



COMMON PRE-BOARD EXAMINATION 2024-25

Subject: APPLIED MATHEMATICS (241)



Date:

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Use of calculators is not allowed.

SECTION-A

(This section comprises of multiple-choice questions (MCQs) of 1 mark each) Select the correct option (Question 1 - Question 18): [1x18=18]

1. The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3 \left(\frac{dy}{dx}\right)^3 + y = 0$ is

- a) 0 b) 1 c) 2 d) 3

2. $(15 - 53) \pmod{4}$ is

- a) 1 b) 4 c) 2 d) 3

3. If $A = [a_{ij}]$ given by $A = \begin{bmatrix} -2 & 1 & 2 \\ 0 & -3 & 2 \\ -1 & 3 & -1 \end{bmatrix}$, then cofactor of a_{23} .

- a) -5 b) -3 c) 6 d) -4

4. If the area of the triangle with vertices $P(-3,0)$, $Q(3,0)$ and $R(0,k)$ is 9 sq.units, then the value(s) of k will be:

- a) ± 3 b) ± 9 c) 6 d) 0

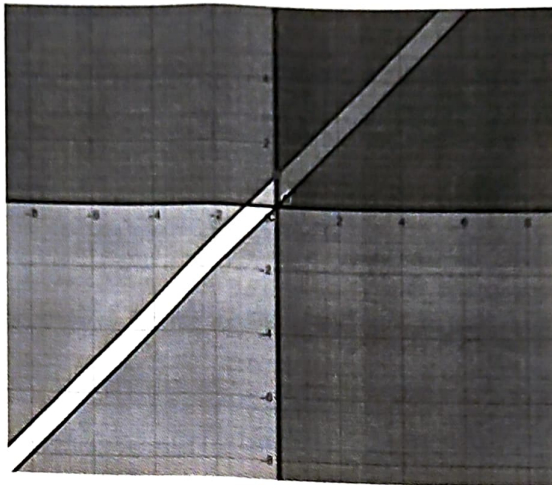
5. If $x \in R$, $|x + 1| \leq 9$, then

- a) $-9 \leq x \leq 9$ b) $-8 \leq x \leq 8$ c) $x \geq 9$ d) $-10 \leq x \leq 8$

6. The curve $y = x^{\frac{1}{5}}$ has at (0,0)

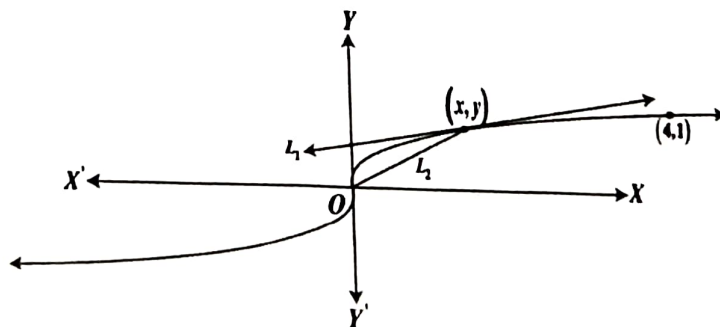
- a) A vertical tangent (parallel to y-axis) b) A horizontal tangent (parallel to X-axis)
 c) no tangent d) an oblique tangent

7. The feasible region represented by the inequation :
 $x - y \leq -1, x - y \geq 0, x \geq 0, y \geq 0$ is



- a) bounded b) unbounded c) do not exist d) triangular region

8. Shown below is a curve.



L_1 is the tangent to any point (x, y) on the curve.
 L_2 is the line that connects the point (x, y) to the origin.
 The slope of L_1 is one third of the slope of L_2 .
 Then the differential equation, using the given conditions is:

- a) $\frac{dy}{dx} = \frac{3y}{x}$ b) $\frac{dy}{dx} = \frac{x}{3y}$ c) $\frac{dy}{dx} = \frac{y}{3x}$ d) $\frac{dy}{dx} = \frac{y}{x}$

9. For a standard normal variate, the value of standard deviation is

- a) 1 b) 0 c) ∞ d) $\frac{1}{2}$

10. A machine makes car wheels and in a random sample of 26 wheels, the test statistic is found to be 3.07. As per the t-distribution test (of 5% level of significance), what can you say about the quality of wheels produced by the machine? (Use $t_{25}(0.05) = 2.06$)

- a) Superior quality b) Inferior quality c) Same quality d) Can't say

11. A sample of 50 bulbs is taken at random. Out of 50 we found 15 bulbs are Bajaj, 17 are of Surya and 18 are of Crompton. What is the point estimate of population proportion of Surya?

