




- c) (40, 15) d) (20, 35)
5. If  $y = Ae^{5x} + Be^{-5x}$ , then  $\frac{d^2y}{dx^2}$  is equal to [1]
- a)  $-25y$  b)  $15y$   
 c)  $5y$  d)  $25y$
6. If  $X$  follows a binomial distribution with parameters  $n = 8$  and  $p = \frac{1}{2}$ , then  $P(|X - 4| \leq 2)$  equals [1]
- a)  $\frac{117}{128}$  b)  $\frac{118}{128}$   
 c)  $\frac{119}{128}$  d)  $\frac{116}{128}$
7. A box contains 20 identical balls of which 10 balls are white and 10 balls are red. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is [1]
- a)  $\frac{1}{2}$  b)  $\frac{27}{32}$   
 c)  $\frac{5}{64}$  d)  $\frac{5}{32}$
8. Integrating factor of the differential equation  $\frac{dy}{dx} + y = \frac{x^3 + y}{x}$  is [1]
- a)  $e^x$  b)  $\frac{e^x}{x}$   
 c)  $x e^x$  d)  $\frac{x}{e^x}$
9. A man rows  $d$  km upstream and back again in  $t$  hours. If he can row in still water at  $u$  km/hr and the rate of stream is  $v$  km/hr, then  $t =$  [1]
- a)  $\frac{u^2 - v^2}{d}$  b)  $\frac{2ud}{u^2 + v^2}$   
 c)  $\frac{2ud}{u^2 - v^2}$  d)  $\frac{uv}{d}$
10. The number of arbitrary constants in the particular solution of a differential equation of third order is: [1]
- a) 0 b) 2  
 c) 3 d) 1
11. If  $100 \equiv x \pmod{7}$ , then the least positive value of  $x$  is: [1]
- a) 2 b) 3  
 c) 6 d) 4
12. The linear inequality representing the solution set given in figure is [1]
- 
- a)  $|x| \geq 5$  b)  $|x| > 5$   
 c)  $|x| < 5$  d)  $|x| \leq 5$
13. In a kilometer race, A beats B by 50 meters or 10 seconds. The time taken by A to complete the race is: [1]
- a) 190 seconds b) 90 seconds  
 c) 120 seconds d) 200 seconds
14. If the objective function for an L.P.P. is  $Z = 3x - 4y$  and the corner points for the bounded feasible region are (0, [1]



2002	1.9
2003	2
2004	1.4
2005	2.1
2006	1.3
2007	1.8
2008	1.1
2009	1.3

Determine the trend of rainfall by 3-year moving average.

22. A man borrows ₹ 3,00,000 at 6% per annum compound interest and promises to pay off the debt in 20 annual instalments beginning at the end of the first year. Find the amount of annual instalment. [Given:  $(1.06)^{-20} = 0.312$ ] [2]

OR

The effective annual rate of interest corresponding to normal rate of 6% p.a. payable half yearly is \_\_\_\_\_.

23. Evaluate:  $\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$  [2]

24. An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule. [2]

OR

If the cash equivalent of the perpetuity of ₹1,200 payable at the end of each quarter is ₹96,000, find the rate of interest convertible quarterly.

25. Evaluate:  $(9 + 23) \bmod 12$  [2]

### Section C

26. The rate of increase of bacteria in a culture is proportional to the number of bacteria present and it is found that the number doubles in 6 hours. Prove that the bacteria becomes 8 times at the end of 18 hours. [3]

OR

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal.

- i. If the interest is compounded continuously at 5% per annum, in how many years will ₹ 100 double itself?
  - ii. At what interest rate will ₹ 100 double itself in 10 years? ( $\log_e 2 = 0.6931$ )
  - iii. How much will ₹ 1000 be worth at 5% interest after 10 years? ( $e^{0.5} = 1.648$ )
27. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 6,50,000. She paid ₹ x as down payment and the balance in 20 monthly equal instalments of ₹ 21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x. [Given that:  $(1.0075)^{-20} = 0.86118985$ ] [3]
28. If the marginal revenue function for output x is given by  $MR = \frac{6}{(x+2)^2} + 5$ , find the total revenue function and the demand function. [3]
29. A bag contains 8 red and 5 white balls. Two successive draws of all 3 balls are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and second draw yields 3 red balls. [3]

OR

Let  $X$  denote the no of hours you study during a randomly selected school day. The probability that  $X$  can take the values  $x$ , has the following form where  $K$  is some unknown constant

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1, \text{ or } 2 \\ K(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

- i. Find the value of  $K$ .
- ii. What is the probability that you study at least two hours? Exactly two hours. At most two hours.

30. From the following time series obtain trend value by 3 yearly moving averages. [3]

Year	Sales (in ₹ 000)	Year	Sales (in ₹ 000)
2008	8	2014	16
2009	12	2015	17
2010	10	2016	14
2011	13	2017	17
2012	15		
2013	12		

31. Ten cartons are taken at random from an automatic filling machine. The mean net weight of the cartons is 11.8 kg and the standard deviation 0.15 kg. Does the sample mean differ significantly from the intended weight of 12 kg? [Given that for d.f. = 9,  $t_{0.05} = 2.26$ ] [3]

#### Section D

32. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5.00 per kg to purchase food I and ₹7.00 per kg to produce food II. Determine the minimum cost to such a mixture. Formulate the above as an LPP and solve it. [5]

OR

A small manufacturer has employed 5 skilled men and 10 semi-skilled men and makes an article in two qualities deluxe model and an ordinary model. The making of a deluxe model requires 2 hrs. work by a skilled man and 2 hrs. work by a semi-skilled man. The ordinary model requires 1 hr by a skilled man and 3 hrs. by a semi-skilled man. By union rules no man may work more than 8 hrs per day. The manufacturers clear profit on deluxe model is ₹15 and on an ordinary model is ₹10. How many of each type should be made in order to maximize his total daily profit.

33. Solve the system of inequations graphically:  $2x + y \geq 8$ ,  $x + 2y \geq 8$ ,  $x + y \leq 6$  [5]
34. A die is tossed twice. Success is defined as getting an odd number on a random toss. Find the mean and variance of the number of successes. [5]

OR

Let  $X$  denote the number of hours a person watches television during a randomly selected day. The probability that  $X$  can take the values  $x_i$  has the following form, where  $k$  is some unknown constant.

