NAVODAYA VIDYALAYA SAMITI

SHILLONG REGION

SAADHANA PROGRAM

2024-2025

PRACTICE PAPERS

for

CLASS XII

SUBJECT: MATHEMATICS

FOREWORD

It gives me immense pleasure to introduce this practice paper book under Sadhana program, which has been meticulously prepared by a team of experienced teachers under the able supervision of the Principals, with the aim of enhancing students' performance in the upcoming CBSE Class X and Class XII examinations. As we navigate through an era of academic challenges and evolving pedagogical approaches, it becomes imperative that we equip our students with the right tools and resources to excel in their exams.

This book is a culmination of hard work and dedication by teachers who have vast experience in guiding students through the intricacies of the CBSE curriculum. The practice papers included here are designed to provide a comprehensive revision of key concepts and an opportunity for students to familiarize themselves with the format and style of questions they are likely to encounter in the board examinations.

By engaging with these practice papers, students will not only gain confidence in their ability to tackle exam questions but will also develop critical thinking and time-management skills, which are essential for success. The papers cover a wide range of topics, ensuring a holistic preparation strategy for all subjects. The feedback and analysis sections that follow each paper will also serve as an invaluable resource for identifying areas of improvement.

As a Deputy Commissioner of NVS Shillong, I encourage all students to make the most of this resource, taking a disciplined and focused approach to their studies. The efforts put forth by the teachers in creating this book reflect a shared commitment to the academic growth of every student, and I am confident that with consistent practice and dedication, our students will emerge victorious in their forthcoming examinations.

I extend my best wishes to all students and teachers, and I hope that this practice paper book proves to be a stepping stone towards their success.

Aditya Prakash Singh
Deputy Commissioner
Navodaya Vidyalaya Samiti,
Shillong

Development Committee

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7	Sh. T.V.S.Prakasa Rao Assistant Commissioner (Acad)	Overall Guidance

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Unit Wise Practice Questions Paper

NAVODAYA VIDYALAYA SAMITI – RO SHILLONG
CLASS: XII
SUBJECT: MATHEMATICS (041)
SESSION:2024-25
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: RELATION-FUNCTION, MATRICES, DETERMINANTS)

Time: 3 Hours Max. Marks: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION A [1×20 = 20]

(This section comprises of Multiple –choice questions (MCQ) of 1 mark each.)

Select the correct option (Question 1 - Question 18):

Q1. A function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2 + x^2$ is

(A) not one-one (B) one-one (C) not onto (D) neither one-one nor onto

Q2. The area of the triangle whose vertices are $(3,8), (-4,2)$ and $(5,1)$ is

(A) 60 (B) 61 (C)$61/2$ (D) 30

Q3. How many matrices are possible of order 2×2 with the numbers 0, 1 and 2 such that each element of the matrix is non-zero

(A)3^4 (B)2^4 (C)4^2 (D)4^3

Q4. If a relation R on a set $\{1,2,3\}$ be defined by $R = \{(1,2)\}$ then R is

(A) Reflexive (B) Transitive(C) Symmetric (D) None of these

Q5. The value of the determinant $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ is

(A) 1 (B) -1(C) 2(D) 0

Q6. If A is a skew matrix of odd order n then

(A) $|A| = 0$ (B)$|A| = -1$ (C) $|A| = |A'|$ (D)None of These

Q7. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer

(A) $(2,4) \in \mathbb{R}$ (B)$(3,8) \in \mathbb{R}$ (C)$(6,8) \in \mathbb{R}$ (D)$(8,7) \in \mathbb{R}$

Q8. If the diagonal elements of a diagonal matrix are all equal then the matrix is called

(A) Row Matrix (B) Scalar Matrix (C) Rectangular Matrix (D) None of these

Q9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -5 & 6 \end{bmatrix}$ then det(A) will be

(A) 2 (B) 0 (C) -2 (D) doesn't exist

Q10. If $f: \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^3 + 3$ then $f^{-1}(x)$ is equal to

(A) $x^{1/3} - 3$ (B)$x^{1/3} + 3$ (C)$(x - 3)^{1/3}$ (D)$x + 3^{1/3}$

Q11. If A is a square matrix such that $A^2 = I$ then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(A)A (B)$I - A$ (C)$I + A$ (D)$3A$

Q12. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$ then

(A)$fog(x) = 2x$ (B)$fog(x) = 8x$ (C)$gof(x) = 2x^{\frac{1}{3}}$ (D)$gof(x) = x^{1/3}$

Q13. Matrices A and B will be inverse of each other if and only if

(A)$AB = BA$ (B)$AB = BA = 0$ (C)$AB = 0, BA = I$ (D)$AB = BA = I$

Q14. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to

(A) 6 (B)± 6 (C)-6 (D) 0

Q15. If $A = \{a, b, c, d\}$ and $f = \{(a, b)(b, d)(c, a)(d, c)\}$ then f^{-1} is

(A) $\{(a, b)(d, b)(a, c)(c, d)\}$ (B)$\{(b, a)(d, b)(a, c)(c, d)\}$

(C) $\{(a, b)(b, d)(c, a)(d, c)\}$ (D) does not exist

Q16. Let L is the collection of straight lines in a plane and a relation R defined as $R = \{(L_1, L_2): L_1 \parallel L_2\}$ then relation R is

(A) reflexive only (B) symmetric only (C) transitive only (D) Equivalence relation

Q17. Minor of an element of a determinant order $n (n \geq 2)$ is a determinant of order

(A)n (B)$n - 1$ (C)$n - 2$ (D)$n - 3$

Q18. If A and B are two square matrices of the same order and $AB = 3I$ then A^{-1} is equal to

(A) $3B$ (B) $\frac{1}{3}B$ (C) $3B^{-1}$ (D) $\frac{1}{3}B^{-1}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of(A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Q19. **Assertion:** If A is a skew symmetric matrix then A^2 is also a skew symmetric matrix.

Reason: If A is a skew symmetric matrix then $A' = -A$.

Q20. **Assertion:** Let L be the collection of all lines in a plane and R is a relation on L defined as $R = \{(L_1, L_2): L_1 \perp L_2\}$.

Reason: A relation R is said to be symmetric if $(a, b) \in \mathbb{R} \Rightarrow (b, a) \in \mathbb{R}$.

SECTION B **[2× 5 = 10]**

(This section comprises of 5 very short answer (VSA) type-questions of 2 marks each.)

Q21. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y\ is\ divisible\ by\ x\}$ is symmetric or transitive.

Q22. If A and B are symmetric matrices prove that $AB - BA$ is a skew symmetric matrix.

Q23. If $A = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$ then show that $|2A| = 8|A|$.

Q24. If $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{x} \forall x \in \mathbb{R}$. Then check whether it is a function or not.

Q25. Find the value of $x - y$ if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

SECTION – C **[3× 6 = 18]**

(This section comprises of 6 short answer (SA) type questions of 3 marks each)

Q26. Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b): a \leq b\}$ is reflexive and transitive but not symmetric.

Q27. If the area of the triangle with vertices $A(x, 4)$ $B(-2,4)$and $C(2, -6)$ is 35 sq units. Find the value of x.

Q28. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Q29. Prove that the greatest integer function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = [x]$ is neither one-one nor onto.

Q30. Using co-factors of elements of 3rd column evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Q31. Construct a matrix of order 3×2 whose elements are given by $a_{ij} = e^{ix} \sin jx$.

SECTION – D **[5× 4 = 20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q32. Given a non-empty set X define the relation R in $P(X)$ as follows:
For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive and transitive but not symmetric.

Q33. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ then show that $A^3 - 4A^2 - 3A + 11I = 0$.

Q34. Evaluate the product AB where-

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Hence solve the system of linear equations:

$$x - y = 3$$

$$2x + 3y + 4z = 17$$

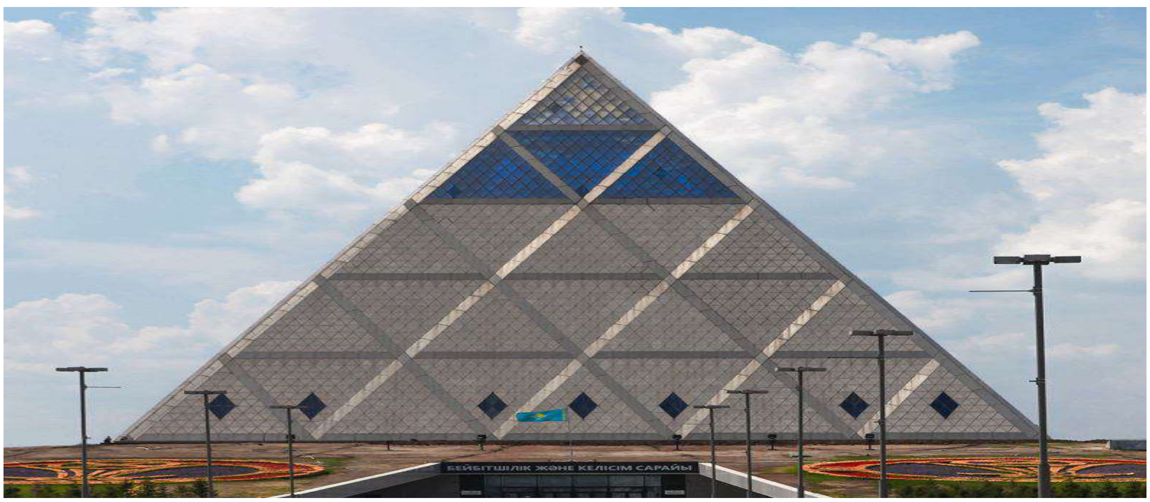
$$y + 2z = 7$$

Q35. Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one.

SECTION – E **[4× 3 = 12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts)

Q36. The Palace of Peace and Reconciliation, also known as the Pyramid of peace and Accord is a 62-meterhigh Pyramid in Mursultan, the capital of Kazakhstan that serves as a non-denominational national spiritual centre and an event venue. It is designed by faster and partners with a stained glass apex. It has 25 smaller equilateral triangles as shown in the figure.



(i) If the vertices of one triangle are $(0, 0), (3, \sqrt{3})$ *and* $(3, -\sqrt{3})$ then find the area. [1 Mark]

(ii) Find the area of face of the Pyramid. [1 Mark]

(iii) Find the length of an altitude of a smaller equilateral triangle. [2 Mark]

Q37. To promote the usage of house toilets in villages especially for women, an organization tried to generate awareness among the villagers through (i) house calls (ii) letters and (iii)

announcements

The cost for each mode per attempt is (i) Rs 50 (ii) Rs 20 (iii)Rs40 respectively. The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the chance of making of toilets corresponding to one attempt of given modes is:
(i) 2% (ii) 4% (iii) 20%
Let A, B, C be the cost incurred by organization in three villages respectively. Based on the above information answer the following questions
(A) Form a required matrix on the basis of the given information. [1 Mark]

(B) Form a matrix, related to the number of toilets expected in villagers X, Y, Z after the promotion campaign. [1 Mark]

(C) What is total amount spent by the organization in all three villages X, Y and Z [2 Marks]

Q38. Maths-teacher started the lesson Relations and Functions in Class XI. He explained the following topics:
Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., (a, b) = (c, d) ⇒ a = c and b = d.

Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A x B is the set of all ordered pairs of elements from sets A and B.
In symbolic form, it can be written as $A \times B = \{(a, b) : a \in A, b \in B\}$.
Based on the above topics, answer the following questions.
(i) If $(a - 3, b + 7) = (3, 7)$, then find the value of a and b. [1 Mark]

(ii) If $(x + 6, y - 2) = (0, 6)$, then find the value of x and y. [1 Mark]

(iii) If $(x + 2, 4) = (5, 2x + y)$, then find the value of x and y. [1 Mark]

(iv) Find x and y, if $(x + 3, 5) = (6, 2x + y)$. [1 Mark]

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UNIT WISE PRACTICE QUESTIONS PAPER
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Time: 3 Hours | Max. Marks: 80

Marking Scheme

Q.No.	Ans.	Hints/Solution
1.	(D)	$f(x) = 2 + x^2$ For one-one; let $f(x) = f(y) \Rightarrow 2 + x^2 = 2 + y^2 \Rightarrow x = \pm y$ So not one-one. For onto; there are so many elements in co-domain like -3, -4 etc. which are not mapped with any element of domain so not onto.
2.	(C)	$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix} = \frac{61}{2}$ sq units
3.	(A)	For all the 4 elements of the matrix there are three choices so 3^4.
4.	(B)	Reflexive: $(1,1) \notin R$ so not reflexive. Symmetric: $(1,2) \in R$ but $(2,1) \notin R$ so not symmetric. Transitive: It is not violating the rule of being transitive so transitive.
5.	(A)	$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x^2 - (x^2 - 1) = 1$
6.	(A)	Since A is a skew matrix so $A' = -A \Rightarrow \lvert A' \rvert = \lvert -A \rvert \Rightarrow \lvert A' \rvert = (-1)^n \lvert A \rvert$ $\lvert A \rvert = (-1)^n \lvert A \rvert \Rightarrow$ Since n is odd so $\lvert A \rvert = -\lvert A \rvert \Rightarrow 2\lvert A \rvert = 0 \Rightarrow \lvert A \rvert = 0$
7.	(C)	Only $(a, b) \in R$ If $a = b - 2$ and $b > 6$ so $(6,8) \in R$.
8.	(B)	If all the diagonal elements of a diagonal matrix are equal then it is called scalar matrix.
9.	(D)	Since the given matrix is not a square matrix so det(A) doesn't exist.
10.	(C)	Let $f^{-1}(x) = y \Rightarrow x = f(y) \Rightarrow x = y^3 + 3 \Rightarrow x - 3 = y^3 \Rightarrow y = (x-3)^{1/3}$
11.	(A)	$(A - I)^3 + (A + I)^3 - 7A$ $A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$ $2A^3 + 6AI^2 - 7A$ $2A + 6A - 7A$ A
12.	(B)	$f(x) = 8x^3$ and $g(x) = x^{1/3}$ $fog(x) = f(g(x)) = 8x$
13.	(D)	Two matrices are inverse of each other iff$AB = BA = I$.
14.	(B)	$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ $x^2 - 36 = 36 - 36$ $x = \pm 6$
15.	(B)	$A = \{a, b, c, d\}$ $f = \{(a, b)(b, d)(c, a)(d, c)\}$ $f^{-1} = \{(b, a)(d, b)(a, c)(c, d)\}$

16.	(D)	Reflexive: Since each line parallel to itself i.e. $(L, L) \in R$ so reflexive. Symmetric: Let $(L_1, L_2) \in R \Rightarrow L_1 is$ parallel to $L_2 \Rightarrow L_2$ is parallel to $L_1 \Rightarrow (L_2, L_1) \in R$ So symmetric. Transitive: Let $(L_1, L_2) \& (L_2, L_3) \in R \Rightarrow L_1 \parallel L_2 \& L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3 \Rightarrow (L_1, L_3) \in R$ So transitive So R is an equivalence relation.
17.	(B)	Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n - 1$.
18.	(B)	$AB = 3I$ $A^{-1} AB = 3A^{-1}I$ $B = 3A^{-1}$ $A^{-1} = \frac{B}{3}$
19.	(C)	(A) is incorrect and (R) is correct.
20.	(D)	(A) is false, but (R) is true.

SECTION – B

21.	$A = \{1, 2, 3, 4, 5, 6\}$ $R = \{(x, y) : y \textit{ is divisible by } x\}$ <u>Symmetric</u>: $(1,2) \in R$ but $(2,1) \notin R$ because 1 is not divisible by 2. So not symmetric. <u>Transitive</u>: let $(x, y) \in R \Rightarrow y$ is divisible by $x \Rightarrow y = \lambda x$ let $(y, z) \in R \Rightarrow z$ is divisible by $y \Rightarrow z = \mu y \Rightarrow z = \mu . \lambda . x \Rightarrow z$ is divisible by x $\Rightarrow (x, z) \in R \Rightarrow$ So transitive.	1 1
22.	Given A and B are Symmetric matrices so $A' = A$ and $B' = B$ Now $(AB - BA)' \Rightarrow (AB)' - (BA)'$ $\Rightarrow B'A' - A'B'$ $\Rightarrow BA - AB$ $\Rightarrow -(AB - BA)$ So $AB - BA$ is a skew symmetric matrix.	½ ½ ½ ½
23.	$A = \begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$ $\lvert A \rvert = 6(4) - 0 + 0 = 24$ $\lvert 2A \rvert = \begin{vmatrix} 12 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{vmatrix}$ $\lvert 2A \rvert = 12(16) - 0 + 0 = 192 = 8 \times 24 = 8\lvert A \rvert$	½ 1 ½
24.	Given that $f(x) = \frac{1}{x} \forall\, x \in \mathbb{R}$ But $0 \in \mathbb{R}$ for which $f(0)$ is not defined Hence $f(x)$ is not a function.	2

25.	$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ $\begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ $2+y=5\ \&\ 2x+2=8$ $y=3\ \&\ x=3$ $x-y=0$	1 1/2 1/2

SECTION – C

26.	Clearly $a \leq a\ \forall\ a \in \mathbb{R} \Rightarrow (a,a) \in\ \mathbb{R} \Rightarrow$ So reflexive. Let $(a,b)\&(b,c) \in\ \mathbb{R} \Rightarrow a \leq b\ \&\ b \leq c \Rightarrow a \leq c \Rightarrow (a,c) \in\ \mathbb{R} \Rightarrow$ So transitive But not symmetric because $(1,2) \in \mathbb{R}$ but $(2,1) \notin \mathbb{R}$	1 1 1
27.	$A(x,4)\ B(-2,4)$ and $C(2,-6)$ $\Delta = \frac{1}{2}\begin{vmatrix}x & 4 & 1\\-2 & 4 & 1\\2 & -6 & 1\end{vmatrix} = 5x+10$ $5x+10 = \pm 35$ $x = 5$ or -9	2 1
28.	$AB = \begin{bmatrix}2 & 3\\1 & -4\end{bmatrix}\begin{bmatrix}1 & -2\\-1 & 3\end{bmatrix} = \begin{bmatrix}-1 & 5\\5 & -14\end{bmatrix}$ $(AB)^{-1} = \begin{bmatrix}-1 & 5\\5 & -14\end{bmatrix}^{-1} = \frac{1}{11}\begin{bmatrix}-14 & -5\\-5 & -1\end{bmatrix}$ Further $\lvert A\rvert = -11$ and $\lvert B\rvert = 1$ $A^{-1} = -\frac{1}{11}\begin{bmatrix}-4 & -3\\-1 & 2\end{bmatrix}$ and $B^{-1} = \begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$ $B^{-1}A^{-1} = -\frac{1}{11}\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix} \times \begin{bmatrix}-4 & -3\\-1 & 2\end{bmatrix} = -\frac{1}{11}\begin{bmatrix}14 & 5\\5 & 1\end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1}$	1 1 1
29.	$f: \mathbb{R} \to \mathbb{R}$ $f(x) = [x]$ $\exists\ 1, 1.6\ \in\ \mathbb{R}$(domain) For which $f(1) = (1.6) = 1$ So not one-one There are so many elements in co-domain (like 2.5,7.3 etc.) which are not image of any element of domain so it is not onto	1½ 1½
30.	$\Delta = \begin{vmatrix}1 & x & yz\\1 & y & zx\\1 & z & xy\end{vmatrix}$ Co-factors of elements of 3rd column are: $A_{13} = z-y$; $A_{23} = -(z-x)$; $A_{33} = -(x-y)$ $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$ $\Delta = yz(z-y) - zx(z-x) - xy(x-y)$ $\Delta = (x-y)(y-z)(z-x)$	1 2
31.	$a_{ij} = e^{ix}\sin jx$	3

	$A=\begin{bmatrix} e^x\sin x & e^x\sin 2x \\ e^{2x}\sin x & e^{2x}\sin 2x \\ e^{3x}\sin x & e^{3x}\sin 2x \end{bmatrix}$	

SECTION – D

32.	Let $A\in P(X)$ then $A\subset A\Rightarrow (A,A)\in R\Rightarrow$ So R is reflexive. Let $(P,X)\in P(X)$ such that $P\subset X$ Hence $(P,X)\in R$ but $X\not\subset P\Rightarrow (P,X)\notin R$ So R is not Symmetric. Let $A,B,C\in P(X)$ such that $(A,B)(B,C)\in R\Rightarrow A\subset B,\ B\subset C\Rightarrow A\subset C\Rightarrow (A,C)\in R$ Hence R is transitive.	1 2 2
33.	$A=\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}$ $A^2=A\times A=\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}\times\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}=\begin{bmatrix}9&7&5\\1&4&1\\8&9&9\end{bmatrix}$ $A^3=\begin{bmatrix}9&7&5\\1&4&1\\8&9&9\end{bmatrix}\times\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}=\begin{bmatrix}28&37&26\\10&5&1\\35&42&34\end{bmatrix}$ Now $A^3-4A^2-3A+11I$ $\begin{bmatrix}28&37&26\\10&5&1\\35&42&34\end{bmatrix}-4\begin{bmatrix}9&7&5\\1&4&1\\8&9&9\end{bmatrix}-3\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	 2 2 1
34.	$A=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$ $\|A\|=1$ $Adj\ A=\begin{bmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{bmatrix}$ $A^{-1}=\begin{bmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{bmatrix}$ The given system can be written as $\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}10\\8\\7\end{bmatrix}$ $A'X=B$ $X=(A')^{-1}B=(A^{-1})'B$ $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-3&2&2\\-2&1&1\\-4&2&3\end{bmatrix}\begin{bmatrix}10\\8\\7\end{bmatrix}=\begin{bmatrix}0\\-5\\-3\end{bmatrix}$ $x=0, y=-5\ \&\ z=-3$	½ 1½ ½ 1 1½

35.	$$f(x)=\frac{4x+3}{6x-4}$$ Let $f(x)=f(y)\Rightarrow\frac{4x+3}{6x-4}=\frac{4y+3}{6y-4}\Rightarrow(4x+3)(6y-4)=(4y+3)(6x-4)$$ $$24xy+18y-16x-12=24xy+18x-16y-12$$ $$34x=34y$$ $$x=y$$ So f is One-one function.	2 2 1

SECTION – E

36.	(i) Required Area= $\left\|\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 3 & \sqrt{3} & 1\\ 3 & -\sqrt{3} & 1\end{vmatrix}\right\| = \frac{6\sqrt{3}}{2}=3\sqrt{3}$ sq. units (ii) Since, a face of the Pyramid consists of 25 smaller equilateral triangles. ∴ Area of a face of the Pyramid = $25\times 3\sqrt{3}=75\sqrt{3}$ sq. units (iii) Area of equilateral triangle $=\frac{\sqrt{3}}{4}(side)^2$ $3\sqrt{3}=\frac{\sqrt{3}}{4}(side)^2\Rightarrow$ side $=2\sqrt{3}$ units Let h be the length of the altitude of a smaller equilateral triangle $$\frac{1}{2}\times base\times h=3\sqrt{3}$$ $$\frac{1}{2}\times 2\sqrt{3}\times h=3\sqrt{3}$$ $$h=3 \text{ units}$$	1 1 1 1
37.	(A) Rs A, Rs B and Rs C are the cost incurred by the organization for villages X, Y, Z respectively, therefore matrix equation will be $$\begin{bmatrix}400 & 300 & 100\\ 300 & 250 & 75\\ 500 & 400 & 150\end{bmatrix}\begin{bmatrix}50\\ 20\\ 40\end{bmatrix}=\begin{bmatrix}A\\ B\\ C\end{bmatrix}$$ (B) Let number of toilets expected in villagers X,Y,Z be x,y,z respectively Therefore required matrix is $$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}400 & 300 & 100\\ 300 & 250 & 75\\ 500 & 400 & 150\end{bmatrix}\begin{bmatrix}2\\ 4\\ 20\end{bmatrix}$$ (C) $$\begin{bmatrix}A\\ B\\ C\end{bmatrix}=\begin{bmatrix}400 & 300 & 100\\ 300 & 250 & 75\\ 500 & 400 & 150\end{bmatrix}\begin{bmatrix}50\\ 20\\ 40\end{bmatrix}=\begin{bmatrix}30000\\ 23000\\ 39000\end{bmatrix}$$ Total money spent = 30000 + 23000 + 39000 = 92000 Rs	1 1 2
38.	(i) We know that, two ordered pairs are equal, if their corresponding elements are equal. $(a-3, b+7)=(3,7)$ $\Rightarrow a-3=3$ and $b+7=7$ [equatingcorresponding elements] ⇒ a = 3 + 3 and b = 7 – 7 $\Rightarrow a=6$ and $b=0$ (ii) $(x+6, y-2)=(0,6)$	1

	$\Rightarrow x + 6 = 0$ $\Rightarrow x = -6$ and $y - 2 = 6$ $\Rightarrow y = 6 + 2 = 8$ (iii) $(x + 2, 4) = (5, 2x + y)$ $\Rightarrow x + 2 = 5$ $\Rightarrow x = 5 - 2 = 3$ *and* $4 = 2x + y$ $\Rightarrow 4 = 2 \times 3 + y$ $\Rightarrow y = 4 - 6 = -2$ (iv) $x + 3 = 6,$ $2x + y = 5$ $\Rightarrow x = 3, y = 1$	1 1 1

NAVODAYA VIDYALAYA SAMITI – RO SHILLONG
CLASS: XII
SUBJECT: MATHEMATICS (041)
SESSION:2024-25
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: Continuity and differentiability, Application of derivatives)

Time: 3 Hours **Max. Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION A **[1×20 = 20]**

(This section comprises of Multiple –choice questions (MCQ) of 1 mark each.)

1. If $f(x) = x^2 \sin\frac{1}{x}$ where $x \neq 0$, then the value of the function f at x = 0, so that the function f(x) is continuous at x = 0, is

a) 1 b) – 1 c) 0 d) 2

2. The function f(x) = [x] , where [x] denotes the greatest integer function, is continuous at

a) 4 b) 2 c) 1 d) 1.5

3. The relation between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, & if \quad x \leq 3 \\ bx+3, if & x > 3. \end{cases}$$ continuous at x = 3 is

a) $3a = 2 - 3b$ b) $3a - 3b = 2$ c) $3a + 3b = 2$ d) $3b - 3a = 2$.

4. The value of the constant k so that the function f defined by

$$f(x) = \begin{cases} \frac{1-cos4x}{8x^2}, & x \neq 0, \\ k, & x = 0 \end{cases}$$ continuous at $x = 0$ is

a) 0 b) 1 c) 2 d) 3

5. If $e^x + e^y = e^{x+y}$, then dy/dx

a) e^{y-x} b) $-e^{y-x}$ c) e^{x+y} d) e^{x-y}

6. Derivative of $\sin(\tan^{-1} e^x)$ is

a) $\cos(\tan^{-1} e^x) . e^x$ b) $\frac{e^x \cos(tan^{-1} e^x)}{1+e^{2x}}$ c) $-(\cos(\tan^{-1} e^x) . e^x)/ 1+ x^2$

d) $-\frac{e^x \cos(tan^{-1} e^x)}{1+e^{2x}}$

7. If $y = A \sin x + B \cos x$, then which of the following is correct ?

a) $D^2y + y = 0$ b) $D^2y - y = 0$ c) $D^2y + 2y = 0$ d) $D^2y - 2y = 0$.

8. Derivative of $\sin^2 x$ w r t $e^{\cos x}$ is

a) $-2 \cos x . e^{-\cos x}$ b) $2\cos x\, e^{\cos x}$ c) $2 \sin x\, e^{\cos x}$ d) $-2\sin x\, e^{-\cos x}$

9. If $f(x) = (\sin x)^{\sin x}$, for all $0< x < \pi$, then $f'(x)$ is equal to

a) $(1 - \log(\sin x))(\sin x)^{\sin x} \cos x$ b) $(1 + \log(\sin x))(\sin x)^{\sin x} \cos x$

c) $(1 - \log(\cos x))(\sin x)^{\sin x} \cos x$ d) $(1 + \log(\cos x))(\sin x)^{\sin x} \cos x$

10 If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then $\frac{d^2y}{dx^2}$ is

a) $\cos^3 t/ at$ b) $\sec^3 t/ at$ c) $\sin^3 t / at$ d) $at \sec^3 t$.

11. If $y = tan^{-1} \frac{cosx}{1+sinx}$ then dy/dx =

a) 1 b) 0 c) ½ d) – ½

12. The total revenue in Rupees from the sale of x units of a product is given by $R(x)= 3x^2 +36x +5$. Then the marginal revenue when x = 5 is

a) ₹44 b) ₹66 c)₹ 360 d) ₹88

13. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is

a) 10π b) 12π c) 8π d) 11π

14. The function $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is increasing in

a) $\left[0, \frac{\pi}{3}\right]$ b) $\left[0, \frac{\pi}{4}\right]$ c) $\left[0, \frac{\pi}{6}\right]$ d) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

15. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x – 1$ is strictly decreasing in

a) $[-1 , \propto]$ b) $(-2, -1)$ c) $(- \propto , -2)$ d) $[- 1 ,1]$

16. The absolute maximum and minimum values of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1,5] respectively are

a) 56 and 28 b) 56 and 29 c) 29 and 24 d) 56 and 24

17. Let f have second derivative at c such that $f'(c) = 0$, $f''(c) \geq 0$, then c is a point of

a)local minima b) local maxima c) extreme value of f d) neither maxima nor minima.

18. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

a) two points of local maximum b) two points of local minimum

c) one maxima and one minima d) no maxima or minima.

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each.

Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. Assertion (A): If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of (x/y) is 2.

Reason (R): If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of (x/y) is 1/5 .

20. Assertion (A): Minimum value of (x - 5)(x -7) is -1.

Reason (R): Minimum value of $ax^2 + bx + c$ is $\frac{4ac- \ \ ^2}{4a}$

SECTION B **[2× 5 = 10]**

(This section comprises of 5 very short answer (VSA) type-questions of 2 marks each.)

21. If for $f(x) = \begin{cases} \frac{1-cos4x}{x^2}, if \quad x < 0 \\ a, \quad if \ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}, \quad if \ x > 0 \end{cases}$, f is continuous at x = 0 , find a.

22. Find dy/dx. Where, $xy = e^{x-y}$.

23. Find the intervals in which the function f given by $f(x) = x^2 – 4x + 6$ is

a. increasing b. Decreasing

24. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

25. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate 0.05 cm/s. Find the rate at which its area increasing when radius is 3.2 cm .

SECTION – C **[3× 6 = 18]**

(This section comprises of 6 short answer (SA) type questions of 3 marks each)

26. If x = a(cost + log tan$\frac{t}{2}$) , , y = a sin t, then evaluate d^2y/dx^2 at $t = \frac{\pi}{3}$.

27 .If $x\sqrt{1+y}$ + $y\sqrt{1+x}$ = 0 , for – 1<x<1 , prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

28. Find *dy/dx* of the function $(\cos x)^y = (\cos y)^x$.

29. Find the maximum profit that a company can make , if the profit function is given by p(x)= 41 – 72x - $18x^2$.

30. Prove that the perimeter of a right triangle of given hypotenuse is maximum when the triangle is isosceles.

SECTION – D **[5× 4 = 20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

31. Water is dripping out at a steady rate of 1 cu cm / sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical . When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height , where the vertical angle of the conical vessel is $\frac{\pi}{6}$.

