \cap B)$ $\therefore$ A is not true.

but R is true. $\therefore$ option (d)

SECTION-B

A21. $\sin\left(\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \sin\left(\frac{\pi}{6} + \sin^{-1}\frac{\sqrt{3}}{2}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = \sin\frac{\pi}{2} = 1$ Ans

OR

$y = \sin^{-1}\left(\frac{x + \sqrt{1-x^2}}{\sqrt{2}}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$\therefore y = \sin^{-1}\left(\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$

$\Rightarrow y = \frac{\pi}{4} + \theta \Rightarrow y = \frac{\pi}{4} + \sin^{-1}x$

A22. $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x+1)(x+2)$

for inc/dec, $f'(x) = 0 \Rightarrow x = -1, -2$

$-\infty \quad - \quad -2 \quad + \quad -1 \quad - \quad \infty$

$\therefore f$ is dec. when $x \in (-\infty, -2) \cup (-1, \infty)$ & inc. when $x \in (-2, -1)$

A23. $\vec{r} = \frac{4(2\hat{i}+3\hat{j}+4\hat{k}) - 1(-\hat{i}+\hat{j}+\hat{k})}{4-1} \Rightarrow \vec{r} = \frac{8\hat{i}+12\hat{j}+16\hat{k}+\hat{i}-\hat{j}-\hat{k}}{3}$

$\therefore \vec{r} = \frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$ or $3\hat{i} + \frac{11}{3}\hat{j} + 5\hat{k}$

OR

Given: $|\vec{a}| = |\vec{b}| = 1$; also $|\vec{a}+\vec{b}| = 1$

Now, $|\vec{a}+\vec{b}| = 1 \Rightarrow |\vec{a}+\vec{b}|^2 = 1 \Rightarrow (\vec{a}+\vec{b}).(\vec{a}.\vec{b}) = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 1$

$\Rightarrow 1+1+2\vec{a}.\vec{b} = 1 \Rightarrow \vec{a}.\vec{b} = -\frac{1}{2}$

Now, $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$= 1+1-2\left(-\frac{1}{2}\right) = 3 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{3}$, Hence shown.

A24. $\vec{d_1} = 2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{d_2} = 3\hat{i}-4\hat{j}-\hat{k}$

ar(||gm) $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

Now, $\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -6 \\ 3 & -4 & -1 \end{vmatrix} = \hat{i}(-3-24) - \hat{j}(-2+18) + \hat{k}(-8-9)$

$= -27\hat{i} - 16\hat{j} - 17\hat{k}$

$\therefore |\vec{d_1} \times \vec{d_2}| = \sqrt{(-27)^2 + (-16)^2 + (-17)^2} = \sqrt{1274}$

$\therefore$ Req. area $= \frac{1}{2}\sqrt{1274}$ sq. units

A25. $I = \int \frac{\sin 3x}{\sin x}dx$

$= \int \frac{3\sin x - 4\sin^3 x}{\sin x}dx$

$= \int (3 - 4\sin^2 x)dx = \int \left[3 - 4\left(\frac{1-\cos 2x}{2}\right)\right]dx = \int (1 + 2\cos 2x)dx$

$= x + 2 \cdot \frac{\sin 2x}{2} + C$

$= x + \sin 2x + C$

SECTION-C

A26. $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let $x = \sin A$, $y = \sin B$ $\Rightarrow$ $A = \sin^{-1}x$ and $B = \sin^{-1}y$

$\therefore \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow \cancel{2}\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) = a\left[\cancel{2}\,\cancel{\cos}\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow$ $A - B = 2\cot^{-1}a$

$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx} = 0$ $\Rightarrow$ $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$, H.P.

OR

$y = e^{\sin x} + (\tan x)^x$

$\Rightarrow y = u + v$, where, $u = e^{\sin x}$, $v = (\tan x)^x$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ——— (1)

Now, $u = e^{\sin x}$ $\Rightarrow$ $\frac{du}{dx} = \cos x . e^{\sin x}$ ——— (2)

Now, $v = (\tan x)^x$ $\Rightarrow$ $\log v = x.\log(\tan x)$

$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x.\frac{1}{\tan x}\times\sec^2 x + \log(\tan x).1$

$\Rightarrow \frac{dv}{dx} = (\tan x)^x\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$ ——— (3)

from (1), (2) and (3),

$\frac{dy}{dx} = \cos x . e^{\sin x} + (\tan x)^x\left[\frac{x\sec^2 x}{\tan x} + \log(\tan x)\right]$ Ans

A27. Let $I = \int \frac{e^x}{(1+e^x)(2+e^x)}dx$

Put $e^x = t$ $\Rightarrow$ $e^x dx = dt$

$\therefore I = \int \frac{dt}{(t+1)(t+2)} = \int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$

$= \log|t+1| - \log|t+2| + C$

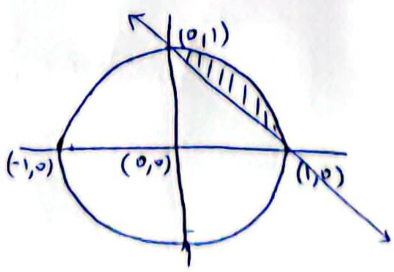
$= \log\left|\frac{t+1}{t+2}\right| + C = \log\left|\frac{e^x+1}{e^x-1}\right| + C$

A28. $R = \{(x,y) : x^2+y^2 \le 1 \le x+y\}$

$x^2+y^2 = 1$; $x+y = 1$

Point of intersection :

$(0,1)$ and $(1,0)$



Required area $= \int_0^1 (y \text{ of circle})\,dx - \int_0^1 (y \text{ of line})\,dx$