- a) 0.3 b) 0.34 c) 0.36 d) 0.4

12. If $\int_0^a x^2 dx = \frac{1}{3}$, then value of a is _____.

- a) 0 b) 1 c) 2 d) 3

13. If $y = x^{2x}$, then find $\frac{dy}{dx}$.

- a) x^{2x-1} b) $2x^{2x} \log x$ c) $2x^{2x}(1 + \log x)$ d) $2x^{2x}(1 - \log x)$

14. If $x = e^t$, $y = \log t^3$, $\frac{dy}{dx} =$ _____.

- a) $\frac{e^t}{3t}$ b) $\frac{3}{te^t}$ c) $\frac{3}{e^t}$ d) $\frac{3t}{e^t}$

15. If X is a poisson Variable such that $P(X = 1) = 2 P(X = 2)$, then $P(X = 0)$ is _____

- a) e b) e^2 c) 1 d) $\frac{1}{e}$

16. If the average cost (AC) of producing x units of an item is given by $AC = 3x^2 - 7x + 5 - \frac{11}{x}$, then find the the marginal cost (MC) of producing 5 units.

- a) ₹150 b) ₹160 c) ₹250 d) ₹225

17. Find the present value of a perpetuity ₹24,000 payable at the end of each 6 months, if the money is worth 8% compounded semi-annually.

- a) ₹6,00,000 b) ₹ 60,000 c) ₹8,00,000 d) ₹80,000

18. If $\sum y = 484$, $\sum x = 1$, $\sum x^2 = 51$, $\sum xy = 120$ and $n=7$, then the values a and b in the equation of the straight line trend is _____.

- a) $a = 71.1, b = 1.2$ b) $a = 69.14, b = 2.35$ c) $a = 70, b = 2$ d) $a = 68, b = 3$

ASSERTION-REASON BASED QUESTIONS

(Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below) [1x2=2]

- (A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19. Assertion (A): The effective rate which is equivalent to nominal rate of 8% p.a compounded half-yearly is 8.16%.

Reason (R): If nominal rate is r% compounded k times in a year, then the effective rate of

$$\text{interest } r_e \text{ is given by } r_e = \left[1 + \frac{r}{100k} \right]^k - 1.$$

20. Assertion (A): For a square matrix of order 3, if $|adj A| = 49$, then $|A| = \pm 7$.

Reason (R): For a square matrix of order n , $|adj A| = |A|^n$.

Section B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

[2x5=10]

21. A) Solve the inequality: $\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in R$.

OR

B) A and B can cover a 200 m race in 22 seconds and 25 seconds respectively. When A finished the race, then B is at what distance from the finishing line?

22. A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 hours and 10 hours respectively. If all the three pipes are opened simultaneously, how long will it take to fill the tank?

23. In what ratio does a grocer mix two varieties of pulses worth ₹85 per kg and ₹100 per kg respectively so as to get a mixture worth ₹92 per kg?

24. A) It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains

- no defective screw.
- One defective screw

[Given: $e^{-2} = 0.14$]

OR

B) If the standard deviation of a Poisson Variable X is $\sqrt{3}$, then find $P(X > 1)$.
[Use: $e^{-3} = 0.05$]

25. Find the values of $2a + 3b - c$, if $\begin{bmatrix} 0 & -1 & 28 \\ a-8 & 0 & 3b \\ -c+2 & -2 & 0 \end{bmatrix}$ is skew-symmetric.

Section C

(This section comprises of 6 short answer (SA) type questions of 3 marks each)

[3x6=18]

26. A) The incidence of an occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers, 4 or more will catch the disease? Also find mean and variance.

OR

B) The lifetime of an item produced by a machine has a normal distribution with a mean of 12 months and a standard deviation of 2 months. Find the probability that an item produced by this machine will last

i) less than 7 months

ii) between 7 and 14 months. [Given $P(Z < \frac{5}{2}) = 0.9938$ and $P(Z < 1) = 0.8413$]

27. A) A factory produces PVC pipes with mean inner diameter of 4cm. A sample of 17 pipes gives a mean inner diameter of 4.02cm and a standard deviation of 0.09cm. Is the difference in the value of means significant or not? Test at 5% level of significance.
[Use $t_{16}(0.05) = 2.120$]

OR

- B) For the following data, examine if the means of two random samples differ significantly at 5% level of significance. [use $t_{25}(0.05) = 2.060$]

	size	mean	Standard deviation
Sample I	10	200	10
Sample II	17	220	8

28. Find the remainder when 17^{113} is divided by 3.

29. Formulate the following problem as an LPP:

A firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.