32. Find the area greatest rectangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

33 . Find dy/dx, if y = $e^{sin^2x}(2tan^{-1}\sqrt{\frac{1-x}{1+x}}$) . + $cot^{-1}\{\frac{\sqrt{1+si} \quad +\sqrt{1-sin}}{\sqrt{1+sin} \quad -\sqrt{1-sin}}\}$

34. .Find the intervals in which the function f(x) = sinx + cosx , $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

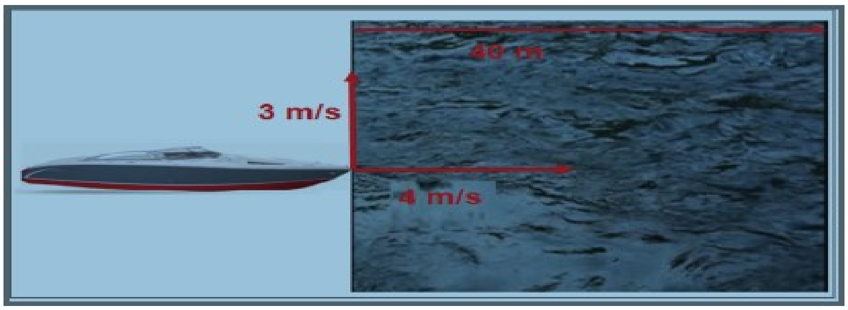
35. An open box with a square base is to be made out of a given quantity of cardboard of area c^2square unit. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic unit.

SECTION – E **[4× 3 = 12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts)

36. **Read the following text carefully and answer the questions that follow:**

Once Ramesh was going to his native place at a village near Agra. From Delhi and Agra he went by flight, In the way, there was a river. Ramesh reached the river by taxi. Then Ramesh used a boat for crossing the river. The boat heads directly across the river 40 m wide at 4 m/s. The current was flowing downstream at 3 m/s.



i. What is the resultant velocity of the boat? (1)

ii. How much time does it take the boat to cross the river? (1)

iii. How far downstream is the boat when it reaches the other side? (2)

OR

If speeds of boat and current were 1.5 m/s and 2.0 m/s then what will be resultant velocity?

(2)

37 **Read the following text carefully and answer the questions that follow:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

i. Find the volume of the open box formed by folding up the cutting each corner with x cm. (1)

ii. Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (1)

iii. Verify that volume of the box is maximum at x = 3 cm by second derivative test? (2)

OR

Find the maximum volume of the box. (2)

38. A mason wants to put a ladder on a wall. It is 5 m long and leaning against a wall. The bottom of the ladder is pulled by the man along the ground away from the wall at the rate of 2m/s.

If x is the distance of the bottom and top of the ladder then

i) write a relation between x and y. (1)

ii) How fast is its height on the wall decreasing ? (1)

iii) Find the rate of decreasing the height on the wall decreases when the foot of the ladder is 4 m away from the wall ? (2)

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Time: 3 Hours **Max. Marks: 80**

Marking Scheme

1. c) 0 , since ,
   $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0, as \sin\left(\frac{1}{x}\right)$ $is\ \ always\ a\ definite\ value\ \ as$ $-1 \le sinx \le 1$
2. d) 1.5, since greatest integer function [x] is discontinuous at all integral values of x . Hence at x = 1.5 is continuous.

3. b) Since the function is continuous at x = 3 , hence
$$\lim_{x\to 3-} f(x) = \lim_{x\to 3+} f(x)$$
$$\lim_{x\to 3-} (ax+1) = \lim_{x\to 3+} (bx+3)$$
$$3a + 1 = 3b + 3$$
$$3a - 3b = 2$$
4. b) 1 . Given , f is continuous at x = 0 , hence
$$\lim_{x\to 0} f(x) = \quad f(0)$$
$$\lim_{x\to 0} \frac{1-cos4x}{8x^2} = k$$
$$\lim_{x\to 0} \frac{2sin^2 2x}{8x^2} = k$$
$$\lim_{x\to 0} (\frac{sin2x}{2x})^2 = k$$
$$K=1$$
5. b) $-e^{y-x}$. Here, $e^x + e^y = e^{x+y}$, differentiating both sides w r t x , we have,
$$e^x + e^y \frac{dy}{dx} = e^{x+y} \left(1 + \frac{dy}{dx} \right)$$
$$(e^y - e^{x+y}) \frac{dy}{dx} = e^{x+y} - e^y$$
$$dy/dx = \frac{e^{x+y}-e^y}{e^y-e^{x+y}} = \frac{(e^x + e^y-e^y}{e^y-e^x-e^y} = - e^{y-x}$$
6. b) . By using chain rule , $\frac{d}{dx}(\sin (\tan^{-1}e^x)) = \frac{e^x \cos (tan^{-1}e^x)}{1+e^{2x}}$

7. a) , y = A sinx + Bcosx, Dy = A cosx – Bsinx ---------(i)

$D^2y = -A\sin x - B\cos x = -y$

$D^2y + y = 0.$

8. a) , Let, $u = \sin^2x$, $v = e^{\cos x}$, $du/dx = 2\sin x\cos x$, $dv/dx = -\sin x\, e^{\cos x}$

$du/dv = -2\cos x / e^{\cos x} = -2\cos x\, e^{-\cos x}$.

9. b) . Here, $y = (\sin x)^{\sin x}$

$\log y = \sin x \log(\sin x)$

Differentiating w r t x : $1/y\, dy/dx = \sin x/\sin x\ .\ \cos x + \log(\sin x)(\cos x)$

$dy/dx = (1 + \log(\sin x))\cos x\, (\sin x)^{\sin x}$

10. b). , Here, $dx/dt = a(-\sin t + t\cos t + \sin t) = at\cos t$

$dy/dt = a(\cos t - \cos t + t\sin t) = at\sin t$.

$dy/dx = dy/dt / dx/dt = \tan t$

Hence, $\frac{d^2y}{dx^2} = \sec^2 t\ dt/dx = \sec^2 t\ .\ 1/at\cos t = \sec^3 t/at$.

11. d) -1/2 , $y = \tan^{-1}\left(\frac{\cos x}{1+\sin}\right) = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{(\sin\frac{x}{2} + \cos\frac{x}{2})^2}\right)$

$= \tan^{-1}\left(\frac{\cos\frac{x}{2} - s\ \frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x)}{2}}\right) = \tan^{-1}\left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \frac{\pi}{4} - \frac{x}{2}$

$dy/dx = -½$

12. b) 66. Here, Marginal Revenue, $dR/dx = 6x + 36$ at $x = 5$,

$dR/dx = 30 + 36 = 66$

13. b) 12π Area of a circle , $A = \pi r^2$,

$dA/dr = 2\pi r$

dA/dr at $r = 6$ is 12π

14. c) $\left[0, \frac{\pi}{6}\right]$

Here, $f(x) = \sin 3x$

$f'(x) = 3\cos 3x$.

Now , $f'(x) = 0$ gives, $\cos 3x = 0$, => $3x = \frac{\pi}{2}$, $\frac{3\pi}{2}$ => $x = \frac{\pi}{6}, \frac{\pi}{2}$

Hence the intervals are, $[0, \frac{\pi}{6}]$ and $[\frac{\pi}{6}, \frac{\pi}{2}]$

Now, $f'(x) > 0$ in $[0, \frac{\pi}{6}]$. Hence $f(x)$ is increasing in $[0, \frac{\pi}{6}]$.

15. b) . $f(x) = 2x^3 + 9x^2 + 12x - 1$

$f'(x) = 6x^2 + 18x + 12$

$f'(x) = 6(x^2 + 3x + 2) = 6(x+2)(x+1)$

$f'(x) = 0 \Rightarrow x = -2, -1$.

Hence the intervals are , $(-\propto, -2)$, $(-2, -1)$, $(-1, \propto)$.
In (-2,-1) , $f'(x)$ = (+ve)(-ve) = - ve, hence decreasing in (-2 , -1)

16. d) . We have $f(x) = 2x^3 - 15x^2 + 36x + 1$
$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$
$= 6(x-3)(x-2)$.
$f'(x) = 0 \Rightarrow x = 2, 3$
Hence, f(2)= 29 , f(3) = 28 , f(1)= 24 , f(5) = 56.
Thus absolute maximum value is 56 and absolute minimum value is 24.
17. a) local minima.
18. c) one maxima and one minima.
19. b) Both A and R correct but R is not the correct explanation of A.
20. a) (a) Both A and R are true and R is the correct explanation of A. Explanation: We have,
$(x-5)(x-7) = x^2 - 12x + 35$ We know that, $ax^2 + bx + c$ has minimum value . Here, a = 1, b = -12 and c = 35 Minimum value of (x - 5)(x - 7) is -1 .
21. Here, f(0) = a,
Left hand limit at x = 0, $\lim_{x \to o-} f(x) = \lim_{x \to o-} \frac{1 - c0s4x}{x^2} = \lim_{x \to o-} \frac{2sin^2 2x}{x^2}$
$= \lim_{x \to o-} 8(\frac{sin2x}{2x})^2 = 8 \text{ x} 1 = 8$.
Thus , a = 8 .
22. Now , $xy = e^{x-y}$
Log xy = x – y,
Logx + logy = x – y
Differentiating w r t x we get, 1/x + 1/y dy/dx = 1 - dy/dx
(1/y + 1) dy/dx = 1 – 1/x
Dy/dx = $\frac{x-1}{y+1}(\frac{y}{x})$
23. $f(x) = x^2 - 4x + 6$, $f'(x) = 2x - 4$
$f'(x) = 0 \Rightarrow x = 2$. Thus the intervals are, $(-\propto, 2)$ *and* $(2, \propto)$

$f'(x) > 0$ in $(2, \propto)$ and $f'(x) < 0$ in $(-\propto, 2)$.
Hence f(x) is increasing in $(2, \propto)$ and f(x) is decreasing in $(-\propto, 2)$.

24. Here, $f(x) = x^3 - 3x^2 + 6x - 100$. $f'(x) = 3x^2 - 6x + 6 = 3(x^2 - 2x + 2)$
$= 3\{(x-1)^2 + 1\} > 0$ for all $x \in R$
Thus f(x) is increasing function.

25. Let r be the radius of the given disc and A be its area. Then $A = \pi r^2$
dA/dt = $2\pi r$ dr/dt.
Given, dr/dt = 0.05 cm/s,

Thus rate of change of area when r = 3.2 cm is dA/dt = $2\pi . 3.2(0.05)$
$= 0.320\pi\ cm^2$/s.

26. X =a (cost + log tan$\frac{t}{2}$) => dx/dt = a(- sint + ½ $\frac{sec^2\frac{t}{2}}{tan\frac{t}{2}}$) = a (- sint + 1/sint) = a cott . cost

Y = a sint => dy/dt = a cost
dy/dx = (dy/dt)/(dx/dt) = a cost/ (a cot t . cost) = tant
$d^2y/dx^2 = sec^2t\ dt/dx. = sec^2t . \frac{1}{a\cot t .cost}$ =1/a . sec^4t .sint
When, t = $\frac{\pi}{3}$, $d^2y/dx^2 = 1/a\ .\ 2^4 . \sqrt{3}/2 = \frac{8\sqrt{3}}{a}$

27. Here , $x\sqrt{1+y} + y\sqrt{1+x} = 0$
$x\sqrt{1+y} = - y\sqrt{1+x}$
squaring both sides, $x^2(1+y) = y^2(1+x)$
⇨ $X^2 - y^2 + x^2 y - y^2x = 0$
⇨ (x-y)(x+y +xy) = 0
⇨ X=y or x + y +xy = 0, here $x \neq y$.
⇨ X+y +xy = 0
⇨ Y(1+x) = - x
⇨ Y = $\frac{-x}{1+x}$
⇨ Differentiating w r t x , dy/dx = - $\frac{1}{(1+x)^2}$
⇨

28. Here, $(cosx)^y = (cosy)^x$
⇨ Taking log on both sides, y log cosx = x log cosy
⇨ Differentiating both sides wrt x , y.$\frac{-sinx}{cosx}$ + log cosx dy/dx = x$\frac{-siny}{cosy}\frac{dy}{dx}$ + log cosy
⇨ dy/dx (log cosx + x tany) = y tanx + log cosy
⇨ dy/dx = $\frac{ytanx+\log co}{logcosx+x\ tany}$

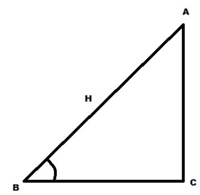
29. Here ,profit function is $p(x)= 41 - 72x - 18x^2$
⇨ $p^{/}(x) = - 72 - 36x$,
⇨ $p^{//}(x) = - 36$

For extreme values of P(x) , $p^{/}(x) = 0$, => - 72 – 36x = 0

x = - 2

At x = -2 , $p^{//}(2) = - 36 < 0$, hence p(x) is maximum at x = - 2 , Hence max profit is P(-2) = 41 + 144 – 72 = 113 .

30.



Let H be the hypotenuse AC and θ be the angle between the hypotenuse and the base BC of the right angles triangle ABC .

Then BC= H cos θ , AC = H sin θ

- ⇨ P= Perimeter of the right triangle
- ⇨ P = H + H cos θ + H sin θ

For extreme value of P , dP/d θ = 0 => H(- sin θ + cos θ) = 0

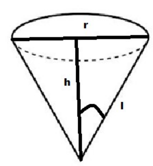
- ⇨ $\sin\theta = cos\theta$
- ⇨ tan θ = 1
- ⇨ $\theta = \frac{\pi}{4}$

Now, $d^2P/d\theta^2$ = - H cos θ - H sin θ and is – ve for $\theta = \frac{\pi}{4}$.

Hence, P is maximum at $\theta = \frac{\pi}{4}$.

At , $\theta = \frac{\pi}{4}$, BC = AC = H/$\sqrt{2}$, Thus triangle ABC is isosceles triangle.

31.



Given that dv/dt = 1 cm^3/s, where v is the volume of water in the conical vessel..

From the fig, l = 4cm , h = lcos $\frac{\pi}{6} = \frac{\sqrt{3}}{2}l$, and r = $l sin\frac{\pi}{6} = \frac{l}{2}$

Now, v = $\frac{1}{3}\pi r^2 h = \frac{\pi}{3}\frac{l^2}{4}\frac{\sqrt{3}}{2}l = \frac{\sqrt{3}\,\pi}{24}l^3$

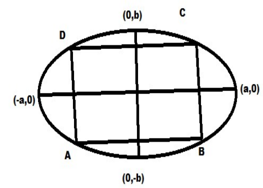
- ⇨ Dv/dl = $\frac{3\frac{\sqrt{3}\,\pi}{24}l^2 dl}{dt} = \frac{\sqrt{3}\,\pi}{8}l^2\frac{dl}{dt}$

But, dv/dt = 1 cm^3/s when l = 4 cm,

Hence, 1 = $\frac{\sqrt{3}\,\pi}{8}4^2\frac{dl}{dt}$ => $\frac{dl}{dt} = \frac{1}{2\sqrt{3}\pi} cm/s$

Thus rate of decrease of slant height , $\frac{dl}{dt} = \frac{1}{2\sqrt{3}\pi} cm/s$

32.



Let ABCD be the rectangle of maximum area of sides AB = 2x and BC = 2y, where C(x,y) be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Now area of the rectangle A = 4xy. Or, $A^2 = 16\, x^2\, y^2$ = S(say)

$S = 16\, x^2\, y^2 = 16x^2 \left(1 - \frac{x^2}{a^2}\right)b^2 = 16\,\frac{b^2}{a^2}(a^2x^2 - x^4)$

$ds/dx = 16\,\frac{b^2}{a^2}(2a^2x - 4x^3)$

Now , ds/dx = 0 => $x = \frac{a}{\sqrt{2}}$ $\quad and \quad y = \frac{b}{\sqrt{2}}$

$d^2s/dx^2 = 16\,\frac{b^2}{a^2}(2a^2 - 12x^2)$. For , $x = \frac{a}{\sqrt{2}}$, $d^2s/dx^2 = 16\,\frac{b^2}{a^2}(2a^2 - 6a^2) = 16\,\frac{b^2}{a^2}(-4a^2) < 0$

Thus area is maximum at $x = \frac{a}{\sqrt{2}}$ $\quad and \quad y = \frac{b}{\sqrt{2}}$

Maximum area is , $A = 4xy = 4\,\frac{a}{\sqrt{2}}.\frac{b}{\sqrt{2}}$ =2ab.

33. Here, $y = e^{sin^2x}\left(2tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) + cot^{-1}\left\{\frac{\sqrt{1+sinx}+\sqrt{1-sin}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right\}$

. For, $x = \cos\theta$, $tan^{-1}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2}\, cos^{-1}x$, and

$cot^{-1}\left\{\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right\} = \cot^{-1}\left\{\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right\} = \cot^{-1}(\cot x/2) = x/2$

Thus, $y = e^{sin^2x}\, cos^{-1}x + x/2$

$\Rightarrow dy/dx = e^{sin^2x}.\frac{-1}{\sqrt{1+x^2}} + cos^{-1}x\, e^{sin^2x}.2sinx.cosx. + ½$

34. Here, f(x) = sinx + cosx , $f^{/}(x)$ = cosx – sinx ,

For critical points of f , $f^{/}(x)$ = 0. => cosx = sinx

⇨ Tanx = 1
⇨ X= $\frac{\pi}{4,}, \frac{5\pi}{4}, as\ \ 0 \leq x \leq 2\pi$.

Hence, intervals are $(0, \frac{\pi}{4,}), \left(\frac{\pi}{4,}, \frac{5\pi}{4,}\right), \left(5\frac{\pi}{4,}, 2\pi\right)$.

For, $(0, \frac{\pi}{4,})$, $f^{/}(x)$ >0, cosx > sinx for $x \in (0, \frac{\pi}{4,})$,

.For, $\left(\frac{\pi}{4,}, \frac{5\pi}{4,}\right)$, $f^{/}(x)$ < 0 , cosx < sinx , $x \in \left(\frac{\pi}{4,}, \frac{5\pi}{4,}\right)$,

For, $\left(5\frac{\pi}{4,}, 2\pi\right)$. $f^{/}(x)$ >0 , cosx > sinx for $x \in \left(5\frac{\pi}{4,}, 2\pi\right)$,

Hence , given function f(x) I strictly increasing in $(0, \frac{\pi}{4,}) \cup \left(5\frac{\pi}{4,}, 2\pi\right)$

And decreasing in $\left(\frac{\pi}{4,}, \frac{5\pi}{4,}\right)$.

35.Let length , breadth and height of open box with square base be , x,x and h respectively. If V be the volume of box then , V =x.x.h => v = x^2h

Also, $c^2 = x^2 + 4xh$ => h = $(c^2 - x^2)/ 4x$.

Hence, v = x^2h => V = $x^2 (c^2 - x^2)/ 4x = \frac{c^2 x}{4} - \frac{x^3}{4}$

dV/dx = $\frac{c^2}{4} - \frac{3x^2}{4}$, for maximum or minimum value of V, dV/dx = 0

$\frac{c^2}{4} - \frac{3x^2}{4} = 0 => \frac{3x^2}{4} = \frac{c^2}{4}$ => $x^2 = \frac{c^2}{3}$ => x = $c/\sqrt{3}$

Now, d^2V/dx^2= - 6x/4 ,

At, x= $c/\sqrt{3}$, d^2V/dx^2= -6c/4$\sqrt{3}$ <0,

Hence , V is maximum at $x = \frac{c}{\sqrt{3}}$, Now, h = $\frac{c^2-x^2}{4x} = \frac{c^2-\frac{c^2}{3}}{4\,c/\sqrt{3}} = \frac{c}{2\sqrt{3}}$.

Hence max volume is $x^2h = \frac{c^3}{6\sqrt{3}}$.

36. i. Resultant velocity of boat= $\sqrt{3^2+4^2}$ = 5 m/s

ii. Time taken by boat to cross the river = width of the river / resultant velocity of the river =distance travelled / speed = 8 sec

iii. Downstream distance travelled by boat = downstream speed time taken by boat to cross the river

= 3x 8

= 24 m

OR

Resultant velocity of boat = $\sqrt{(1.5)^2+2^2}$ = 2.5 m/s

37.

i. Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm. The volume V(x) of the box is given by

$V(x) = x(18 - 2x)^2$

dv.dx = x 2(18 –2 x)(-2) + $(18 - 2x)^2$

dv/dx= 0 => (18 –2x)(- 4x + (18- 2x)) \

⇨ x = 9 or x =3

⇨ here x =9 is not possible, Hence, x = 3 cm

Thus the length of the square to be cut off is 3 cm.

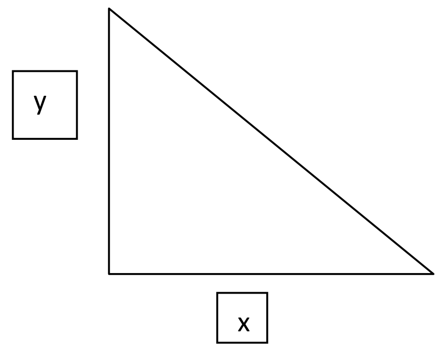
iii) dv/dx = (18 – 2x)(18 – 6x)

$d^2v/dx^2 = -2(18-6x) + (-6)(18-2x) = -144+24x$

At , x = 3 , $d^2v/dx^2 = -72 < 0$, hence V is maximum.

Or, Max volume , V(x=3) = 432 cm^3 .

38. i) By Pythagoras theorem,



$x^2 + y^2 = 25$ ……………(i)

ii) When, x = 4 m, y = 3 m

Differentiating (i) w r t 'x;'

2x dx/dt+ 2y dy/dt=0

2x 2cm/s + 2y dy/dt = 0

Dy/dt = - -2x/y

iii) At x = 4m ,y = 3m , dy/dx = - 8/3 cm/s

Thus the rate of decrease its height is 8/3 cm/s.

NAVODAYA VIDYALAYA SAMITI – RO SHILLONG
CLASS: XII
SUBJECT: MATHEMATICS (041)
SESSION:2024-25
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: INTEGRATION, APPLICATION OF INTEGRATION)

Time: 3 Hours **Max. Marks: 80**

General Instructions:

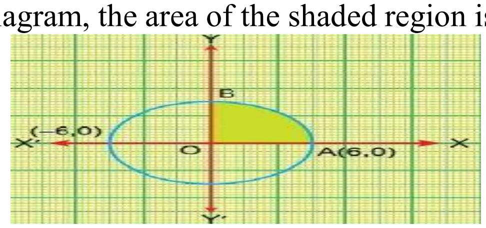
Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION – A

(Multiple Choice Questions)
Each question carries One Mark

Q.1 For the below given diagram, the area of the shaded region is



(A) $\int_{-6}^{6}\sqrt{x^2-6^2}\,dx$ (B) $\int_{0}^{6}\sqrt{x^2-6^2}\,dx$ (C) $4\int_{0}^{6}\sqrt{x^2-6^2}\,dx$ (D) None of these

Q.2 The area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$, $x-$axis is

(A) 2 sq.units (B) 4sq. units (C) 6sq. units (D) 8sq.units

Q.3 If f(x) is an odd function, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\cos^3 x\, dx$ equals:

(A) $2\int_0^{\frac{\pi}{2}} f(x)\cos^3 x\, dx$ (B) 0 (C) $2\int_0^{\frac{\pi}{2}} f(x)\, dx$ (C) $2\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$.

Q.4 $\int x^2 e^{x^3} dx$ is equal to

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^4} + C$ (C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{2}e^{x^2} + C$

Q.5 $\int \frac{e^x(1+x)}{\cos^2(xe^x)}$ dxis equal to

(A) $\tan(xe^x) + C$ (B) $\cot(xe^x) + C$ (C) $\cot(e^x) + C$

(D) $\tan[e^x(1+x)] + C$

Q.6 $\int_{-1}^{1} \frac{|x-2|}{x-2}$ dx, x≠0 is equal to

(A) 1 (B) -1 (C) 2 (D) -2

Q.7 If $f'(x) = x + \frac{1}{x}$, then f(x) is

A. $\log(x^2) + C$ B. $\frac{x^2}{2} + \log|x| + C$. C. $\frac{1}{x} + \log x + C$ D. None of these.

Q.8 Value of $\int \sin x^0 dx$ is

A. $-\cos x^0 + C$ B. $\cos x + C$ C. $-\frac{180}{\pi}\cos\frac{\pi x}{180} + C$ D. None of these

Q.9 Value of $\int \log_5 x\, dx$ is

A. $\frac{1}{\log_e 5}(x\log_e x - x) + C$ B. $x\log_e x - x + C$ C. $\frac{1}{x} + C$ D. None of these

Q.10 $\int \frac{x}{x^2+1} dx$ is

A. $\log(x^2+1) + C$ B. $2\log(x^2+1) + C$ C. $\frac{1}{2}\log(x^2+1) + C$

D. None of these

Q.11 Area of region bounded by curve y = x+1 and the lines x=2 and x= 3 is

A. $\frac{7}{2}$sq. unit B. $\frac{13}{2}$ sq. unit C. $\frac{9}{2}$ sq. unit D. $\frac{11}{2}$

Q.12 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ is

A. 0 B. -2 C. 2 D. 1

Q.13 $\int \frac{\cos(\log x)}{x} dx$ is

A. logx +C B. sin(logx) + C C. log(sinx) + C D. log(cosx) + C

Q.14 $\int e^{5\log x} dx$ is

A. x + C B. $e^{\log x}$ + C C. $\frac{x^6}{6}$ + C D. None of these

Q.15 $\int e^x . \frac{(1+\sin x)}{(1+\cos x)} dx$ is

A. e^x tanx +C B. tan(sinx) + C C. $e^x \tan\frac{x}{2}$ + C D. None of these

Q.16 $\int \cos x . \cos 2x \, dx$ is

A. $\frac{1}{2}(\frac{\sin 3x}{3} + \sin x) + C$ B. $2(\frac{\sin}{3} + \sin x)$ C. sinx.sin2x + C D. None of these

Q.17 $\int e^x (1 - \cot x + \mathrm{Cosec}^2 x) dx$ is

A. e^xcotx + C B. e^x(1-cotx) + C C. e^x(1+cosecx) + c D. None of these

Q.18 $\int \frac{\sec^2(\log x)}{x} dx$ is

A. tan(logx) + C B. cot(logx) + C C. log(tanx) + C D. None of these

ASSERTION-REASON BASED QUSETIONS

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true

Q.19 **Assertion(A)**: $\int_0^3 (x^2 + x + 1)dx = \frac{33}{2}$

Reason(R): $\int_0^2 4x^3 dx = \frac{81}{4}$

Q.20 **Assertion(A)**: $\int 3x^2 (\cos x^3 + 8)dx = \sin x^3 + 8x^3 + C$

Reason(R): The above integration is solved using substitution method.

SECTION B

(Each question carries 2 marks)

Q.21 Evaluate: $\int \frac{dx}{x^2-6x+13}$

Q.22 $Evaluate$: $\int \frac{cos2x+2si\ \ ^2x}{cos^2x}$ dx

Q.23 $Evaluate$: $\int sinx.logcosx\ dx$

Q.24 Evaluate : $\int \frac{sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}$ dx

Q.25 Evaluate : $\int \frac{sinx+cosx}{\sqrt{1+sin}}$ dx

SECTION C

(Each question carries 3 marks)

Q.26 Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}}$

Q.27 Evaluate : $\int_0^{\pi} \frac{x}{a^2cos^2x+b^2sin^2x}$ dx

Q.28 Evaluate : $\int \frac{6x+7}{(6x-5)(x-4)}$ dx

Q.29 Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}$ dx

Q.30 Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{sinx}}$ dx

Q.31 Evaluate : $\int \frac{2x^2+1}{x^2(x^2+1)}$ dx

SECTION D

(Each question carries 5 marks)

Q.32 Find the area of region bounded by ellipse, $\frac{x^2}{25}+\frac{y^2}{16}=1$.