$= \int_0^1 \sqrt{1-x^2}\,dx - \int_0^1 (1-x)\,dx$

$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$

$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

A29. Given diff. eq'n is:

$$\frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

eq'n becomes, $v + x\frac{dv}{dx} = \frac{2x\sin v - vx\cos v}{vx - x\cos v}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v}$

$\Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{2\sin v - v^2}{v - \cos v} \Rightarrow \int \frac{v - \cos v}{2\sin v - v^2}\,dv = \int \frac{dx}{x}$

Put $2\sin v - v^2 = t \Rightarrow (2\cos v - 2v)\,dv = dt \Rightarrow (v - \cos v)\,dv = -\frac{dt}{2}$

$\therefore -\frac{1}{2}\int \frac{dt}{t} = \int \frac{dx}{x} \Rightarrow -\frac{1}{2}\log|t| = \log|x| + \log C$

$\Rightarrow \log\left(\frac{1}{2\sin v - v^2}\right) = \log(cx)^2$

$\Rightarrow 2\sin v - v^2 = \frac{1}{c^2x^2}$

$\Rightarrow 2\sin\left(\frac{y}{x}\right) - \frac{y^2}{x^2} = \frac{c'}{x^2}$

OR

$(x^2+1)\cdot\frac{dy}{dx} - 2xy = (x^2+1)^2\cdot\cos x \quad ; \quad y(0) = 0$

$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2+1}\cdot y = (x^2+1)\cdot\cos x$

$\text{I.F.} = e^{\int \frac{-2x}{x^2+1}dx} = \frac{1}{x^2+1}$

sol'n is given by,

$y\cdot\frac{1}{x^2+1} = \int \frac{(x^2+1)\cdot\cos x}{x^2+1}\,dx \Rightarrow \frac{y}{x^2+1} = \int \cos x\,dx$

$\Rightarrow \frac{y}{x^2+1} = \sin x + C$

$\Rightarrow y = (x^2+1)\cdot\sin x + C(x^2+1)$

when $x=0, y=0 \Rightarrow 0 = 0 + C \Rightarrow C = 0$

$\therefore y = (x^2+1)\sin x$ Ans.

A30. $P(\text{success}) = P(S) = \frac{1}{2}$, $P(\text{failure}) = P(F) = \frac{1}{2}$

$P(A) = P(S) + P(FFS) + P(FFFFS) + \ldots \infty$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} + \ldots \infty$

$a = \frac{1}{2}$, $r = \frac{1}{4}$

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$

$\therefore P(B) = 1 - \frac{2}{3} = \frac{1}{3}$

A31. Min. $Z = 5x + 10y$

$x + 2y = 120$

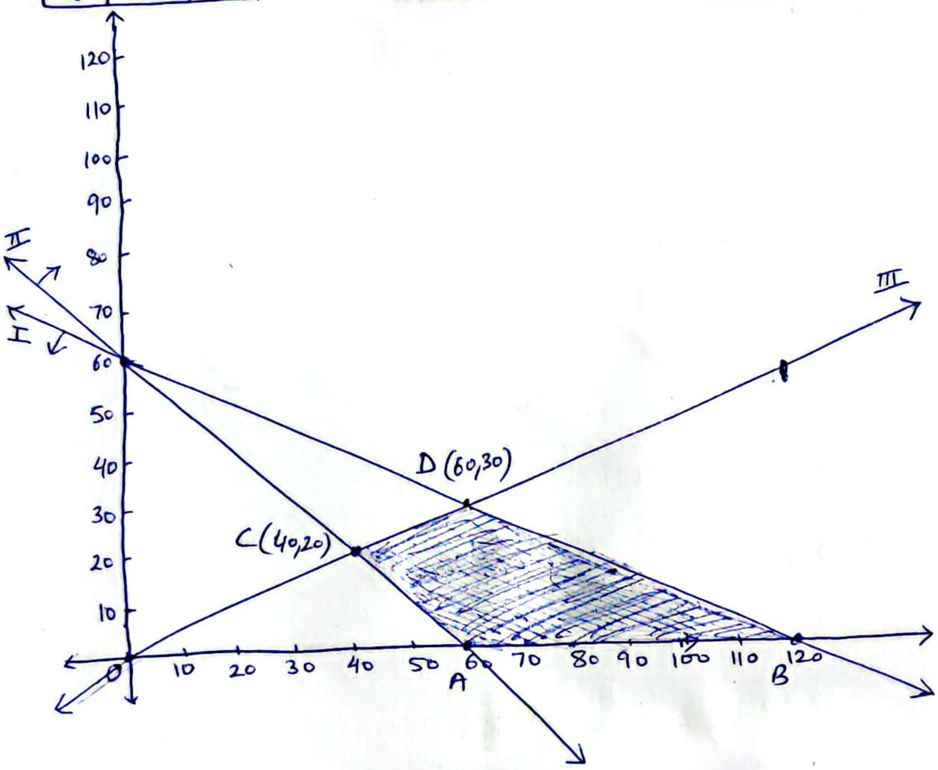
x	0	120
y	60	0

$x + y = 60$

x	0	60
y	60	0

$x = 2y$

x	0	120
y	0	60



Corner Pt.	$Z = 5x + 10y$
A (60,0)	300
B (120,0)	600
C (40,20)	400
D (60,30)	600

→ min.

$Z_{min} = 300$ when $x = 60$, $y = 0$

SECTION-D

A32. $A=\begin{bmatrix}2 & 3 & 4\\ 3 & -2 & 2\\ 4 & 2 & -3\end{bmatrix}$ $\Rightarrow |A| = 2(6-4)-3(-9-8)+4(6+8) = 111 \neq 0$

$\Rightarrow A^{-1}$ exists.

