PACIFIC WORLD SCHOOL

PREBOARD-2 EXAMINATION (2024-25)
CLASS - XII
SUBJECT - MATHEMATICS (CODE - 041)

No. of Questions : 38	No. of Printed sides : 08	Date : 6-Jan-2024
Max. Marks : 80		Time : 3 hours

General Instructions:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

SECTION A

(Question numbers 01 to 20 carry 1 mark each.)

01. The area under the curve $x^2 = y$ between the line $x = 0$ and $x = k$ is 9 square units. Which of the following could be the correct value of k?

(a) $\frac{3}{2}$ (b) 9 (c) 3 (d) $\frac{9}{2}$

02. If $\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \dots\dots\dots$ is :

(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

03. Function f defined by $f(x) = e^{-x}$ is strictly increasing when

(a) $x \in \phi$ (b) $x \in (-\infty, 0)$ (c) $x \in [0, \infty)$ (d) $x \in (-\infty, \infty)$

04. If A, B and C are square Matrices of order 3 and $\det(BC) = 2\det(A)$, then what is the value of $\det(2A^{-1}BC)$?

(a) 1 (b) 2^2 (c) 2^3 (d) 2^4

05. The value of '**n**', such that the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{x^n}$; where $x, y \in R^+$ is homogeneous, is:

(a) 0 (b) 1 (c) 2 (d) 3

06. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing (1,3) is:

(a) 1 (b) 3 (c) 2 (d) 4

07. If A is a square matrix of order n, then the number of minors in the determinant of A are

(a) n (b) $n-1$ (c) n^2 (d) n^n

08. Delhi Metro is highly popular mode of transport among the commuters. The Metro connects numerous stations across Delhi NCR. Among them are Dwarka and Hauz Khas. Arnav takes the Metro scheduled at 8:25 AM from Dwarka station to Hauz Khas station every morning. The probability that the Metro is late is $\frac{3}{4}$ and, the probability that Arnav gets a seat in the Metro is $\frac{1}{15}$. The probability that the Metro is on time and he gets a seat in it is:

(a) $\frac{1}{20}$ (b) $\frac{1}{4}$ (c) $\frac{1}{60}$ (d) $\frac{7}{10}$

09. There are two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}.\vec{b} = 0$. Then the projection of $\vec{a}$ on $\vec{b}$ is

(a) 1 (b) 0 (c) 2 (d) can not be determined

10. Let $y = f(x)$ be a real function such that its first-order derivative is same as its second-order derivative. Then the value of f(x) is :

(a) x (b) 2^x (c) e^x (d) no such function exists

11. If $f(x)$ is continuous for all real values of x, then $\int_{\frac{a}{4}}^{\frac{b}{4}} f(4x)dx$ equals

(a) $4\int_a^b f(x)dx$ (b) $\frac{1}{4}\int_{4a}^{4b} f(x)dx$ (c) $\frac{1}{4}\int_a^b f(x)dx$ (d) $4\int_{4a}^{4b} f(x)dx$

12. The domain of the function $cos^{-1}(2x-1)$ is:

(a) [0,1] (b) [-1,1] (c) $[0,\pi]$ (d) $(-1,1)$

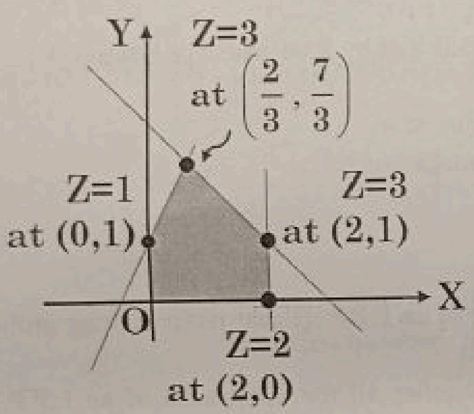
13. A differential equation has an order of 3 and a degree of 2. Which of the following could this differential equation be?

(a) $\frac{d^3y}{dx^3}-\left(\frac{d^2y}{dx^2}\right)^2=0$ (b) $\tan\left(\frac{d^2y}{dx^2}\right)+\left(\frac{d^3y}{dx^3}\right)^2=0$

(c) $\left(\frac{d^3y}{dx^3}\right)^2+\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3=0$ (d) $\left(\frac{d^3y}{dx^3}\right)^3+\left(\frac{dy}{dx}\right)^2=0$

14. Shown below is the feasible region of a linear programming problem (L.P.P.) whose objective function is : Maximize $Z=x+y$.

A student Drishti claimed that there exists **no optimal solution** for the L.P.P. as there is no unique maximum value at the corner points of its feasible region. Based on her statement, choose most appropriate option.



(a) Her claim is correct, as there are two corner points of its feasible region at which maximum value of Z occurs.

(b) Her claim is false, as there are exactly two corner points i.e., $\left(\frac{2}{3},\frac{7}{3}\right)$ and (2, 1) at which the maximum value of Z occurs, which is 3.

(c) Her claim is false, as every point on the line joining $\left(\frac{2}{3},\frac{7}{3}\right)$ and (2, 1) gives the maximum value of Z, which is 3.