Q.33 Evaluate : $\int_{-1}^{\frac{3}{2}} |xsin\pi x|$dx

OR

$\int_0^{\frac{3}{2}} |x\cos\pi x|\,dx$

Q.34 Find the area of region bounded by parabola, $y^2 = 8x$ and line, $x = 2$

Q.35 Evaluate: $\int e^x . \sin 2x\, dx$

SECTION- E **[4x3=12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case study-I

Q.36 Read the following text and answers the following questions on the basis of the same:
Reena and Sapna practice the problems based on integrals. They will try to evaluate the integrals based upon $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$.
Reena first explains the steps to solve this type of integrals.
Step 1 : Obtain the integral, $I = \int \frac{f'(x)}{f(x)}\,dx$
Step 2 : Put $f(x) = t$ and replace $f'(x)dx$ by dt to obtain $I = \int \frac{1}{t}dt$
Step 3 : Evaluate integral obtained in step II to obtain $I = \log|t| + C$
Step 4 : Replace t by f(x) in step III to get $I = \log|f(x)| + C$

i. Evaluate : $\int \frac{2x+5}{x^2+5x-7}dx$
ii. Evaluate : $\int \frac{1}{x(3+\log x)}\,dx$
iii. Evaluate: $\int \frac{1}{1+e^{-x}}\,dx$
Or
Evaluate : $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

Case study-II

Q.37 Read the following text and answer the following questions on the basis of the same:
Ram & Lakshman were discussing integration, in with following points:
a. In question like $\int e^x(f(x) + f'(x))\,dx$, the result is $e^x f(x) = C$
b. If we have irrational terms in integration question, then we should try doing rationalizing these terms.
On the basis of the above information answer the following:
i. Evaluate : $\int \frac{1}{\sqrt{3x+4}-\sqrt{3x+1}}dx$

ii. Evaluate : $\int e^{x}(tanx + logsecx)$ dx

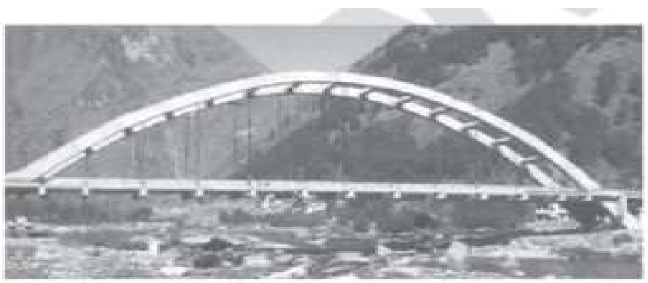
iii. Evaluate : $\int \frac{1}{\sqrt{1-2x}+\sqrt{3-2x}}$dx

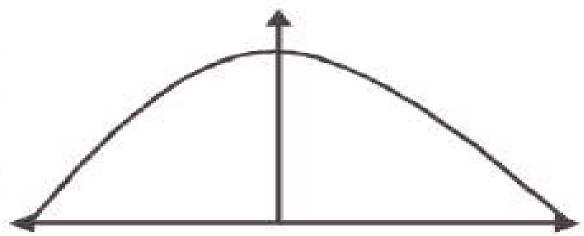
Or

Evaluate :$\int e^{x}\left(\frac{1-sinx}{1-cosx}\right) dx$ dx

Case study-III

Q.38 The bridge connects 2 hills 100ft apart. The arch on the bridge is in a parabolic form. The highest
point on the bridge is 10ft above the road at the middle of the bridge as seen in the figure :





i. The equation of the parabola:

A. $x^2 = 250y$ B. $x^2 = -250y$ C. $y^2 = 250x$ D. $y^2 = -250x$

ii. The value of integral $\int_{-50}^{50} \frac{x^2}{250}$dx is

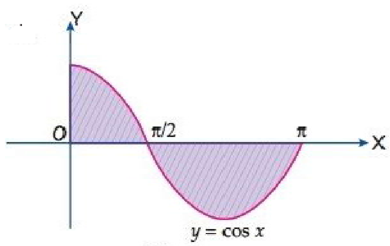
A. $\frac{1000}{3}$ B. $\frac{250}{3}$ C. 1200 D. 0

NVS RO SHILLONG
SADHANA PROGRAM MODEL PAPER (2024-2025)
MARKINGSCHEME CLASS XII
MATHEMATICS (CODE-041)

CHAPTER/UNITS: INTEGRATION, APPLICATION OF INTEGRATION

SECTION:A

(Solution of MCQs of 1 Mark Each)

Q.NO.	ANS	SOLUTION
1.	(D)	Equation of circle is $x^2 + y^2 = 6^2 \Rightarrow y = \sqrt{6^2 - x^2}$. So, area of the shaded region =area of the region bounded by the circle between the y-axis i.e. x=0 and x=6(radius) upto x-axis. $=\int_0^6 \sqrt{6^2 - x^2}\, dx$
2.	(A)	 Area $=\int_0^{\frac{\pi}{2}} cosx\, dx + \left\|\int_{\frac{\pi}{2}}^{\pi} cosx\, dx\right\|$= 1+1 =2 sq units
3.	(B)	$f(x)cos^3x$ is an odd function since f(x) as odd
4.	(A)	$$\int x^2 e^{x^3} dx$$ Putting $z = x^3 \Rightarrow dz=3x^2\, dx$ $$\int \frac{1}{3} e^z dz = \frac{1}{3} e^z$$
5.	(A)	$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}\, dx$$ Putting $z = xe^x \Rightarrow dz=(xe^x + e^x)dx \Rightarrow dz=e^x(x+1)dx$ $$\int \frac{1}{\cos^2 z} dz = \int \sec^2 z dz = \tan z + C = \tan(xe^x) + C$$

6.	(D)	$$\int_{-1}^{1} \frac{\lvert x-2\rvert}{x-2} dx$$ $= \int_{-1}^{1}(-1)\,dx$, since $\lvert x-2\rvert = -(x-2)in[-1,1]$ $= -[x]_{-1}^{1}$ = -2
7.	(B)	Given: $f'(x) = x + \frac{1}{x} \Rightarrow f(x) = \int \left(x+\frac{1}{x}\right) dx = \frac{x^2}{2} + \log\lvert x\rvert + C$
8.	(C)	$\int \sin x^0 dx = \int \sin\frac{\pi x}{180}\, dx = -\frac{180}{\pi}\cos\frac{\pi x}{180} + C$
9.	(A)	$$\int \log_5 x\, dx = \int \frac{\log_e x}{\log_e 5}\, dx = \frac{1}{\log_e 5}\int \log_e x\, dx$$ $= \frac{1}{\log_e 5}[(\log_e x)x - \int \frac{1}{x}.x\, dx]$, (by parts) $=\frac{1}{\log_e 5}(x\log_e x - x) + C$
10.	(C)	$\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx = \frac{1}{2}\log(x^2+1) + C$, by $\frac{f\prime(x)}{f(x)}$ form.
11.	(A)	Area= $\int_2^3 (x+1)dx = \left[\frac{(x+1)^2}{2}\right]_2^3 = \frac{7}{2}$
12.	(A)	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$, as $\sin^7 x$ an odd function
13	(B)	$$\int \frac{\cos(\log x)}{x} dx = \sin(\log x) + C, \text{by putting } Z = \log x$$
15	(C)	$$\int e^{5\log x}\, dx$$ $$= \int e^{\log x^5}\, dx$$ $$= \int x^5\, dx$$ $$= \frac{x^6}{6} + C$$
16	(A)	$\int e^x.\frac{(1+\sin x)}{(1+\cos x)}\, dx = \int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx = \int e^x\left\{\frac{1}{2}sec^2\frac{x}{2} + tan\frac{x}{2}\right\} dx$ $= e^x\tan\frac{x}{2} + C$, by $\int e^x\{f'(x) + f(x)\}\, dx$
17	(B)	$$\int \cos x.\cos 2x\, dx = \frac{1}{2}\int (cos3x + cosx)\, dx, \text{by} \cos(A+B) + \cos(A-B)$$

		$=\frac{1}{2}\left(\frac{Sin3x}{3}+\ sinx\right)+C$
17	(C)	$\int e^x(1-\cot x+\ \mathrm{cosec}^2 x)dx$ $= e^x(1\text{-cotx}) + C$, by $\int e^x\{f'(x)+f(x)\}\,dx$ where $f(x)=1-\cot x$
18	(A)	$\int \frac{\sec^2(\log x)}{x}\,dx = \tan(\log x) + C,\ by\ putting\ Z = logx$
19	(C)	$\int_0^2 4x^3 dx=16$, So (A) is true but (R) is false
20	(A)	Assertion (A)and Reason(R) both are correct, ***(R) is the correct explanation of (A)***

Section–B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21.	$\int\frac{dx}{x^2-6x+13} = \int\frac{dx}{(x-3)^2+4} \ = \ \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$
22.	$\int\frac{cos2x+2sin^2x}{cos^2x}\ dx$ Using relation, $\cos 2x = 1\text{-}\sin^2 x$ $\int\frac{1-2sin^2x+2sin^2x}{cos^2x}\ dx = \tan x + C$
23	$= \int sinx.Logcosx\ dx$ putting z =cosx $I= -\int 1.logzdz$ $=\cos x\{1-\log(\cos x)\} + C$
24	$I = \int\frac{sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$ Put $z= \sin^{-1}x$ $dz = \frac{1}{\sqrt{1-x^2}}dx$ $I = \int zsec^2zdz$ $=\frac{x}{\sqrt{1-x^2}}\sin^{-1}x + \log\left\|\sqrt{1-x^2}\right\| + C$
25	$\int\frac{sinx+cosx}{\sqrt{1+sin2x}}\,dx = \int\frac{sinx+cosx}{sinx+cosx}dx =\int dx= x +C$

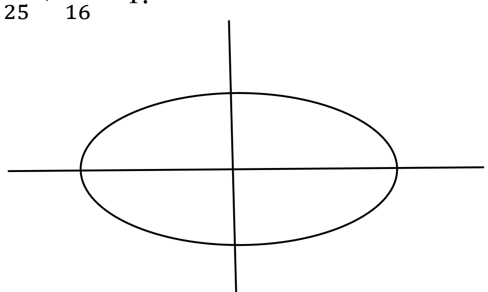
SECTION C (Each question carries 3 marks)

26	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}$ dx ………..1 Using Property, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}$ dx ……………………2 (1) + (2) $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$ $I = \frac{\pi}{12}$
27	$I = \int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x}$ dx ………………..(1) Use prop, $\int_0^a f(x) = \int_0^a f(a-x)dx$ $I = \int_0^{\pi} \frac{\pi - x}{a^2cos^2x + b^2sin^2x}$ dx ………………..(2) (1) + (2) $2I = \int_0^{\pi} \frac{\pi}{a^2cos^2x + b^2sin^2x}$ dx $I = \frac{\pi}{2}\int_0^{\pi} \frac{1}{a^2cos^2x + b^2sin^2x}$ dx $= \frac{\pi}{2}\int_0^{\pi} \frac{sec^2x}{a^2 + b^2tan^2x}$ dx
28	Use Linear = A$\frac{d}{dx}$Quadratic + B Solution is $6\sqrt{x^2 - 9x + 20}$ $+34\log\left\|x - \frac{9}{2} + \sqrt{x^2 - 9x + 20}\right\|$ +C
29.	$\int \frac{x^4}{(x-1)(x^2+1)}$ dx = $\int \left\{x + 1 + \frac{1}{(x-1)(x^2+1)}\right\}$ dx Apply partial Fraction in the above $I = \int (x+1)dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{x+1}{x^2+1}$ dx $= \frac{x^2}{2} + x + \frac{1}{2}\log\|x-1\| - \frac{1}{4}\log\|x^2+1\| - \frac{1}{2}\tan^{-1}x + C$
30	$I = \int_0^{2\pi} \frac{1}{1+e^{Sinx}}$ dx ………….(1) Use property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

	I=$\int \int_0^{2\pi} \frac{1}{1+e^{-sinx}}$ dx $=\int_0^{2\pi} \frac{e^{sinx}}{1+e^{sinx}}$ dx ....(2) (1)+(2) 2I = $\int_0^{2\pi} \frac{1+e^{sinx}}{1+e^{sinx}}$ dx $= 2\pi$ I = π
31	I = $\int \frac{2x^2+1}{x^2(x^2+1)}$ dx Replace $x^2 = z$ Apply Partial Fraction Soln is $\frac{-1}{4x}+\frac{7}{8}\tan^{-1}\frac{x}{2}+C$

SECTION D

(Each question carries 5 marks)

32	Equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$. Find y = $\frac{4}{5}\sqrt{5^2-x^2}$  Area of ellipse = 4 x area of ellipse in 1st Quadrant $= 20\pi$ sq. unit
33	$\int_{-1}^{\frac{3}{2}} \lvert x sin\pi x\rvert$dx When xsinx =0 x=n$\pi$ x= 0,1$\in (-1,\frac{3}{2})$ $\int_{-1}^{\frac{3}{2}} \lvert x sin\pi x\rvert dx = \int_{-1}^{1} x sin\pi x\, dx + \int_{1}^{\frac{3}{2}} -x sin\pi x\, dx$ $= \frac{1+3\pi}{\pi^2}$ OR
33.(OR)	$\int_{0}^{\frac{3}{2}} \lvert x cos\pi x\rvert$dx When $x cos\pi x = 0$

	$X = (2n-1)\frac{\pi}{2}$ $X = 1/2 \in (\frac{0,3}{2})$ $\int_0^{\frac{3}{2}} \lvert x\cos\pi x \rvert dx = \int_0^{1/2} x\cos\pi x dx + \int_{1/2}^{3/2} x\cos\pi x dx$ $= \frac{5}{2\pi} - \frac{1}{\pi^2}$
34	Equation of parabola is $y^2 = 8x$ & equation of line, $x = 2$ Area = $2\int_0^2 y dx$ $= \frac{32}{3}$ sq unit.
35	$\int e^x . \sin 2x dx$ Apply Integration By Part Soln is $\frac{1}{5}(e^x\sin 2x - 2e^x\cos 2x) + c$

<u>Section –E</u>

(This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36	i. $\int \frac{2x+5}{x^2+5x-7} dx = \log(x^2+5x-7) + c$ ii. $\int \frac{1}{x(3+\log x)} dx = \log(3+\log x) + C$ iii. $\int \frac{1}{1+e^{-x}} dx = \log(1+e^x) + C$ Or $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \log(e^x + e^{-x}) + c$
37	i. $\int \frac{1}{\sqrt{3x+4} - \sqrt{3x+1}} dx = \frac{2}{27}\{(3x+4)^{3/2} + (3x+1)^{3/2}\} + C$ $\int e^x(\tan x + \log\sec x)\, dx = e^x \log\sec x + C$ ii. $\int \frac{1}{\sqrt{1-2x} + \sqrt{3-2x}} dx = \frac{1}{6}(1-2x)^{\frac{3}{2}} - \frac{1}{6}(3-2x)^{\frac{3}{2}} + C$ Or $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) x = - e^x \mathrm{lot}\frac{x}{2} + C$
38	i. (B) $x^2 = -250y$ ii. (A) $\frac{1000}{3}$

Navodaya Vidyalaya Samiti, RO Shillong
UNIT WISE PRACTICE QUESTION PAPER
(2024-25)
Class-XII
Subject: Mathematics (041)

CHAPTER/UNITS: DIFFERENTIAL EQUATION, VECTOR AND 3D GEOMETRY

Time:3 Hours **Maximum Marks:80**

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. Use of calculators is **not** allowed.

SECTION – A
(Multiple Choice Questions)
Each question carries One Mark

Q.1 If $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is
(A) 9 (B) 16 (C) 3 (D) None of these

Q.2 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\sqrt{3}\vec{a} - \vec{b}$ is also an unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is
(A) 30^0 (B) 45^0 (C) 60^0 (D) None of these

Q.3 If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then, the angle between $\vec{a}$ and $\vec{b}$ is
(A) 90^0 (B) 45^0 (C) 60^0 (D) None of these

Q.4 Let $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + \lambda\hat{k}$. The value of λ if $|-5\vec{a}| = 25$ is
(A) 0 (B) $\pm 2\sqrt{3}$ (C) ± 1 (D) ± 12

Q.5 The value of λ for which the two vectors $2\,\hat{\imath} - \hat{\jmath} + 2\,\hat{k}$ *and* $3\,\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are orthogonal (i.e.perpendicular) is
(A) 2 (B) 4 (C) 6 (D) 8

Q.6 If $\overrightarrow{AB} = 2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ and $A(1,2,-1)$ is the given point, then the coordinates of B are
(A) $(3,-3,4)$ (B) $(3,3,4)$ (C) $(-3,-3,4)$ (D) $(3,3,-4)$

Q.7 The integrating factor of the differential equation :

$$(1-y^2)\frac{dx}{dy}+yx=ay,(-1<y<1)\text{ is}$$

(A) $\frac{1}{\sqrt{1-y^2}}$ (B) $\frac{1}{1-y^2}$ (C) $\frac{1}{\sqrt{y^2-1}}$ (D) $\frac{1}{y^2-1}$

Q.8 Familyy $= Ax + A^3$ of curves will corresponds to a differential equation of order

(A) 3 (B) 2 (C) 1 (D) not defined

Q.9 If m and n are the order and the degree of the differential equation

$1+\left(\frac{d^3y}{dx^3}\right)^2=\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$, then the value of $4m-3n$ is

(A) 0 (B) 2 (C) 1 (D) not defined

Q.10 The number of solution of $\frac{dy}{dx}=\frac{y+1}{x-1}$wheny(1)=2 is

(A) none (B) one (C) two (D) infinite

Q.11 Which of the following is not a homogeneous function of x and y ?

(A) x^2+2xy (B) $2x-y$ (C) $\cos^2\left(\frac{y}{x}\right)+\frac{y}{x}$ (D) $\sin x-\cos y$

Q.12 The solution of the differential equation is $\frac{dx}{x}+\frac{dy}{y}=0$is

(A) $\frac{1}{x}+\frac{1}{y}=C$ (B) $\log x-\log y=C$ (C) xy=C (D) x +y=C

Q.13 Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ are

(A) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (B) $\frac{2}{7},-\frac{3}{7},-\frac{6}{7}$ (C) $\frac{2}{\sqrt{157}},-\frac{3}{\sqrt{157}},\frac{12}{\sqrt{157}}$ (D) $\frac{2}{7},-\frac{3}{7},\frac{6}{7}$

Q.14 The coordinates of the foot of the perpendicular drawn from the point $(2,-3,4)$ on the y-axis is

(A) $(2,3,4)$ (B) $(-2,-3,-4)$ (C) $(0,-3,0)$ (D) $(2,0,4)$

Q.15 If a line makes angles α,β and γ with the axes respectively,then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$

(A) -2 (B) -1 (C) 1 (D) 2

Q.16 The distance of the point P(a, b, c) from y-axis is

(A) b (B) $|b|$ (C) $|b|+|c|$ (D) $\sqrt{a^2+c^2}$

Q.17 If the line through the points $(1,-1,2)$,$(3,4,-2)$ is perpendicular to the line through points $(\lambda,3,2)$,$(3,5,6)$, then the value of λ is

(A) 0 (B) 2 (C) 1 (D) not defined

Q.18 The value of p for which the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles is

(A) $-\frac{11}{70}$ (B) $\frac{70}{11}$ (C) $\frac{70}{\sqrt{11}}$ (D) $-\frac{70}{\sqrt{11}}$

ASSERTION-REASON BASED QUSETIONS

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true

Q.19 **Assertion(A)**: If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ then its vector form is

$$\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$$

Reason(R): The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Q.20 **Assertion(A)**: A line in space cannot be drawn perpendicular to x,y and z axes simultaneously.

Reason(R): For any line making angles α,β and γ with the positive directions of x,y and z- axes respectively, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

SECTION B

(Each question carries 2 marks)

Q.21 If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$,
Then prove that $|\vec{b}|=\sqrt{2}|\vec{a}|$

(OR)

The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a triangle ABC.Find the length of the median through A.

Q.22 Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$, externally in the ratio 1:2 .Also, show that P is the mid point the line segment RQ.

(OR)

Find a vector of magnitude 9 unit perpendicular to both the vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$

Q.23 Find the general solution of $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Q.24 Given that $\frac{dy}{dx} = ye^x$ and x=0, y=e. Find the value of y when x=1

Q.25 Find the angle between the lines
$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\vec{r} = 2\hat{\jmath} - 5\hat{k} + \mu(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

SECTION C

(Each question carries 3 marks)

Q.26 The two adjacent sides of a parallelogram are $2\,\hat{\imath} - 4\,\hat{\jmath} - 5\,\hat{k}$ *and* $2\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}$.
Find the two-unit vectors parallel to its diagonals. Using the diagonals vectors, find the area of the parallelogram.

(OR)

If A,B,C,D are the points with position vectors $\hat{\imath} + \hat{\jmath} - \hat{k}$, $2\,\hat{\imath} - \hat{\jmath} + 3\,\hat{k}$, $2\,\hat{\imath} - 3\,\hat{k}$ and $3\,\hat{\imath} - 2\,\hat{\jmath} + \hat{k}$ respectively, find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

Q.27 Find the position vector of a point P in space such that $\overrightarrow{OP}$ is inclined at 60^0 to OX and at 45^0 to OY and $|\overrightarrow{OP}|$=10 units.

Q.28 Find the angle between any two diagonals of a cube.

Q.29 Solve: $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when $x = 1$

Q.30 Find the particular solution of $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y = 0$ when $x = 1$

Q.31 let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal tothe projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}$,$\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

OR The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\,\hat{k}$ is equal to $\sqrt{2}$ .Find the value of λ.

SECTION D

(Each question carries 5 marks)

Q.32 By using vectors, in a ΔABC, prove that, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
where a,b.c are represent the magnitude of the sides opposite to the vertices A,B,C respectively.

(OR)

If $\vec{a}$ and $\vec{b}$are two unit vectors inclined an angle Θ, then prove that:

(i) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

(ii) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$

(iii) $\tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$.

Q.33 Find the coordinates of the foot of perpendicular drawn from the point A(1, 8, 4) to the line

joining the points B(0,-1,3) and C(2,-3,-1) . Also find
(i) length of the perpendicular
(ii) image of A in the line through B and C

(OR)

Find the equation of a line l_2 which is the mirror image of the linel_1 with repect to the line l: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that the line l_1 passes through the point P(1, 6, 3) and parallel to line l. **(CBSE 2024 65/1/3)**

Q.34 Find the shortest distance between the lines whose vector equations are:
$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k})$.

Q.35 Solve:$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

SECTION- E **[4x3=12]**

(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)*

Case study-I

Q.36 Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10).

Based on the above information, answer the following questions

(i) What are the components to the two edge vectors defined by $\vec{\mathbf{A}}$ = PV of P_2 – PV of P_1
and $\vec{\mathbf{B}}$ = PV of P_4– PV of P_1 ? (where PV stands for position vector). 1

(ii) Find the vector $\vec{\mathbf{N}}$, perpendicular to $\vec{\mathbf{A}}$and $\vec{\mathbf{B}}$ and the surface of the roof? 1

(iii) (a) If the flow of solar energy is given by the vector $\vec{\mathbf{F}} = \mathbf{6\hat{i} - 2\hat{j} + 3\hat{k}}$ what is the dot product of vectors $\vec{\mathbf{F}}$ with $\vec{\mathbf{N}}$. 2

OR

(b) What is the angle between vectors $\vec{\mathbf{N}}$and $\vec{\mathbf{F}}$?

Case study-II

Q.37 Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.

Base on the above information, answer the following

(i) Write the Cartesian equation of the line along which the motorcycle A is running. 1

(ii) Find the direction cosines of the line along which motorcycle B is running 2

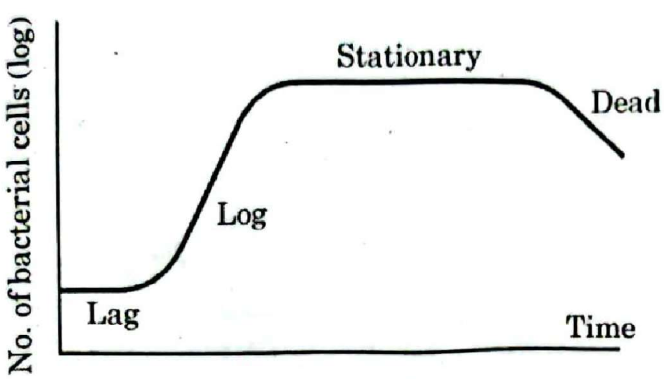
(iii) (a) Find shortest distance between the given lines 2

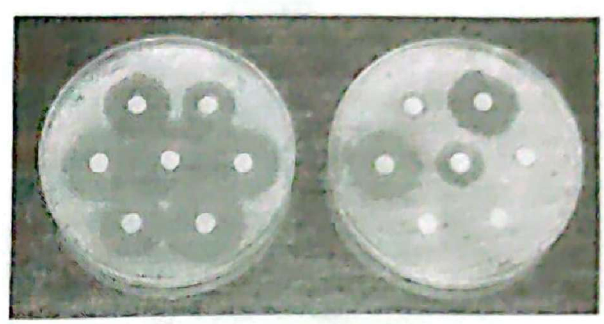
OR

(b) Find the angle between the given lines

Case study-III

Q.38 A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth, the rate of growth of this sample bacteria are calculated.





The differential equation representing the growth of bacteria is given as:

$\frac{dP}{dt} = \mathbf{kP}$,where P is the population of bacteria at any time 't'.

Base on the above information, answer the following questions

(i) Obtain the general solution of the differential equation and express it as an exponential function of ‘t’. 2

(ii) If the population of bacteria is 1000 at t=0, and 2000 at t=1,find value of k. 2

NVS RO SHILLONG

UNIT WISE PRACTICE QUESTION PAPER

(2024-2025)

MARKING SCHEME CLASS XII

MATHEMATICS(CODE-041)

CHAPTER/UNITS: DIFFERENTIAL EQUATION, VECTOR AND 3D GEOMETRY

SECTION:A

(Solution of MCQs of 1 Mark each)

Q.NO.	ANS	SOLUTION
1.	(C)	Given: $\left\|\vec{a}.\vec{b}\right\|^2 + \left\|\vec{a}\times\vec{b}\right\|^2 = 144 \& \|\vec{a}\| = 4$ By Langrange's identity, $\left\|\vec{a}.\vec{b}\right\|^2 + \left\|\vec{a}\times\vec{b}\right\|^2 = \|\vec{a}\|^2\left\|\vec{b}\right\|^2$ $\Rightarrow 144 = 4^2\left\|\vec{b}\right\|^2$ $\Rightarrow \left\|\vec{b}\right\|^2 = 9$ $\Rightarrow \left\|\vec{b}\right\| = 3$
2.	(A)	Given: $\sqrt{3}\vec{a} - \vec{b}$ is an unit vector $\Rightarrow \left\|\sqrt{3}\vec{a} - \vec{b}\right\| = 1$ Squaring on both sides, $\left\|\sqrt{3}\vec{a} - \vec{b}\right\|^2 = 1$ $\Rightarrow \left\|\sqrt{3}\vec{a}\right\|^2 - 2\sqrt{3}\vec{a}.\vec{b} + \left\|\vec{b}\right\|^2 = 1$ $\Rightarrow \left\|\sqrt{3}\right\|^2\|\vec{a}\|^2 - 2\sqrt{3}\|\vec{a}\|\left\|\vec{b}\right\|\cos\Theta + \left\|\vec{b}\right\|^2 = 1$ $\Rightarrow 3 - 2\sqrt{3}\cos\Theta + 1 = 1 \quad \left(\text{since } \|\vec{a}\| = 1 \& \left\|\vec{b}\right\| = 1\right)$ $\therefore \cos\theta$ $= \frac{\sqrt{3}}{2} \Rightarrow \theta$ $= \frac{\pi}{6} = 30^0$
3.	(A)	Given: $\left\|\vec{a} + \vec{b}\right\| = \left\|\vec{a} - \vec{b}\right\|$ Squaring on both sides, $\left\|\vec{a} + \vec{b}\right\|^2 = \left\|\vec{a} - \vec{b}\right\|^2$ $\Rightarrow \|\vec{a}\|^2 + \left\|\vec{b}\right\|^2 + 2\,\vec{a}.\vec{b} = \|\vec{a}\|^2 + \left\|\vec{b}\right\|^2 - 2\,\vec{a}.\vec{b}$ $\Rightarrow 4\,\vec{a}.\vec{b} = 0$ $\Rightarrow 4\,\|\vec{a}\|\left\|\vec{b}\right\|\cos\Theta = 0$ $\therefore \cos\theta = 0 \Rightarrow \theta = 90^0$

4		Given: $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + \lambda\hat{k}$ and $\lvert -5\vec{a}$ $\rvert = 25$ $\Rightarrow \lvert -5\rvert \lvert\vec{a}\rvert = 25$ $\Rightarrow 5\sqrt{2^2 + 3^2 + \lambda^2} = 25$ $\Rightarrow \sqrt{13 + \lambda^2}$ $\Rightarrow 13 + \lambda^2 = 25 \Rightarrow \lambda^2 = 12 \Rightarrow \lambda = \pm 2\sqrt{3}$
5	(D)	Given: $2\,\hat{\imath} - \hat{\jmath} + 2\,\hat{k}$ *and* $3\,\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular $\Rightarrow (2\,\hat{\imath} - \hat{\jmath} + 2\,\hat{k}).(3\,\hat{\imath} + \lambda\hat{\jmath}$ $+ \hat{k})$ $\Rightarrow 2.3$ $\Rightarrow \lambda = 8$
6	(D)	Given: $\overrightarrow{AB} = 2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ and A $(1,2,-1)$ $\Rightarrow \text{p.v. of B} - \text{p.v. of A} = 2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ $\Rightarrow \text{p.v. of B} - (\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ $\Rightarrow \text{p.v. of B} = 3\,\hat{\imath} + 3\,\hat{\jmath} - 4\,\hat{k}$
7	(A)	Given: $(1 - y^2)\frac{dx}{dy} + yx = ay$ $\Rightarrow \frac{dx}{dy} + \frac{y}{(1-y^2)}x = \frac{ay}{(1-y^2)}$, which is in the form of $\frac{dx}{dy} + Px = Q$ $\therefore \text{I.F.} = e^{\int P dy} = e^{\int \frac{y}{1-y^2} dy}$ $= e^{\frac{-1}{2}\int \frac{-2y}{1-y^2} dy}$ $= e^{\frac{-1}{2}\log(1-y^2)}$ $= e^{\log(1-y^2)^{\frac{-1}{2}}}$ $= (1-y^2)^{\frac{-1}{2}} = \frac{1}{\sqrt{1-y^2}}$
8	(C)	$y = Ax + A^3$ is corresponds to the D.E. of order one as it is involved only one arbitrary constant i.e. A
9	(A)	$1 + \left(\frac{d^3y}{dx^3}\right)^2$ $= \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ Squaring on both sides, $\left\{1 + \left(\frac{d^3y}{dx^3}\right)^2\right\}^2 = \left(\frac{d^2y}{dx^2}\right)^3$ Highest derivative is $\frac{d^3y}{dx^3} \Rightarrow m = 3, n = 4$

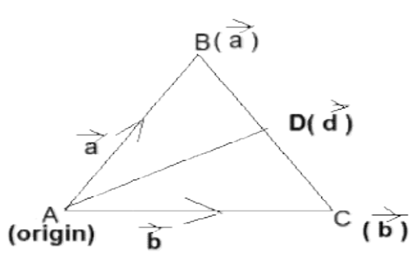
		$\therefore 4m - 3n = 4.4 - 3.4 = 0$
10	(B)	Concept: Solution of a D.E. under an initial given condition is a particular solution. And, a particular solution of a D.E. of order one and degree one have only one solution. Explanation: Given: $\frac{dy}{dx} = \frac{y+1}{x-1}$wheny(1)=2 (initial condition) $\Rightarrow \frac{1}{y+1} dy = \frac{1}{x-1} dx \Rightarrow \int \frac{1}{y+1} dy = \int \frac{1}{x-1} dx \Rightarrow \log(y+1)$ $= \log(x-1) + C$ $\Rightarrow \log(y+1) - \log(x-1) = C$ $\Rightarrow \log\left(\frac{y+1}{x-1}\right) = \log A$ $\Rightarrow \frac{y+1}{x-1} = A$ $\Rightarrow y + 1 = A(x-1)$.........(i) But, when x=1, y=2 ; (i) $\Rightarrow 2 + 1 = A(0-1) \Rightarrow A = -3$ Hence ,from(i), the P.S. is $y + 1 = -3(x-1)$ $\Rightarrow y + 3x = 2$
11	(D)	Here,in option (D) the degree of x and y not defined. A homogeneous function is a function that has the same degree of the polynomial in each variablesHomogeneous function of x and y is a function that can be expressed in the form of either $f\left(\frac{x}{y}\right)$ or $f\left(\frac{y}{x}\right)$.
12	(C)	Given:$\frac{dx}{x} + \frac{dy}{y} = 0$ $\Rightarrow \frac{1}{x} dx$ $\Rightarrow \int \frac{1}{x} dx$ $+ \int \frac{1}{y} dy$ $\Rightarrow \log x$ $\Rightarrow \log xy$ $\Rightarrow xy = C$

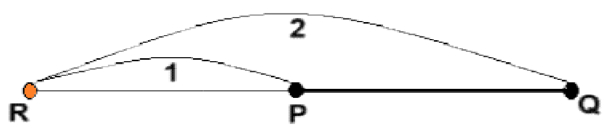
13	(D)	Given: $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ Standard form: $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-\frac{1}{2}}{6}$ Direction ratios are: $\langle 2,-3,6\rangle$ Direction cosines are: $\langle\frac{2}{\sqrt{2^2+(-3)^2+6^2}},\frac{-3}{\sqrt{2^2+(-3)^2+6^2}},\frac{6}{\sqrt{2^2+(-3)^2+6^2}}\rangle$ i.e. $\langle\frac{2}{7},\frac{-3}{7},\frac{6}{7}\rangle$
14	(C)	The x and z co-ordinates of a point on y-axis are 0. Therefore, required point on the y-axis = (0,-3,0)
15	(B)	We know that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ Now, $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=(2\cos^2\alpha-1)+(2\cos^2\beta-1)+(2\cos^2\gamma-1)$ $=2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)-3$ $=-1$
16	(D)	The required distance is the distance of P(a,b,c) from Q(0,b,0) $=\sqrt{a^2+c^2}$
17	(A)	Direction ratios of the through $(1,-1,2)\&(3,4,-2)$ are $\langle 2,5,-4\rangle$ Direction ratios of the through $(\lambda,3,2)\&(3,5,6)$ are $\langle 3-\lambda,2,4\rangle$ Since, the lines are perpendicular we have $a_1a_2+b_1b_2+c_1c_2=0$ $\Rightarrow 2(3-\lambda)+5.2+(-4).4=0$ $\Rightarrow 6-2\lambda+10-16=0$ $\Rightarrow \lambda=0$
18	(B)	Given lines: $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ Standard forms of the lines: $\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$ and $\frac{x-1}{-\frac{3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$ Since, the lines are perpendicular we have $a_1a_2+b_1b_2+c_1c_2=0$ $\Rightarrow -3.(-\frac{3p}{7})+\frac{2p}{7}.1+2.(-5)=0$ $\Rightarrow \frac{11p}{7}=10$ $\Rightarrow p=\frac{70}{11}$
19	(C)	The assertion(A) is true as, for the Cartesian equation $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-y_1}{c}$, the vector equation is $\vec{r}=x_1\hat{\imath}+y_1\hat{\jmath}+y_1\hat{k}+\lambda(a\hat{\imath}+b\hat{\jmath}+c\hat{k})$. The reason (R) is not true because the correct equation of the line

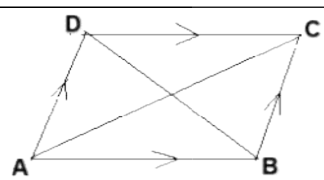
		passing through (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$
20	(A)	The assertion(A) that a line in space cannot be drawn perpendicular to x , y and z-axes simultaneously is true, as a line can only be perpendicular to a single axis or lie in a plane that is perpendicular to two axes,but not all three simultaneously. The reason (R) is also true as well as (R) is the correct explaination of (A) as, suppose the line is perpendicular to the 3 axes simultaneously then $\cos^2 90^0+\cos^2 90^0+\cos^2 90^0=0\neq 1$

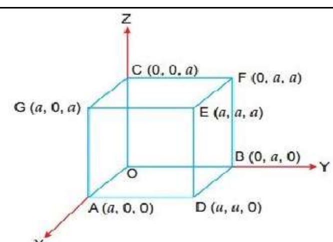
<u>Section–B</u>

[This section comprises of solution of very short answer type questions **(VSA)of** 2 marks **ea**ch]