Now, $A_{11}=2$, $A_{12}=17$, $A_{13}=14$

$A_{21}=17$, $A_{22}=-22$, $A_{23}=8$

$A_{31}=14$, $A_{32}=8$, $A_{33}=-13$

$\text{adj}A = \begin{bmatrix}2 & 17 & 14\\ 17 & -22 & 8\\ 14 & 8 & -13\end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{111}\begin{bmatrix}2 & 17 & 14\\ 17 & -22 & 8\\ 14 & 8 & -13\end{bmatrix}$

Given eqn's can be written as $AX=B$

where, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}17\\ 11\\ 8\end{bmatrix}$

Now, $AX=B \Rightarrow X = A^{-1}.B$

$= \frac{1}{111}\begin{bmatrix}2 & 17 & 14\\ 17 & -22 & 8\\ 14 & 8 & -13\end{bmatrix}\begin{bmatrix}17\\ 11\\ 8\end{bmatrix}$

$= \frac{1}{111}\begin{bmatrix}333\\ 111\\ 222\end{bmatrix} = \begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$ $\therefore$ $x=3, y=1, z=2$

A33. $(a,b)\ R\ (c,d) \Leftrightarrow a+d = b+c$

for reflexive: $(a,b)\ R\ (a,b) \Rightarrow a+b = b+a$, always true

$\therefore$ R is reflexive

for symmetric: If $(a,b)\ R\ (c,d) \Rightarrow a+d = b+c$

$\Rightarrow d+a = c+b$

$\Rightarrow c+b = d+a \Rightarrow (c,d)\ R\ (a,b)$

$\Rightarrow$ R is symm.

for transitive: let $(a,b), (c,d), (e,f) \in A\times A$

If $(a,b)\ R\ (c,d) \Rightarrow a+d = b+c$

and $(c,d)\ R\ (e,f) \Rightarrow c+f = d+e$

adding, $a+d+c+f = b+c+d+e$

$\Rightarrow a+f = b+e \Rightarrow (a,b)\ R\ (e,f) \Rightarrow$ R is transitive

Since R is reflexive, symmetric & transitive $\Rightarrow$ R is equivalence.

for Equivalence class:

$(2,5)\ R\ (a,b) \Rightarrow 2+b = 5+a \Rightarrow b-a=3$

$\therefore$ $[(2,5)] = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

OR

$f: R-\{3\} \to R-\{1\}$

$f(x) = \frac{x-2}{x-3}$.

For <u>one-one</u>. Let $x_1, x_2 \in R-\{3\}$.

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow (x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one

for <u>onto</u>

Let $y = f(x)$

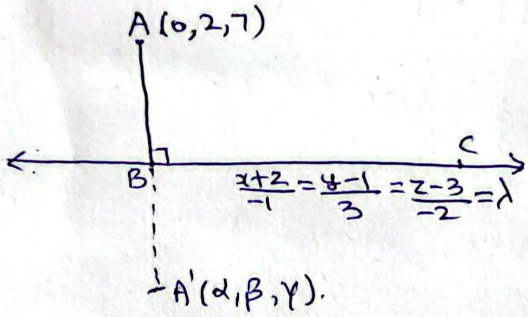
$\Rightarrow y = \frac{x-2}{x-3} \Rightarrow (x-3)\cdot y = x-2 \Rightarrow xy - 3y = x-2$

$\Rightarrow xy - x = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$

clearly, $y-1 \neq 0 \Rightarrow y \neq 1$ ie. $y \in R-\{1\}$ ie. R_f = co-domain

$\Rightarrow f$ is onto

A34.



Any point on BC is $(-\lambda-2, 3\lambda+1, -2\lambda+3)$

Let $B(-\lambda-2, 3\lambda+1, -2\lambda+3)$

Now,

d.r. of AB $= \langle -\lambda-2, 3\lambda-1, -2\lambda-4 \rangle$

d.r. of BC $= \langle -1, 3, -2 \rangle$

as, $AB \perp BC \Rightarrow -1(-\lambda-2) + 3(3\lambda-1) - 2(-2\lambda-4) = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

$\therefore$ Point $B\left(-\frac{3}{2}, -\frac{1}{2}, 4\right)$

Let image of A is $A'(\alpha, \beta, \gamma)$

Now, $AB = \sqrt{\left(-\frac{3}{2}-0\right)^2 + \left(-\frac{1}{2}-2\right)^2 + (4-7)^2} = \frac{\sqrt{70}}{2}$ units

(iii) B is mid-point on AA'.

Using mid-point formula,

$\left(\frac{0+\alpha}{2}, \frac{2+\beta}{2}, \frac{7+\gamma}{2}\right) = \left(-\frac{3}{2}, -\frac{1}{2}, 4\right) \Rightarrow \alpha = -3, \beta = -3, \gamma = 1$

$\therefore$ image $A'(-3, -3, 1)$ Ans.

OR

Given lines in vector form are:

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ or $\vec{r} = (3\hat{i}+3\hat{j}-5\hat{k}) + \mu'(2\hat{i}+3\hat{j}+6\hat{k})$

Here, $\vec{a_1} = \hat{i}+2\hat{j}-4\hat{k}$, $\vec{a_2} = 3\hat{i}+3\hat{j}-5\hat{k}$, $\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$

$$S.D. = \frac{|(\vec{a_2}-\vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

here, $\vec{a_2}-\vec{a_1} = 2\hat{i}+\hat{j}-\hat{k}$

$$(\vec{a_2}-\vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i}-14\hat{j}+4\hat{k}$$

$\therefore |(\vec{a_2}-\vec{a_1} \times \vec{b}| = \sqrt{81+196+16} = \sqrt{293}$

also, $|\vec{b}| = \sqrt{4+9+36} = 7$

$\therefore$ Req. distance $= \frac{\sqrt{293}}{7}$ units.