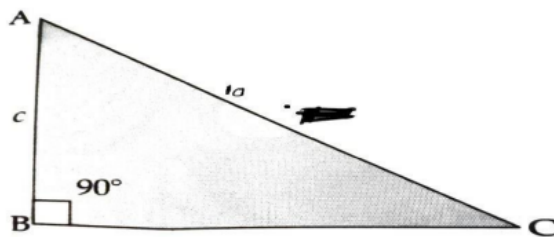
$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

- Find the value of k.
  - What is the probability that the person watches two hours of television on a selected day?
  - What is the probability that the person watches at least two hours of television on a selected day?
  - What is the probability that the person watches at most two hours of television on a selected day?
  - Calculate mathematical expectation.
  - Find the variance and standard deviation of random variable X.
35. A person amortizes a loan of ₹ 1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly. Find [5]
- the equated monthly installment
  - the principal outstanding at the beginning of 40th month.
  - the interest paid in 40<sup>th</sup> payment.
- [Given  $(1.01)^{96} = 2.5993$ ,  $(1.01)^{57} = 1.7633$ ]

### Section E

36. Read the text carefully and answer the questions: [4]

The sum of the length of hypotenuse and a side of a right-angled triangle is given by  $AC + BC = 10$



- Base BC = ?
- If S be the area of the triangle, then find the value of  $\frac{dS}{dc}$ ?
- What is the values of c when  $\frac{ds}{dc} = 0$ ?

OR

Find the value of  $\frac{d^2S}{dc^2}$  at  $C = \frac{10\sqrt{3}}{3}$ ?

37. Read the text carefully and answer the questions: [4]

#### What Is a Sinking Fund?

A sinking fund contains money set aside or saved to pay off a debt or bond. A company that issues debt will need to pay that debt off in the future, and the sinking fund helps to soften the hardship of a large outlay of revenue. A sinking fund allows companies that have floated debt in the form of bonds gradually save money and avoid a large lump-sum payment at maturity.

#### Example:

- Cost of Machine: ₹2,00,000/-
  - Effective Life: 7 Years
  - Scrap Value: ₹30,000/-
  - Sinking Fund Earning Rate: 5%
  - The Expected Cost of New Machine: ₹3,00,000/-
- What is the money required for a new machine after 7 years?

- (b) What is the value of A, i and n here?  
 (c) What formula will you use to get the requisite amount?

**OR**

What amount should the company put into a sinking fund earning 5% per annum to replace the machine after its useful life?

38. An automobile company uses three types of steel  $S_1$ ,  $S_2$  and  $S_3$  for producing three types of cars  $C_1$ ,  $C_2$  and  $C_3$ . [4]

Steel requirements (in tons) for each type of cars are given below:

	Cars		
Steel	$C_1$	$C_2$	$C_3$
$S_1$	2	3	4
$S_2$	1	1	2
$S_3$	3	2	1

Using Cramer's rule, find the number of cars of each type which can be produced using 29, 13 and 16 tonnes of steel of three types respectively.

**OR**

Express the following matrices as the sum of symmetric and skew-symmetric matrices:

$$\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

# Solution

## Section A

1. (a) B

**Explanation:**  $AB = A \dots(i)$

$BA = B \dots(ii)$

From equation (ii)

$B \times (AB) = B$

$B^2A = B$

From equation (ii)

$B^2A = BA$

$B^2 = B$

Which is the required solution.

2.

(d) Sample Statistic

**Explanation:** Sample Statistic

3.

(b) future value of annuity due

**Explanation:** future value of annuity due

4.

(c) (40, 15)

**Explanation:**

Given objective function is  $Z = x + y$

Constraints are:

$$x + 2y \leq 70$$

$$2x + y \leq 95, x, y \geq 0$$

Let us consider these constraints as equations for a while, then we will have,

$$x + 2y = 70 \dots(i)$$

$$2x + y = 95 \dots(ii)$$

Now, graph the equations, by transforming the equations to intercept form of line.

Equation (i) dividing throughout by 70

$$\frac{x}{70} + \frac{2y}{70} = \frac{70}{70}$$

$$\frac{x}{70} + \frac{y}{35} = 1$$

The line  $x + 2y = 70$  can be plot in the graph as a line passing through the points, (70, 0) and (0, 35) as 70 and 35 are the intercepts of the line on the x-axis and y-axis respectively.

Similarly equation (ii) can be divided 95 to get

$$\frac{2x}{95} + \frac{y}{95} = \frac{95}{95}$$

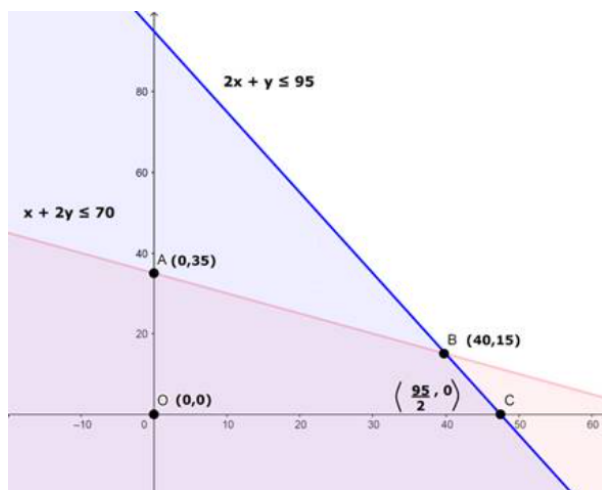
$$\frac{x}{\frac{95}{2}} + \frac{y}{95} = 1$$

The line  $2x + y = 95$  can be plot in the graph as a line passing through the points,  $(\frac{95}{2}, 0)$  and (0, 95) as  $\frac{95}{2}$  and 95 are the intercepts of the line on the x-axis and y-axis respectively.

By considering the constraints  $x, y \geq 0$ , this clearly shows that the region can only be in the 1<sup>st</sup> quadrant.

The graph of the inequations will look like,





The points OABC is the feasible region of the LPP.