21.	Given: $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$ $\Rightarrow(\vec{a}+\vec{b}).\vec{a}=0$ and $(2\vec{a}+\vec{b}).\vec{b}=0$ $\Rightarrow\|\vec{a}\|^2+\vec{a}.\vec{b}=0 \ldots.(\text{i})$ and $2\vec{a}.\vec{b}+\|\vec{b}\|^2$	$\frac{1}{2}$ 1 $\frac{1}{2}$
21 (OR	 	$\frac{1}{2}$

	Given .$\vec{a}$=p.v. of B=$\overrightarrow{AB}$ =$\hat{j}+\ \hat{k}$ and $\vec{b}$ = p.v. of C = $\overrightarrow{AC} = 3\,\hat{\imath} - \ \hat{j} +$ $4\,\hat{k}$ $\therefore$ Median $\overrightarrow{AD} = \vec{d}$ i.e. by mid point formula, $\vec{d} = \dfrac{\vec{a}+\vec{b}}{2}$ $= \dfrac{3}{2}\hat{\imath} + 0\hat{j} + \dfrac{5}{2}\hat{k}$ $\therefore$ Length of the median $\overrightarrow{AD} = \left\|\vec{\square}\right\| = \sqrt{\left(\frac{3}{2}\right)^2 + 0^2 + \left(\frac{5}{2}\right)^2} = \dfrac{\sqrt{34}}{2}$	1 $\frac{1}{2}$
22.	 $(2\vec{a}+\vec{b})$ $(\vec{a}-3\vec{b})$ By section formula ,position vector of R= $\frac{2(2\vec{a}+\vec{b})-1(\vec{a}-3\vec{b})}{2-1} = 3\vec{a}+5\vec{b}$ Again,mid-point of RQ= $\frac{(3\vec{a}+5\vec{b})+(\vec{a}-3\vec{b})}{2} = 2\vec{a}+\vec{b}$ = Position vector of P.Hence,P is the mid-point of RQ	1 1
22. (OR)	The vector of magnitude 9 unit perpendicular to both the vectors $\hat{\imath} - \ \hat{j}$ and $\hat{\imath} + \ \hat{j}$ $= 9.\left[\dfrac{(\hat{\imath} - \ \hat{j}) \times (\hat{\imath} + \ \hat{j})}{\left\|(\hat{\imath} - \ \hat{j}) \times (\hat{\imath} + \ \hat{j})\right\|}\right]$ $= 9.\left[\dfrac{\hat{\imath}\times\hat{\imath} + \hat{\imath}\times\hat{j} - \hat{j}\times\hat{\imath} - \hat{j}\times\hat{j}}{\left\|\hat{\imath}\times\hat{\imath} + \hat{\imath}\times\hat{j} - \hat{j}\times\hat{\imath} - \hat{j}\times\hat{j}\right\|}\right]$ $= 9.\left[\dfrac{0+\hat{k}-(-\hat{k})-0}{\left\|0+\hat{k}-(-\hat{k})-0\right\|}\right]$ by right handed system $= 9.\left[\frac{2\hat{k}}{\|2\hat{k}\|}\right]$=$9\hat{k}$ (since $\|2\hat{k}\| = 2\|\hat{k}\| = 2$)	1 $\frac{1}{2}$ $\frac{1}{2}$

23.	Given: $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ $\Rightarrow \frac{dy}{dx} = \frac{e^x}{e^y} + \frac{x^2}{e^y}$ $\Rightarrow e^y \ dy = (e^x + x^2) \ dx$ $\Rightarrow \int e^y \ dy = \int (e^x + x^2) \ dx$ $\Rightarrow e^y = e^x + \frac{x^3}{3} + C$	$\frac{1}{2}$ 1 $\frac{1}{2}$
24.	Given that: $\frac{dy}{dx} = ye^x$ and x=0, y=e $\Rightarrow \frac{1}{y} \ dy = e^x \ dx$ (variables separation) $\Rightarrow \int \frac{1}{y} \ dy = \int e^x \ dx$ $\Rightarrow \log y = e^x + C$……….(i) But $y = e$ when $x = 0$, (i) gives $\log e = e^0 + C \Rightarrow 1 = 1 + C \Rightarrow C = 0$ From (i), now $\log y = e^x$ When$x = 1$, $\log y = e^1 = e \Rightarrow y = e^e$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25	Given lines: $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = 2\hat{j} - 5\hat{k} + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$ Direction ratios of the line are $\langle 2,1,2 \rangle$ and $\langle 6,3,2 \rangle$ If θ ia an angle between the lines, then $\cos \theta = \frac{2.6 + 1.3 + 2.2}{\sqrt{2^2 + 1^2 + 2^2}\sqrt{6^2 + 3^2 + 2^2}} = \frac{19}{\sqrt{9}\sqrt{49}} = \frac{19}{21}$ $\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right)$	1 1
	SECTION C **(Each question carries 3 marks)**	
26	 Let the parallelogram be ABCD with $\overrightarrow{AB} = \overrightarrow{DC} = 2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}$ and $\overrightarrow{AB} = \overrightarrow{BC} = 2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}$ Now, $\overrightarrow{d_1} = \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}) + (2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}) = 4\,\hat{i} - 2\,\hat{j} - 2\,\hat{k}$ $\overrightarrow{d_2} = \overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{AD} = -(2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}) + (2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}) = 0\,\hat{i} + 6\,\hat{j} + 8\,\hat{k}$ Also, $\lvert\overrightarrow{d_1}\rvert = \sqrt{4^2 + 2^2 + (-2)^2} = \sqrt{24}$ and $\lvert\overrightarrow{d_2}\rvert = \sqrt{0^2 + 6^2 + 8^2} = \sqrt{100} = 10$	1 1 1

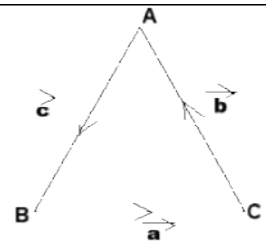
	Thus, $\widehat{d_1} = \frac{\overrightarrow{d_1}}{\lvert\overrightarrow{d_1}\rvert} = \frac{4\,\hat{i}-2\,\hat{j}-2\,\hat{k}}{\sqrt{24}}$ and $\widehat{d_2} = \frac{\overrightarrow{d_2}}{\lvert\overrightarrow{d_2}\rvert} = \frac{6\,\hat{j}+8\,\hat{k}}{10}$ Area of parallelogram$= \frac{1}{2}\lvert\overrightarrow{d_1}\times\overrightarrow{d_2}\rvert = \frac{1}{2}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & -2 & -2\\ 0 & 6 & 8\end{vmatrix} =$ $\frac{1}{2}\lvert -4\,\hat{i} - 32\,\hat{j} + 24\,\hat{k}\rvert = \frac{1}{2}\sqrt{1616}$	
26. (OR)	Given : A, B, C, D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\,\hat{i} - \hat{j} + 3\,\hat{k}$, $2\,\hat{i} - 3\,\hat{k}$ and $3\,\hat{i} - 2\,\hat{j} + \hat{k}$ respectively. Now, $\overrightarrow{AB} = p.v.of\ B - p.v.of\ A = (2\,\hat{i} - \hat{j} + 3\,\hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = \hat{i} - 2\,\hat{j} + 4\,\hat{k}$ $\overrightarrow{CD} = p.v.of\ D - p.v.of\ C = (3\,\hat{i} - 2\,\hat{j} + \hat{k}) - (2\,\hat{i} - 3\,\hat{k}) = \hat{i} - 2\,\hat{j} + 4\,\hat{k}$ Hence, projection of$\overrightarrow{AB}$ along $\overrightarrow{CD} = \frac{\overrightarrow{AB}\,.\,\overrightarrow{CD}}{\lvert\overrightarrow{CD}\rvert} = \frac{1.1+(-2).(-2)+4.4}{\sqrt{1^2+(-2)^2+4^2}} = \frac{21}{\sqrt{21}} =$ $\sqrt{21}$sq.units	1 2
27.	Given :$\alpha = 60^0, \beta = 45^0$ We know,$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\Rightarrow \cos^2 60^0 + \cos^2 45^0 + \cos^2\gamma = 1$ $\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$ $\Rightarrow \cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$ $\therefore\ \cos\gamma = \frac{1}{2} \Rightarrow\ \gamma = 60^0$ $\therefore l = \cos 60^0 = \frac{1}{2}, m = \cos 45^0 = \frac{1}{\sqrt{2}}, n = \cos 45^0 = \frac{1}{2}$ $\therefore\ \overrightarrow{OA} = \lvert\overrightarrow{OA}\rvert(l\hat{\imath} + m\hat{\jmath} + n\hat{k})$ $= 10\left(\frac{1}{2}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath} + \frac{1}{2}\hat{k}\right) = 5\hat{\imath} + 5\sqrt{2}\hat{\jmath} + 5\hat{k}$	1 1 1
28	 Diagonals are OE,AF,BG,CD. Direction ratios of OE are $\langle a - 0, a - 0, a - 0\rangle$ $i.e. \langle a, a, a\rangle$ $\therefore$ direction cosines of OE are	1 1

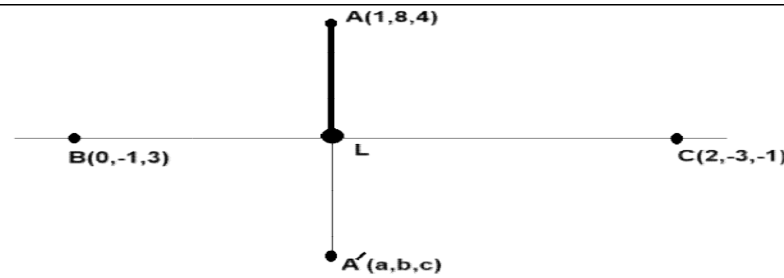
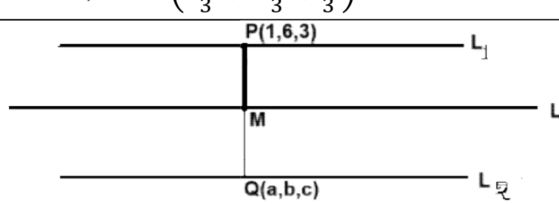
	$\langle\frac{a}{\sqrt{a^2+a^2+a^2}},\frac{a}{\sqrt{a^2+a^2+a^2}},\frac{a}{\sqrt{a^2+a^2+a^2}}\rangle$ $i.e.\langle\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\rangle$ Similarly, direction cosines of AF,BG,CD are $\langle\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\rangle,\langle\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}\rangle,\langle\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}\rangle$ respectively Let α be the angle between the two diagonals OE& AF. We have, $cos\alpha = l_1l_2+m_1m_2+n_1n_2$ $=\frac{1}{\sqrt{3}}.\frac{-1}{\sqrt{3}}+\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}=\frac{-1}{3}+\frac{1}{3}+\frac{1}{3}$ $=\frac{1}{3}$ $\therefore \alpha = \cos^{-1}\left(\frac{1}{3}\right)$ Similarly ,we can prove that angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$	1
29.	Given, $\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0; y=\frac{\pi}{4}$ when $x=1$ $\Rightarrow \frac{\left[x\sin^2\left(\frac{y}{x}\right)-y\right]}{x}+\frac{dy}{dx}=0$ $\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$(It is homogeneous differential equation) Putting $y = vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\Rightarrow v+x\frac{dv}{dx}=v-\sin^2 v$ $\Rightarrow x\frac{dv}{dx}=-\sin^2 v$ $\Rightarrow \frac{dv}{\sin^2 v}=-\frac{dx}{x}$ Integrate on both sides $\Rightarrow \int \mathrm{cosec}^2 v\, dv = -\int\frac{1}{x}\,dx$ $\Rightarrow -\cot v = -\log x + C$ $\Rightarrow \log x - \cot\left(\frac{y}{x}\right) = C \ldots\ldots\ldots (i)$ $y=\frac{\pi}{4}$ when $x=1$, (i)$\Rightarrow \log 1-\cot\left(\frac{\frac{\pi}{4}}{1}\right)=C\Rightarrow 0-1=C\Rightarrow C=-1$ Hence, the reqd. particular solution is,$\log x-\cot\left(\frac{y}{x}\right)=-1$	1 1 1
30.	Given:$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$; $y=0$ when $x=1$ $\Rightarrow\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{(1+x^2)^2}$,which is in the form of $\frac{dy}{dx}+Py=$ Q i.e. linear D.E. in y. Now,$\mathrm{I.F}=e^{\int P\,dx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$ Hence,the solution of the D.E.:$y.(\mathrm{I.F})=\int Q(\mathrm{I.F})dx+C$	1 1

	$\therefore y.(1+x^2)=\int\frac{1}{(1+x^2)^2}.(1+x^2)dx+C$ $=\int\frac{1}{(1+x^2)}dx+C$ $\therefore y.(1+x^2)=\tan^{-1}x+C$ ……(i) By question $y=0$ when $x=1$,(i) $\Rightarrow 0.(1+1^2)=\tan^{-1}1+C \Rightarrow 0=\frac{\pi}{4}+C \Rightarrow C=-\frac{\pi}{4}$ Hence, the reqd. particular solution is, $y.(1+x^2)=\tan^{-1}x-\frac{\pi}{4}$	1
31.	Given, projection of $\vec{b}$ along $\vec{a}$ = the projection of $\vec{c}$ along $\vec{a}$ $\Rightarrow\frac{\vec{b}.\vec{a}}{\lvert\vec{b}\rvert}=\frac{\vec{c}.\vec{a}}{\lvert\vec{b}\rvert}$ $\Rightarrow\vec{b}.\vec{a}=\vec{c}.\vec{a}$ ....(i) Also, given $\vec{b}\perp\vec{c}\Rightarrow\vec{b}.\vec{c}=0$ ……(ii) Then, $\lvert 3\vec{a}-2\vec{b}+2\vec{c}\rvert^2=9\lvert\vec{a}\rvert^2+4\lvert\vec{b}\rvert^2+4\lvert\vec{c}\rvert^2-12\,\vec{a}.\vec{b}-8\vec{b}.\vec{c}+12\vec{c}.\vec{a}$ $=9.1^2+4.2^2+4.3^2-12\,\vec{a}.\vec{b}-0+12\,\vec{a}.\vec{b}$, (by(i) & (ii)) $=9+16+36=31$	1 1 1
31. (OR)	Let $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+2\hat{\jmath}+3\hat{k}$. According to question, $\lvert\vec{a}\times\hat{p}\rvert=\sqrt{2}$, where $\hat{p}=\frac{\vec{b}+\vec{c}}{\lvert\vec{b}+\vec{c}\rvert}=\frac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{\imath}+6\hat{\jmath}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=$ $\therefore\frac{1}{\sqrt{\lambda^2+4\lambda+44}}\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\1&1&1\\2+\lambda&6&-2\end{vmatrix}=\sqrt{2}$ $\Rightarrow\lvert(-2-6)\hat{\imath}-\{-2-(2+\lambda)\}\hat{\jmath}+\{6-(2+\lambda)\}\hat{k}\rvert$ $=\sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow\lvert-8\hat{\imath}+(4+\lambda)\hat{\jmath}+(4-\lambda)\hat{k}\rvert=\sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow\sqrt{(-8)^2+(4+\lambda)^2+(4-\lambda)^2}=\sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow\lambda=1$ (after squaring on both sides)	1 1 1

SECTION D

(Each question carries 5 marks)

32.	 Here, by triagle law of vector addition $\vec{a}+\vec{b}=-\vec{c}$ ……….(i) By pre cross multiplication of (i) by $\vec{a}$, we get $\vec{a}\times\vec{a}+\vec{a}\times\vec{b}=-\vec{a}\times\vec{c}$ $\Rightarrow\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$………(ii) By post cross multiplication of (i) by $\vec{b}$, we get $\vec{a}\times\vec{b}+\vec{b}\times\vec{b}=-\vec{c}\times\vec{b}$ $\Rightarrow\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$………(iii) From (ii) and (iii) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ $\Rightarrow\lvert\vec{a}\times\vec{b}\rvert=\lvert\vec{b}\times\vec{c}\rvert=\lvert\vec{c}\times\vec{a}\rvert$ $\Rightarrow\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin(\pi-C)=\lvert\vec{b}\rvert\lvert\vec{c}\rvert\sin(\pi-A)=\lvert\vec{c}\rvert\lvert\vec{a}\rvert\sin(\pi-B)$ $\Rightarrow ab\sin C=bc\sin A=ca\sin B$ Dividing by abc, we get $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{si}{c}$	0.5 0.5 1 1 2
32.(OR)	Here, $\lvert\vec{a}+\vec{b}\rvert^2=\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+2\,\vec{a}\,.\vec{b}$ $=1^2+1^2+2\,\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\Theta$ $=1^2+1^2+2\,.1.1.\cos\Theta$ $=2+2\cos\Theta=2(1+\cos\Theta)=2.2\cos^2\frac{\theta}{2}=4\cos^2\frac{\theta}{2}$ $\therefore\ \lvert\vec{a}+\vec{b}\rvert^2=4\cos^2\frac{\theta}{2}$ $\Rightarrow\lvert\vec{a}+\vec{b}\rvert=2\cos\frac{\theta}{2}$…….(i) And , $\lvert\vec{a}-\vec{b}\rvert^2=\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2-2\,\vec{a}\,.\vec{b}$ $=1^2+1^2-2\,\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\Theta$ $=1^2+1^2-2\,.1.1.\cos\Theta$ $=2-2\cos\Theta=2(1-\cos\Theta)=2.2\sin^2\frac{\theta}{2}=4\sin^2\frac{\theta}{2}$ $\therefore\ \lvert\vec{a}-\vec{b}\rvert^2=4\sin^2\frac{\theta}{2}$ $\Rightarrow\lvert\vec{a}-\vec{b}\rvert=2\sin\frac{\theta}{2}$…….(ii) Dividing (ii) by (i)	2 2 1

	$$\tan\frac{\theta}{2}=\frac{\lvert\vec{a}-\vec{b}\rvert}{\lvert\vec{a}+\vec{b}\rvert}$$	
33.	 Equation of BC:$\frac{x-0}{2-0}=\frac{y+1}{-3+1}=\frac{z-3}{-1-3}$ i.e. $\frac{\boldsymbol{x-0}}{\mathbf{2}}=\frac{\boldsymbol{y+1}}{\mathbf{-2}}=\frac{\boldsymbol{z-3}}{\mathbf{-4}}=\lambda$, with dirction ratios$\langle 2,-2,-4\rangle$ Now , coordinates of L point on the line BC are $(2\lambda,\ -2\lambda-1,\ -4\lambda+3)$. Thus, direction ratios of AL are $\langle 2\lambda-1,-2\lambda-1-8,\ -4\lambda+3-4\rangle$ i.e.$\langle 2\lambda-1,-2\lambda-9,\ -4\lambda-1\rangle$ L is the foot of the perpendicular drawn from A on BC. Therefore, AL is perpendicular to BC. So, we have $2(2\lambda-1)+(-2)(-2\lambda-9)+(-4)(-4\lambda-1)=0$. $\Rightarrow 4\lambda-2+4\lambda+18+16\lambda+4=0$ $\Rightarrow 24\lambda+20=0 \Rightarrow \lambda=\frac{-5}{6}$ Hence , $L=(2\lambda,-2\lambda-1,-4\lambda+3)=\left(2\left(\frac{-5}{6}\right),\ -2\left(\frac{-5}{6}\right)-1,-4\left(\frac{-5}{6}\right)+3\right)=\left(\frac{-5}{3},\ \frac{2}{3},\frac{19}{3}\right)$ The length of the perpendicular AL$=\sqrt{\left(1+\frac{5}{3}\right)^2+\left(8-\frac{2}{3}\right)^2+\left(4-\frac{19}{3}\right)^2}=\frac{\sqrt{597}}{3}$ If $A'(a,b,c)$ be the image of A(1,8,4) in the line through B and C, then L is the mid pint of AA' Therefore, $\frac{-5}{3}=\frac{1+a}{2},\frac{2}{3}=\frac{8+b}{2},\frac{19}{3}=\frac{4+c}{2}$ $\Rightarrow$ a$=\frac{-13}{3},\ b=\frac{-20}{3},\ \ c=\frac{26}{3}$ Hence, $A'=\left(\frac{-13}{3},\ \frac{-20}{3},\frac{26}{3}\right)$	1 1 1 1 1
33.(OR)		1

	$L: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$, with direction ratios $\langle 1,2,3 \rangle$ Coordinates of any point M on the L i.e. M= $(\lambda, 2\lambda+1, 3\lambda+2)$. Now, direction ratios of line PM are $\langle \lambda-1, 2\lambda+1-6, 3\lambda+2-3 \rangle$ i.e.$\langle \lambda-1, 2\lambda-5, 3\lambda-1 \rangle$ If M is the foot of the perpendicular drawn from P on the line L. Then PM is perpendicular to L $\therefore 1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow 14\lambda-14=0 \Rightarrow \lambda=1$ Hence, M= $(\lambda, 2\lambda+1, 3\lambda+2) = (1, 3, 5)$ Let Q(a, b, c) be the image of P(1, 6, 3) in the line L (but on the line L_2). Then, M is the mid-point of PQ (as object distance from the mirror is equal to the image distance from the mirror) Therefore, $1 = \frac{1+a}{2}, 3 = \frac{6+b}{2}, 5 = \frac{3+c}{2}$ $\Rightarrow$ a= 1, $b=0$, $c=7$ Thus , a point on the line L_2 is Q(1,0,7) Hence the equation of the line L_2 is $\frac{x-1}{1} = \frac{y}{2} = \frac{z-7}{3}$(since the lines are parallel , directions ratios are remain same)	1 1 1 1
34.	Given lines are $L_1: \vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ $= \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$ $= (\hat{i} - 2\hat{j} + 3\hat{k}) - t(\hat{i} - \hat{j} + 2\hat{k})$. $L_2: \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$. $= s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$. $= (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$ Now,$\overrightarrow{a_2} - \overrightarrow{a_1} = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = 0\hat{i} + \hat{j} - 4\hat{k}$ Also, $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(2-4) - \hat{j}(-2-2) + \hat{k}(2+1)$ $= -2\hat{i} + 4\hat{j} + 3\hat{k}$. Hence, shortest distance between two lines L_1 and L_2= $\left\lvert \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert} \right\rvert$ $= \left\lvert \frac{0.(-2)+1.4+(-4).3}{\sqrt{(-2)^2+4^2+3^2}} \right\rvert = \frac{8}{\sqrt{29}}$ units.	1 1 1 2

35.

Given: $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$ 1

$$\Rightarrow y + x\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow x\frac{dy}{dx} + 2y = x(\sin x + \log x)$$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$ which is a linear D.E. in the form $\frac{dy}{dx} + Py = Q$

Now, $I.F = e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = e^{2lo} = e^{\log x^2} = x^2$. 1

Hence, the solution of the D.E.: $y.(I.F) = \int Q(I.F)dx + C$

$\therefore y.x^2 = \int x^2(\sin x + \log x)dx + C.$

$$\therefore y.x^2 = \int x^2 \sin x\,dx + \int x^2 \log x\,dx + C$$

$$= x^2(-\cos x) - \int \{2x\,(-\cos x)\}\,dx + \log x.\frac{x^3}{3} - \int \left\{\frac{1}{x}.\frac{x^3}{3}\right\}dx + C$$

$$= -x^2\cos x + 2\int x.\cos x\,dx + \frac{x^3}{3}.\log x - \frac{1}{3}\int x^2 dx + C$$

$$= -x^2\cos x + 2\left[x.\sin x - \int 1.\sin x\,dx\right] + \frac{x^3}{3}.\log x - \frac{1}{3}.\frac{x^3}{3} + C$$ 3

$$= -x^2\cos x + 2[x.\sin x - (-\cos x)] + \frac{x^3}{3}.\log x - \frac{x^3}{9} + C$$

Section –E

(This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36. 1

Corners of the roof are P_1 (6,8,4), P_2 (21,8,4), P_3 (21,16,10) and P_4 (6,16,10).

(i) Here, $\vec{A}$ = PV of P_2 – PV of P_1 = $(21\hat{\imath} + 8\hat{\jmath} + 4\hat{k}) - (6\hat{\imath} + 8\hat{\jmath} + 4\hat{k}) = 15\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

and $\vec{B}$ = PV of P_4 – PV of P_1 = $(6\hat{\imath} + 16\hat{\jmath} + 10\hat{k}) - (6\hat{\imath} + 8\hat{\jmath} + 4\hat{k}) = 0\hat{\imath} + 8\hat{\jmath} + 6\hat{k}$ 1

Therefore, components of $\vec{A}$ and $\vec{B}$ are 15 , 0 , 0 and 0 . 8 , 6

(ii) $\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix} = \hat{\imath}(0-0) - \hat{\jmath}(90-0) + \hat{k}(120-0) = 0\hat{\imath} - 90\hat{\jmath} + 120\hat{k}$ 2

(iii) Given: $\vec{F} = 6\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Now, $\vec{F}.\vec{N} = 6 \times 0 + 2 \times 90 + 3 \times 120 = 540$

OR

$$\cos\theta = \frac{\vec{F}\,.\,\vec{N}}{|\vec{F}||\vec{N}|} = \frac{540}{\sqrt{(6)^2 + (-2)^2 + (3)^2}\sqrt{(0)^2 + (-90)^2 + (120)^2}} = \frac{540}{7 \times 150} = \frac{18}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{18}{35}\right)$$

37.	Given, line for motorcycle A, L_1: $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ And, line for motorcycle B, L_2: $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ (i) L_1: $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ $\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ $\therefore x=\lambda$, $y= 2\lambda$, $z= -\lambda$ $\therefore \frac{x}{1} = \lambda, \quad \frac{y}{2} = \lambda, \quad \frac{z}{-1} = \lambda,$ $\therefore \frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = \lambda$, which is the reqd. cartesian equation. (ii) L_2: $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ Direction ratios are $\langle 2, 1, 1 \rangle$ $\therefore$ direction cosines of OE are $\langle \frac{2}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}} \rangle$ $i.e. \langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ (iii) Now,$\overrightarrow{a_2} - \overrightarrow{a_1} = (3\hat{i} + 3\hat{j}) - (0\hat{i} + 0\hat{j} + 0\hat{k}) = 3\hat{i} + 3\hat{j}$ Also, $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(2+1) - \hat{j}(1+2) +$ $\hat{k}(1-4) = 3\hat{i} - 3\hat{j} - 3\hat{k}$. Hence, shortest distance between two lines L_1 and $L_2 = \left\| \frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{\|\overrightarrow{b_1}\times\overrightarrow{b_2}\|} \right\|$ $= \left\| \frac{3.(3)+3.(-3)+0.(-3)}{\sqrt{3^2+(-3)^2+3^2}} \right\| = 0$ units OR $\cos\theta = \frac{\overrightarrow{b_1} . \overrightarrow{b_2}}{\|\overrightarrow{b_1}\|\|\overrightarrow{b_2}\|} = \frac{1.2 + 2.1 + (-1).1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}\sqrt{(2)^2 + (1)^2 + (1)^2}} = \frac{3}{6} = \frac{1}{2}$ $\therefore \theta = 60^0$	1 1 2
38.	(i) Given :$\mathbf{\frac{dP}{dt} = kP}$ $\Rightarrow \frac{dP}{P} = k\,dt$ Integrate on both sides $\Rightarrow \int \frac{1}{P}\,dP = \int k\ \ dt$ $\Rightarrow \log P = kt + C$ $\Rightarrow \log P = kt + \log C$ $\Rightarrow \log P - \log C = kt$ $\Rightarrow \log \frac{P}{C} = kt$ $\therefore \frac{P}{C} = e^{kt}$ $\therefore P = C.e^{kt}$............(i) (ii) According to question, at t=0,P=1000 In this case,(i) $\Rightarrow 1000 = C.e^0$ $\Rightarrow C = 1000$	2 2

	Hence, (i) $\Rightarrow P = 1000\ e^{kt}$……….(ii) Again, at t=1,P=2000 In this case,(ii)) $\Rightarrow 2000 = 1000\ e^{k}$ $\Rightarrow 2 = e^{k} \therefore k = \log 2$	

NAVODAYA VIDYALAYA SAMITI – RO SHILLONG
CLASS: XII
SUBJECT: MATHEMATICS (041)
SESSION: 2024-25
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: Inverse Trigonometric functions, LPP and Probability)

Time: 3 Hours **Max. Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION A [1×20 = 20]

<u>This section contains multiple choice question (MCQ) 1 mark each</u>

1.The value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

2.$\tan^{-1}\sqrt{3}$ - $\cot^{-1}(-\sqrt{3})$ is equal to

A) π B) $\frac{-\pi}{2}$
C) 0 D) $2\sqrt{3}$

3.The value of $\cot(\sin^{-1} x)$ is

(A) $\frac{\sqrt{1+x^2}}{x}$ (B) $\frac{x}{\sqrt{1+x^2}}$
(C) $\frac{1}{x}$ (D) $\frac{\sqrt{1-x^2}}{x}$

4.The value of $2\sec^{-1}\sqrt{2} + \sin^{-1}\frac{1}{2}$ is

(A) $\frac{7\pi}{6}$ (B) $\frac{\pi}{6}$

(C) $\frac{5\pi}{6}$ (D) $\frac{2\pi}{3}$

5.If $\sin^{-1} x$= y

(A) $0\leq y \leq \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0<y<\pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

6.Which of the following corresponds to the principal value branch of $\tan^{-1} x$

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (D) $(0, \pi)$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

7.The domain of the function defined by f(x)=$\sin^{-1} \sqrt{x-1}$ is

(A)[1,2] (B)[-1 , 1]

(C)[0,1] (D)[-1,0]

8.The corner points of the feasible region in the graphical representation of a LPP are (2,72),(15,20) and (40,15).If Z=18x+9y be the objective function, then

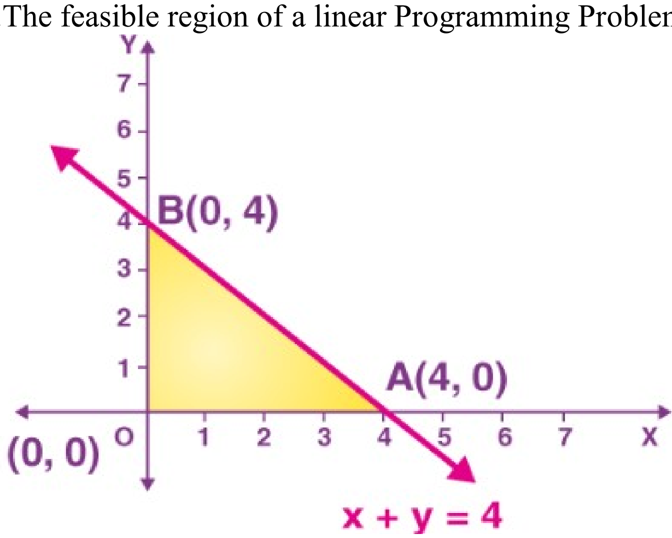
(A) Z is maximum at (2,72),minimum at (15,20)

(B) Z is maximum at (15,20),minimum at (40,15)

(C) Z is maximum at (40,15),minimum at (15,20)

(D) Z is maximum at (40,15),minimum at (2,72)

9.The feasible region of a linear Programming Problem is shown in the figure below



Which of the following are the possible constraints?

(A) $x + y > 4, x \geq 0, y \geq 0$

(B) $x + y \leq 4, x < 0, y \geq 0$

(C) $x + y \leq 4, x \geq 0, y \geq 0$

(D) $x + y \leq 4, x \geq 0, y < 0$

10.In a linear programming problem, feasible region is the region where
(A)All possible solutions satisfying all the constraints of the problems exist.