A35. $I = \int_{-5}^{0} (|x|+|x+2|+|x+5|)\,dx$

critical points are $x = 0, -2, -5$

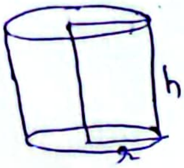
$\therefore I = \int_{-5}^{-2} [-x-(x+2)+(x+5)]\,dx + \int_{-2}^{0} [-x+(x+2)+(x+5)]\,dx$

$= \int_{-5}^{-2} (-x+3)\,dx + \int_{-2}^{0} (x+7)\,dx$

$= \left[-\frac{x^2}{2}+3x\right]_{-5}^{-2} + \left[\frac{x^2}{2}+7x\right]_{-2}^{0} = \left(-8+\frac{55}{2}\right) + (12) = \frac{63}{2}$

SECTION-E

A36.



$75\pi = 2\pi rh + \pi r^2 \Rightarrow h = \frac{75\pi - \pi r^2}{2\pi r}$

(i) $V = \pi r^2 h \Rightarrow V = \pi r^2\left(\frac{75\pi - \pi r^2}{2\pi r}\right)$

$\Rightarrow V = \frac{r}{2}(75\pi - \pi r^2)$ or $\frac{75}{2}\pi r - \frac{\pi}{2} r^3$

(ii) $\frac{dV}{dr} = \frac{75\pi}{2} - \frac{3}{2}\pi r^2$

(iii) for critical point, $\frac{dV}{dr} = 0 \Rightarrow 75\pi = \frac{3}{2}\pi r^2 \Rightarrow r = 5$ cm

Now, $\frac{d^2V}{dr^2} = -3\pi r \Rightarrow \left.\frac{d^2V}{dr^2}\right]_{r=5cm} = -15\pi < 0 \therefore$ V is max. when $r = 5$cm

Also, $h = \frac{75\pi - \pi r^2}{2\pi r} = \frac{75\pi - 25\pi}{10\pi} = 5$ $\quad \therefore h = r$

$\therefore h > r$ is false.

A37. Given: $\frac{dr}{dt} = 1$ cm/s

(i) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\Rightarrow \left.\frac{dV}{dt}\right]_{r=6} = 4\pi (6)^2 \times 1 = 144\pi \text{ cm}^3/\text{s}$

(ii) $\frac{dV}{dt} = 400\pi$ cm³/s (given)

$\Rightarrow 4\pi r^2 \cdot \frac{dr}{dt} = 400\pi \Rightarrow \frac{dr}{dt} = \frac{100}{r^2}$ $\therefore \frac{100}{r^2} = 1 \Rightarrow r^2 = 100$

$\Rightarrow r = 10$ cm

Now, $A = 4\pi r^2$

$\Rightarrow \frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$= 8\pi (10)(1) = 80\pi \text{ cm}^2/\text{s}$

A38. Let E_1: seed is of type A_1

E_2: seed is of type A_2

E_3: seed is of type A_3

A: seed will germinate

(a) $P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{49}{100}$

(b) $P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{49}{100}} = \frac{24}{49}$

SECTION – A

A1. $2A+B=0 \Rightarrow B=-2A \Rightarrow B=-2\begin{bmatrix}3 & 4\\ 5 & 2\end{bmatrix}=\begin{bmatrix}-6 & -8\\ -10 & -4\end{bmatrix}$ $\therefore$ option (b)

A2. $|\text{adj}A|=64 \Rightarrow |A|^{3-1}=64 \Rightarrow |A|=\pm 8$ $\therefore$ option (d)

A3. here, $x=-3,\ y=-1,\ z=4$ $\therefore x+y+z=0$ $\therefore$ option (d)

A4. $f(x)=2x^3+9x^2+12x-1 \Rightarrow f'(x)=6x^2+18x+12=6(x+1)(x+2)$

$f'(x)=0 \Rightarrow x=-1,-2$

$-\infty \quad + \quad -2 \quad - \quad -1 \quad + \quad \infty$

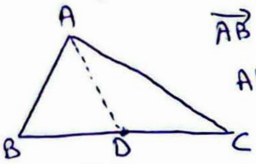
$\therefore x\in(-2,-1)$ $\therefore$ option (b)

A5. $n(A)=5,\ n(B)=6$ Since $n(A)<n(B)$ $\therefore$ option (d)

A6. order $=3$, degree $=2$ $\therefore$ sum $=3+2=5$ $\therefore$ option (a)

A7. clearly bounded region will be formed $\therefore$ option (d)

A8.