Now from the points O, A, B and C the vertices of the polygon formed by the constraints, one of the points will provide the maximum solution to the function  $Z = x + y$

Now checking the points, O, A, B and C by substituting in  $Z = x + y$

Z at O(0, 0)	$Z = 0 + 0 = 0$
Z at A (0, 35)	$Z = 0 + 35 = 35$
Z at B(40, 15)	$Z = 40 + 15 = 55$
Z at C $\left(\frac{95}{2}, 0\right)$	$Z = \frac{95}{2} + 0 = \frac{95}{2} = 47.5$

From the above values, it is clear that Z maximized at point B(40, 15).

5.

(d) 25 y

**Explanation:** We have,

$$y = ae^{5x} + be^{-5x}$$

On differentiating w.r.t x, we get

$$\frac{d^2y}{dx^2} = 5ae^{5x} - 5be^{-5x}$$

$$\frac{d^2y}{dx^2} = 25(ae^{5x} + be^{-5x})$$

$$\frac{d^2y}{dx^2} = 25y$$

Hence, this is the answer.

6.

(c)  $\frac{119}{128}$

**Explanation:**  $n = 8$ ,  $p = \frac{1}{2} = q$

$$P(|x - 4| \leq 2)$$

$$\Rightarrow -2 \leq x - 4 \leq 2$$

$$\Rightarrow 4 - 2 \leq x \leq 2 + 4$$

$$\Rightarrow 2 \leq x \leq 6$$

$$P(2 \leq x \leq 6) = P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(2 \leq x \leq 6) = {}^8C_2 \left(\frac{1}{2^8}\right) + {}^8C_3 \left(\frac{1}{2^8}\right) + {}^8C_4 \left(\frac{1}{2^8}\right) + {}^8C_5 \left(\frac{1}{2^8}\right) + {}^8C_6 \left(\frac{1}{2^8}\right)$$

$$= \frac{119}{128}$$

7.

(d)  $\frac{5}{32}$

**Explanation:**  $\frac{5}{32}$

8.

(b)  $\frac{e^x}{x}$

**Explanation:**  $\frac{dy}{dx} + (1 - \frac{1}{x})y = x^2$ , which is linear in y.

$$\text{I.F.} = e^{\int (1 - \frac{1}{x}) dx} = e^{x - \log x} = \frac{e^x}{e^{\log x}} = \frac{e^x}{x}$$

9.

(c)  $\frac{2ud}{u^2 - v^2}$

**Explanation:** upstream speed = (u - v) km/hr

downstream speed = (u + v) km/hr

$$t_{\text{downstream}} = \frac{d}{u+v}$$

$$t_{\text{upstream}} = \frac{d}{u-v}$$

$$t = t_{\text{downstream}} + t_{\text{upstream}}$$

$$t = \frac{d}{u+v} + \frac{d}{u-v}$$

$$t = \frac{(u-v)d + (u+v)d}{(u^2 - v^2)}$$

$$t = \frac{d[u-v+u+v]}{(u^2 - v^2)}$$

$$t = \frac{2ud}{u^2 - v^2}$$

10. (a) 0

**Explanation:** 0, because the particular solution is free from arbitrary constants.

11. (a) 2

**Explanation:**  $100 \equiv x \pmod{7} \Rightarrow 100 - x$  is divisible by 7

Putting  $x = 1, 2, 3, \dots$

For  $x = 1, 100 - 1 = 99$  which is not divisible by 7

For  $x = 2, 100 - 2 = 98$  which is divisible by 7.

Hence, the least positive value of  $x$  is 2.

12. (a)  $|x| \geq 5$

**Explanation:** The given figure is highlighted between  $-\infty$  to 5 and 5 to  $\infty$

So,  $x \in (-\infty, 5] \cup [5, \infty)$

$\Rightarrow x \leq -5$  and  $x \geq 5$

$\Rightarrow |x| \geq 5$

13. (a) 190 seconds

**Explanation:** In a 1000 m race

50 m = 10 sec

$$1 \text{ m} = \frac{10}{50}$$

1000 m of race will take =  $\frac{10}{50} \times 1000 = 200$  sec

$\therefore$  Time taken by 'A' to complete 1000 m =  $(200 - 10) = 190$  sec

14. (a) (5, 0)

**Explanation:** (5, 0)

15.

(b)  $q = 3p$

**Explanation:** Given the vertices of the feasible region are:

Q(0, 0)

A(5, 0)

B(3, 4)

C(0, 5)

Also given the objective function is  $Z = px + qy$

Now substituting O, A, B and C in Z

Z at O(0, 0)	$Z = p(0) + q(0) = 0$
Z at A(5, 0)	$Z = p(5) + q(0) = 5p + 0 = 5p$
Z at B(3, 4)	$Z = p(3) + q(4) = 3p + 4q$
Z at C(0, 5)	$Z = p(0) + q(5) = 0 + 5q$

As per the condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5)

Then we can equate Z values at B and C, this gives

$$3p + 4q = 5q$$

$$3p = 5q - 4q$$

$$3p = q$$

16.

**(d)**  $n_1 + n_2 - 2$

**Explanation:**  $n_1 + n_2 - 2$

17.

**(c)**  $e^x f(x) + C$

**Explanation:**  $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$t = e^x f(x)$$

$$\frac{dt}{dx} = e^x \cdot \frac{d}{dx}(f(x)) + f(x) \frac{d}{dx}(e^x)$$

$$= e^x f'(x) + f(x) \cdot e^x$$

$$dt = e^x (f'(x) + f(x)) dx$$

$$\int e^x \{f(x) + f'(x)\} dx = \int dt = t + C$$

$$= e^x f(x) + C$$

18.

**(a)**  $\frac{a+c+d+e}{4}$

**Explanation:**  $\frac{a+c+d+e}{4}$

19.

**(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** The given matrices are  $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$\text{Then, } A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

20.