(B)Only optimal solution exist

(C)Only non-negative solutions exist

(D)None of these

11.In an LPP, if the objective function Z=ax+by has the same maximum value on two corner points of the feasible region ,then the number of points of which Z_{max} occurs is
(A) 0 (B) 2 (C) Finite (D) Infinite

12.Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0).Let z=px+qy,where p,q>0.Condition on p and qso that the minimum of Z occurs at (3,0) and (1,1) is
(A) p=2q (B) p=$\frac{q}{2}$ (C) p=3q (D) p=q

13.The corner points of the feasible region of an LPP are (0,4),(0.6,1.6) and (3,0). The minimum value of the objective function z = 4x + 6y occurs at
(A) (0.6,1.6) only
(B) (3,0) only
(C) (0.6,1.6) and (3,0) only
(D) at every point of the line segment joining points (3,0) and (0.6,1.6)

14.In a single throw of a die, A = event of getting odd numbers and B = event of getting prime numbers,
A) A and B are independent events
B) A and B are not independent events
C) $\boldsymbol{P(A|B)} = \frac{1}{3}$
D) None of these

15.If for any two events A and B ,P(A)=$\frac{4}{5}$ and P(A∩B) = $\frac{7}{10}$,then P(B/A) is equal to
(A)$\frac{1}{10}$
B) $\frac{1}{8}$
(C)$\frac{7}{8}$
D) $\frac{17}{20}$

16.A bag contains 3 white,4 black and 2 red balls. If 2 balls are drawn at random (without replacement),then the probability that both the balls are white is
(A)$\frac{1}{18}$
B) $\frac{1}{36}$
(C)$\frac{1}{12}$
D) $\frac{1}{24}$

17.Two dice are thrown together. Let A be the given event 'getting 6 on the first die' and B be the event 'getting 2 on the second die', then P$(\boldsymbol{A} \cap \boldsymbol{B})$ is
(A)$\frac{1}{36}$
B) $\frac{7}{4}$
(C)$\frac{9}{20}$
D) None of these

18. Assume that in a family ,each child is equally likely to be a boy or a girl.A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is

(A)$\frac{1}{2}$ B) $\frac{1}{3}$

(C)$\frac{2}{3}$ D) $\frac{4}{7}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19.Assertion(A): Two coins are tossed simultaneously.The probability of getting two heads,if it is known that at least one head comes up,is$\frac{1}{3}$.

Reason(R) : Let E and F be two events with a random experiment then P(F/E)=$\frac{P(E\cap F)}{P(E)}$.

20.Assertion(A): Function f:R→ $\boldsymbol{R}$ given by f(x)= sinx is not a bijection.

Reason(R):A function f:A→ $\boldsymbol{B}$ is said to be bijection if it is one – one and on to.

Section B **5 x 2 = 10**

<u>This section contains 5 very short answer type (VSA) of 2 marks each</u>

21.Simplify: $\tan^{-1}\frac{1-\sin\theta}{\cos\theta}$

22. $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

23.Find the domain of y=$\sin^{-1}(x^2-4)$

OR,

Find the range of f(x) = 2$\sin^{-1}x+\frac{3\pi}{2}$, where x$\in[-1,1]$

24.Maximize Z=3x+4y

Subject to the constraints x+y≤ 4,x≥ 0, y≥ 0.

OR,

Minimize Z=-3x+4y

Subject to the constraints x+2y≤ 8,3x+2y≤ $\mathbf{12}$,,x≥ 0, y≥ 0.

25.An unbiased die is thrown twice.Let the event A be 'odd number on the first throw'and B be the event 'odd number on the second throw'.Check the independence of event A and B.

Section C **6 x 3 = 18**

<u>This section contains 6 short answer type (SA) of 3 marks each</u>

26. Solve for x $\mathbf{\sin^{-1}(1-x)}$-2$\mathbf{\sin^{-1} x} = \frac{\pi}{2}$

27. Show that: $\mathbf{\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)} = \frac{4-\sqrt{7}}{3}$

28. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

OR,

The probability of two students A and B coming to school in time are $\frac{2}{7}$ and $\frac{4}{7}$,respectively. Assuming that the events 'Acoming on time' and 'B coming on time' are independent, Find the probability of only one of them coming to school on time.

29.Probability that at least one of the two events A and B occurs is 0.6.If A and B occur simultaneously with probability 0.3,evaluate P($\overline{A}$) + P($\overline{B}$)

OR,

If A and B are two independent events ,then the probability of occurrence of at least one of A and B is given by 1- P($\overline{A}$)P($\overline{B}$)

30.Determine the maximum value of z =11x+7y subject to the constraints

$$2x + y \leq 6,\ x \leq 2,\ x \geq 0,\ y \geq 0$$

31.Two dice are thrown together and the total score is noted. The events E,F and G are 'a total score of 4', 'a total score of 9 or more' and 'a total score divisible by 5' respectively.

Calculate P(E), P(F) and P(G) and decide which pairs of events are independent.

OR,

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits the target ,is $\frac{2}{5}$.If both try to hit the target independently ,find the probability that the target is hit.

Section D **4 x 5 = 20**

<u>This section contains 4long answer type (LA) of 5 marks each</u>

32. It is believed that the the smoke from the candles on the birth day cake would carry wishes and prayers to the Gods and serves as a centrepiece during the parties and acts as a focal point for the celebration. Sharing a slice of cake with friends and family fosters a sense of togetherness and strengthens social bonds.

On the birthday ceremony of Meenu ,Her parent ordered two kinds of cake in a bakery. The ingredients required for baking these cakes are as follows-

one kind of cake requires 200gm of flour and 25 gm of fat, another kind of cake requires 100 gm of flour and 50 gm of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

33. As the Bond investing can effectively mitigate risk and offer ones investment portfolio fixed income, capital preservation, and diversification benefits. Therefore, A retired person wants to invest an amount of ₹ 50000. His broker recommends investing in two types of bonds A' and 'B' yielding 10% and 9% return respectively on the invested amount

.
He decides to invest at least ₹ 20000 in bond A' and at least ₹ 10000 in bond 'B'.
He also wants to invest at least as much in bond A' as in bond 'B'.
Solve this linear programming problem graphically to maximise his returns.

.

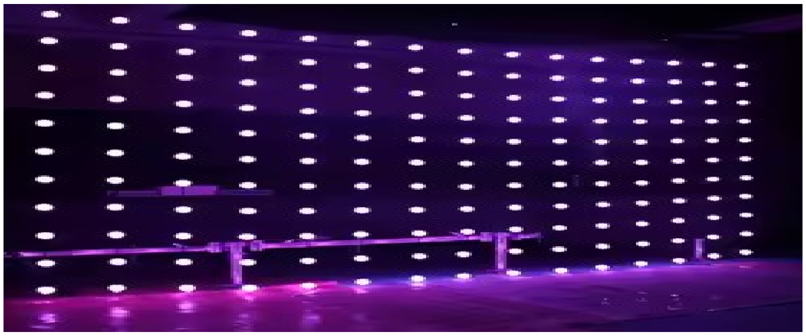
34. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning ,if A starts first

OR,

Manisha,a girl of JNV, likes to decorate her house premise in almost every festivals. This year, in the festival of Deepawali she was decorated her house with lighting the tinny electric bulbs of different four colours purchased from the market.

The coloured of electric balls were packed in four different boxes as shown in the following table:

Box	Green	Yellow	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5



A box is selected at random and then an electric bulb is randomly drawn from the selected box.The colour of the bulb is green, what is the probability that the bulb drawn is from the box III?

35. A card from a Pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

OR,

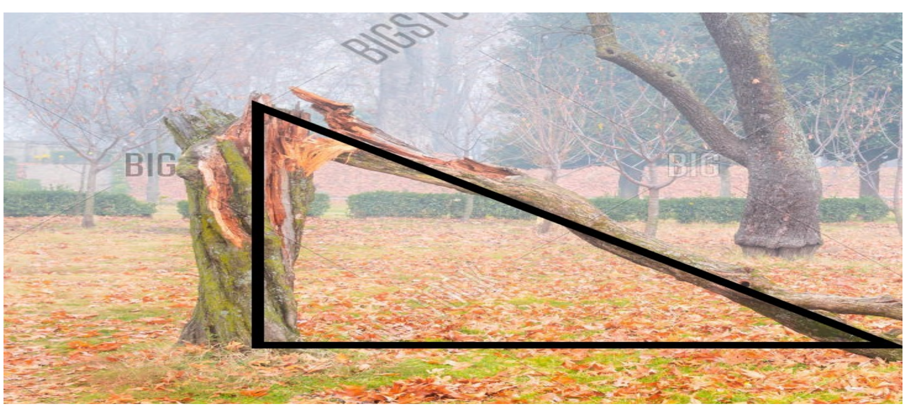
A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Section E **3 x 4 = 12**

This section contains 3 case study based questions of 4 marks each

36. ***Case study-1***

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle θ with it .The distance between the foot of the tree to the point where the top touches the grounds is'a' metre.The height of the tree after breaking is 'b' metre



Based on the given information, solve the following questions:

(A)Find the angle $\boldsymbol{\theta}$ in terms of $\boldsymbol{sin^{-1}}$. 1

(B) Find the angle $\boldsymbol{\theta}$ in terms of $\boldsymbol{cos^{-1}}$. 1

(C) If the distance between the foot of the tree where the top touches the ground 5 m and the height of the tree after breaking is 2m. Find the angle made by the broken part of the tree which touches the ground with the standing part of the tree $\boldsymbol{\theta}$ in $\boldsymbol{sin^{-1}}$ 2

Or

If the distance between the foot of the tree where the top touches the ground 5 m and the height of the tree after breaking is 2m. Find the angle made by the broken part of the tree which touches the ground with the standing part of the tree in $\boldsymbol{cos^{-1}}$ 2

37. Case study – 2:

A fruit vendor wants sale a box of oranges which contained 12 good oranges and 3 bad oranges to a customer out of his stall. The box is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise it is rejected.

Based on the given information, answer the following questions:

Based on the given information, answer the following questions:

(A) In how many ways 3 oranges (at a time) can be drawn out of the total oranges? 1

(B) How many arrangements of 3 oranges out of the total arrangement contain only good oranges. 1

(C) Find the probability that the box is approved for sale. 2

Or

Find the probability that the box is not approved for sale. 2

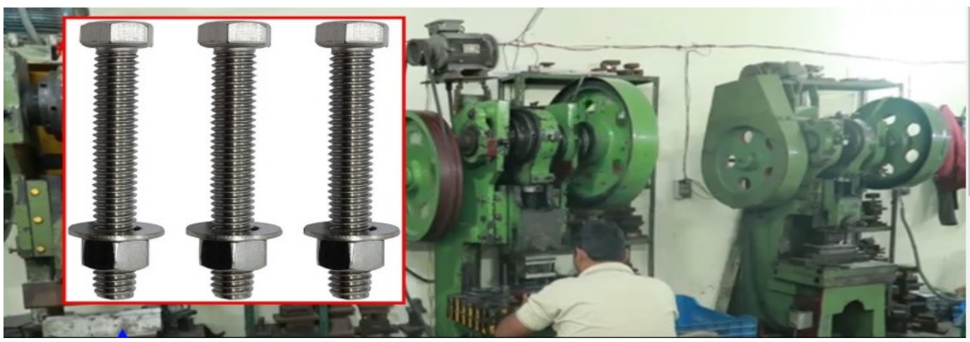
38. Case study-3

Nitish is a manufacturer of nuts and bolts. Recently he has installed two latest versions of machines viz machine A and machine B for producing nuts and bolts in his factory. The capacity of doing work of the machines is as follows:

It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts.

How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

Based on the given information, answer the following questions:



(A)If the manufacturer produce x package of nuts and y package of bolts, then write the constraints. 1

(B) If he earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts, then express the profit z in terms of x and y. 1

(C) How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day? Find graphically. 2

Or

What is the maximum profit if he operates his machines for at the most 12 hours a day? Find graphically. 2

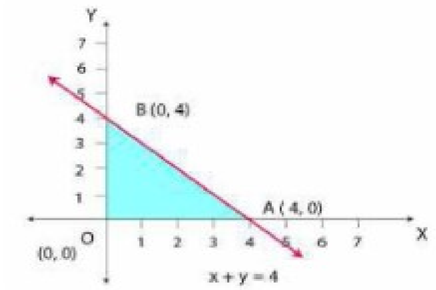
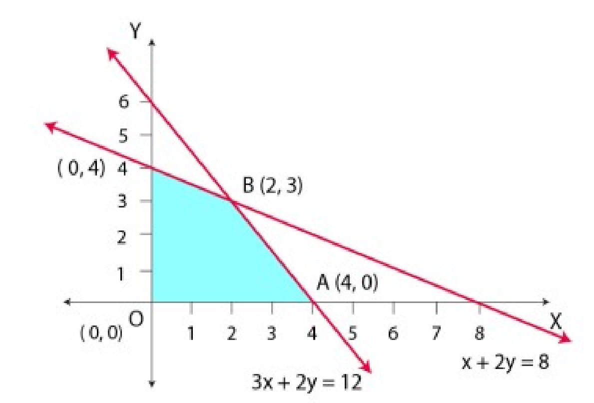
NAVODAYA VIDYALAYA SAMITI – RO SHILLONG
CLASS: XII
SUBJECT: MATHEMATICS (041)
SESSION: 2024-25
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: Inverse Trigonometric functions, LPP and Probability)

Time: 3 Hours **Max. Marks: 80**

Marking Scheme

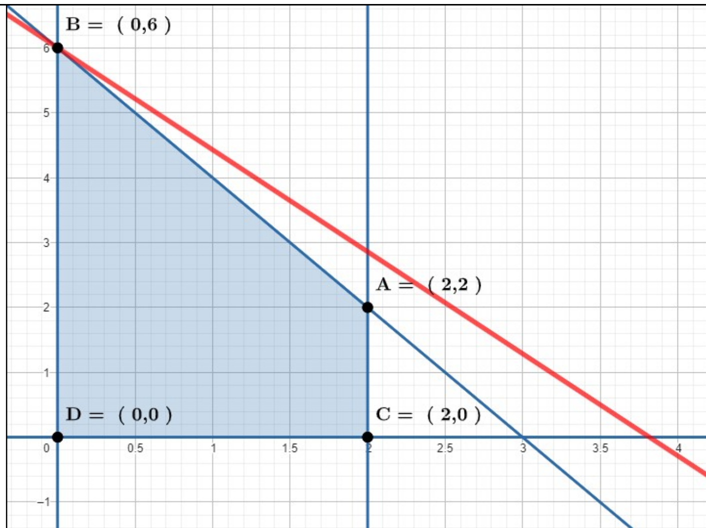
Q.NO	ANSWER	MARKS
1	(B) $\frac{5\pi}{6}$	1
2	(B) $\frac{-\pi}{2}$	1
3	**(D)** $\frac{\sqrt{1-x^2}}{x}$	1
4	(C) $\frac{5\pi}{6}$	1
5	(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	1
6	(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	1
7	(A)[1,2]	1
8	**(C) Z is maximum at (40,15),minimum at (15,20)**	1
9	**(C) x + y ≤ 4, x ≥ 0, y ≥ 0**	1
10	**(A)All possible solutions satisfying all the constraints of the problems exist.**	1
11	(D) Infinite	1
12	(B) p=$\frac{q}{2}$	1
13	D) at every point of the line segment joining points (3,0) and (0.6,1.6)	1
14	**A) B) A and B are not independent events**	1
15	(C)$\frac{7}{8}$	1
16	(C)$\frac{1}{12}$	1
17	(A)$\frac{1}{36}$	1
18	D) $\frac{4}{7}$	1
19	**(A). Both (A) and (R) are true and (R) is the correct explanation**	1

	of (A).	
20	**(A). Both (A) and (R) are true and (R) is the correct explanation of (A)**	1
21	$\tan^{-1}\frac{1-sin\theta}{cos\theta}=\tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)}\right)$ $=\tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right)$ $=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)=\frac{\pi}{4}-\frac{\theta}{2}$	1 1
22	$\mathbf{\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)}$ $=\frac{\pi}{4}+(\pi-\frac{\pi}{3})+\left(-\frac{\pi}{6}\right)$ $=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$ $=\frac{3\pi+8\pi-2\pi}{12}$ $=\frac{9\pi}{12}$ $=\frac{3\pi}{4}$	1 1
23	**Given:** $y=\sin^{-1}(x^2-4)$ $\Rightarrow -1\leq x^2-4\leq 1$ $\Rightarrow 3\leq x^2\leq 5$ $\Rightarrow\ x^2\geq 3\ and\ x^2\leq 5$ $\Rightarrow x\leq -\sqrt{3}$, $x\ \geq\sqrt{3}$ **and** $-\sqrt{5}\leq x\leq\sqrt{5}$ **The domain of y is** $[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},\sqrt{5}]$	1 $\frac{1}{2}$ $\frac{1}{2}$
23(OR)	**Given:** $f(x)=2\sin^{-1}x+\frac{3\pi}{2}$ $-\frac{\pi}{2}\leq\ \sin^{-1}x\leq\frac{\pi}{2}$ $\Rightarrow -\pi\leq\ 2\sin^{-1}x\leq\pi$ $\Rightarrow -\pi+\frac{3\pi}{2}\leq\ 2\sin^{-1}x+\frac{3\pi}{2}\leq\pi+\frac{3\pi}{2}$ $\Rightarrow -\frac{\pi}{2}\leq\ 2\sin^{-1}x+\frac{3\pi}{2}\leq\frac{5\pi}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

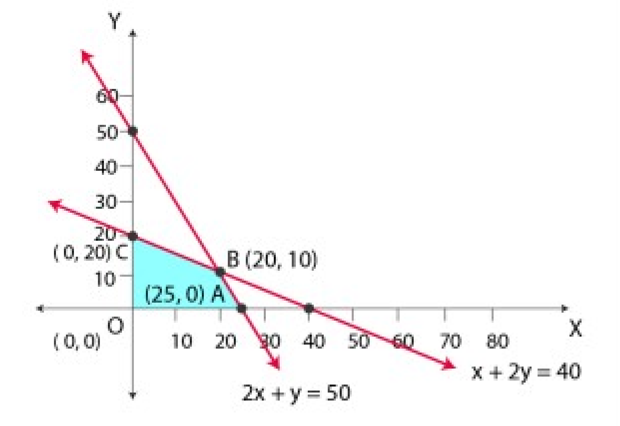
	The range of f(x) is $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$	$\frac{1}{2}$
24	 **The corner points are O(0,0),A(4,0) and B(0,4)** **Maximum Value of Z is 16 attained at B(0,4)**	For figure-1 1
24 (OR)	 **The corner points are O(0,0),A(4,0) ,B(2,3) and C(0,4)** **Minimum Value of Z is -12 attained at A(4,0)**	For figure-1 1
25	We have P(A)=$\frac{18}{36}=\frac{1}{2}$ P(B)=$\frac{18}{36}=\frac{1}{2}$ Also P($A\cap B$)=P(Odd number on both throws) $=\frac{9}{36}=\frac{1}{4}$ Now P(A)P(B)=$\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$ P($A\cap B$)= P(A)P(B) A and B are independent events	$\frac{1}{2}$ $\frac{1}{2}$ 1
26	**Given**: $\sin^{-1}(1-x)$-2$\sin^{-1}x=\frac{\pi}{2}$ $\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$ $\Rightarrow$1-x=$\sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$ $\Rightarrow$1-x=$\cos(2\sin^{-1}x)$.........(i)	1

	Let , $\sin^{-1} x = \theta$ $\Rightarrow$x=sinθ From(i),we get 1-x=cos2θ $\Rightarrow$1-x=1-2$sin^2\theta$ $\Rightarrow$1-x=1-2x^2 $\Rightarrow$ $2x^2$-x= 0$\Rightarrow$ $x(2x-1)=0$ $\Rightarrow$ x=0 or x=$\frac{1}{2}$ Putting x = $\frac{1}{2}$ in the given equation ,we get $\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$ =$\frac{\pi}{6}$ - 2$\times\frac{\pi}{6} \neq \frac{\pi}{2}$ So,$x \neq \frac{1}{2}$ $\therefore$x=0	$\frac{1}{2}$ 1 $\frac{1}{2}$
27	To show that: $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)= \frac{4-\sqrt{7}}{3}$ Let $\sin^{-1}\frac{3}{4}=\theta$ $\Rightarrow$ sinθ $=\frac{3}{4}$ $\Rightarrow \frac{2\tan\frac{\theta}{2}}{1+tan^2\frac{\theta}{2}}= \frac{3}{4}$ $\Rightarrow$3+3$tan^2\frac{\theta}{2}$=8$\tan\frac{\theta}{2}$ $\Rightarrow$3$tan^2\frac{\theta}{2}$-8$\tan\frac{\theta}{2}$ + 3=0 $\Rightarrow \tan\frac{\theta}{2} = \frac{8\pm\sqrt{64-36}}{6}$ $\Rightarrow \tan\frac{\theta}{2} = \frac{8\pm2\sqrt{7}}{6}$ $\Rightarrow \tan\frac{\theta}{2} = \frac{4\pm\sqrt{7}}{3}$ $\therefore$ $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)= \frac{4-\sqrt{7}}{3}$	1 1 1
28	Total number of questions=1400 n(S)=1400 Let E=Selected questions is easy F= Selected questions is M.C.Q E∩F= Selected questions is Easy and M.C.Q n($\mathbf{E} \cap \mathbf{F}$)=500+400=900	1

	$\therefore P(\mathbf{E} \cap \mathbf{F}) = \frac{n(E\cap F)}{n(S)} = \frac{500}{1400}$ $\therefore P(\mathbf{F}) = \frac{n(F)}{n(S)} = \frac{900}{1400}$ Required Probability=P(E/F) = $\frac{P(E\cap F)}{P(S)} = \frac{5}{9}$	1 1
28 (OR)	P(A) = $\frac{2}{7}$ i.e A coming on time, P(B) = $\frac{4}{7}$ i.e B coming on time, $P(\overline{A}) = 1- \frac{2}{7} = \frac{5}{7}$ $P(\overline{B}) = 1- \frac{4}{7} = \frac{3}{7}$ ∴Probability of only one of them coming to school on time $=P(A)\,P(\overline{B}) + P(\overline{A})\,P(B)$ $=\frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7}$ $=\frac{26}{49}$	1 1 1
29	We know that ,A∪ B denotes the occurrence of atleast one of A and B and A∩B denotes the occurrence of both A and B simultaneously Thus,$P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$ $\therefore P(A \cup B) = 0.6$ $\Rightarrow P(A) + P(B) - P(A \cap B)=0.6$ $\Rightarrow P(A) + P(B) =0.6 + 0.3 =0.9$ $\Rightarrow [1 - P(\overline{A})] + [1 - P(\overline{B})] = 0.9$ $\Rightarrow P(\overline{A}) + P(\overline{B}) = 2-0.9=1.1$	$1\frac{1}{2}$ $1\frac{1}{2}$
29 (OR)	We have P(atleast one of A and B) = P(A∪B) $= P(A) + P(B) - P(A \cap B)$ $=P(A) + P(B) - P(A)P(B)$ $=P(A) + P(B)[1 - P(A)]$ $=P(A) + P(B)P(\overline{A})$ $=1-P(\overline{A}) + P(B)P(\overline{A})$ $=1-P(\overline{A})[1 - P(B)]$ $=1- P(\overline{A})P(\overline{B})$	1 1 1

30	 The corner points are O(0,0),C(2,0),a(2,2) and B(0,6) The maximum value of Z is 42 and which attained at B(0,6)	For graph 2 1
31	When two dice are thrown together, the number of possible outcomes=36 Given E:event of outcomes whose total score is 4 ={(1,3)(2,2)(3,1)} F: event of outcomes whose total score is 9 or more {(3,6)(4,5),(4,6)(5,4),(5,5),(5,6),(6,3),(6,4),(6,5)} G: event of outcomes whose total score is divisible by 5 {(1,4)(2,3),(3,2),(4,1),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)} : $P(E) = \frac{1}{12}$, $P(F) = \frac{1}{4}$, $P(G) = \frac{5}{18}$ $E\cap F$={}, $E\cap G$={},$F\cap G$={(4,6),(5,5),(6,4)} $P(E\cap F) = 0$ $P(E)XP(F)=\frac{1}{48}$ $P(E\cap F) \neq P(E)XP(F)$ E and F are not independent $P(E\cap G) = 0$ $P(E)\times P(G)=\frac{1}{12}\times\frac{5}{18}=\frac{5}{216}$ $\therefore P(E\cap G) \neq P(E)\times P(G)$ E and G are not independent	1 1

	$P(F\cap G)=\frac{1}{12}$ $P(F)\times P(G)=\frac{1}{4}\times\frac{5}{18}=\frac{5}{72}$ $\therefore P(F\cap G)\neq P(F)\times P(G)$ F and G are not independent No pairs are independent	1
31(or)	Probability that A hits the target, $P(A)=\frac{1}{3}$ Probability that B hits the target, $P(B)=\frac{2}{5}$ Probability that A does not hit the target, $P(\bar{A})=1-\frac{1}{3}=\frac{2}{3}$ Probability that B does not hit the target, $P(\bar{B})=1-\frac{2}{5}=\frac{3}{5}$ Probability that the target is hit=At least one of them hit the target $=1-P(\bar{A})\,P(\bar{B})$ $=1-\frac{2}{3}\times\frac{3}{5}$ $=\frac{3}{5}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
32	Let the first kind of cake be x and second kind of cakes be y. Hence, $x\geq 0$ and $y\geq 0$ The total number of cakes $z=x+y$ The mathematical formulation of the given problem can be written as Maximise, $z = x + y$ subject to the constraints, $2x + y \leq 50$, $x + 2y \leq 40$, $x, y \geq 0$	1



Graph-2

OABC represent the feasible region.

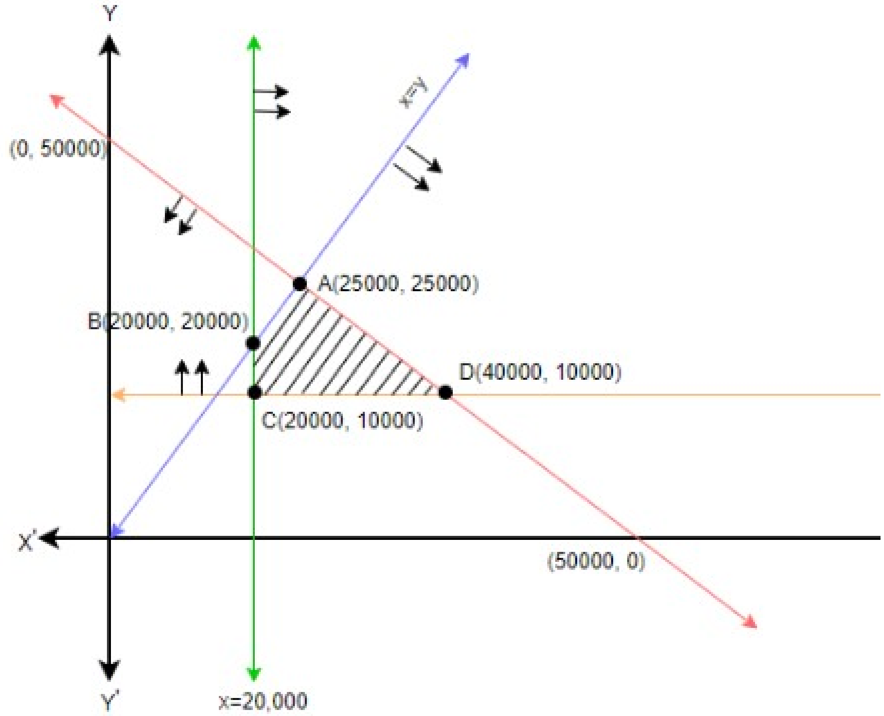
The corner points are A (25, 0), B (20, 10), O (0, 0) and C (0, 20).
We now find the value of Z at the corner points A (25, 0), B (20, 10), O (0, 0) and C (0, 20). $\frac{1}{2}$

Corner Points	Z=x+y	Conclusion
A(25,0)	25	
B(20,10)	30	
O(0,0)	0	
C(0,20)	20	

1

Hence, the maximum numbers of cakes that can be made are 30 (20 cakes of one kind and 10 cakes of other kind). $\frac{1}{2}$

33 Let the amounts invested by the person in bonds A and B are respectively Rsx and Rsy.
We now have the following mathematical model for the given problem.
Maximise, Z = 10% of x + 9% of y =0.1x+0.09y 1
Subject to the constraints $x+y \leq 50{,}000$
$x \geq y \Rightarrow x - y \geq 0$ and $x \geq 20{,}000$, $y \geq 10{,}000$



Graph-2

$\frac{1}{2}$

ABCD is the feasible region ,which is bounded.
The corner points are A(25000,25000), B(20000,20000) ,C(20000,10000)and D(40000,10000)
We now find the value of Z at the corner points A(25000,25000), B(20000,20000) ,C(20000,10000)and D(40000,10000) 1

Corner Points	Z=0.1x+0.09y	Conclusion
A(25000,25000)	2500+2250=4750	
B(20000,20000)	2000+1800=3800	
C(20000,10000)	2000+900=2900	
D(40000,10000)	4000+900=4900	Maximum

$\frac{1}{2}$

So, in order to get the maximum return the man has to invest Rs. 40000 in bond A and Rs. 10000 in bond B and the maximum return will be Rs. 4900.

34 Let S denote the success(getting a 6) and F denote the failure (not getting a 6)

Thus P $(S) = \frac{1}{6}$, P(F)= $\frac{5}{6}$ 1

P(A wins the first throw) = P $(S) = \frac{1}{6}$

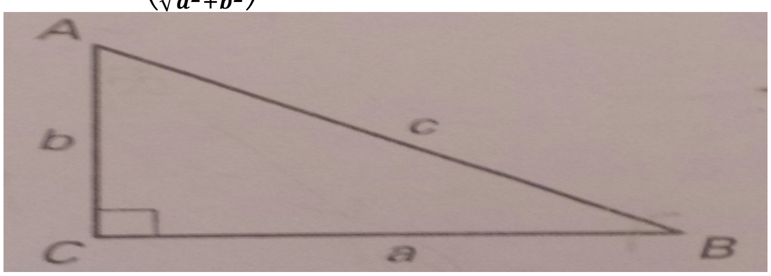
A gets the third throw ,when the first throw by A and second throw by B result into failures.