$\vec{AB}+\vec{BC}=\vec{AC} \Rightarrow \vec{BC}=2\hat{i}-2\hat{j}+2\hat{k}$ $\therefore \vec{BD}=\hat{i}-\hat{j}+\hat{k}$

Also, $\vec{AB}+\vec{BD}=\vec{AD} \Rightarrow \vec{AD}=2\hat{i}+3\hat{k}$ $\therefore$ option (d)

A9. $I=\int_0^{\pi/6}\sec^2\left(\frac{x}{8}-\frac{\pi}{6}\right)dx=\left[\tan\left(x-\frac{\pi}{6}\right)\right]_0^{\pi/6}=\tan 0-\tan\left(-\frac{\pi}{6}\right)=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

$\therefore$ option (a)

A10. $A^2-3A+I=0$ (given). $\Rightarrow I=-A^2+3A$ —— ①

Now, $A^{-1}=xA+yI \Rightarrow AA^{-1}=xA^2+yIA \Rightarrow I=xA^2+yA$ —— ②

from ① + ②,, $x=-1,\ y=3$ $\therefore x+y=2$ $\therefore$ option (b)

A11. $(\hat{i}\times\hat{j})\cdot\hat{j}+(\hat{j}\times\hat{i})\cdot\hat{k}=\hat{k}\cdot\hat{j}-\hat{k}\cdot\hat{k}=-1$ $\therefore$ option (d)

A12. option (c)

A13. $|4A^{-1}|=4^2\times|A^{-1}|=16\times\frac{1}{|A|}=16\times\frac{1}{2}=8$ $\therefore$ option (c)

A14. $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{8}}{1-\frac{3}{4}}=\frac{1}{2}$ $\therefore$ option (a)

A15. $\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\theta=1 \Rightarrow \cos^2\theta=0 \Rightarrow \theta=\frac{\pi}{2}$ $\therefore$ option (c)

A16. $x\,dy-(1+x^2)dx=dx \Rightarrow x\,dy=(2+x^2)dx \Rightarrow dy=\left(\frac{2}{x}+x\right)dx$

$\Rightarrow y=2\log x+\frac{x^2}{2}+C$ $\therefore$ option (d)

A17. div. by $\cos x$, $y=\frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)$

$\therefore \frac{dy}{dx}=-\sec^2\left(\frac{\pi}{4}-x\right)$ $\therefore$ option (a)

A18. line is $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-1/2}{6}$; d.r. $=\langle 2,-3,6\rangle$

d.c. $=\left\langle \frac{2}{7},-\frac{3}{7},\frac{6}{7}\right\rangle$ $\therefore$ option (d)

A19. $y = \sin^{-1}x + 2\cos^{-1}x = \sin^{-1}x + \cos^{-1}x + \cos^{-1}x = \frac{\pi}{2} + \cos^{-1}x$

Now, $\cos^{-1}x \in [0, \pi]$ $\therefore$ $\frac{\pi}{2} + \cos^{-1}x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ $\therefore$ A is false.

but, R is true. $\therefore$ option (d).

A20. d.r. of line I $= \langle -2, -4, -4 \rangle = \langle 1, 2, 2 \rangle$

d.r. of line II $= \langle 2, 4, 4 \rangle = \langle 1, 2, 2 \rangle$

as d.r's are proportional, lines are parallel. $\therefore$ A is true.

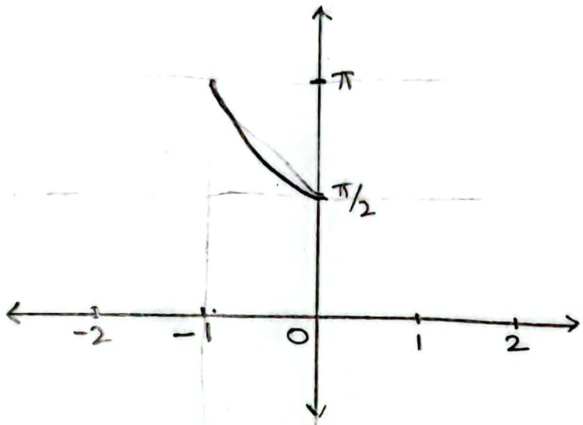
but R is false. $\therefore$ option (c)

SECTION-B

A21. $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}(\cos\pi) + \tan^{-1}1$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{4}\right)\right) + \pi + \frac{\pi}{4} = \frac{\pi}{4} + \pi + \frac{\pi}{4} = \frac{3\pi}{2}$

OR



Range $= \left[\frac{\pi}{2}, \pi\right]$

A22. $\cancel{p = (7\hat{i} - \hat{j} + 8\hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}$

$\frac{(\hat{i}+\hat{j}+\hat{k}).(p\hat{i}+\hat{j}-2\hat{k})}{\sqrt{p^2+1+4}} = \frac{1}{3} \Rightarrow 8p^2 - 18p + 4 = 0 \Rightarrow p = 2$ or $\frac{1}{4}$

A23. Differentiating $3y = ax^3 + 1$ gives $3.\frac{dy}{dx} = 3ax^2$

when $x = 1$, $\frac{dy}{dx} = 2$ $\therefore$ $3(2) = 3a(1)^2 \Rightarrow a = 2$

OR

$f(x) = \frac{16\sin x}{4 + \cos x} - x$

$\Rightarrow f'(x) = \frac{16[4 + \cos x].\cos x + 16\sin^2 x}{(4 + \cos x)^2} - 1$

$= \frac{\cos x(56 - \cos x)}{(4 + \cos x)^2}$

when $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$; $56 - \cos x > 0$ and $(4 + \cos x)^2 > 0$

$\therefore$ $f'(x) < 0 \Rightarrow f$ is st. dec. in $\left(\frac{\pi}{2}, \pi\right)$