**(a)** Both A and R are true and R is the correct explanation of A.

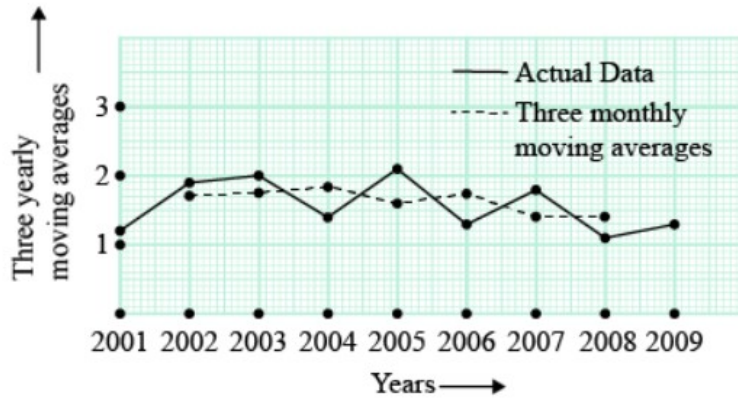
**Explanation:** Both A and R are true and R is the correct explanation of A.

**Section B**

21.

Year	Rainfall (in mm)	Three Yearly Moving total	Three yearly Moving Average
2001	1.2		
2002	1.9	5.1	1.7
2003	2	5.3	1.76
2004	1,4	5.5	1.83
2005	2.1	4.8	1.6
2006	1.3	5.2	1.73
2007	1.8	4.2	1.4
2008	1.1	4.2	1.4
2009	1.3		

The points are joined by a line segment to obtain the graph to understand the trend.



22. Here,  $V = ₹ 3,00,000$ ,  $r = 6\%$  and  $n = 20$

We know  $V = \frac{A}{r} [1 - (1 + r)^{-n}]$

Thus  $3,00,000 = \frac{A}{0.06} [1 - (1 + 0.06)^{-20}]$

$$\Rightarrow A = \frac{300000 \times 0.06}{[1 - (1 + 0.06)^{-20}]}$$

$$\Rightarrow A = \frac{18000}{[1 - (1.06)^{-20}]}$$

$$\Rightarrow A = ₹ 26,162.79$$

hence, the amount of annual instalment is ₹ 26,162.79

OR

Let principal be ₹ 100

Rate = 3% half yearly

$$\therefore \text{Interest for 1st half year} = \frac{100 \times 3 \times 1}{100} = ₹ 3$$

$$\text{Interest for 2nd half year} = \frac{103 \times 3 \times 1}{100} = ₹ 3.09$$

$\therefore$  Total yearly interest = ₹ 6.09

Let effective rate of interest be  $r\%$

$$\therefore 6.09 = \frac{100 \times r \times 1}{100} \Rightarrow r = 6.09\%$$

$$\text{Using formula } \left(1 + \frac{3}{100}\right)^2 - 1 = (1.03)^2 - 1$$

$$= 1.0609 - 1 = 0.0609$$

$\therefore$  Effective rate % = 6.09%

$$\begin{aligned} 23. \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx &= \left[ 4 \cdot \frac{x^4}{4} - 5 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^2}{2} + 9x \right]_1^2 \\ &= \left[ x^4 - \frac{5}{3}x^3 + 3x^2 + 9x \right]_1^2 = \left( 16 - \frac{40}{3} + 12 + 18 \right) - \left( 1 - \frac{5}{3} + 3 + 9 \right) \\ &= 46 - \frac{40}{3} - 13 + \frac{5}{3} = 33 - \frac{35}{3} = \frac{64}{3} \end{aligned}$$

24. Here  $C = ₹ 4,50,000$

$S = ₹ 1,00,000$

and  $n = 5$  years.

$$\text{Annual depreciation } D = \frac{C - S}{n} = ₹ 70,000$$

Thus, yearly depreciation schedule is as follows:

Years	Book value at the beginning of the year (in ₹)	Depreciation (in ₹)	Book value at the end of the year (in ₹)
1	4,50,000	70,000	3,80,000
2	3,80,000	70,000	3,10,000
3	3,10,000	70,000	2,40,000
4	2,40,000	70,000	1,70,000
5	1,70,000	70,000	1,00,000

OR

Let the rate of interest be  $r\%$  converted quarterly. Then,  $i = \frac{r}{400}$

It is given that the present value of a perpetuity of ₹1,200 payable at the end of each quarter is ₹96,000

i.e.,  $P = ₹96,000$ ,  $R = ₹1,200$  and  $i = \frac{r}{400}$

$$\therefore P = \frac{R}{i} \Rightarrow 96,000 = \frac{1,200}{\frac{r}{400}} \Rightarrow 96,000 = \frac{1,200 \times 400}{r} \Rightarrow r = \frac{1,200 \times 400}{96,000} = 5$$

Hence, the rate of interest is 5% convertible quarterly.

25. To find  $(9 + 23) \bmod 12$ , let us divide  $9 + 23$  i.e. 32 by 12

$$\begin{array}{r} 12 \overline{) 32} \quad 2 \\ \underline{24} \\ 8 \end{array} \rightarrow \text{Remainder}$$

So,  $(9 + 23) \bmod 12 = 8$

### Section C

26. Let  $A$  be the quantity of bacteria present in culture at any time  $t$  and initial quantity of bacteria is  $A_0$

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\frac{dA}{A} = \lambda dt$$

$$\int \frac{dA}{A} = \int \lambda dt$$

$$\log A = \lambda t + c \dots (i)$$

Initially,  $A = A_0$ ,  $t = 0$

$$\log A_0 = 0 + c$$

$$\log A_0 = c$$

Now equation (i) becomes,

$$\log A = \lambda t + \log A_0$$

$$\log \left( \frac{A}{A_0} \right) = \lambda t \dots (ii)$$

Given  $A = 2 A_0$  when  $t = 6$  hours

$$\log \left( \frac{A}{A_0} \right) = 6\lambda$$

$$\frac{\log 2}{6} = \lambda$$

Now equation (ii) becomes,

$$\log \left( \frac{A}{A_0} \right) = \frac{\log 2}{6} t$$

Now,  $A = 8 A_0$

$$\text{so, } \log \left( \frac{8A_0}{A_0} \right) = \frac{\log 2}{6} t$$

$$\log 2^3 = \frac{\log 2}{6} t$$

$$3 \log 2 = \frac{\log 2}{6} t$$

$$18 = t$$

Hence, Bacteria becomes 8 times in 18 hours.