(d) Her claim is false, as the maximum value of Z occurs at (2, 0), which is 2.

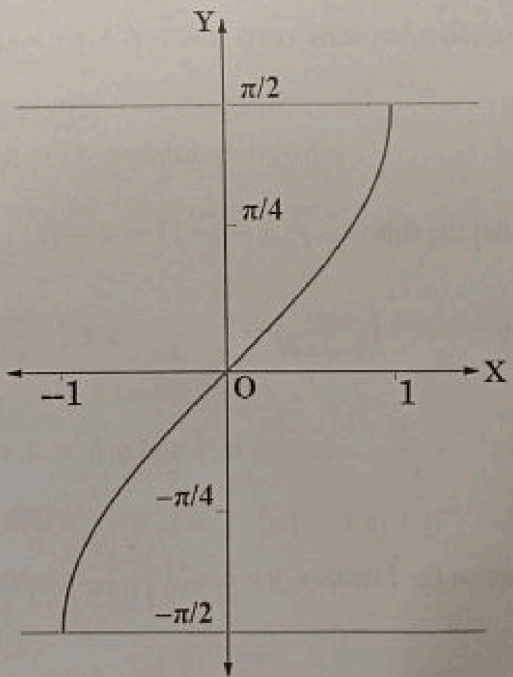
15. The graph drawn below depicts

(a) $y=\cos^{-1}x$

(b) $y=\text{cosec}^{-1}x$

(c) $y=\sin^{-1}x$

(d) $y=\cot^{-1}x$



16. The set of all points where the function $f(x) = x + |x|$ is differentiable, is:

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

17. If [.] denotes the greatest integer function, then $f(x) = [x]$ is discontinuous at

(a) infinite points, in $2 < x < 5$ (b) only two points, in $2 < x < 5$

(c) only three points, in $2 < x < 5$ (d) no point, in $2 < x < 5$

18. If for a square matrix P, $P.(\text{adj}.P) = \begin{bmatrix} -2025 & 0 & 0 \\ 0 & -2025 & 0 \\ 0 & 0 & -2025 \end{bmatrix}$, then $|P| + |\text{adj}.P| =$

(a) $2025^2 \times 2024$ (b) $(-2025) + 1$ (c) 2025×2024 (d) $(-2025)^2 + 2025$

Followings are **Assertion-Reason based questions**.

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A) :** All the points in the feasible region of an L.P.P. (linear programming problem) are optimal solutions to the problem.

Reason (R) : Every point in the feasible region satisfies all the constraints of an L.P.P. (linear programming problem).

20. **Assertion (A) :** If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p}.\vec{q} < 0$.

Reason (R) : Value of $\cos\theta$ lies in $(-1, 0)$, when $90^\circ < \theta < 180^\circ$.

SECTION B

(Question numbers 21 to 25 carry **2 marks** each.)

21. Show that the function $f: R - \{-1\} \to R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective.

22. Prove that: $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

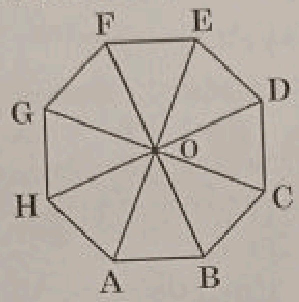
23. If $y = ax^{n+2} + \frac{b}{x^{n+1}}$, where $n \in N$ and $a, b \in R$, then prove that $x^2 \frac{d^2y}{dx^2} = (n+1)(n+2)y$.

OR

Differentiate the function $y = \cos^{-1}\left(\frac{1-3^{2x}}{1+3^{2x}}\right)$ with respect to x.

24. A bird is sitting on an electric wire (assuming that the wire has no slack). If the equation of wire is given by $\frac{x+1}{-3} = y-2 = z$ and the position of bird is at a point P such that the distance between P and Q(−1, 2, 0) is $6\sqrt{11}$ units, then find the position of bird (coordinates of point P).

25. Shown below is a regular octagon ABCDEFGH, with centre O.



Show that $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AD} - \overrightarrow{BC})$.

OR

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of λ.

SECTION C

(Question numbers 26 to 31 carry **3 marks** each.)

26. A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change such that the cylindrical shape is retained and the volume (V) of the disk remains constant. What is the ratio of the rate of change of height to the rate of change of radius in terms of R?

27. Arpit bought a luxury car for Rs y. The price P(t) of the car depreciates as $\frac{dP(t)}{dt} = -a(T-t)$, after 't' years; where 'T' is the total life of the car (in years) and 'a' is an arbitrary constant. Find the value of the car when the car has been used for 'T' years, in terms of 'y'.

OR

The first derivative of a function y with respect to x is given by $-\frac{1}{x^2(1+x^2)}$. Find the function, if it is given that $y = \frac{\pi}{4}$, when $x = 1$.

28. If vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

29. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

OR

Evaluate: $\int_{-5}^{1} f(x)dx$, where $f(x) = |x| + |x+3| + |x+6|$.