Therefore, P(A wins the third throw) =P(FFS) = P(F)P(F)P(S)=$\frac{5}{6} \times \frac{5}{6}$ 1

	$\times \frac{1}{6}$ $=\left(\frac{5}{6}\right)^2 \times \frac{1}{6}$	1
	P(A wins the 5^{th} throw) =P(FFFFS) = P(F)P(F)P(F)P(F)P(S) $=\left(\frac{5}{6}\right)^4 \times \frac{1}{6}$ and so on	1
	Hence P(A wins) = $\frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \ldots..$ $= \frac{\frac{1}{6}}{1-\frac{25}{36}}$ $= \frac{6}{11}$ P(B wins) = 1- $\frac{6}{11} = \frac{5}{11}$	1
34 (OR)	Let A, E_1, E_2, E_3 and E_4 be the events as defined below: A:a Green bulb is selected E_1:Box I is selected E_2: Box II is selected E_3: Box III is selected E_4: Box IV is selected	1
	Since the boxes are chosen at random P(E_1) = P(E_2) = P(E_3) = P(E_4) $= \frac{1}{4}$	1
	Also, P(A/E_1) = $\frac{3}{18}$, P(A/E_2) = $\frac{2}{8}$, P(A/E_3) = $\frac{1}{7}$ and P(A/E_4) = $\frac{4}{13}$	1
	P(box III is selected given that the drawn bulb is green)=P(E_3/A) By the Bays' Theorem P(E_3/A) = $\frac{P(E_3).P(A/E_3)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+P(E_3).P(A/E_3)+P(E_4).P(A/E_4)}$	1 1

	$= \dfrac{\frac{1}{4}\times\frac{1}{7}}{\frac{1}{4}\times\frac{3}{18}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{4}{13}}$ $= 0.165$	
35	Let A, $\boldsymbol{E_1}$and $\boldsymbol{E_2}$be the events as defined below: $\boldsymbol{E_1}$:The lost card is a spade card $\boldsymbol{E_2}$: The lost card is not a spade card A:drawing three spade cards from the remaining cards. P($\boldsymbol{E_1}$) = $\frac{13}{52} = \frac{1}{4}$, P($\boldsymbol{E_2}$) = $\frac{39}{52} = \frac{3}{4}$ $P(A/E_1) = \frac{12_{C3}}{51_{C3}} = \frac{12\times11\times10}{51\times50\times49}$, $P(A/E_2) = \frac{13_{C3}}{51_{C3}} = \frac{13\times12\times11}{51\times50\times49}$ By the Bays' Theorem $P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$ $= \dfrac{\frac{1}{4}\times\frac{12\times11\times10}{51\times50\times49}}{\frac{1}{4}\times\frac{12\times11\times10}{51\times50\times49}+\frac{3}{4}\times\frac{13\times12\times11}{51\times50\times49}}$ $=\frac{10}{49}$	1 1 1 1 1
35 (OR)	Let A, $\boldsymbol{E_1}$and $\boldsymbol{E_2}$be the events as defined below: $\boldsymbol{E_1}$:Event that 6 occurs . $\boldsymbol{E_2}$: Event that 6 does not occurs A:The man reports that 6 occurs. P($\boldsymbol{E_1}$) = $\frac{1}{6}$, P($\boldsymbol{E_2}$) $= \frac{5}{6}$ Also, $P(A/E_1) = \frac{3}{4}$, $P(A/E_2) = 1- \frac{3}{4} = \frac{1}{4}$ By the Bays' Theorem	1 1 1

	$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$ $= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}$ $=\frac{1}{8}\times\frac{24}{8}$ $=\frac{3}{8}$	1 1
36	By the Pythagoras theorem $C=\sqrt{a^2+b^2}$ (A)Sin$\angle ABC = \frac{AC}{AB} = \frac{b}{\sqrt{a^2+b^2}}$ $\Rightarrow \angle ABC = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ $\theta = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$	1



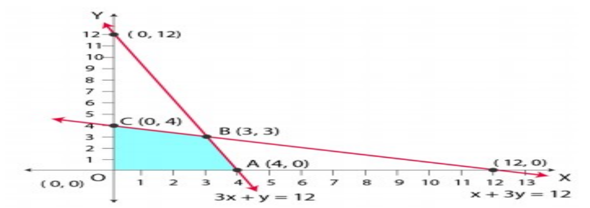
	(B) Cos$\angle ABC = \frac{BC}{AB} = \frac{a}{\sqrt{a^2+b^2}}$ $\Rightarrow \angle ABC = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$ $\therefore \theta = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ (C)Given, A=5m ,b=2m $c=\sqrt{5^2+2^2}=\sqrt{29}$ $\therefore \sin\angle CAB = \frac{BC}{AB}$ $\Rightarrow \angle CAB = \sin^{-1}\frac{5}{\sqrt{29}}$ **OR** (iii)Given, a=5m ,b=2m $C=\sqrt{5^2+2^2}=\sqrt{29}$	1 2 2

	$\cos\angle CAB=\frac{AC}{AB}$ $\Rightarrow \theta=\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$	
37	(A) No. of total oranges=15 3 oranges out of 15 oranges can be arranged in 15_{C3} ways The total number of ways that can be arranged taking 3 oranges at once out of 15 oranges=15_{C3} $=\frac{15\times14\times13}{3!}$ =455 (B) No. of good oranges=12 The number of arrangement that contained only god oranges =The number of ways that can be arranged taking 3 oranges at once out of 12 oranges=12_{C3} $=\frac{12\times11\times10}{3!}$ =220 (C) The probability that the box is approved for sale=Probability that all the three oranges drawn are good. $=\frac{220}{455}$ $=\frac{44}{91}$ OR, The probability that the box is not approved for sale=Probability that all the three oranges drawn are bad one. $= 1- \frac{44}{91}$ $=\frac{47}{91}$	1 1 2 2
38	(A) The constraints are x + 3y ≤ 12(constraint related to machine-A) , 3x + y ≤ 12(constraint related to machine B) x≥0 and y≥0	1

(B) The total profit is Z = 17.5x + 7y

1

(C)



OABC Is the feasible region

The corner points are O(0,0) ,A (4, 0), B (3, 3) and C (0, 4)

We now find the value of Z at the corner points O(0,0), A (4, 0), B (3, 3),
and C (0, 4).

Graph($1\frac{1}{2}$)

Corner Points	Z=17.5x+7y	Conclusion
O(0,0)	0	
A(4,0)	70	
B(3,3)	73.5	
C(0,4)	28	

$\frac{1}{2}$

Hence, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit

Or

Graph($1\frac{1}{2}$)
The graph is same as above

The maximum profit Rs 73.50.

$\frac{1}{2}$

WHOLE SYLLABUS PRACTICE PAPER

NVS RO-SHILLONG
WHOLE SYLLABUS PRACTICE QUESTION PAPER SET-I
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)

TIME: 3 HRS **MAX MARKS:80**

BLUE PRINT

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

CHAPTERS	MCQ (1M)	A & R (1M)	VSA (2M)	SA (3M)	LA (5M)	CSQ (4M)	TOTAL
Relations & Functions Inverse Trigonometric Functions	---	1	1	---	1	---	8
Matrices & Determinants	5	---	---	---	1	---	10
Continuity & Differentiability	2	---	1	---	---	---	4
Application of Derivatives	---	---	1	---	---	2	10
Integrals	2	---	---	3	---	---	11
Application of Integrals	---	---	---	---	1	---	5
Differential Equations	2	---	---	1	---	---	5
Vector Algebra	3	---	1	---	---	---	5
Three-Dimensional Geometry	1	1	1	---	1	---	9
Linear Programming Problem	2	---	---	1	---	---	5
Probability	1	----	---	1	---	1	8
	18 (1M)	2 (1M)	5 (2M)	6 (3M)	4 (5M)	3 (4M)	80 M

Navodaya Vidyalaya Samiti, RO Shillong
WHOLE SYLLABUS PRACTICE PAPER SET-I
(2024-25)
Class-XII
Subject: Mathematics (041)

Time:3 Hours **Maximum Marks:80**

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7.Useofcalculatorsis**not**allowed.

SECTION – A

(Multiple Choice Questions)
Each question carries One Mark

Q.1 **If $\begin{bmatrix} 10 & 0 \\ 6 & 12 \end{bmatrix}$= P + Q, where P is symmetric and Q is a skew symmetric matrix, then Q is equal to**

(A) $\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix}$(C) $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q,2 If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is

(A) 0 (B) -1 (C) 2 (D) 1

Q.3 **A is a 2×2 matrix whose elements are given by $a_{ij} = \begin{cases} 1, \text{if } i \neq j \\ 0, \text{if } i = j \end{cases}$**
Then value of A^2 is

(A)$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.4 A is a matrix of order 2×3 and B is a matrix of order 3×2. If $C=AB$ and $D=BA$, then order of CD is

(A) 3×3 (B) 2×2 (C) 3×2 (D) CD not defined

Q.5 If the matrix $\begin{bmatrix}1 & k & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of A and $|A| = 4$, then the value of k is

(A) 11 (B) 8 (C) 12 (D) -11

Q.6 The function $f(x) = [x]$, where $[x]$ denote the greatest integer function, is continuous at

(A) 4 (B) – 2 (C) 1 (D) 1.5

Q.7 If f (x) = |cos x|, then the value $f'\left(\frac{3\pi}{4}\right)$ is

(A)$\frac{1}{2}$ (B)$\frac{-1}{2}$ (C)$\frac{1}{\sqrt{2}}$ (D)$\frac{-1}{\sqrt{2}}$

Q.8 If $\int_0^a \frac{1}{1+4x^2} = \frac{\pi}{8}$, then the value of a is

(A)$\frac{1}{2}$ (B)$\frac{-1}{2}$ (C)$\frac{1}{4}$ (D)$\frac{1}{8}$

Q.9 The area of the region bounded by the lines $y = mx$, $x = 1$, $x = 2$ and x -axis is 6 sq. units, then $m$is equal to

(A) 3 (B) 1 (C) 2 (D) 4

Q.10 The integrating Factor of the differential equation: $x\frac{dy}{dx} - y = 2x^2$is

(A) e^x (B) e^{-x} (C) $\frac{1}{x}$ (D) x

Q.11 The number arbitrary constants involved in the general solution of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^5 + \left(\frac{d^3y}{d3^4}\right)^6 - \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 25$$

(A) 6 (B) 5 (C) 2 (D) 4

Q.12 If $\vec{a}$and $\vec{b}$ are two vectors, such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then find the angle between $\vec{a}$and $\vec{b}$

(A)$\frac{\pi}{2}$ (B)$\frac{\pi}{4}$ (C)$\frac{\pi}{3}$ (D)$\frac{\pi}{6}$

Q.13 Position vector of the mid-point of line segment AB is $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.If the position vector of

the point A is $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$,then the position vector of the point B is

(A)$\frac{5\hat{\imath}}{2} + \frac{5\hat{\jmath}}{2} - \frac{7\hat{k}}{2}$(B) $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (C) $5\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ (D)$\frac{\hat{\imath}}{2} - \frac{\hat{\jmath}}{2} + \frac{\hat{k}}{2}$

Q.14 If $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=3, |\vec{b}|=4$ and $|\vec{c}|=\sqrt{37}$,then the angle between $\vec{a}$and $\vec{b}$ is

(A)$\frac{\pi}{2}$ (B)$\frac{\pi}{4}$ (C)$\frac{\pi}{3}$ (D)$\frac{\pi}{6}$

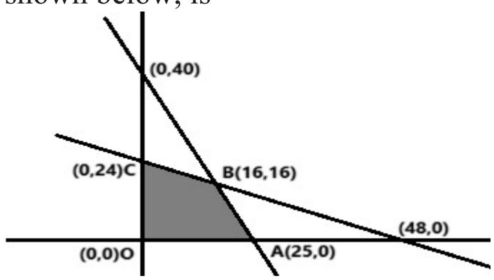
Q.15 If a line makes an angle of $\frac{\pi}{4}$with each of *y* and *z* axis, then the angle which it makes with *x*-axis is

(A)$\frac{\pi}{2}$ (B)$\frac{\pi}{4}$ (C)$\frac{\pi}{3}$ (D)$\frac{\pi}{6}$

Q.16 The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at(3, 2). Which of the following is true?

(A) $a = 9, b = 1$ (B) $a = 5, b = 2$ (C) $a = 3, b = 5$ (D) $a = 5, b = 3$

Q.17 The maximum value of Z = 4x+3y, if the feasible region for an LPP is as shown below, is



(A) 112 (B) 72 (C) 100 (D) 110

Q.18 The probability that A speaks the truth is $\frac{4}{5}$and that of B speaking the truth is $\frac{3}{4}$

The probability that they contradict each other in stating the same fact is

(A)$\frac{7}{20}$ (B)$\frac{1}{5}$ (C)$\frac{3}{20}$ (D)$\frac{4}{5}$

<u>ASSERTION-REASON BASED QUSETIONS</u>

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

A) Both A and R are true and R is the correct explanation of A.
B) Both A and R are true but R is not the correct explanation of A.
C) A is true but R is false.
D) A is false but R is true

Q.19 Assertion (A): The value of $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$is 17

Reason (R): $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

Q.20 Assertion(A): A line through the points (4,7,8) and (2,3.4) is parallel to a line through the points (-1,-2,1) and (1,2,5)

Reason (R): lines$\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$are parallel if $\overrightarrow{b_1}\cdot\overrightarrow{b_2} = 0$

SECTION B

(Each question carries 2 marks)

Q.21 Let $A = \{1,2,3,4\}$.Let R be the equivalence relation on A × A defined by $(a,b)R_{(c,d)}$ if $a + d = b + c$. Find the equivalence class $[(1,3)]$.

Q.22 If $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & -1 \le x < 0 \\ \frac{2x+1}{x-1}, & 0 \le x < 1 \end{cases}$ is continuous at x=0,find the value of k.

Q.23 Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$, is increasing or decreasing in the interval$\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

(OR)A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x- coordinate.

Q.24 Find the area of the parallelogram whose one of the sides and one diagonal are given by the vectors$3\hat{\imath} + \hat{\jmath} + 4\hat{k}$and $4\hat{\imath}$+$5\hat{\jmath}$ respectively.

Q.25 Find the direction ratios and direction cosines of the line whose equation is
$6x - 12 = 3y + 9 = 2z - 2$
(OR)
Find the angle between any two diagonals of a cube.

SECTION C

(Each question carries 3 marks)

Q.26 Find the value of $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

(OR)

Evaluate: $\int \frac{\sin^{-1}x - \cos^{-1}x}{\sin^{-1}x + \cos^{-1}x}\,dx$

Q.27 Find the value of $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}\,dx$

Q.28 Find $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}\, dx.$

Q.29 Find the general solution of $x\,\mathrm{dy} - \mathrm{y}\,\mathrm{dx} - \sqrt{(\mathrm{x}^2+\mathrm{y}^2)}\,\mathrm{dx} = 0.$

(OR)

Find the general solution of the differential equation: $\frac{d}{dx}(xy^2) = 2y(1+x^2)$

Q.30 Solve the L.P.P graphically: Maximize and Minimize Z=5x+10y subject to constraints x+2y≤120, x+y≥60, x-2y≥0, x,y ≥ 0.

Q.31 A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

SECTION D

(Each question carries 5 marks)

Q.32 Sketch the graph of $y = |x+3|$ and then evaluate the area under the curve $y = |x+3|$ above x-axis and between $\mathrm{x} = -6$ to $\mathrm{x} = 0.$

(OR)

Using integration, find the area bounded by the curve $y = x\,|x|$,x-axis and the ordinates $x = -1$ and$x = 1$.

Q.33 If$\mathrm{A} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -6 & 9 \\ 10 & 5 & -20 \end{bmatrix}$, find A^{-1} and then using A^{-1}solve the following system of equations:$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 2$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 5$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z} = -4$$

Q.34 Show that the function f: **R** → **R**define by$f(x) = \frac{x}{x^2+1}$, for all x ∈ **R,**

is neither one-one nor onto.

OR

Each of the following defines relations on **N:**

(i) x is greater than y, x, y ∈ **N**

(ii) xy is square of an integerx, y ∈ **N**

(iii) x+4y=10,x, y ∈ **N**

Determine which of the above relations are reflexive, symmetric and transitive**.**

Q.35 Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

OR

Find the vector and cartesian equations of the line through the point (1,2,-4) and perpendicular to the lines

$$\vec{r} = \left(8\hat{\imath} - 9\hat{\jmath} + 10\hat{k}\right) + \lambda\left(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}\right)$$

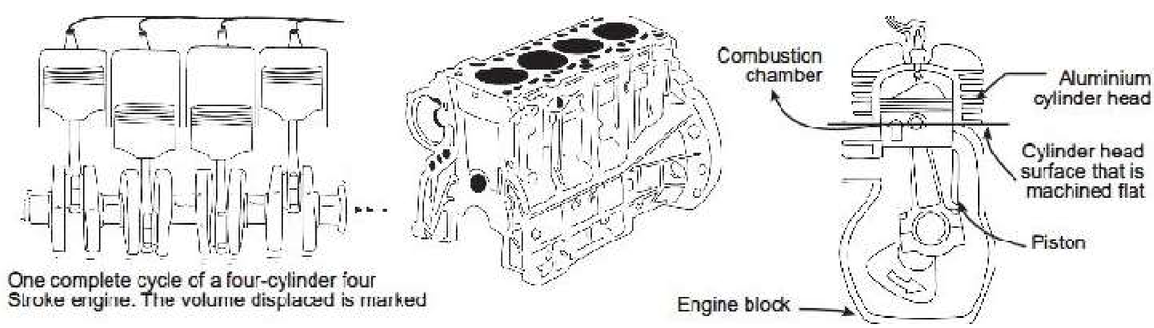
and $\vec{r} = \left(15\hat{\imath} - 29\hat{\jmath} + 5\hat{k}\right) + \mu\left(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}\right)$

SECTION- E [4x3=12]

(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)*

Case Study-1

Q.36 Read the following passage and answer the following questions.
Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston
engine. The piston moves inside the cylinder bore,



One complete cycle of a four-cylinder four Stroke engine. The volume displaced is marked

The cylinder bore in the form of circular cylinder open at the top is to made from a metal sheet of area 75π sq. cm

(i) If the radius of cylinder is r cm and height is h cm,then write the volume of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

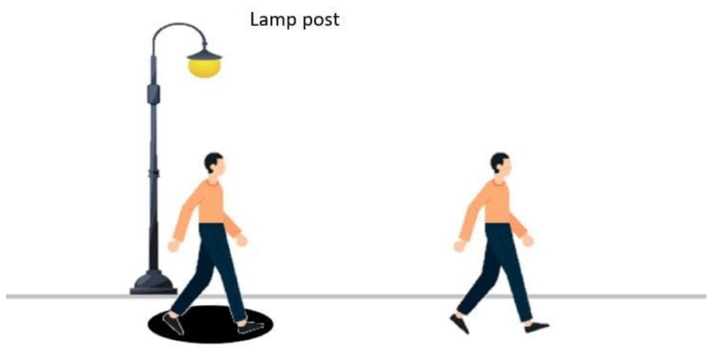
OR

(b) For maximum volume,h $> r$.State true or false,justify.

Case Study-2

Q.37 Read the following passage and answer the following questions.

A man , 2 m tall, walks at the rate of $1\frac{2}{3}$ metre per second towards a street light which is $5\frac{1}{3}$ metreabove the ground.If x and y are the distance of the man from the foot of the lamp post and length of his shadow on the ground at the time t ,then



(i) Find y in terms of x.

(ii) At what rate is the length of the shadow changing when he is $3\frac{1}{3}$m from the base of the light.

(iii) At what rate is the tip of his shadow moving ?

Case Study-3

Q.38 Read the following passage and answer the following questions.

Three bags contain a number of red and white balls as follows:
Bag I :3 red balls
Bag II: 2 red balls and 1 white ball
Bag III:3 white ball
The probability that bag *i* will be chosen and a ball is selected from it is $\frac{i}{6},\ i = 1,2,3.$

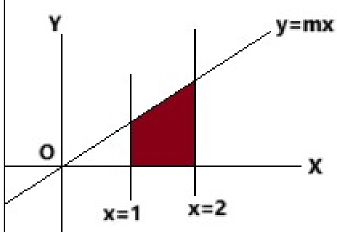
(i) What is the probality that a red ball will be selected ?
(ii) What is the probality that a white ball is selected ?
(iii) If a white ball is selected, what is the probability that it came from Bag II ?

NVS RO SHILLONG
WHOLE SYLLABUS PRACTICE PAPER SET I
(2024-2025)
MARKING SCHEME
CLASS XII
MATHEMATICS(CODE-041)

SECTION:A

(Solution of MCQs of 1 Mark each)

Q.NO.	ANS	HINTS/SOLUTION
1.	(C)	Let $A = \begin{bmatrix} 10 & 0 \\ 6 & 12 \end{bmatrix} \therefore A' = \begin{bmatrix} 10 & 6 \\ 0 & 12 \end{bmatrix}$ Now $A + A' = \begin{bmatrix} 10 & 0 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 6 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 20 & 6 \\ 6 & 24 \end{bmatrix} \therefore \frac{1}{2}(A + A') = \begin{bmatrix} 10 & 3 \\ 3 & 12 \end{bmatrix} = P$ $A - A' = \begin{bmatrix} 10 & 0 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 6 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 6 & 0 \end{bmatrix} \therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = Q$
2.	(A)	$\lvert A\rvert = \lvert kA\rvert$ and n=2 $\lvert A\rvert = k^2\lvert A\rvert$ ($\because \lvert kA\rvert = k^n\lvert A\rvert$) $\Rightarrow k^2 = 1 \Rightarrow k = \pm 1 \Rightarrow$ Sum of all values of k = $+1 - 1 = 0 \therefore$ **Correct option is (A).**
3.	(D)	$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \therefore A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4.	(D)	O(C)=2×2 and O(D)=3×3. The number of columns of C not equal to number of rows of B. Therefore, CD not defined Option: (D) CD not defined
5.	(A)	**Order of A is 3×3** **Therefore, $\lvert adj\ A\rvert = \lvert A\rvert^2$** $\Rightarrow \begin{vmatrix} 1 & k & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 4^2$ $\Rightarrow 1(9-9) - k(4-6) + 3(4-6) = 16$ $\Rightarrow 2k = 22 \Rightarrow k = 11$
6.	(D)	The greatest integer function $[x]$ is discontinuous at all integral values of x. Thus **(D) is the correct answer.**
7.	(C)	$\boldsymbol{x = \frac{3\pi}{4}}$ is lie in the second quadrant.

		i.e. $\frac{\pi}{2} < x < \pi \Rightarrow cosx < 0$ $\Rightarrow$ f(x) $\therefore$ f′(x) $\Rightarrow f'\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
8.	**(A)**	$\int_0^a \frac{1}{1+4x^2}$ $= \frac{\pi}{8}$ $\Rightarrow$ $\frac{1}{2}\left[\tan^{-1}\left(\frac{2x}{1}\right)\right]_0^a =$ $\frac{\pi}{8}$ $\Rightarrow \tan^{-1}(2a) = \frac{\pi}{4} \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$
9.	**(D)**	 **Area=** $\int_1^2 mx\,dx$ $\Rightarrow 6 = m\left[\frac{x^2}{2}\right]_1^2 \Rightarrow 6 = \frac{3}{2}m \Rightarrow m=4$
10.	**(C)**	$x\frac{dy}{dx} - y = 2x^2 \Rightarrow \frac{dy}{dx} - \frac{1}{x}.y = 2x$ I.F.$= e^{\int P\,dx} = e^{-\int \frac{1}{x}dx} = e^{-logx} = e^{log\frac{1}{x}} = \frac{1}{x}$
11.	**(D)**	The number of arbitrary constants in the general solution of a differential equation is determined by its order, not by its degree. Since the order of the given equation is 4, it will have only 4 arbitrary constants. Note: No arbitrary constants are involved in the Particular Solution of a D.E.
12.	**(B)**	$\lvert\vec{a}\cdot\vec{b}\rvert = \lvert\vec{a}\times\vec{b}\rvert$ $\Rightarrow \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta =$ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta$ $\Rightarrow$ tanθ =1, therefore θ $= \frac{\pi}{4}$

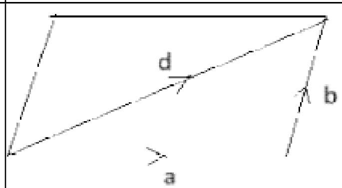
13.	**(B)**	$A(2\hat{i}+3\hat{j}-4\hat{k})B(x\hat{i}+y\hat{j}+z\hat{k})$ $M(3\hat{i}+2\hat{j}-3\hat{k})$ M is the mid- point of AB $\therefore \frac{2+x}{2}=3, \frac{3+y}{2}=2, \frac{-4+z}{2}=-3$ $\Rightarrow \mathrm{x}=4, \mathrm{y}=1, \mathrm{z}=-2$ Hence, $\mathrm{B}=4\hat{\imath}+\hat{\jmath}-2\hat{\mathrm{k}}$
14.	**(C)**	$\vec{a}+\vec{b}+\vec{c}=0$ $\Rightarrow \vec{a}+\vec{b}=-\vec{c} \Rightarrow \lvert\vec{a}+\vec{b}\rvert=\lvert\vec{c}\rvert \Rightarrow \lvert\vec{a}+\vec{b}\rvert^2=\lvert\vec{c}\rvert^2 \Rightarrow \lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+2\vec{a}.\vec{b}=\lvert\vec{c}\rvert^2$ $\Rightarrow \lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\Theta=\lvert\vec{c}\rvert^2$ $\Rightarrow 3^2+4^2+2.3.4.\cos\Theta=(\sqrt{37})^2$ $\Rightarrow \cos\Theta=\frac{1}{2}$, therefore $\Theta=\frac{\pi}{3}$
15.	**(A)**	Let the line be make angle α with *x*-axis. Then $\cos^2\alpha+\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}=1$ which after simplification gives $\alpha=\frac{\pi}{2}$
16.	**(C)**	According to Question, $42=4a+6b$ and $19=3a+2b$ Solving above equations, we get $a=3$ and $b=5$. **Thus (C) is correct option.**
17.	**(A)**	<table><tr><td>Corner Point</td><td>Value of Z=4x+3y</td></tr><tr><td>O (0, 0)</td><td>4 (0) + 3 (0) = 0</td></tr><tr><td>A (25, 0)</td><td>4 (25) + 3 (0) = 100</td></tr><tr><td>B (16, 16)</td><td>4 (16) + 3 (16) = 112→(Max.)</td></tr><tr><td>C (0, 24)</td><td>4 (0) + 3 (24) = 72</td></tr></table>
18.	**(A)**	Required Probability $=\mathrm{P(A)P(\overline{B})+P(\overline{A})P(B)}=\frac{4}{5}\left(1-\frac{3}{4}\right)+\left(1-\frac{4}{5}\right)\frac{3}{4}=\frac{1}{5}+\frac{3}{20}=\frac{7}{20}$
19.	**(D)**	$\sec^2(\tan^{-1}2)+\mathrm{cosec}^2(\cot^{-1}3)$ $=1+\tan^2(\tan^{-1}2)+1+\cot^2(\cot^{-1}3)$ $=1+2^2+1+3^2=15$ **Therefore, Assertion (A) is false but Reason (R) is true.**
20.	**(C)**	line through the points (4,7,8) and (2,3.4) is $\frac{x-4}{2-4}=\frac{y-7}{3-7}=\frac{z-8}{4-8} \Rightarrow \frac{x-4}{-2}=\frac{y-7}{-4}=$

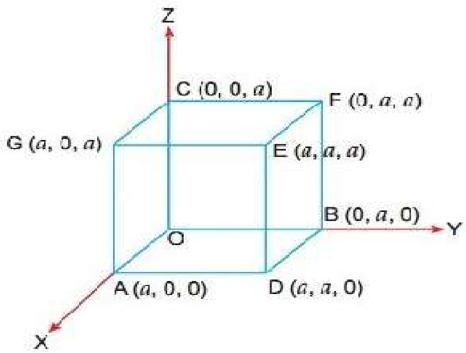
		$\frac{z-8}{-4}$ $\Rightarrow \vec{r} = \left(4\hat{\imath} + 7\hat{\jmath} + 8\hat{k}\right) + \lambda\left(-2\hat{\imath} - 4\hat{\jmath} - 4\hat{k}\right)$ line through the points (-1,-2,1) and (1,2.5) is $\frac{x+1}{1+1} = \frac{y+2}{2+2} = \frac{z-1}{5-1} \Rightarrow \frac{x+1}{2} = \frac{y+2}{4} = \frac{z-1}{4}$ $\Rightarrow \vec{r} = \left(-\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) + \mu\left(2\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right)$ Observed that $\frac{-2}{2} = \frac{-4}{4} = \frac{-4}{4} \Rightarrow$ the lines are parallel ∴ Assertion (A) is true but the Reason (R) is false.

Section–B

[This section comprises of solution of very short answer type questions (VSA) of marks each]

21.	$[(1,3)]$ $= \{(x, y)$ $\in A$ $= \{(x, y) \in A$ $= \{(x, y)$ $= \{(3,1).(4,2)\}$	$\frac{1}{2}$ $\frac{1}{2}$ **1**
22.	At the point x=0, $f(0) = \dfrac{2 \times 0 + 1}{0 - 1} = -1$ $\mathrm{R.H.L} = \lim\limits_{x \to 0^+} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$ $= \lim\limits_{x \to 0^+} \dfrac{(1+kx) - (1-kx)}{x(\sqrt{1+kx} + \sqrt{1-kx})}$ $= k$ Since, f(x) is continuous at $x = 0$, L.H.L=R.H.L. = f(0) ⇒ k= -1	$\frac{1}{2}$ $\frac{1}{2}$ **1**
23.	$f(x) = \cos\left(2x + \frac{\pi}{4}\right) \Rightarrow f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right)$ Given $\frac{3\pi}{8} < x < \frac{5\pi}{8} \Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4} \Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2} \Rightarrow \sin\left(2x + \frac{\pi}{4}\right) < 0$ (3^{rd} quadrant) Therefore, $f'(x) < 0 \Rightarrow$ f(x) is increasing in $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$	$\frac{1}{2}$ **1** $\frac{1}{2}$

23. (OR)	By question $\frac{dy}{dt} = 2.\frac{dx}{dt}$ ---------(i) Given curve: $y = \frac{2}{3}x^3 + 1$ ----------(ii) Differentiating with respect to t $\frac{dy}{dt} = \frac{2}{3} \cdot 3x^2 \frac{dx}{d\,t} \Rightarrow 2.\frac{dx}{dt} = 2x^2 \frac{dx}{d\,t}$ by (i) $\Rightarrow x = 1, -1$ From (ii) , $y = \frac{5}{3}, \frac{1}{3}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
24.	 Given, $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{d} = 4\,\hat{i} + 5\hat{j}$ By parallelogram law of vectors addition, $\vec{d} = \vec{a} + \vec{b}$ Therefore, $\vec{b} = \vec{d} - \vec{a} = (4\,\hat{i} + 5\hat{j}) - (3\hat{i} + \hat{j} + 4\hat{k}) = \hat{i} - \hat{j} + \hat{k}$ Thus, area of the parallelogram= $\lvert\vec{a} \times \vec{b}\rvert = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \lvert(1+4)\hat{i} - (3-4)\hat{j} + (-3-1)\hat{k}\rvert$ $= \lvert 3\hat{i} + \hat{j} + 4\hat{k}\rvert$ $= \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{42}$ sq. units	$\frac{1}{2}$ $\frac{1}{2}$ 1
25.	Line: $6x - 12 = 3y + 9 = 2z - 2$ Standard form of the line : $\frac{x-2}{\frac{1}{6}} = \frac{y+3}{\frac{1}{3}} = \frac{z-1}{\frac{1}{2}}$ Direction ratios are $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ Direction Cosines are $\frac{\frac{1}{6}}{\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{2}\right)^2}}, \frac{\frac{1}{3}}{\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{2}\right)^2}}, \frac{\frac{1}{2}}{\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{2}\right)^2}}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25 (OR)		$\frac{1}{2}$



$\frac{1}{2}$

Diagonals are OE,AF,BG,CD

Direction ratios of the diagonal OE are a,a,a⇒direction cosines are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

Direction ratios of the diagonal AF are -a,a,a⇒direction cosines are $\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

Now, $\cos\alpha = \frac{1}{\sqrt{3}}\left(\frac{-1}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}=\frac{1}{3} \quad \therefore \alpha=\cos^{-1}\left(\frac{1}{3}\right)$ **1**

SECTION C

(Each question carries 3 marks)

26. $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}.\cos^2 x}\,dx=$ **2**

$\int \frac{\sqrt{\tan x}}{\tan x}.\sec^2 x\,dx$ **1**

Putting $z=\sec x \Rightarrow dz = \sec^2 x\,dx$

$= \int \frac{\sqrt{z}}{z}\,dz = \int \frac{1}{\sqrt{z}}\,dz = \sqrt{z}+C = \sqrt{\tan x}+C$

26 (OR) $\int \frac{\sin^{-1}x-\cos^{-1}x}{\sin^{-1}x+\cos^{-1}x}\,dx$

$$=\int \frac{\sin^{-1}x-\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\frac{\pi}{2}}\,dx$$ **1**

$$=\frac{4}{\pi}\int \sin^{-1}x\;dx$$ **2**

$$-\int dx$$

Continue by using integration by parts

27. $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}\,dx$

$=\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}\,dx+\int_{-1}^{1}\frac{|x|+1}{x^2+2|x|+1}\,dx=0+2\int_{-1}^{1}\frac{|x|+1}{(|x|+1)^2}\,dx$(Since, odd function + even function) **1**

$=2\int_{-1}^{1}\frac{x+1}{(x+1)^2}\,dx=2\log 2$ **2**

28. $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}\, dx$

Putting z= $\sqrt{x} \Rightarrow dz = \frac{1}{2\sqrt{x}}dx$ **1**

$= 2\int \frac{1}{(z+1)(z+2)}$ dz (Apply Partial fraction Method) **2**

29. Given, $x\,dy - y\,dx - \sqrt{(x^2+y^2)}\,dx = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ (It is homogeneous differential equation) **1**

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x} = v + \sqrt{1+v^2}$$ **1**

$$\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2}$$ **1**

$$\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrate on both sides **1**

29. (OR)

$$\frac{d}{dx}(xy^2) = 2y(1+x^2)$$ **1**

$$\Rightarrow x.\frac{d}{dx}y^2 + y^2\frac{d}{dx}.x = 2y(1+x^2)$$

$$\Rightarrow 2xy\frac{dy}{dx} + y^2 = 2y(1+x^2)$$ **1**

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2x}.y = \frac{1+x^2}{x}$$

It is linear D.E of the form $\frac{dy}{dx} + Py = Q$.