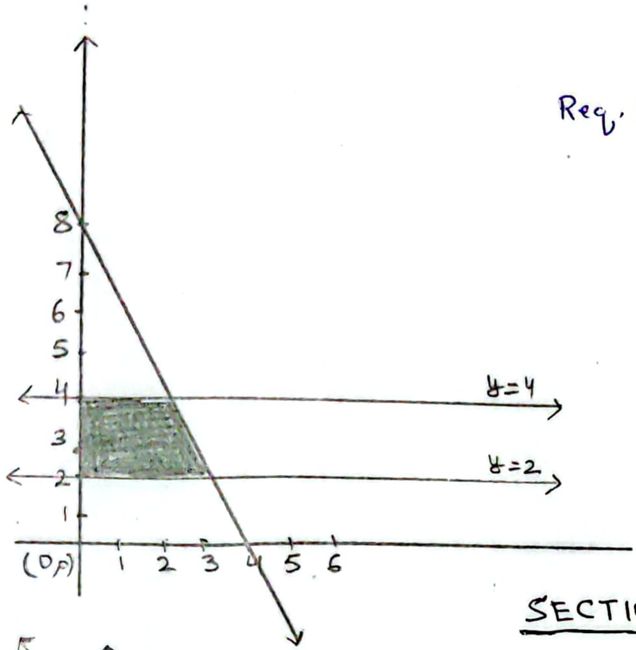
A24. General point on line is $P(\lambda, 2\lambda+1, 2\lambda-1)$.

Now, $OP = \sqrt{11} \Rightarrow OP^2 = 11$

$\Rightarrow \lambda^2 + (2\lambda+1)^2 + (2\lambda-1)^2 = 11 \Rightarrow k = \pm 1$

$\therefore$ co-ordinates of points are $(1,3,1)$ and $(-1,-1,-3)$

A25.



$$\text{Req. Area} = \int_2^4 x \cdot dy$$

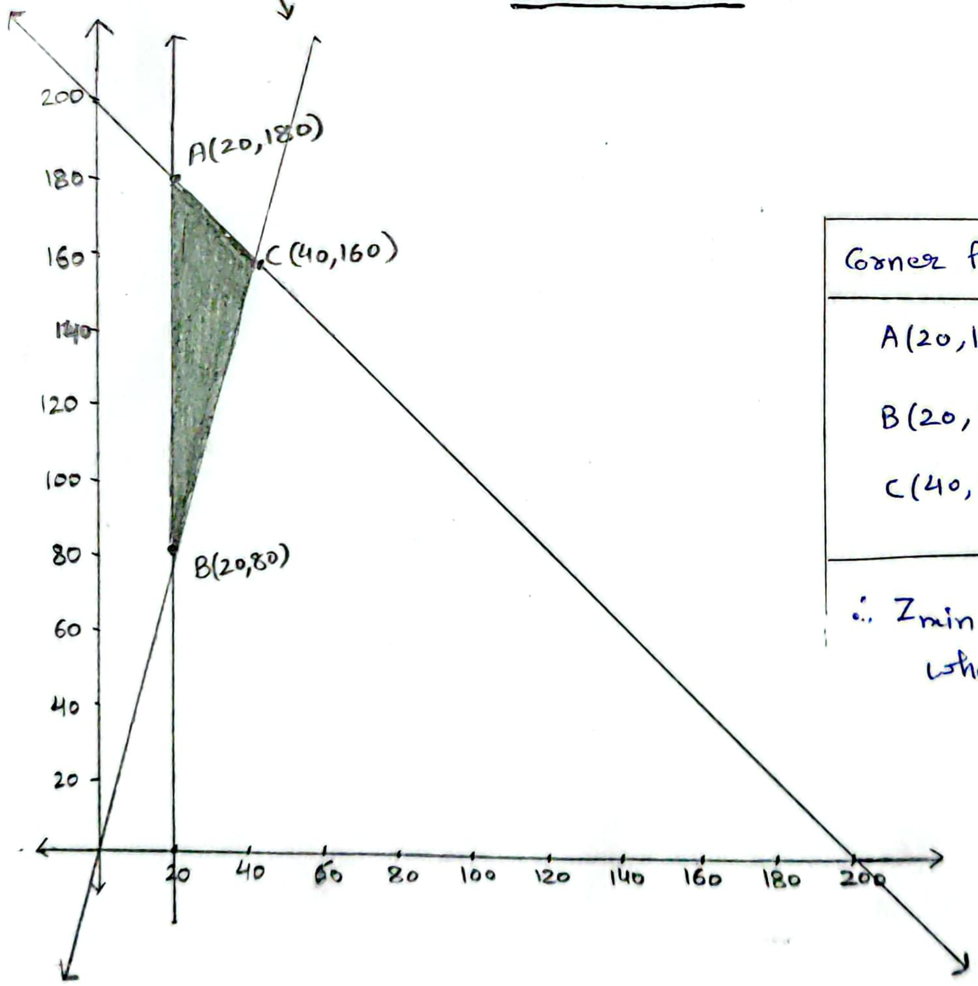
$$= \int_2^4 \frac{1}{2}(8-y)\,dy$$

$$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4$$

$$= 5 \text{ sq. units}$$

SECTION-C

A26.



Corner Point	$Z = 500x + 400y$
A(20,180)	82000
B(20, 80)	42000
C(40,160)	84000

$\therefore Z_{min} = 42000$

when $x = 20,\ y = 80$

A27. X : No. of red balls out of the two balls drawn.

then $X = 0, 1, 2$

Probability Distribution table is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Mean $= 0\times\frac{1}{4} + 1\times\frac{1}{2} + 2\times\frac{1}{4} = 1$ Ans.