OR

If  $P$  denotes the principal at any time  $t$  and the rate of interest be  $r\%$  per annum compounded continuously, then according to the law given in the problem, we get

$$\frac{dP}{dt} = \frac{Pr}{100}$$

$$\Rightarrow \frac{dP}{P} = \frac{r}{100} dt$$

$$\Rightarrow \int \frac{1}{P} dP = \frac{r}{100} \int dt$$

$$\Rightarrow \log P = \frac{rt}{100} + C \dots (i)$$

Let  $P_0$  be the initial principal i.e. at  $t = 0$ ,  $P = P_0$

Putting  $P = P_0$  in (i), we get

$$\log P_0 = C$$

Putting  $C = \log P_0$  in (i), we get

$$\log P = \frac{rt}{100} + \log P_0$$

$$\Rightarrow \log\left(\frac{P}{P_0}\right) = \frac{rt}{100} \dots(ii)$$

i. In this case, we have

$$r = 5, P_0 = ₹ 100 \text{ and } P = ₹ 200 = 2P_0$$

Substituting these values in (ii), we have

$$\log 2 = \frac{5}{100}t \Rightarrow t = 20 \log_e 2 = 20 \times 0.6931 \text{ years} = 13.862 \text{ years.}$$

ii. In this case, we have

$$P_0 = ₹ 100, P = ₹ 200 = 2P_0 \text{ and } t = 10 \text{ years.}$$

Substituting these values in (ii), we get

$$\log 2 = \frac{10r}{100}t \Rightarrow r = 10 \log 2 = 10 \times 0.6931 = 6.931$$

Hence,  $r = 6.931\%$  per annum.

iii. In this case, we have

$$P_0 = ₹ 1000, r = 5 \text{ and } t = 10$$

Substituting these values in (ii), we get

$$\log\left(\frac{P}{1000}\right) = \frac{5 \times 10}{100} = \frac{1}{2} = 0.5 \Rightarrow \frac{P}{1000} = e^{0.5} \Rightarrow P = 1000 \times 1.648 = 1648$$

$$P = ₹ 1648$$

27. Madhu paid the balance in 20 monthly installments of ₹ 21000 each

Let Principle =  $P$ ,  $i = \frac{9}{1200} = 0.0075$ ,  $n = 20$  and  $E = 21000$

$$E = \frac{Pi}{1 - (1+i)^{-n}}$$

$$\Rightarrow 21000 = \frac{P \times (0.0075)}{1 - (1.0075)^{-20}}$$

$$\Rightarrow 21000 = \frac{P \times (0.0075)}{1 - 0.8611}$$

$$\Rightarrow 21000 = \frac{P \times (0.0075)}{0.1389}$$

$$\Rightarrow 21000 \times 0.1389 = P \times (0.0075)$$

$$\Rightarrow P = 388920$$

Thus, the balance is ₹ 388920

Madhu exchanged her old car valued at ₹ 150000 and a new one priced at ₹ 650000.

So, Madhu had ₹ 500000 after the exchange.

She paid approximately ₹ 388920 in the form of monthly installments.

Therefore, the down payment  $x = 500000 - 388920 = 111080$ .

Hence, the value of  $x$  is 111080.

$$28. MR = \frac{6}{(x+2)^2} + 5$$

$$R = \int (MR) dx + C$$

$$= \int \left( \frac{6}{(x+2)^2} + 5 \right) dx + C$$

$$= 6 \int \frac{1}{(x+2)^2} dx + 5 \int dx + C$$

$$= 6 \frac{-1}{(x+2)} + 5x + c$$

$$= \frac{-6}{(x+2)} + 5x + c$$

When  $R = 0$  and  $x = 0$

$$0 = \frac{-6}{(0+2)} + 5(0) + C$$

$$C = 3$$

$$\therefore R = \frac{-6}{(x+2)} + 5x + 3$$

and demand function is given by,

$$p = \frac{R}{x}$$

$$p = \frac{\frac{-6}{(x+2)} + 5x + 3}{x}$$

$$p = \frac{-6}{x(x+2)} + 5 + \frac{3}{x}$$

Where  $p$  is the price, when number of units sold  $x$ .

29. Let E : Event that 3 balls in the first draw are all white.

F : Event that 3 balls in the second draw are all red.

Now, 3 balls can be drawn out of 13 in  ${}^{13}C_3$  ways and 3 white balls can be drawn out of 5 in  ${}^5C_3$  ways

$$P(E) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{5!}{3! \times 2!} \times \frac{3! \times 10!}{13!} = \frac{5}{143}$$

Since, 3 balls are not replaced before the second draw, we are left with 8 red and 2 white balls.

Now, 3 balls can be drawn in  ${}^{10}C_3$  ways and 3 red balls can be drawn in  ${}^8C_3$  ways.

$$P\left(\frac{F}{E}\right) = \frac{{}^8C_3}{{}^{10}C_3}$$

$$= \frac{8!}{3! \times 5!} \times \frac{3! \times 7!}{10!} = \frac{7}{15}$$

$$\therefore P(E \cap F) = P(E) \cdot P\left(\frac{F}{E}\right) = \frac{5}{143} \times \frac{7}{15}$$

$$= \frac{7}{429}$$

OR

The probability distribution of x is

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

i.  $\sum_{i=1}^n pi = 1$

$$0.1 + K + 2K + 2K + K = 1$$

$$K = 0.15$$

ii. p (study atleast two hours) = p (x ≥ 2)

$$= 2K + 2K + K$$

$$= 5K$$

$$= 5 \times 0.15$$

$$= 0.75$$

p (Study exactly two hours) = p(x = 2)

$$= 2K$$

$$= 2 \times 0.15$$

$$= 0.3$$

p (Study at most two hours) = p(x ≤ 2)

$$= p(x=0) + p(x=1) + p(x=2)$$

$$= 0.1 + k + 2k$$

$$= 0.1 + 3k = 0.1 + 3(0.15)$$

$$= 0.1 + 0.45 = 0.55$$

30. Calculating of trend values by three yearly moving average method.

Year	Sales (Thousand ₹)	Three-yearly Moving Totals	Three-yearly Moving Average (Trend value)
2008	8		
2009	12	(8 + 12 + 10) = 30	10.00
2010	10	(12 + 10 + 13) = 35	11.67
2011	13	(10 + 13 + 15) = 38	12.67
2012	15	(13 + 15 + 12) = 40	13.33
2013	12	(15 + 12 + 16) = 43	14.33
2014	16	(12 + 16 + 17) = 45	15.00
2015	17	(16 + 17 + 14) = 47	15.67
2016	14	(17 + 14 + 17) = 48	16.00
2017	17		

31.  $\mu =$  Population mean = 12 Kg

$\bar{X} =$  Sample mean = 11.8 Kg

$n = 10$

Sample standard deviation =  $s = 0.15$

Null Hypothesis  $H_0 =$  There is no significance between the sample mean

$\bar{X}$  and the population mean  $\mu$ .