30. Consider the following Linear Programming Problem.

Maximize $Z = x + 2y$

Subject to $2x + 3y \geq 6$, $4x + y \geq 4$; $x, y \geq 0$.

Show graphically that the maximum value of Z will not occur.

31. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

OR

There are four cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Find the probability distribution of the sum of the numbers on the two cards drawn.

SECTION D

(Question numbers 32 to 35 carry **5 marks** each.)

32. Draw a rough sketch of the given curve $y = 1 + |x+1|$, $x = -3$, $x = 3$, $y = 0$ and find the area of the region bounded by them, using integration.

33. The curve $y = ax^2 + bx + c$; (where $a, b, c \in R$ and $a \neq 0$) passes through the points $(-1, 0)$, $(2, 12)$ and $(3, 20)$. Use matrix method to determine the values of a, b and c by solving the system of linear equations in a, b and c. Find the equation of the curve. If $y = ax^2 + bx + c = 0$, then write the real roots of quadratic equation (if possible).

34. The vertices of a ΔABC are A (1, 1, 0), B (1, 2, 1) and C (–2, 2, –1). Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid.

OR

Find the image of the point (3,-1, 11) on $\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4}$

35. Show that the function $f(x) = |x-3|, x \in R$ is continuous but not differentiable at $x = 3$.

OR

If $x^{16}y^9 = (x^2+y)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$

SECTION E

(Question numbers 36 to 38 carry **4 marks** each.)

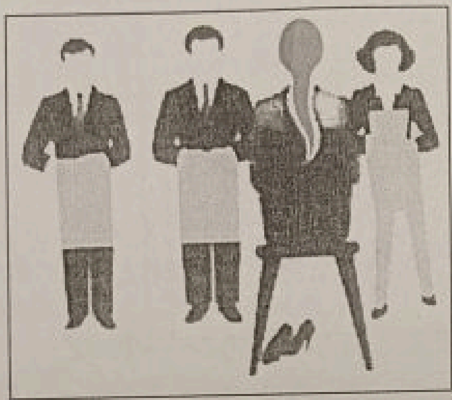
This section contains **three Case-study / Passage based questions.**

First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.

Third question has **two sub-parts** (i) and (ii) of **2 marks** each.

36. **CASE STUDY I :**

Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions:

(i) What is the probability that at least one of them is selected?

(ii) Find $P(\frac{G}{\overline{H}})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected.

(iii) (a) Find the probability that exactly one of them is selected.

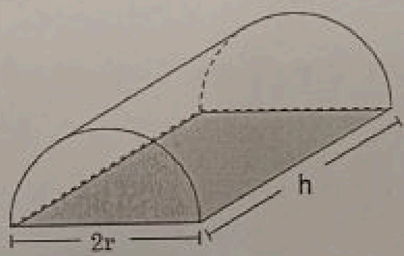
OR

(iii) (b) Find the probability that exactly two of them are selected.

37. **CASE STUDY –II**

A company deals in casting and molding of metal on order received from its clients.

A given quantity of metal (1000 cubic units) is to be cast into a half cylinder with a rectangular base and semicircular ends.



Using the information given above, answer the following.

(i) Write an express for 'h', in terms of 'r'.

(ii) Express the total surface area (A) of the half-cylinder, in terms of 'r'.

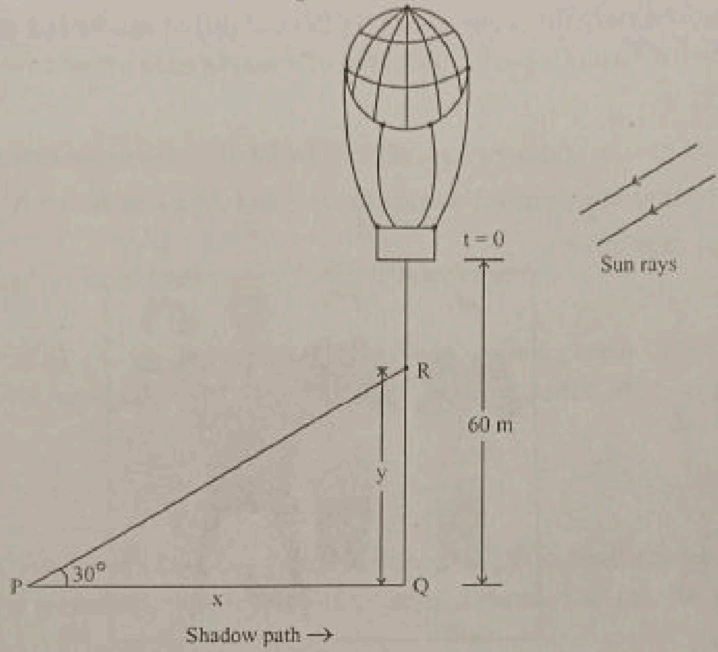
(iii) (a) For what value of r, the total surface area (A) will be minimum?

OR

(iii) (b) What is the value of $h : (2r)$?

38. **CASE STUDY – III**

A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\,t^2$, where y is the height of the sandbag above the ground and t is the time in seconds. On the basis of the above information, answer the following questions :

(i) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

(ii) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

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