OR

Let $P(A) = x$, $P(B) = y$

$P(A)\cdot P(\bar{B}) = \frac{1}{4} \Rightarrow x(1-y) = \frac{1}{4}$ —— ①

$P(\bar{A})\cdot P(B) = \frac{1}{6} \Rightarrow (1-x)\cdot y = \frac{1}{6}$ —— ②

solving, $x - y = \frac{1}{12}$

eleminating y, we get $12x^2 - 13x + 3 = 0 \Rightarrow x = \frac{1}{3}, \frac{3}{4}$

$\therefore P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ or $P(A) = \frac{3}{4}, P(B) = \frac{2}{3}$

A28. $I = \int \frac{x^4}{(x-1)(x^2+1)} dx$

$= \int \left[x + 1 + \frac{1}{(x-1)(x^2+1)}\right] dx$

$= \int \left(x + 1 + \frac{1}{2(x-1)} - \frac{1}{2}\cdot\frac{x+1}{x^2+1}\right) dx$

$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$

OR

$I = \int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}} d\theta$

Put $\sin\theta = t$, $\cos\theta\, d\theta = dt$

$\therefore I = \int \frac{\cos\theta}{\sqrt{\sin^2\theta - 3\sin\theta + 2}} d\theta$

$= \int \frac{dt}{\sqrt{t^2 - 3t + 2}} = \int \frac{dt}{\sqrt{\left(t - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$= \log\left|\left(t - \frac{3}{2}\right) + \sqrt{t^2 - 3t + 2}\right| + C$

$= \log\left|\left(\sin\theta - \frac{3}{2}\right) + \sqrt{\sin^2\theta - 3\sin\theta + 2}\right| + C$

A29. Given diff. eqn is : $\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\sqrt{x^2+4}}{1+x^2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$

soln is given by : $y\cdot(1+x^2) = \int \frac{\sqrt{x^2+4}}{1+x^2}\cdot(1+x^2)\cdot dx = \int \sqrt{x^2+4}\, dx$

$\therefore y\cdot(1+x^2) = \frac{x\sqrt{x^2+4}}{2} + 2\log\left|x + \sqrt{x^2+4}\right| + C$

OR

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2} \quad \text{—— ①}$$

Put $y = vx$ —— ②

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— ③}$$

using ② & ③ from ①.

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow \int \frac{1+v^2}{v^3}\,dv = -\int \frac{dx}{x}$$

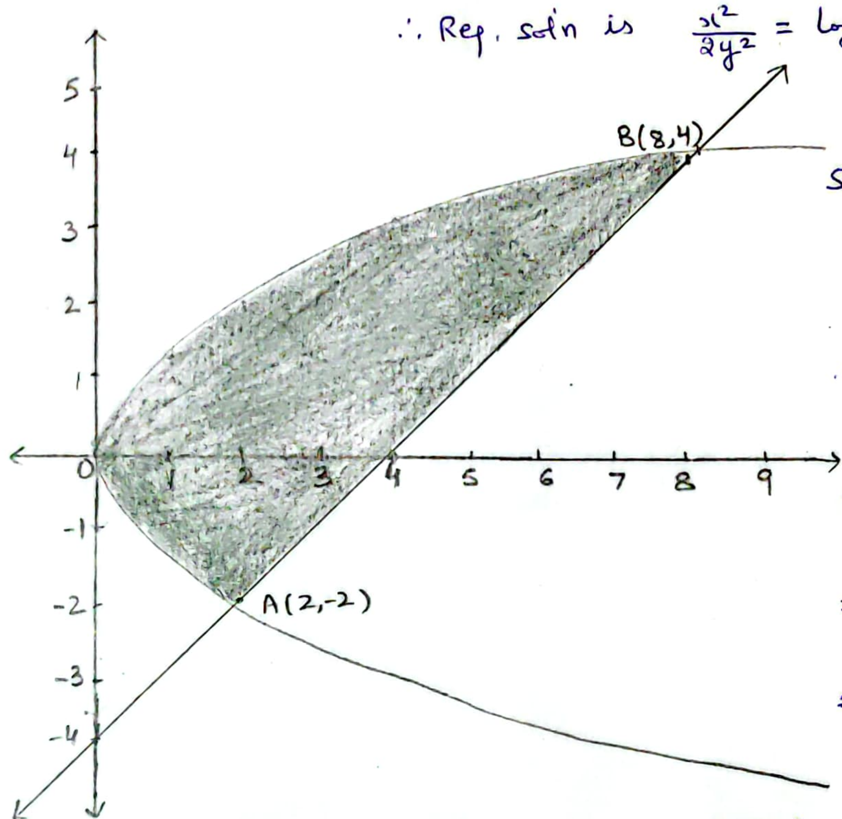
$$\Rightarrow -\frac{1}{2v^2} + \log|v| = -\log|x| + \log C$$

$$\Rightarrow -\frac{x^2}{2y^2} = \log\left|\frac{c}{y}\right|$$

When $x = 0$, $y = 1$ gives $c = 1$

∴ Req. soln is $\frac{x^2}{2y^2} = \log|y|$

A30.



Solving $y^2 = 2x$ and $y = x-4$ we get $A(2,-2)$, $B(8,4)$

Req. Area, A

$$= \int_{-2}^{4} (y \text{ of Line})\,dx - \int_{-2}^{4} (y \text{ of Par.})\,dx$$

$$= \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) dy$$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$$

$= 18$ sq. units.

A31. Let $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ and $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$$\therefore u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right) = \sec^{-1}(\sec\theta) = \theta = \sin^{-1}x \qquad \therefore \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Again, $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \sin^{-1}\left(2\sin\theta.\sqrt{1-\sin^2\theta}\right)$

$$= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$$

$$\therefore \frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{1}{2}$$

Note: If the substitution made is $x = \cos\theta$, answer will be $-\frac{1}{2}$.