Alternate Hypothesis  $H_1 =$  There is significance between the sample mean  $\bar{X}$  and the population mean  $\mu$

Let  $t$  be the test statistic given by

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$t = \left( \frac{11.8 - 12}{0.15} \right) \times 3$$

$= -4$

The test statistic  $t$  follows student  $t$ -distribution with  $(10-1)=9$  degrees of freedom

It is given that  $t_{0.05} = 2.26$

We observe that,

$$|t| = 4 > 2.26$$

$\implies$  Calculate  $|t| >$  tabulated  $t_9(0.05)$

So, the null hypothesis is rejected at a 5% level of significance.

Hence there is a significance between the sample mean  $\bar{X}$  and the population mean  $\mu$ .

#### Section D

32. Let the dietician mix  $x$  kg of food I with  $y$  kg of food II. Then, the mathematical model of the LPP is as follows:

Minimize  $Z = 5x + 7y$

Subject to  $2x + y \geq 8$

$x + 2y \geq 10$

and,  $x, y \geq 0$

To solve this LPP graphically, we first convert the inequations into equations to obtain the following lines.

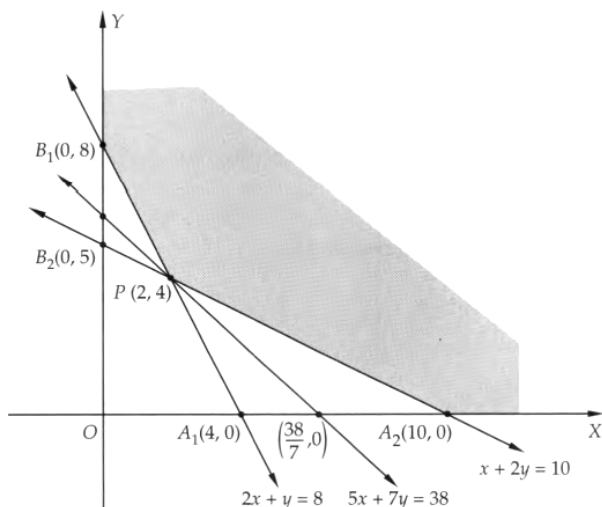
$2x + y = 8$ ,  $x + 2y = 10$ ,  $x = 0$ ,  $y = 0$

The line  $2x + y = 8$  meets the coordinate axes at  $A_1(4, 0)$  and  $B_1(0, 8)$ . Join these points to obtain the line represented by  $2x + y = 8$ . The region not containing the origin is represented by  $2x + y \geq 8$ .

The line  $x + 2y = 10$  meets the coordinate axes at  $A_2(10, 0)$  and  $B_2(0, 5)$ . Join these points to obtain the line represented by  $x + 2y = 10$ . Clearly,  $O(0, 0)$  does not satisfy the inequation  $x + 2y \geq 10$ . So, the region not containing the origin is represented by this inequation.

Clearly,  $x \geq 0$  and  $y \geq 0$  represent the first quadrant.

Thus, the shaded region in figure is the feasible region of the LPP. The coordinates of the corner-points of this region are  $A_2(10, 0)$ ,  $P(2, 4)$  and  $B_1(0, 8)$ .



The point  $P(2, 4)$  is obtained by solving  $2x + y = 8$  and  $x + 2y = 10$  simultaneously.

The values of the objective function  $Z = 5x + 7y$  at the corner points of the feasible region are given in the following table:



Point (x, y)	Value of the objective function $Z = 5x + 7y$
$A_2(10, 0)$	$Z = 5 \times 10 + 7 \times 0 = 50$
$P(2, 4)$	$Z = 5 \times 2 + 7 \times 4 = 38$
$B_1(0, 8)$	$Z = 5 \times 0 + 7 \times 8 = 56$

Clearly,  $Z$  is minimum at  $x = 2$  and  $y = 4$ . The minimum value of  $Z$  is 38. We observe that open half-plane represented by  $5x + 7y < 38$  does not have points in common with the feasible region. So,  $Z$  has a minimum value equal to 38 at  $x = 2$  and  $y = 4$ .

Hence, the optimal mixing strategy for the dietician will be to mix 2 kg of food I and 4 kg of food II. In this case, his cost will be minimum and the minimum cost will be ₹ 38.00

OR

Let  $x$  articles of deluxe model and  $y$  articles of an ordinary model be made

Number of articles cannot be negative

Therefore,  $x, y \geq 0$

According to the question, the making of a deluxe model requires 2 hrs. work by a skilled man and the ordinary model requires 1 hr by a skilled man

$$2x + y \leq 40$$

The making of a deluxe model requires 2 hrs. work by a semi-skilled man ordinary model requires 3 hrs. work by a semi-skilled man