30. Maximize and Minimize Z=5x+10y

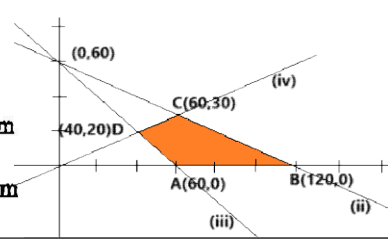
Constrains : x, y ≥ 0 .....(i)

x+2y ≤ 120 .....(ii)

x+y≥ 60 .....(iii)

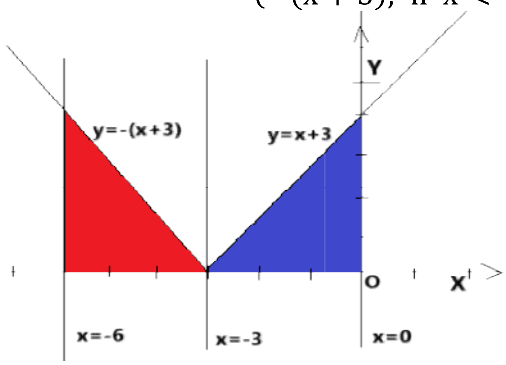
x-2y ≥0 .....(iv)

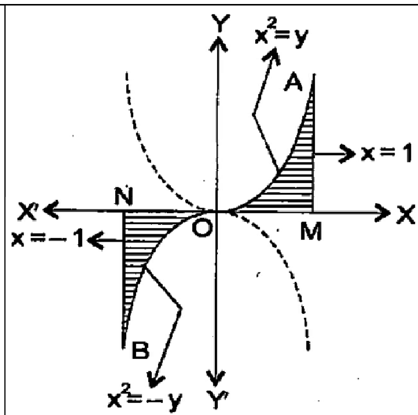
Corner Point	Z = 5x + 10y	
A(60, 0)	300 = m	← Minimum
B(120, 0)	600	
C(60, 30)	300 + 300 = 600 = M	← Maximum
D(40, 20)	400	



$1\frac{1}{2}$

$\frac{1}{2}$

31.	Let E_1= Event that ball transferred from the first bag is white. E_2= Event that ball transferred from the first bag is black. & E = Event that the ball drawn from the second bag is white. By total probability theorem, $$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)$$ $$= \frac{4}{9}.\frac{10}{17} + \frac{5}{9}.\frac{9}{17} = \frac{5}{9}$$	**1** **1** **1**
	SECTION D (Each question carries 5 marks)	
32. **32. (OR)**	Given, $y = \|x+3\| = \begin{cases} x+3, & \text{if } x \geq -3 \\ -(x+3), & \text{if } x < -3 \end{cases}$  The shaded region in the graph is the graph of $y = \|x+3\|$. The area of the shaded region $= \int_{-6}^{0} \|x+3\|\,dx$ $$= \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$ $$= -\left[\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0} = -\left[0 - \frac{9}{2}\right] + \left[\frac{9}{2} - 0\right] = 9 \text{ sq. units}$$ Given, $y = x\|x\| = \begin{cases} x^2, & \text{if } x \geq 0 \ \ldots\ldots\ldots(i) \\ -x^2, & \text{if } x < 0 \ \ldots\ldots\ldots(ii) \end{cases}$ Required Area = Area ONBO + Area OMAO	**0.5** **1** **1.5** **1** **1**

	 $= \left\| \int_{-1}^{0} -x^2\,dx \right\| + \int_0^1 x^2\,dx = \frac{2}{3}$ sq.units	1
33.	Here, $\|A\| = 1200$ Co-factors are $\left.\begin{matrix} A_{11} = 75, & A_{12} = 150, & A_{13} = 75 \\ A_{21} = 110, & A_{22} = -100 & A_{23} = 30 \\ A_{31} = 72 & A_{32} = 0 & A_{33} = -24 \end{matrix}\right\}$ $A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$ Given system of equation in the matrix form is, $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$ $\Rightarrow A'X = B$ $\Rightarrow X = (A')^{-1}B$ $\therefore \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = (A^{-1})'B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$ Hence, $x = 2, y = -3, z = 5$	1 1 1 1 1

34.	Consider $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$ $\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$ $\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$ $\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$ $\therefore\ x_1 = x_2$ or $x_1x_2 = 1$ We observe that there are point, $x_1$and $x_2$with $x_1 \neq x_2$and $f(x_1) = f(x_2)$. For instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, we have $f(x_1) = \frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ $\therefore x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$ Hence, f is not one-one. Also, f is not onto for if so then for $1 \in R$ $\exists$ $x \in R$ such that $f(x) = 1$ $\Rightarrow \frac{x}{x^2+1} = 1 \Rightarrow x^2 - x + 1 = 0$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.	**1** **2** **2**
34 (OR)	(i) Given, $R_1 = \{(x, y): x > y\ , x, y \in N\}$ If $(x,x) \in R_1$,then $x > x$, *which is not true for any* $x \in N$ So,R is not reflexive. Let $(x,y) \in R_1 \Rightarrow x > y \Rightarrow y > x$, *which is not true for any* $x\ , y \in N$ So,R is not symmetric. Let $(x,y),(y,z) \in R_1 \Rightarrow x > y$ *and* $y > z \Rightarrow x > z$, *for any* $x\ , y \in N$ So,R_1 is transitive. (ii) Given, $R_2 = \{(x, y)$: xy is a square of an integer , $x, y \in N\}$ If $(x,x) \in R_2$,then $x.x = x^2$ ·, which is a square of an integer for any $x \in N$ So,R is reflexive. Let $(x,y) \in R_2 \Rightarrow xy = m^2$ and $yx = m^2 \Rightarrow xz = (y, x) \in R_2$ So,R is symmetric. Let $(x,y),(y,z) \in R_3 \Rightarrow$ $xy = m^2$ and $yz = n^2 \Rightarrow xz = \frac{m^2n^2}{y^2}$, which is square of integer So,R_2 is transitive. (iii) Given, $R_3 = \{(x, y): x + 4y = 10; x, y \in N\}$ Then, $R_3 = \{(2,2), (6,1)\}$ Clearly,$(1,1) \notin R_3 \Rightarrow R_3$is not reflexive. $(6,1) \in R_3$ but$(1,6) \notin R_3 \Rightarrow R_3$is not symmetric. Suppose $(x,y) \in R_3 \Rightarrow x + 4y = 10$ And $(y,z) \in R_3 \Rightarrow y + 4z = 10 \Rightarrow x - 16z = -30 \Rightarrow (x, z) \notin R_3$ So, R_3 is transitive.	$1\frac{1}{2}$ $1\frac{1}{2}$ **2**
35	Given: $3l + m + 5n = 0 \Rightarrow m = -5n - 3l$ --------(i) $6mn - 2nl + 5lm = 0$ -----(ii) Substitute *m* from (i) in (ii) $6n(-5n - 3l) - 2nl + 5l(-5n - 3l) = 0$ $\Rightarrow -30n^2 - 18nl - 2nl - 25ln - 15l^2 = 0$ $\Rightarrow -30n^2 - 45nl - 15l^2 = 0$ $\Rightarrow 2n^2 + 3nl + l^2 = 0$	$1\frac{1}{2}$

$\Rightarrow 2n^2 + 2nl + nl + l^2 = 0$
$\Rightarrow 2n(n + l) + l(n + l) = 0$
$\Rightarrow (n + l)(2n + l) = 0$
$\Rightarrow$ either $l = -n$ or $l = -2n$ **$1\frac{1}{2}$**
Now if $l = -n$, then from (i) $m = -2n$
and if $l = -2n$, then from (ii) $m = n$.
Thus the direction ratios of two lines are proportional to $-n, -2n, n$ and $-2n, n, n$,
i.e. -1, -2, 1 and –2, 1, 1.
i.e. 1,2,-1 and -2,1,1.
Let α be the angle between the lines. **2**

Now, $\cos\alpha = \frac{1.(-2)+2.1+(-1).1}{\sqrt{1^2+2^2+(-1)^2}\sqrt{(-2)^2+1^2+1^2}} = \frac{-1}{6}$

$$\therefore \alpha = \cos^{-1}\left(\frac{-1}{6}\right)$$

35 (OR)

1
Let the direction ratios of the line be $\langle a, b, c\rangle$
Then, $\mathbf{L}:\ \frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \Rightarrow$ DRs are $\langle a, b, c\rangle$ **1**
A/Q the line **L** is perpendicular to the lines
$\mathbf{L_1}: \vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \Rightarrow$ DRs are $\langle 3, -16, 7\rangle$
$\mathbf{L_2}: \vec{r} = (15\hat{i} - 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \Rightarrow$ DRs are $\langle 3, 8, -5\rangle$ **1**
Now, L is perp. to $L_1 \Rightarrow 3a - 16b + 7c = 0$ -------(i)
L is perp. to $L_2 \Rightarrow 3a + 8b - 5c = 0$ --------(ii)
Cross multiplying (i) and (ii)

$$\begin{array}{ccc} a & b & c \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{array}$$

2

$$\frac{a}{80-56} = \frac{-b}{-15-2} = \frac{c}{24+48} \Rightarrow \frac{a}{24} = \frac{-b}{-36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$

Hence, $\mathbf{L}:\ \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

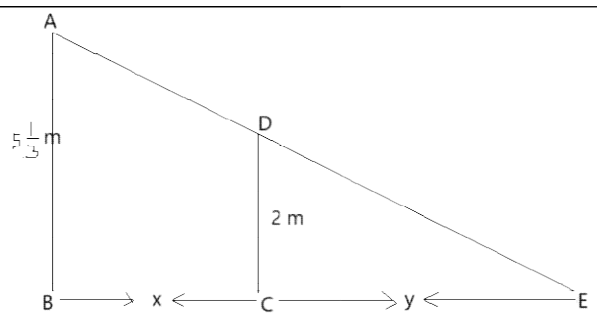
Section –E

(This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36 (i) We have,

$$2\pi rh + \pi r^2 = 75\pi$$

$$\Rightarrow \quad 2rh + r^2 = 75 \qquad \Rightarrow 2rh = 75 - r^2$$

	$\Rightarrow \quad h = \frac{75-r^2}{2r}$ $\therefore$ Volume of cylinder $V = \pi r^2.h = \pi r^2 \times \frac{(75-r^2)}{2r}$ $= \frac{\pi r(75-r^2)}{2} = \frac{\pi}{2}\times(75r-r^3)$ (ii) $\frac{dV}{dr} = \frac{\pi}{2}\frac{d}{dr}(75r-r^3) = \frac{\pi}{2}\times(75-3r^2) = \frac{3\pi}{2}(25-r^2)$ (iii) (a) For volume to be maximum, $\frac{dV}{dr}=0$. $\Rightarrow \quad \frac{3\pi}{2}(25-r^2)=0 \quad \Rightarrow 25-r^2=0$ $\Rightarrow r^2=25 \quad \Rightarrow \quad r=5$ cm $\frac{d^2V}{dr^2} = -3\pi r$ $\therefore \quad \left.\frac{d^2V}{dr^2}\right\|_{r=5} = -3\pi\times 5 = -15\pi < 0$ $\therefore$ Volume is maximum when $r=5$ cm **OR** (iii) (b) Volume V is maximum when $r=5$ cm $h = \frac{75-r^2}{2r} = \frac{75-(5)^2}{2\times 5} = \frac{75-25}{10} = \frac{50}{10} = 5$ $\Rightarrow \quad h = 5$ cm Here, $\quad h=r$ $\therefore \quad h>r$ is false.	1 1 2
37	 Let AB= the height of the street light post $= 5\frac{1}{3}$ m $= \frac{16}{3}$ m DC= the height of the man=2 m Given, $\frac{dx}{dt} = -1\frac{2}{3} = -\frac{5}{3}$ m/s (since the man is moving towards the light post) (i) Here, $\Delta ABE \sim \Delta DCE$ [by AAA Similarity] $\therefore \frac{AB}{DC} = \frac{BE}{CE} \Rightarrow \frac{\frac{16}{3}}{2} = \frac{x+y}{y} \Rightarrow \frac{16}{6} = \frac{x+y}{y} \Rightarrow 16y = 6x+6y \Rightarrow 10y = 6x$ $\therefore y = \frac{3}{5}x$ ....(i) (ii) Differentiating (i) w.r.to t	1

	$\frac{dy}{dt}=\frac{3}{5}\frac{dx}{dt}=\frac{3}{5}.\left(-\frac{5}{3}\right)=-1\ \text{m/s}$ Hence, the length of the shadow is decreasing at the rate of 1 m/s. (iii) Let z=x+y Differentiating (i) w.r.to t $\frac{dz}{dt}=\frac{dx}{dt}+\frac{dy}{dt}=-\frac{5}{3}+(-1)=-\frac{8}{3}=-2\frac{2}{3}\,\text{m/s}$ Hence, the tip of the shadow is moving at the rate of $2\frac{2}{3}$ m/s towards to light post.	1 2
38.	<table><tr><td>Bag</td><td>Red Ball</td><td>White Ball</td><td>Total</td></tr><tr><td>I</td><td>3</td><td>0</td><td>3</td></tr><tr><td>II</td><td>2</td><td>1</td><td>3</td></tr><tr><td>III</td><td>0</td><td>3</td><td>3</td></tr></table>Let E_1 = Event that Bag I is selected $\Rightarrow P(E_1)=\frac{1}{6}$ E_2 = Event that Bag II is selected $\Rightarrow P(E_2)=\frac{2}{6}$ E_3 = Event that Bag III is selected $\Rightarrow P(E_3)=\frac{3}{6}$ (i) Let A= Event that a red ball is selected By total probability theorem, $P(A)=P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)$ $=\frac{1}{6}.\frac{3}{3}+\frac{2}{6}.\frac{2}{3}+\frac{3}{6}.\frac{0}{3}=\frac{7}{18}$ (ii) Let B= Event that a white ball is selected By total probability theorem, $P(B)=P(E_1)P\left(\frac{B}{E_1}\right)+P(E_2)P\left(\frac{B}{E_2}\right)+P(E_3)P\left(\frac{B}{E_3}\right)$ $=\frac{1}{6}.\frac{0}{3}+\frac{2}{6}.\frac{1}{3}+\frac{3}{6}.\frac{3}{3}=\frac{11}{18}$ (iii) By Baye's Theorem, we have $P\left(\frac{E_2}{B}\right)=\frac{P(E_2)P\left(\frac{B}{E_2}\right)}{P(E_1)P\left(\frac{B}{E_1}\right)+P(E_2)P\left(\frac{B}{E_2}\right)+P(E_3)P\left(\frac{B}{E_3}\right)}$ $=\frac{\frac{2}{6}.\frac{1}{3}}{\frac{1}{6}.\frac{0}{3}+\frac{2}{6}.\frac{1}{3}+\frac{3}{6}.\frac{3}{3}}==\frac{\frac{2}{18}.}{\frac{11}{18}.}=\frac{2}{11}$	 1 1 2

NVS RO-SHILLONG
WHOLE SYLLABUS PRACTICE QUESTION PAPER SET-II
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)

TIME: 3 HRS **MAX MARKS:80**

BLUE PRINT

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6**. Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

CHAPTERS	MCQ (1M)	A & R (1M)	VSA (2M)	SA (3M)	LA (5M)	CSQ (4M)	TOTAL
Relations & Functions Inverse Trigonometric Functions	---	1	1	---	1	---	8
Matrices & Determinants	5	---	---	---	1	---	10
Continuity & Differentiability	2	---	1	---	---	---	4
Application of Derivatives	---	---	1	---	---	2	10
Integrals	2	---	---	3	---	---	11
Application of Integrals	---	---	---	---	1	---	5
Differential Equations	2	---	---	1	---	---	5
Vector Algebra	3	---	1	---	---	---	5
Three-Dimensional Geometry	1	1	1	---	1	---	9
Linear Programming Problem	2	---	---	1	---	---	5
Probability	1	----	---	1	---	1	8
	18 (1M)	2(1M)	5(2M)	6(3M)	4(5M)	3 (4M)	80 M

NAVODAYA VIDYALAYA SAMITI - RO SHILLONG
WHOLE SYLLABUS PRACTICE PAPER SET II
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)

Time: 3Hours **Max.Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
(ix) Use of calculators is not allowed.

SECTION A **[1×20 = 20]**

(This section comprises of Multiple – choice questions (MCQ) of 1 mark each.)

Select the correct option (Question 1 - Question 18):

Q 1. Let A be a square matrix of order 3 such that $adj(4A) = \lambda\, adj(A)$. Then the value of λ is –
(A) 4(B) 8(C) 12(D) 16

Q 2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric matrix then find the value of a and b.
(A) $a = 2, b = 2$ (B) $a = -2, b = 3$ (C) $a = 2, b = -3$ (D) $a = 2, b = -2$

Q 3. If $f(x) = x^2 + ax + 1$ is monotonically increasing in the interval [1,2] then minimum value of a is
(A) -1 (B) -2 (C) 1 (D) 0

Q 4. There are two values of a which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, the sum of these values is
(A) 4 (B) 5 (C) -4 (D) 9

Q 5. If the integrating factor of the differential equation $x\frac{dy}{dx} + my = x^2e^x$ is $\frac{1}{x^2}$ then value of m is

(A) -1 (B) 1 (C) 2 (D) -2

Q 6. Let A and B be two matrices such that AB is defined. If $AB = 0$, then which one of the following can be definitely concluded?

(A) $A = 0$ or $B = 0$ (B) $A = 0\&B = 0$ (C) A and B are non zero square matrices

(D) A and B can not both non singular

Q 7. Let A is a square matrix of order 3×3 such that $|A| = -3$, then $|-3AA^T|$ equals

(A) 243 (B) -243 (C) -27 (D) -81

Q 8. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(A) $\frac{1}{10}$(B) $\frac{1}{8}$ (C) $\frac{7}{8}$ (D) $\frac{17}{20}$

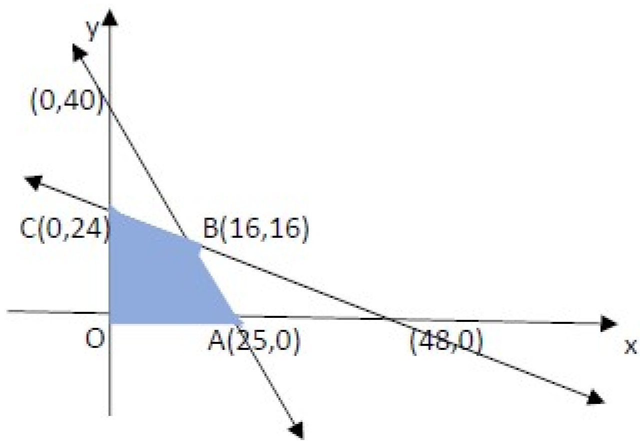
Q 9. If the angle between the vectors $x\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and $x\hat{\imath} - x\hat{\jmath} + 4\hat{k}$ is acute, then in which interval x lies

(A) $(-4,7)$ (B) $[-4,7]$ (C) $\mathbb{R} - [-4,7]$ (D) $\mathbb{R} - (-4,7)$

Q 10. If $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 400$ and $|\vec{a}| = 4$ then $|\vec{b}|$ will be

(A) 2 (B) 3 (C) 4 (D) 5

Q 11. The maximum value of $z = 4x + 3y$, If the feasible region for an LPP shown as in the graph is-



(A) 100 (B) 72 (C) 112(D) None of These

Q 12. $\int \frac{1}{x(x^4+1)} dx$ equals

(A) $\frac{1}{4}\log_e\left(\frac{x^4+1}{x^4}\right) + C$ (B) $\frac{1}{4}\log_e\left(\frac{x^4}{x^4+1}\right) + C(C)\frac{1}{4}\log_e(x^4 + 1) + C$(D) None of These

Q 13. $\int_0^{\frac{\pi}{8}} tan^2 2x\, dx$ is equal to

(A) $\frac{4-\pi}{8}$ (B) $\frac{4+\pi}{8}$ (C) $\frac{4-\pi}{4}$(D) $\frac{4-\pi}{2}$

Q 14. $\tan^{-1} x + \tan^{-1} y = c$ is the general solution of the differential equation

(A) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (B) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$ (C) $(1 + x^2)dy + (1 + x^2)dx = 0$

(D)) $(1 + x^2)dx + (1 + x^2)dy = 0$

Q 15. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0,1)$ then value of x is

(A) 0 (B) $\frac{a}{2}$ (C) a (D) $\frac{2a}{1-a^2}$

Q 16. The corner points of the feasible region determined by the system of linear constraints are: $(0,10), (5,5), (15,15), (0,20)$. Let $z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of z occurs at both the points $(15,15)$ and $(0,20)$ is

(A) $p = q$ (B) $p = 2q$ (C) $q = 2p$ (D) $q = 3p$

Q 17. If the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$ then the value of k is

(A) 0 (B) 1 (C) -1 (D) 2

Q 18. The area enclosed between the curves $y^2 = x$ and $y = |x|$, is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of(A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

Q 19. **Assertion(A):** The function $f(x) = |x - 1| + |x - 2|$ is not differentiable at $x = 1, 2$. However, itis everywhere continuous.

Reason (R): The function $f(x) = |x - a| + |x - b|$, where $a < b$ is everywhere continuous but notdifferentiable at $x = a, b$.

Q 20. Let $X = \{0, 2, 4, 6, 8\}$ and R be a relation on X defined by $R = \{(0,2), (4,2), (4,6), (8,6), (2,4), (0,4)\}$.

Assertion (A): The relation R on set X is a transitive relation.

Reason (R): The relation R has a subset $\{(a, b), (b, c), (a, c)\}$, where $a, b, c \in X$.

SECTION B **[2× 5 = 10]**

(This section comprises of 5 very short answer (VSA) type-questions of 2 marks each.)

Q 21. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then find $\cot^{-1} x + \cot^{-1} y$.

Q 22. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$.

Q 23. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots to \infty}}}$ prove that $(2y - 1)\frac{dy}{dx} = \frac{1}{x}$

OR

If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, $a > 0$ and $-1 < t < 1$. Show that $\frac{dy}{dx} = -\frac{y}{x}$

Q 24. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of x for which the vectors $\vec{\alpha} = (2x + 1)\vec{a} - \vec{b}$ and $\vec{\beta} = (x - 2)\vec{a} + \vec{b}$ are collinear.

.
OR

If a line makes an angle α, β and γ with the co-ordinate axes, find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Q 25. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$represent two adjacent sides of parallelogram, find unitvectors parallel to the diagonals of the parallelogram.

SECTION – C [3× 6 = 18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each)

Q 26. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is $10 m/s$. How fast is the string being let out; when the kite is $250\,m$ away from the boy who is flying the kite? Theheight of boy is $1.5\,m$.

Q 27. The volume of a cube is increasing at a constant rate. Prove that the increase in surface is avaries inversely as the length of the edge of the cube.

Q 28. Find the position vector of a point R which divides the line segment joining P and Q whoseposition vectors are $2\vec{a} + \vec{b}$and$\vec{a} - 3\vec{b}$,externally in the ratio 1: 2. Also, show that P is the midpointof the line segment RQ.

OR

A line passes through $(2, -1, 3)$ and perpendicular to the line$\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$. Fid the equation of line.

Q 29. Find $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx$.

OR

Evaluate: $\int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx$.

Q 30. Solve the Linear Programming Problem graphically:
Maximize $z = 5x + 2y$
subject to the following constraints:

$$x - 2y \leq 2,$$
$$3x + 2y \leq 12,$$
$$-3x + 2y \leq 3,$$
$$x \geq 0, y \geq 0$$

Q 31. A town has two fire extinguishing engines functioning independently. The probability of availability of each engine, when needed, is 0.95. What is the probability that
(i) neither of them is available when needed?
(ii) an engine is available when needed?
(iii) exactly one engine is available when needed?

OR

Suppose that 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

SECTION – D [5× 4 = 20]

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q 32. Draw a rough sketch of the curve$y = \sqrt{x - 1}$in the interval [1, 5]. Find the area underthe curveand between the lines $x = 1$ and $x = 5$.

Q 33. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system ofequations:

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Q 34. If $y^2 = a^2cos^2x + b^2sin^2x$ then prove that $\frac{d^2y}{dx^2} + y = \frac{a^2b^2}{y^3}$.

OR

If $x^m y^n = (x + y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

Q 35. Find the image of the point$(1,6,3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ also write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

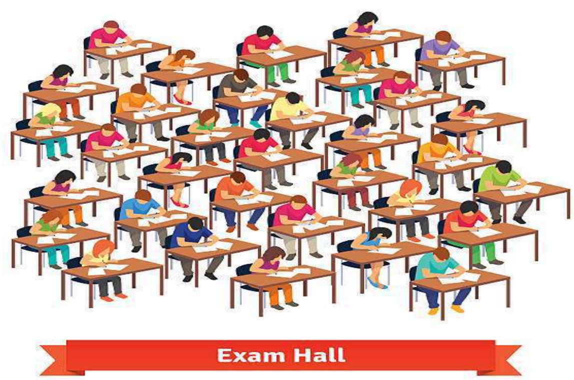
OR

Find the shortest distance between the lines
$\vec{r} = (4\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$.

SECTION – E **[4× 3 = 12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first and third case study questions have three subparts (i), (ii), (iii) of marks 2, 1, 1 respectively. The second case study question has two subparts of 2 marks each)

Q 36. Read the text carefully and answer the questions:
Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

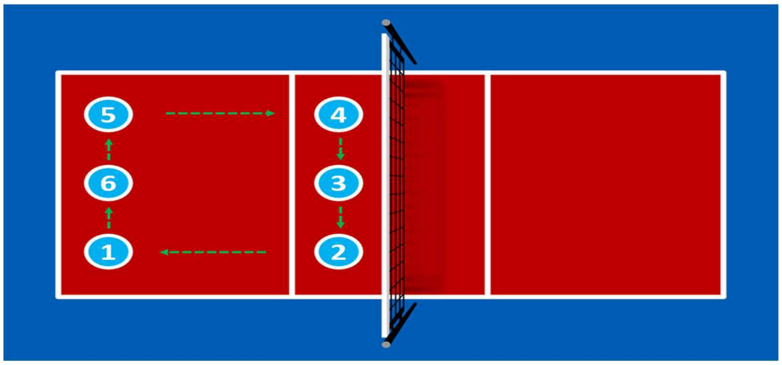


(i) Find the probability that she gets grade A in all subjects. [2 Marks]
(ii) Find the probability that she gets grade A in no subjects.[1 Mark]
(iii) Find the probability that she gets grade A in two subjects.[1 Mark]

OR

(iii) Find the probability that she gets grade A in one subject.[1 Mark]

Q 37. Read the text carefully and answer the questions:
A volleyball player serves the ball which takes a parabolic path given by the equation
$h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where $h(t)$is the height of ball at any time t (in seconds), $(t \geq 0)$



(i)Is h(t) a continuous function? Justify. [2 Marks]

(ii) Find the time at which the height of the ball is maximum. [2 Marks]

Q 38. Read the following passage and answer the following questions.
Ravi and Manish are playing Ludo at home during summer vacation. While rolling the dice, Ravi's sister Jyoti observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible

outcomes.

(i) Let $R: B \to B$ be defined by $R = \{(x, y): y \text{ } is \text{ } divisible \text{ } by \text{ } x\}$ Verify that whether R isreflexive, symmetric and transitive. [2 Marks]

(ii) Jyoti wants to know the number of functions from A to B. Find the number of all possiblefunctions. [1 Mark]

(iii) Let R be a relation on B defined by $R = \{(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)\}$. Then R is which kind of relation? [1 Mark]

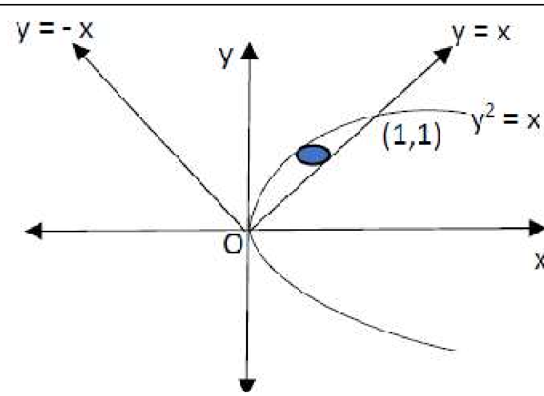
OR

(iii) Jyoti wants to know the number of relations possible from A to B. Find the number of possible relations. [1 Mark]

NAVODAYA VIDYALAYA SAMITI - RO SHILLONG
WHOLE SYLLABUSPRACTICE PAPER SET II
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)
MARKING SCHEME

Q.No.	Ans.	Hints/Solution
1.	(D)	Since A is a square matrix of order 3. Therefore, adj $(4A) = \lambda$ adj $(A) \Longrightarrow 4^2$ adj $(A)\{adj(kA) = k^{n-1}\ adj\ (A)\}$ $\lambda = 4^2$ where $o(A) = n$ $\lambda = 16$
2.	(B)	Since, matrix A is skew symmetric matrix. $\therefore A^T = -A$ $A^T + A = 0$ $a = -2$ and $b = 3$
3.	(B)	We have $f(x) = x^2 + ax + 1$ Therefore, $f'(x) = 2x + a$ Now, the function f is strictly increasing on [1,2] Therefore, $\Longrightarrow f'(x) > 0$ $\Longrightarrow 2x + a > 0$ $\Longrightarrow 2x > -a$ $\Longrightarrow x > -a/2$ Here, we have $1 \leq x \leq 2$ Thus, $-a/2 > 1$ $a > -2$
4.	(A)	Det. is $2a^2 + 8a + 44$ Acc. to given $2a^2 + 8a + 44 = 86$ Sum of roots $= -4$
5.	(D)	$x\dfrac{dy}{dx} + my = x^2e^x$ divide by x $\Longrightarrow \dfrac{dy}{dx} + \dfrac{m}{x}y = xe^x$ $\Longrightarrow$ I.F. $= e^{\int \frac{m}{x}dx} = \frac{1}{x^2}$

		$\Rightarrow e^{mlogx} = \dfrac{1}{x^2}$ $\Rightarrow m = -2$
6.	(C)	Since, AB is defined, neither A nor B is singular i.e., they are non-zero matrix and if AB = 0 both A and B are square matrix.So, A and B are non-zero square matrices.
7.	(B)	$\|-3AA^T\| = (-3)^3\|A\|\|A^T\|$ $= (-3)^3 \times -3 \times -3$ $= -243$
8.	(C)	$P(A) = \dfrac{4}{5}$ $P(A \cap B) = \dfrac{7}{10}$ $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{7/10}{4/5} = \dfrac{7}{8}$
9.	(C)	If angle θ is acute angle then $\vec{a}.\vec{b} > 0$ $(x\hat{\imath} + 3\hat{\jmath} - 7\hat{k}).(x\hat{\imath} - x\hat{\jmath} + 4\hat{k}) > 0$ $x^2 - 3x - 28 > 0$ $(x+4)(x-7) > 0$ So, $x \in \mathbb{R} - [-4,7]$. 
10.	(D)	We know that, $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = \|\vec{a}\|^2\|\vec{b}\|^2 \Rightarrow 400$ $4^2\|\vec{b}\|^2 = 400$ $\|\vec{b}\| = 5$
11.	(C)	Given: $z = 4x + 3y$ z is minimum at B(16,16) $z = 4(16) + 3(16)$ $z = 112$
12.	(B)	$\int \dfrac{1}{x(x^4+1)}dx$ Multiply and divide by x^5 and by substitution we get option B.
13.	(A)	$\mathrm{I} = \int_0^{\frac{\pi}{8}} tan^2\,(2x)dx$ $= \int_0^{\frac{\pi}{8}} \{sec^2(2x) - 1\}dx$ = and operate limit on $\frac{1}{2}(\tan 2x - x)$ we get option A.
14.	(C)	We have ,$\tan^{-1} x + \tan^{-1} y = c$ Diff. w.r.t.x , we get $\frac{1}{1+x^2} + \frac{1}{1+y^2}\frac{dy}{dx} = 0,$ $(1+x^2)dy + (1+y^2)dx = 0$
15.	(D)	We know that $\sin^{-1}\left(\frac{2a}{1+a^2}\right) = 2\tan^{-1} a$ for $-1 \le a \le 1$

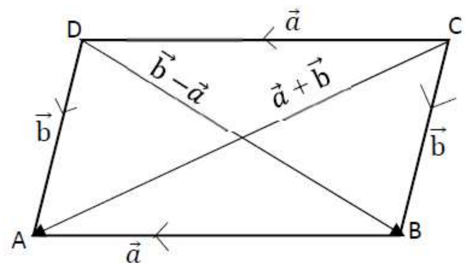
		$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = 2\tan^{-1}a$ for $0 \le a < \infty$ $\tan^{-1}\left(\frac{2a}{1-a^2}\right) = 2\tan^{-1}a$ for $-1 < a < 1$ Using above formula we get $x = \frac{2a}{1-a^2}$.
16.	(D)	Given that maximum $z = px + qy$ occurs at both the points $(15,15)$ and $(0,20)$ $\therefore 15p + 15q = 0 \times p + 20 \times q$ $\Rightarrow 15p = 15q \Rightarrow 3p = q$
17.	(B)	If $f(x)$ is continuous at $x = 0$, then $\lim\limits_{x \to 0} f(x) = f(0)$ $\lim\limits_{x \to 0}\left(\frac{1-\cos 4x}{8x^2}\right) = k$ $\Rightarrow \lim\limits_{x \to 0}\left(\frac{2sin^2 2x}{8x^2}\right) = k \Rightarrow \lim\limits_{x \to 0}\left(\frac{\sin 2x}{2x}\right)^2 = k$ $k = 1^2 \Rightarrow k = 1$
18.	(C)	Area $= \int_0^1(\sqrt{x} - x)dx$ $= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1$ $= \frac{1}{6}$ 
19.	(A)	Both (A) and (R) are true and (R) is the correct explanation of (A). (By continuity and differentiability of Modulus function)
20.	(D)	We find that $(4,2)$ R and $(2,4)$R but $(4,4) \notin$R. So, R is not transitive. Consequently, Assertion is not true. Reason is true as a relation on X is a subset of X×X.