SECTION-D

A32. $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow |A| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1}$ exists

here, $A_{11} = -2$, $A_{12} = -1$, $A_{13} = 3$

$A_{21} = -1$, $A_{22} = 2$, $A_{23} = -1$

$A_{31} = 5$, $A_{32} = -5$, $A_{33} = -5$

$\text{adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

Given system of equations can be written as $AX = B$; $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$

Now, $X = A^{-1} \cdot B$

$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$ $\therefore x = 400,\ y = 300,\ z = 200$

A33. Vector eqn of required line through $(1, 2, -4)$ is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and cartesian eqn is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda$

Equation of line through $A(3,3,-5)$ and $B(1,0,-11)$ is :

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Distance between parallel lines is given by $d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$

Here, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$\therefore (\vec{a_2} - \vec{a_1}) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\Rightarrow d = \frac{\sqrt{293}}{7}$

OR

Let d.r. of required line be a, b, c .

Since it is $\perp$ to two given lines ;

$\therefore a + 2b + 3c = 0$ and $-3a + 2b + 5c = 0$

solving , $a = 4\lambda$, $b = -14\lambda$, $c = 8\lambda$

$\therefore$ Eqn of line is $\frac{x+1}{4\lambda} = \frac{y-3}{-14\lambda} = \frac{z+2}{8\lambda} = \mu$

or $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4} = \lambda'$

vector Eqn of line is $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda'(2\hat{i} - 7\hat{j} + 4\hat{k})$

Distance from origin $= \frac{|(-\hat{i} + 3\hat{j} - 2\hat{k}) \times (2\hat{i} - 7\hat{j} + 4\hat{k})|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|}$

$= \frac{|-2\hat{i} + \hat{k}|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|} = \frac{\sqrt{5}}{\sqrt{69}}$

A34. $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ——— ①

$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right) . \sin\left(\frac{\pi}{2}-x\right) \cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right) + \cos^4\left(\frac{\pi}{2}-x\right)} dx$

$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right) . \cos x . \sin x}{\cos^4 x + \sin^4 x} dx$ ——— ②

adding ① + ②,

$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \pi . \int_0^{\pi/4} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$\Rightarrow I = \frac{\pi}{2} . \int_0^{\pi/4} \frac{\tan x . \sec^2 x}{(\tan^2 x)^2 + 1} dx$ (∵ div. by $\cos^4 x$)

Putting $\tan^2 x = t \Rightarrow 2 \tan x . \sec^2 x \, dx = dt$

$I = \frac{\pi^2}{16}$

OR

$I = \int_0^{\pi/2} 2 \sin x \cos x . \tan^{-1}(\sin x) \, dx$

Let $\sin x = t \Rightarrow \cos x \, dx = dt$

$\therefore I = \int_0^1 2t . \tan^{-1} t \, dt$

$= t^2 . \tan^{-1} t \Big]_0^1 - \int_0^1 \frac{t^2}{1+t^2} dt$

$= \left[t^2 . \tan^{-1} t - t + \tan^{-1} t\right]_0^1 = \frac{\pi}{2} - 1$

A35. $f : [-4,4] \rightarrow [0,4]$

$f(x) = \sqrt{16-x^2}$

for <u>onto</u>. Let $y = f(x) \Rightarrow y = \sqrt{16-x^2} \Rightarrow y^2 = 16 - x^2 \Rightarrow x^2 = 16 - y^2 \Rightarrow x = \pm\sqrt{16-y^2}$

Now, $16 - y^2 \geq 0 \Rightarrow y^2 \leq 16 \Rightarrow -4 \leq y \leq 4 \Rightarrow y \in [-4,4]$;

but $y \geq 0 \Rightarrow y \in [0,4]$ ie. R_f = co-domain

$\Rightarrow f$ is onto.

for <u>one-one</u>

$f(-1) = f(1) = \sqrt{15}$, but $-1 \neq 1 \Rightarrow f$ is not one-one.

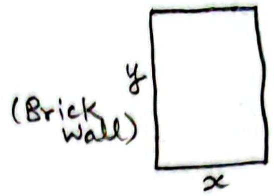
Now, $f(a) = \sqrt{7}$

$\Rightarrow \sqrt{16-a^2} = \sqrt{7} \Rightarrow a = \pm 3$

SECTION-E

A36. (i) (a) $2x + y = 200$

(b) $A(x) = xy = x(200-2x)$



(ii) from (a) & (b) of (i)

$A(x) = x(200-2x)$

$= 200x - 2x^2$

for max./min, $\frac{dA}{dx} = 0 \Rightarrow 200 - 4x = 0 \Rightarrow x = 50$

Now, $\frac{d^2A}{dx^2} = -4 < 0$ $\therefore$ A is maximum at $x = 50$.

Thus, Max. $A(x) = 200(50) - 2(50)^2 = 5000$ sq.m.

A37. (i) $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

(ii) $P(E) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$

$= 0.65 \times 0.35 + 0.35 \times 0.8$

$= 0.35 \times 1.45 = 0.51$

(iii) (a) $P(E_1|E) = \dfrac{P(E_1) \cdot P(E|E_1)}{P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2)} = \dfrac{0.65 \times 0.35}{0.51} = 0.45$

(b) $P(E_2|E) = \dfrac{P(E_2) \cdot P(E|E_2)}{P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2)} = \dfrac{0.35 \times 0.8}{0.51} = 0.55$

A38. (i) for the year 2000, $t = 0$ and $V(0) = -2$, and the number of vehicles can't be negative $\therefore$ the given fn. $V(t)$ can't be used.

(ii) $V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$\Rightarrow V'(t) = \frac{3}{5}t^2 - 5t + 25$

$= \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right] > 0$ always

$\therefore$ $V(t)$ is an increasing fn.