$$2x + 3y \leq 80$$

Total profit =  $Z = 15x + 10y$  which is to be maximised

Thus, the mathematical formulation of the given linear programming problem is

$$\text{Max } Z = 15x + 10y$$

subject to

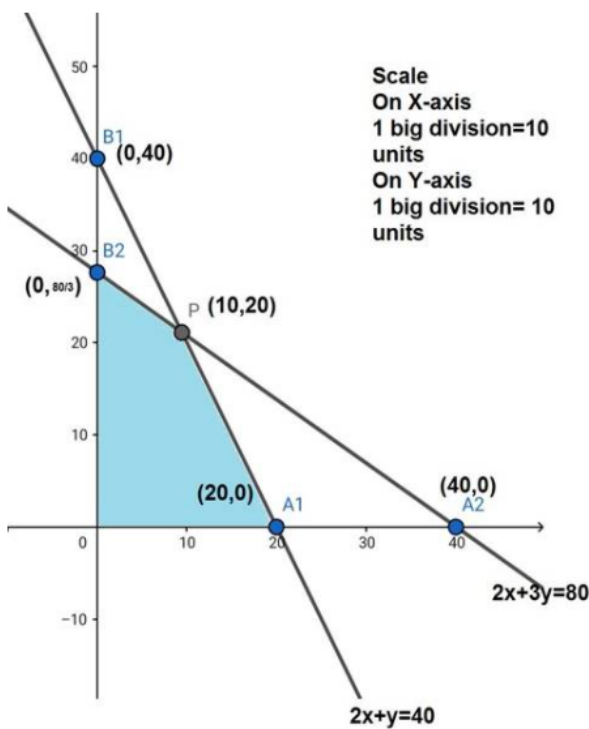
$$2x + y \leq 40$$

$$2x + 3y \leq 80$$

$$x \geq 0$$

$$y \geq 0$$

The feasible region determined by the system of constraints is



The corner points are  $A(0, \frac{80}{3})$ ,  $B(10, 20)$ ,  $C(20, 0)$

The values of  $Z$  at these corner points are as follows

Corner point	$Z = 15x + 10y$
A	$\frac{800}{3}$

B	350
C	300

The maximum value of Z is 300 which is attained at C(20, 0)

Thus, the maximum profit is ₹300 obtained when 10 units of deluxe model and 20 units of ordinary model is produced.

33. First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality. You can choose any value but find the two mandatory values which are at  $x = 0$  and  $y = 0$ , i.e.,  $x$  and  $y$ -intercepts always.

$$2x + y \geq 8$$

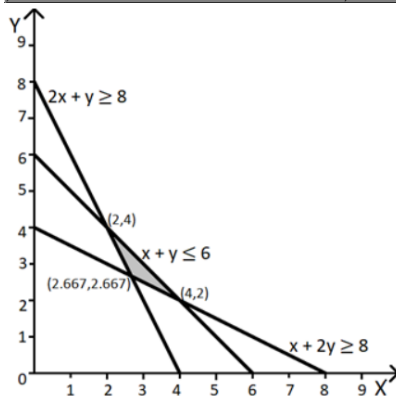
x	0	2	4
y	8	4	0

$$x + 2y \geq 8$$

x	0	4	8
y	4	2	0

$$x + y \leq 6$$

x	0	3	6
y	6	3	0



34. Let  $x$  be the random variable denoting the number of times an odd number (the number of successes) when a die is tossed twice.

Then  $x$  takes the values 0, 1, 2

Let  $P(X = 0)$  be probability of getting no odd number (both times showing even).

$$\therefore P(X = 0) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$$

Let  $P(X = 1)$  be probability of getting odd number once.

$$\therefore P(X = 1) = {}^2C_1 \frac{3}{6} \times \frac{3}{6} = \frac{6}{6} \times \frac{3}{6} = \frac{1}{2}$$

Let  $P(X = 2)$  be probability of getting odd number twice.

$$\therefore P(X = 2) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$$

Thus the probability distribution of  $X$  is given by

$$X = x: x = 0, x = 1, x = 2$$

$$P(X = x) \frac{1}{4} \frac{1}{2} \frac{1}{4}$$

$$\text{We know that mean } E(X) = \sum x_i p_i = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$\therefore E(X) = 0 + \frac{1}{2} + \frac{1}{2} = 1$$

Thus mean  $E(X) = 1$

$$\text{We know that } \text{var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum x_i^2 p_i = 0 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$$

$$\therefore E(X^2) = 0 + \frac{1}{2} + 4 \times \frac{1}{4} = \frac{3}{2}$$

$$\text{Thus } \text{var}(X) = \frac{3}{2} - [1]^2 = \frac{3}{2} - 1 = \frac{1}{2}$$

Hence mean is 1 and variance is  $\frac{1}{2}$

OR

From the given information, we find that the probability distribution of  $X$  is

--	--	--	--	--

X	0	1	2	3
P(X)	0.2	k	2k	2k

a. We know that  $\sum p_i = 1$

$$\Rightarrow 0.2 + k + 2k + 2k = 1$$

$$\Rightarrow 5k = 0.8 \Rightarrow k = \frac{4}{25}$$

b. Probability that the person watches two hours of television

$$= P(X = 2) = 2k = 2 \times \frac{4}{25} = \frac{8}{25}$$

c. P (the person watches at least two hours of television)

$$= P(X \geq 2) = P(X = 2) + P(X = 3)$$

$$= 2k + 2k = 4k$$

$$= 4 \times \frac{4}{25} = \frac{16}{25}$$

d. P (the person watches at most two hours of television)

$$= P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= 0.2 + k + 2k$$

$$= 0.2 + 3k = \frac{1}{5} + \frac{12}{25} = \frac{17}{25}$$

e. We construct the following table:

$x_i$	$P_i$	$P_i x_i$	$P_i x_i^2$
0	0.2	0	0
1	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{4}{25}$
2	$\frac{8}{25}$	$\frac{16}{25}$	$\frac{32}{25}$
3	$\frac{8}{25}$	$\frac{24}{25}$	$\frac{72}{25}$
Total		$\frac{44}{25}$	$\frac{108}{25}$

$$E(X) = \sum p_i x_i = \frac{44}{25} = 1.76$$

f. Variance  $\sigma^2 = \sum p_i x_i^2 - (\sum p_i x_i)^2$

$$= \frac{108}{25} - \left(\frac{44}{25}\right)^2 = \frac{108}{25} - \frac{1936}{625} = \frac{764}{625} = 1.22$$

$$\text{and standard deviation } \sigma = \sqrt{\text{Variance}} = \sqrt{1.22} = 1.1$$