30. The investment of ₹20000 in the mutual fund in 2015 increased to ₹32000 in year 2020, then find CAGR(compound Annual Growth rate). [given $(1.6)^{1/5} = 1.098$]

31. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values "x" has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1 & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x) & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{otherwise} \end{cases}$$

- i) Find the value of k
ii) Find E(X).

SECTION D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

[5x4=20]

32. A) Consider the following data

Year	Production
2003	137
2004	140
2005	34
2006	137
2007	151
2008	121
2009	124
2010	159
2011	157
2012	169
2013	172
2014	150

Calculate the 3 yearly moving average and show these moving averages on a graph.

OR

- B) Fit a straight-line trend by using the method of least squares for the following data and calculate the trend values.

Year	Production (In tonnes)
1962	2
1963	4
1964	3
1965	4
1966	4
1967	2
1968	4
1969	9
1970	7
1971	10
1972	8

33.

A) Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify $AB=6I$.

Use this to solve the system

$$x - y = 3$$

$$2x + 3y + 4z = 17$$

$$y + 2z = 7$$

OR

- B) The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some other (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awards is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category.

34.

A) Find $\int \frac{3x-2}{(x+1)(x-2)^2} dx$.

- B) A tyre manufacturer estimates that (thousand) radial tyres will be purchased i.e. demanded by whole sales when price is $p = D(x) = 90 - \frac{x^2}{10}$ thousand rupees per tyre and the same number of tyres will be supplied when the price is $p = S(x) = \frac{1}{5}x^2 + x + 50$ thousand rupee per tyre.

- i) Find the equilibrium price and the quantity supplied and demanded at that price.
ii) Determine the Consumer surplus at the equilibrium price.

35. In 10 years, a machine costing ₹6,00,000 will have salvage value of ₹25,000. A new Machinery at that time (i.e after 10 years) is expected to cost ₹7,00,000. In order to provide funds for the difference between the replacement cost and the salvage cost, a sinking fund is to set up into which equal payments are placed at the end of each 6 months. If the firm earns interest at the rate of 5% p.a compounded semi-annually: [Given $(1.025)^{20} = 1.637$]

- i) How much should each payment be?
ii) Find the amount of Annual Depreciation of the machinery's value over 10 years.
iii) Find the rate of depreciation (under straight line method).

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each)

[4x3=12]

36. Case Study 1:

A fighter-jet of the enemy is flying along the parabolic path $4y = x^2$. A soldier is located at the point (0,5) and is aiming to shoot down the jet when it is nearest to him.



Based on the above, answer the following questions:

- i) Let (x,y) be the position of the jet at any instant. Express the distance between the soldier and the jet as the function of x . [1M]
- ii) Taking $S = [f(x)]^2$, find $\frac{dS}{dx}$. [1M]
- iii) (a) What will be the position of the jet when the soldier shoots it down? [2M]

OR

- iii) (b) What will be the distance between the soldier and the jet at the instant when he shoots it down? [2M]

37. Case Study 2:

EQUATED MONTHLY INSTALMENTS (EMI): -

Each instalment can be considered as consisting of two parts:

- (i) Interest on the outstanding loan
(ii) Repayment of part of the loan.

Methods of calculation of EMI or Instalment: -

EMI or Installment can be calculated by two methods:

1. Flat Rate Method
2. Reducing-balance method or Amortization of Loan.

In 2020, Mr. Anand took a home loan of ₹30,00,000 from State Bank of India at 7.5% compounded monthly for 20 years.

Based on the above information, answer the following questions:

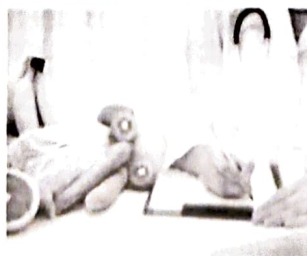
- i) Find the number of payments and rate of interest per month. [1M]
- ii) (a) What are the monthly payments for instalments using the reducing balance method? [2M]

OR

- ii) (b) What are the monthly payments of instalments using the flat rate method? [2M]
- iii) What is the total interest payment made in the process applied to calculate EMI in the above part (37 (ii))? [Use $(1.00625)^{240} = 4.4608$] [1M]

38. Case Study 3:

(Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol.



Based on the above information answer the following questions:

- i) State the objective function and the constraints for the given LPP. [2M]
- ii) Solve graphically to find how many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A? [2M]