35. Given, P = ₹ 1500000,  $i = \frac{12}{12 \times 100} = \frac{1}{100} = 0.01$

and  $n = 8 \times 12 = 96$

i.  $EMI = \frac{1500000 \times 0.01 \times (1.01)^{96}}{(1.01)^{96} - 1}$   
 $= \frac{1500000 \times 0.01 \times 2.5993}{2.5993 - 1}$   
 $= \frac{1500000 \times 0.01 \times 2.5993}{1.5993}$   
 $= ₹ 24,379.10$

ii. Principal outstanding at the beginning of 40<sup>th</sup> month

$$= \frac{EMI \left[ (1+i)^{96-40+1} - 1 \right]}{i(1+i)^{96-40+1}}$$

$$= \frac{24379.10 \times \left[ (1.01)^{57} - 1 \right]}{0.01(1.01)^{57}}$$

$$= \frac{24379.10 \times (1.7633 - 1)}{0.01 \times 1.7633}$$

$$= \frac{24379.10 \times 0.7633}{0.017633}$$

$$= ₹ 1,055,326.20$$

iii. Interest paid in 40<sup>th</sup> payment

$$= \frac{EMI \left[ (1+i)^{96-40+1} - 1 \right]}{(1+i)^{96-40+1}}$$

$$= \frac{24379.10 \left[ (1.01)^{57} - 1 \right]}{(1.01)^{57}}$$

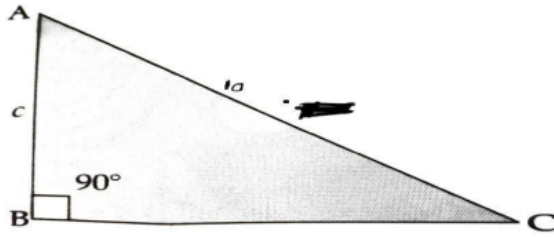
$$= \frac{24379.10 \times 0.7633}{1.7633}$$

$$= ₹ 10553.26$$

### Section E

#### 36. Read the text carefully and answer the questions:

The sum of the length of hypotenuse and a side of a right-angled triangle is given by  $AC + BC = 10$



- (i)  $\frac{100-c^2}{20}$   
 (ii)  $\frac{100-3c^2}{40}$   
 (iii)  $\frac{10\sqrt{3}}{3}$

OR

$$\frac{-\sqrt{3}}{2}$$

#### 37. Read the text carefully and answer the questions:

##### What Is a Sinking Fund?

A sinking fund contains money set aside or saved to pay off a debt or bond. A company that issues debt will need to pay that debt off in the future, and the sinking fund helps to soften the hardship of a large outlay of revenue. A sinking fund allows companies that have floated debt in the form of bonds gradually save money and avoid a large lump-sum payment at maturity.

##### Example:

- Cost of Machine: ₹2,00,000/-
- Effective Life: 7 Years
- Scrap Value: ₹30,000/-
- Sinking Fund Earning Rate: 5%
- The Expected Cost of New Machine: ₹3,00,000/-

(i) Cost of new machine = ₹ 300000

Scrap value of old machine = ₹ 30000

Hence, the money required for new machine after 7 years

$$= ₹ 300000 - ₹ 30000 = ₹ 270000$$

(ii)  $A = ₹ 270000$ ,  $i = \frac{5}{100} = 0.05$ ,  $n = 7$

(iii)  $A = R \left[ \frac{(1+i)^n - 1}{i} \right]$

OR

Cost of new machine = ₹300000

Scrap value of old machine = ₹30000

Hence, the money required for new machine after 7 years

$$= ₹300000 - ₹30000 = ₹270000$$

So, we have  $A = ₹270000$ ,  $i = \frac{5}{100} = 0.05$ ,  $n = 7$

Using formula,  $A = R \left[ \frac{(1+i)^n - 1}{i} \right]$ , we get

$$270000 = R \left[ \frac{(1.05)^7 - 1}{0.05} \right]$$

$$[\text{Let } x = (1.05)^7]$$

$$\Rightarrow \log x = 7 \log 1.05 = 7 \times 0.0212 = 0.1484$$

$$\Rightarrow x = \text{antilog } 0.1484$$

$$\Rightarrow x = 1.407$$

$$\Rightarrow R = \frac{270000 \times 0.05}{(1.05)^7 - 1}$$

$$\Rightarrow R = \frac{13500}{1.407 - 1} = \frac{13500}{0.407}$$

$$\Rightarrow R = 33169.53$$

Hence, the company should deposit ₹33169.53 at the end of each year for 7 years.

38. Let  $x$ ,  $y$  and  $z$  be the number of cars produced by steel type  $C_1$ ,  $C_2$  and  $C_3$  respectively.

Now, we can arrange this model in linear equation system

Thus, we have

$$2x + 3y + 4z = 29$$

$$x + y + 2z = 13$$

$$3x + 2y + z = 16$$

It can be written as  $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$  or  $AX = B$

where  $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 29 \\ 13 \\ 16 \end{bmatrix}$

Here

$$\Rightarrow D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

Expand by  $R_1$

$$\Rightarrow D = 2(1 - 4) - 3(1 - 6) + 4(2 - 3)$$

$$= -6 + 15 - 4$$

$$\Rightarrow D = 5$$

Again, Solve  $D_1$  formed by replacing 1<sup>st</sup> column by B matrices

$$\Rightarrow D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix}$$

$$= 29(1 - 4) - 3(13 - 32) + 4(26 - 16)$$

$$= -87 + 57 + 40$$

$$\Rightarrow D_1 = 10$$

Again, Solve  $D_2$  formed by replacing 2<sup>nd</sup> column by B matrices

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix}$$

$$= 2(13 - 32) - 29(1 - 6) + 4(16 - 39) = -38 + 145 - 92$$

$$\Rightarrow D_2 = 15$$

And, Solve  $D_3$  formed by replacing 3<sup>rd</sup> column by B matrices

$$\Rightarrow D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix}$$

$$= 2(16 - 26) - 3(16 - 39) + 29(2 - 3)$$

$$= -20 + 69 - 29$$

$$\Rightarrow D_3 = 20$$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

$$\Rightarrow x = \frac{10}{5}, y = \frac{15}{5} \text{ and } z = \frac{20}{5}$$

$$\Rightarrow x = 2, y = 3 \text{ and } z = 4$$

Thus the Number of cars produced by type  $C_1$ ,  $C_2$  and  $C_3$  are 2, 3 and 4 respectively.

OR

$$\text{Let } A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$$

$$\therefore A + A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2} (A + A') = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}, \text{ which is a symmetric matrix.}$$

$$A - A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2} (A - A') = \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix}, \text{ which is a skew-symmetric.}$$

$$\text{Since } A = \frac{1}{2} (A + A') + \frac{1}{2} (A - A'),$$

$$\therefore \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & -\frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$