SECTION – B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

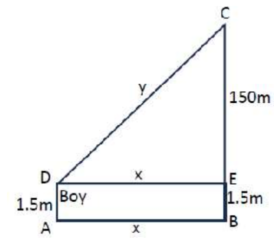
21.	$\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5} \Rightarrow \left(\frac{\pi}{2} - \cot^{-1}x\right) + \left(\frac{\pi}{2} - \cot^{-1}y\right) = \frac{4\pi}{5}$ $\Rightarrow \pi - (\cot^{-1}x + \cot^{-1}y) = \frac{4\pi}{5}$ $\Rightarrow \cot^{-1}x + \cot^{-1}y = \frac{\pi}{5}$	1 1
22.	Since marginal revenue is the rate of change of total revenue with	

	respect to the number of units sold, we have Marginal Revenue $MR = \frac{dR}{dx} = 6x + 36$ When $x = 5$, $MR = 6(5) + 36 = 66$ Hence, the required marginal revenue is 66 Rs.	1 1
23.	$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots to\ \infty}}}$ $y = \sqrt{\log x + (y)}$ On squaring both side $y^2 = \log x + y$ Diff. w.r.t.x $2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Longrightarrow (2y - 1)\frac{dy}{dx} = \frac{1}{x}$ Hence proved. **OR** $x = \sqrt{a^{\sin^{-1} t}}.........(i) y = \sqrt{a^{\cos^{-1} t}}.........(ii)$ Eqn (i) multiply by Eqn (ii), we get $xy = \sqrt{a^{\sin^{-1} t + \cos^{-1} t}}$ $xy = \sqrt{a^{\frac{\pi}{2}}}$ $\because \left\{\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right\}$ Diff. w.r.t.x $x\frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ Hence proved.	1 1 1 1

24.	Given vectors $\vec{\alpha}$ and $\vec{\beta}$ will be collinear,if $\vec{\alpha} = m\vec{\beta}$ for some scalar m $\Rightarrow (2x+1)\vec{a} - \vec{b} = m\{(x-2)\vec{a} + \vec{b}\}$ $\Rightarrow \{(2x+1) - m(x-2)\}\vec{a} - (m+1)\vec{b} = \vec{0}$ $\Rightarrow (2x+1) - m(x-2) = 0$ and $-(m+1) = 0$ $\Rightarrow m = -1$ and $x = \frac{1}{3}$ **OR** We know that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$ $\frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1 \qquad \because cos^2\theta = \frac{1+\cos 2\theta}{2}$ $3 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2$ $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$	1 1 1 ½ ½
25.	Let ABCD be a parallelogram such that$\overrightarrow{AB} = \vec{a}$ and$\overrightarrow{BC} = \vec{b}$ .Then, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ $\Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b} = 3\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$ $\overrightarrow{\lvert AC \rvert} = \sqrt{9+36+4} = 7$ And $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$ $\Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$ $\Rightarrow \overrightarrow{BD} = \vec{b} - \vec{a} = \hat{\imath} + 2\hat{\jmath} - 8\hat{k}$ $\Rightarrow \overrightarrow{\lvert BD \rvert} = \sqrt{1+4+64} = \sqrt{69}$  Unit vector along $\overrightarrow{AC} = \frac{\overrightarrow{AC}}{\overrightarrow{\lvert AC \rvert}} = \frac{1}{7}(3\hat{\imath} + 6\hat{\jmath} - 2\hat{k})$ Unit vector along $\overrightarrow{BD} = \frac{\overrightarrow{BD}}{\overrightarrow{\lvert BD \rvert}} = \frac{1}{\sqrt{69}}(\hat{\imath} + 2\hat{\jmath} - 8\hat{k})$	1 $\frac{1}{2}$ $\frac{1}{2}$

SECTION – C

[This section comprises of solution short answer type questions (SA) of 3 marks each]

<table>
<tr>
<td>26.</td>
<td>
Let the position of the kite at time t be at C.
$\therefore BC = 151.5m$
Let AD be the boy who is flying the kite.
Let AB = x =DE and DC = y
$\therefore CE = BC - BE = 151.5 = 150m$


Now, in $\Delta CDE,$
$y^2 + x^2 = 150^2$
$\Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$
$\Rightarrow \frac{dy}{dt} = \frac{x}{y}10 \qquad \left(\because \frac{dx}{dt} = 10\,m/s\right)$
$\Rightarrow \frac{dy}{dt} = \frac{10\sqrt{y^2 - 150^2}}{y} = \frac{10\sqrt{250^2 - 150^2}}{250}(y = 250m)$
$\Rightarrow \frac{10 \times 200}{250} = 8\,m/s$
</td>
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1

1</td>
</tr>
<tr>
<td>27.</td>
<td>
Let x be the length of each edge of the cube, S be its surface area and V be its volume at any time t,
Then, $S = 6x^2$ and $V = x^3$. It is given that $\frac{dV}{dt} = k$(constant)
Now, $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt}$
$\Rightarrow k = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{k}{3x^2}$

And $S = 6x^2$
$\frac{dS}{dt} = 12x\frac{dx}{dt} = 12x\frac{k}{3x^2} = \frac{4k}{x}$
$\frac{dS}{dt} \propto \frac{1}{x}$
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1

1</td>
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<tr>
<td>28.</td>
<td>
It is given that R divides PQ externally in the ratio 1:2.


$\therefore$ The position vector of $R = \frac{1\times(\vec{a}-3\vec{b})-2\times(2\vec{a}+\vec{b})}{1-2} = 3\vec{a} + 5\vec{b}$
Now,
$\frac{\text{Position vector of } R + \text{Position vector of } Q}{2} = \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2}$
$= 2\vec{a} + \vec{b}(\text{Position vector of } P)$
Hence, P is the midpoint of RQ.

OR
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1

1</td>
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	The required line is perpendicular to the lines which are parallel to vectors $\vec{b}_1 = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b}_2 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ respectively. So, its parallel to the vector $\vec{b} = \vec{b}_1 \times \vec{b}_2$. Now , $\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -6\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ Thus, the required line passes through the points $(2, -1, 3)$ and is parallel to the vector $\vec{b} = -6\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. So, its vector equation is $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(-6\hat{\imath} - 3\hat{\jmath} + 6\hat{k})[using\ \vec{r} = \vec{a} + \lambda\vec{b}]$ Or, $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$, where $\mu = -3\lambda$	1 ½ ½ 1
29.	We have I $= \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx$ ……………. (i) $= \int_2^8 \frac{\sqrt{10-(10-x)}}{\sqrt{10-x} + \sqrt{10-(10-x)}} dx \left(\text{using property} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$ $\text{I} = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} dx$ ……………. (ii) Adding (i) and (ii), we get $2\text{I} = \int_2^8 1dx = 8 - 2 = 6$ Hence , I = 3 **OR** Let I $= \int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x} \right) dx = \int e^{2x} \left(\frac{1+2\sin x \cos}{2 \cos^2 x} \right) dx$ $= \int e^{2x} \left(\frac{1}{2} sec^2 x + \tan x \right) dx$	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1

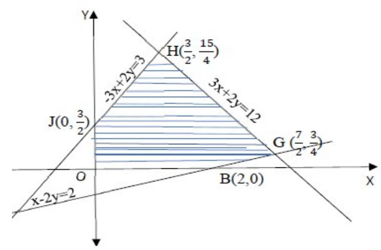
Put $2x = t \implies dx = \frac{1}{2}dt$

$$= \frac{1}{2}\int e^t\left(\frac{1}{2}sec^2\frac{t}{2} + \tan\frac{t}{2}\right)dt$$

$\text{I} = \frac{1}{2}e^t\tan\frac{t}{2} + C = \frac{1}{2}e^{2x}\tan x + C \quad \therefore using \int e^x(f'^{(x)} + f(x)dx = e^x f(x) + C)$

30. The subject of constraints are:

$x - 2y \le 2,$
$3x + 2y \le 12,$
$-3x + 2y \le 3,$
$x \ge 0, y \ge 0$



On solving (i), we have $A(0, -1), B(2, 0)$
On solving (ii), we have $C(0, 6), D(4, 0)$ 2
On solving (iii), we have $E(-1, 0), F(1, 3)$
It is observed that the feasible region OBGHJ is bounded.
Thus, we use corner point method to determine the maximum value of z, where$z = 5x + 2y$

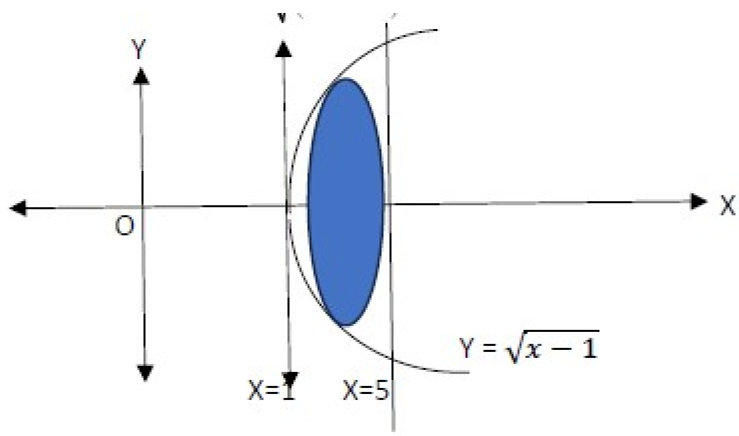
Corner Point	Corresponding value of z=5x+2y
B(2,0)	10
G $(\frac{7}{2}, \frac{3}{4})$	19
H$(\frac{3}{2}, \frac{15}{4})$	15
J$(0, \frac{3}{2})$	3

Hence, Z max = 19 at the point G $\left(\frac{7}{2}, \frac{3}{4}\right)$ 1

31.	Let A denote the event that first engine is available when needed and B, the event that second engine is available when needed. Then, P(A) = P(B) = 0.95. (i) Required probability = $P(\bar{A} \cap \bar{B})$ $= P(\bar{A})P(\bar{B}) = (0.05) \times (0.05) = 0.0025$ $\because (A, B \text{ are independent})$ (ii) Required probability = $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$ $= 1 - (0.05) \times (0.05) = 0.9975$ $\because (A, b \text{ are independent})$ (iii)Required probability = $P(A) + P(B) - 2P(A \cap B)$ $= P(A) + P(B) - 2P(A).P(B)$ $= 0.95 + 0.95 - 2 \times 0.95 \times 0.95 = 0.095$ **OR** Consider the following events: E_1= Person selected is male, E_2 = Person selected is female, A = Person selected is grey haired. Clearly, $P(E_1)=P(E_2)=\frac{1}{2}$, $P\left(\frac{A}{E_1}\right)=\frac{5}{100}$, $P\left(\frac{A}{E_2}\right)=\frac{1}{400}$ $\therefore$ Required probability = $P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$ $=\frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{1}{400}}=\frac{20}{21}$	1 1 1 1 1 1

SECTION – D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.	Given equation of the curve is $y = \sqrt{x-1}$ $y^2 = x - 1$ $\therefore$ Area of shaded region, $A = \int_1^5 (x-1)^{\frac{1}{2}} dx = \left[\frac{2}{3}(x-1)^{\frac{3}{2}}\right]_1^5$ $= \left[\frac{2}{3}(5-1)^{\frac{3}{2}} - 0\right] = \frac{16}{3}$ sq. units 	2 1 2

33.

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$. Then the given product is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\left[\right.$$

$x = 3, y = -$

1

$$CA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$CA = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I_3$$

1

$$\frac{1}{8}CA = I_3 \Rightarrow \left(\frac{1}{8}C\right)A = I_3 \Rightarrow A^{-1} = \frac{1}{8}C$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

1

The given system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} \quad \text{or, } AX = B,$$

1

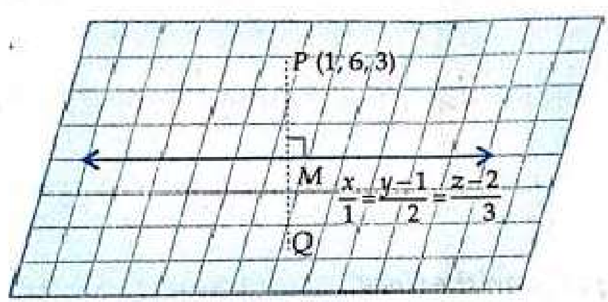
Where $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

The solution of this system of equations is given by

$$X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

1

34.	We have, $y^2 = a^2\cos^2 x + b^2\sin^2 x$ $2y^2 = a^2\left(2\cos^2 x\right) + b^2\left(2\sin^2 x\right) = a^2\left(1+\cos 2x\right) + b^2\left(1-\cos 2x\right)$ $2y^2 = \left(a^2+b^2\right) + \left(a^2-b^2\right)\cos 2x$ ....(i) Differentiating with respect to x, we get $4y\frac{dy}{dx} = -2\left(a^2-b^2\right)\sin 2x \Rightarrow 2y\frac{dy}{dx} = -\left(a^2-b^2\right)\sin 2x$ ....(ii) From (i), we obtain $2y^2 - (a^2+b^2) = (a^2-b^2)\cos 2x$ ....(iii) Squaring (ii) and (iii) and adding, we getb $4y^2\left(\frac{dy}{dx}\right)^2 + \left\{2y^2 - \left(a^2+b^2\right)\right\}^2 = \left(a^2-b^2\right)^2\left(\sin^2 2x + \cos^2 2x\right)$ $4y^2\left(\frac{dy}{dx}\right)^2 + 4y^4 - 4y^2\left(a^2+b^2\right) + \left(a^2+b^2\right)^2 = \left(a^2-b^2\right)^2$ $4y^2\left\{\left(\frac{dy}{dx}\right)^2 + y^2 - \left(a^2+b^2\right)\right\} = \left(a^2-b^2\right)^2 - \left(a^2+b^2\right)^2$ $4y^2\left\{\left(\frac{dy}{dx}\right)^2 + y^2 - \left(a^2+b^2\right)\right\} = -4a^2b^2$ $\left(\frac{dy}{dx}\right)^2 + y^2 - \left(a^2+b^2\right) = -\frac{a^2b^2}{y^2}$ Differentiating both sides with respect to x, we get $2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = \frac{2a^2b^2}{y^3}\frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} + y = \frac{a^2b^2}{y^3}\left(\text{Dividing both sides by } 2\frac{dy}{dx}\right)$	1 1 1 1 1 2

	OR We have, $x^m y^n = (x+y)^{m+n}$ Taking log on both sides, we get m log x + n log y = (m + n) log (x + y) Differentiating both sides with respect to x, we get $\Rightarrow m \times \frac{1}{x} + n \times \frac{1}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\frac{d}{dx}(x+y)$ $\Rightarrow \frac{m}{x} + \frac{n}{y} \times \frac{dy}{dx} = \frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$ $\Rightarrow \left\{\frac{n}{y} - \frac{m+n}{x+y}\right\}\frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{x}$ $\Rightarrow \left\{\frac{nx+ny-my-ny}{y(x+y)}\right\}\frac{dy}{dx} = \left\{\frac{mx+nx-mx-my}{(x+y)x}\right\}$ $\Rightarrow \frac{nx-my}{y(x+y)} \cdot \frac{dy}{dx} = \frac{nx-my}{(x+y)x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$	2 1
35.	Let Q be the image of point $P(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and M be the foot ofperpendicular drawn from P to this line. Then, $PM = MQ$  Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = r$ Then the coordinates of M be given by$(r, 2r+1, 3r+2)$. The direction ratios of PM are proportional to $r-1, 2r-5, 3r-1$. Since PM is perpendicular to the given line. Therefore, $1(r-1) + 2(2r-5) + 3(3r-1) = 0$ $14r - 14 = 0$	1 1 1

	$r = 1.$ So, the coordinates of M are $(1, 3, 5)$. Let(x_1, y_1, z_1) be the coordinates of Q. Since M is the mid-point of PQ $\frac{x_1+1}{2} = 1, \quad \frac{y_1+6}{2} = 3, \quad \frac{z_1+3}{2} = 5$ $x_1 = 1, \quad y_1 = 0, \quad z_1 = 7$ Thus, the coordinates of Q are $(1, 0, 7)$. So, the Cartesian equations of PQ are $\frac{x-1}{1-1} = \frac{y-6}{0-6} = \frac{z-3}{7-3}\, or\, \frac{x-1}{0} = \frac{y-6}{-6} = \frac{z-3}{4}$ And $PQ = \sqrt{(1-1)^2 + (6-0)^2 + (3-7)^2} = 2\sqrt{13}$. **OR** We know that the shortest distance between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by $\boldsymbol{d} = \left\| \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\|\overrightarrow{b_1} \times \overrightarrow{b_2}\|} \right\|$ Comparing the given equations with the equations$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ $\overrightarrow{a_1} = (4\hat{\imath} - \hat{\jmath}), \quad \overrightarrow{b_1} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ $\overrightarrow{a_2} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}), \quad \overrightarrow{b_2} = (2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ $(\overrightarrow{a_2} - \overrightarrow{a_1}) = -3\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$ $\overrightarrow{b_1} \times \overrightarrow{b_2} = 2\hat{\imath} - \hat{\jmath} + 0\hat{k}$ $\|\overrightarrow{b_1} \times \overrightarrow{b_2}\| = \sqrt{5}$ $d = \left\| \frac{(-3\hat{\imath} + 0\hat{\jmath} + 2\hat{k}).(2\hat{\imath} - \hat{\jmath} + 0\hat{k})}{\sqrt{5}} \right\|$ $d = \left\| \frac{-6}{\sqrt{5}} \right\| = \frac{6}{\sqrt{5}}$	1 1 2 2 1

SECTION – E

36.	(i) P(Grade A in Maths) = $P(M) = 0.2$ P(Grade A in Physics) = $P(P) = 0.3$ P(Grade A in Chemistry) = $P(C) = 0.5$ P(not A garde in Maths) = $P(\bar{M}) = 1 - 0.2 = 0.8$ P(not A garde in Physics) = $P(\bar{P}) = 1 - 0.3 = 0.7$ P(not A garde in Chemistry) = $P(\bar{C}) = 1 - 0.5 = 0.5$ P(getting grade A in all subjects) = $P(M \cap P \cap C) = P(M) \times P(P) \times P(C)$	2

	$= 0.2 \times 0.3 \times 0.5 = 0.03$ (ii) P(getting grade A in no subjects)$= P(\bar{M}) \times P(\bar{P}) \times P(\bar{C}) = 0.8 \times 0.7 \times 0.5 = 0.280$ (iii) P(getting grade A in 2 subjects) $\Rightarrow P(M \cap P \cap \bar{C}) + P(\bar{M} \cap P \cap C) + P(M \cap \bar{P} \cap C)$ $\Rightarrow 0.2 \times 0.3 \times 0.5 + 0.3 \times 0.5 \times 0.8 + 0.2 \times 0.5 \times 0.7$ $\Rightarrow 0.03 + 0.12 + 0.07 = 0.22$ P(getting grade A in 2 subjects) $= 0.22$ **OR** P(getting grade A in 1 subjects) $\Rightarrow P(M \cap \bar{P} \cap \bar{C}) + P(\bar{M} \cap P \cap \bar{C}) + P(\bar{M} \cap \bar{P} \cap C)$ $\Rightarrow 0.2 \times 0.7 \times 0.5 + 0.3 \times 0.5 \times 0.8 + 0.5 \times 0.8 \times 0.7$ $\Rightarrow 0.07 + 0.12 + 0.028 = 0.47$ P(getting grade A in 1 subjects) $= 0.47$	1 1 1
37.	(i) Given $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1 (t \geq 0)$ It is a polynomial (in t) so it is a continuous function. Since every polynomial function is continuous function (ii) $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1 (t \geq 0)$ For height to be maximum or minimum $\frac{dh}{dt} = 0$ $\Rightarrow -7t + \frac{13}{2} = 0 \Rightarrow \quad t = \frac{13}{14}$ $\frac{d^2h}{dt^2} = -7 < 0$ $\Rightarrow h(t)$ is maximum at $t = \frac{13}{14}$ sec	2 1 1
38.	(i) Given: $R: B \to B$ be defined by $R = \{(x, y): y \textit{ is divisible by } x\}$ Reflexive: Let $x \in B$, since x always divide x itself$\Rightarrow (x, x) \in R$. So reflexive. Symmetric: $(1,6) \in R$ as 6 is divisible by 1 but $(1,6) \notin R$ So not symmetric. Transitive: let $(x, y) \in R \Rightarrow y$ is divisible by $x \Rightarrow y = \lambda x$ let $(y, z) \in R \Rightarrow z$ is divisible by $y \Rightarrow z = \mu y \Rightarrow z = \mu.\lambda.x \Rightarrow$ z is divisible by x $\Rightarrow (x, z) \in R \Rightarrow$ So transitive. Hence the given relation is Reflexive and Transitive but not Symmetric.	2 1

	(ii)We have, $A = \{S, D\} \Rightarrow n(A) = 2$ and, $B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(B) = 6$ Number of functions from A to B is $6^2 = 36$.	1
	(iii) Given, R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$ R is not reflexive since $(1, 1), (3, 3), (4, 4) \notin R$ R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$ and, R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but$(1, 1) \notin R$ So R is neither reflexive nor symmetric nor transitive **OR** $n(A) = 2, n(B) = 6 \Rightarrow n(A \times B) = 12$ Total number of possible relations from A to B = 2^{12}	1

NVS RO-SHILLONG
WHOLE SYLLABUS PRACTICE QUESTION PAPER SET-III
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)
TIME: 3 HRS **MAX MARKS:80**
BLUE PRINT

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6**. Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Unit	Typology of questions			Total
	Remembering and Understanding	Applying	Analysing, Evaluating and creating	
1.Relations and functions.	1x2 = 2 2x1=2		4x1=4	8
2.Algebra	1x3=3 2x1=2 5x1=5			10
3.Calculus	1x7=7 2x1=2 5x1=5	3x3=9	4x1=4 5x1=5 2x1=2 1x1=1	35
4.Vector and Three dimensional Geometry	1x3=3 3x2=6 5x1=5	5x1=5		14
5.Linear Programming	2x1=2 3x1=3			5
6.Probalbility	1x2=2	4x1=4 2x1=2		8
Total	44	20	16	80

Navodaya Vidyalaya Samiti, RO Shillong
WHOLE SYLLABUS PRACTICE PAPER SET-III
(2024-25)
Class-XII
Subject: Mathematics (041)

Time:3 Hours **Maximum Marks:80**

General Instructions

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7. Use of calculators is **not** allowed.

SECTION-A [1X20=20]

Q1. If A , B are square matrices of order 3 , A in non singular and AB = 0, then B is

a) Non singular b) null matrix c) singular d) unit matrix.

b)

Q2. Let R be a relation in the set N given by R ={ (a,b) : a + 2 =b , b>6 } .Choose the correct answer : a) $(2,4) \in R,$ b) $(3,8) \in R$ c) $(6,8) \in R$ d) $(8,7) \in R.$

Q3. If A is a 2 rowed square matrix and I A I = 6 then A. adj A = ?

a) $\begin{bmatrix} 1/6 & 0 \\ 0 & 1/6 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Q4. If x , y , z are non zero real numbers , then the inverse of matrix A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

a) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $= \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ c) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

d) $\frac{5yz}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q5. The differential equation of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$, is called

a) Non homogeneous differential equation

b) homogeneous differential equation

c) linear differential equation   d) non linear differential equation

Q6. Range of cosec $^{-1}$ x is

a) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ b) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - { 0 }   c) $(\frac{-\pi}{2},\frac{\pi}{2})$ d) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - { 1 }

Q7. $\int_{-\pi}^{\pi} sin^{2025}x\, dx = \ ?$

a) 0   b) $\frac{5\pi}{16}$ c) 2π d) $\frac{3\pi}{4}$

Q8. The Cartesian equation of a line are $\frac{x-1}{2} = \frac{y+3}{3} = \frac{z-5}{-1}$ .Its vector equation is

a) $\vec{r}$ =($\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$) + $\gamma(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$ b) $\vec{r}$ =($\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$) + $\gamma(2\hat{\imath} - 3\hat{\jmath} - \hat{k})$

c) $\vec{r}$ =($\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$) + $\gamma(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$ d) $\vec{r}$ =($\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$) + $\gamma(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$

Q9. The integrating factor of the differential equation x dy/dx + y = x^2 , is

a) X   b) x^2 c) 1/x   d) – x .

b)

Q10. Which of the following function is decreasing in $(0,\frac{\pi}{2})$?

a) Sin2x   b) tanx   c) cosx   d) cos 3x

b)

Q11. $\int \frac{1+tan}{1-tan} dx$ is equal to :

a) Sec^2 $(\frac{\pi}{4} - x)$ + c   b) Sec^2 $(\frac{\pi}{4} + x)$ + c   c) log Isec$(\frac{\pi}{4} - x)$I + c

d) log Isec$(\frac{\pi}{4} + x)$I + c

Q12. The point of discontinuity of the function f(x) = $\begin{cases} 2x+7 & ,if\ x \le 2, \\ 2x-7, & if, x > 2 \end{cases}$ is

a) X=2   b) x = -1   c) x =0   d) x =1

Q13. The three points P(-1,3,2) , Q(-4,2, - 2), and R(5,5,k) are collinear then the value of k is

a) 5   b) 10   c) 8   d) 7

Q14. If A and B are two events such that P(A) = ½ , P(B) = 1/3, P (A/B) = ¼ , then P (A$^{/}$∩ B$^{/}$) equals to

a) 1/12   b) 3/16   c) ¼   d)3/4

Q15. If y = log$\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$, then dy/dx =

a) $\frac{-2}{\sqrt{1+x^2}}$ b) $\frac{2\sqrt{1+x^2}}{x^2}$ c) $\frac{2}{\sqrt{1+x^2}}$ d) none of these.

Q16. Sin(tan^{-1}x) , IxI<1 , is equal to

a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

Q17. Let the vectors $\vec{a}$ and $\vec{b}$ such that I $\vec{a}$ I = 3 and I $\vec{b}$ I = $\frac{\sqrt{2}}{3}$, if $\vec{a}$ x $\vec{b}$ is a unit vector then angle between $\vec{a}$ and $\vec{b}$ *is*

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

Q18. Probability that A speaks truth is 4/5 . A coin is tossed . A reports that a head appears. The probability that it was actually head is

a) 4/5 b) ½ c) 1/5 d) 2/5

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each.

Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Q19. If R is the relation in the set A = {1 ,2 , 3, 4, 5 } given R = { (a, b) : I a – b I is even },

Assertion (A): R is an equivalence relation.

Reason (R): All elements of {1,3,5} are related to all elements of { 2 , 4} .

Q20. Assertion (A): The rate of change of area of a circle with respect to its radius r = 6 cm is 12π cm^2 / cm.

Reason (R): Rate of change of area of a circle with respect to its radius r is dA/dr, where A is the area of the circle.

SECTION B **[2x5=10]**

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

Q21. Write the interval for the principal value of function $\cos^{-1}x$ and draw its graph.

OR

Find the value of $\tan^{-1}(\tan \frac{2\pi}{3})$

Q22. Two dice are thrown together. Let A be the event: Getting 6 on the first die , B be the event: getting 2 on the 2nd die. Are the events A and B are independent ?

Q23. Find the intervals in which the function given by f(x) = sin 3x , $x \in [0,\frac{\pi}{2}]$, is (a) increasing ,(b) decreasing .

OR

Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) increasing , (b) decreasing .

Q24. Integrate : $\int \frac{xe^x}{(1+x)^2}dx$

Q25. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

SECTION C **[3x6=18]**

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

Q26. Evaluate $\int_0^{\frac{\pi}{2}} \frac{cos^2 x}{cosx+sinx} dx$

Q27. Find the shortest distance between the lines $\vec{r} =(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \gamma(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$ and
$\vec{r} =(4\,\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + m(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

Q28. Minimise Z = 3x + 5y subject to the constraints:
X + 2y ≥ 10 , x + y ≥ 6 , 3x + y ≥ 8 , x , y ≥ 0
OR
Solve graphically the following linear programming problem:
Maximise , Z = 6x + 3y , subject to the constraints :
4x + y ≥ 80 , 3x + 2y ≤ 150 , x + 5y ≥ 115, x , y ≥ 0

Q29. Find the area enclosed by the parabola $4y = 3x^2$ and the line 2y = 3x + 12 .

Q30. Find dy/dx, $y = x^{\sin x} + 4^x$.

Q31. If $\vec{a} =(5\,\hat{\imath} - \hat{\jmath} - 3\hat{k})$ and $\vec{b} =(\hat{\imath} + 3\hat{\jmath} - 5\hat{k})$, then check whether $\vec{a} + \vec{b}$ and $\vec{a}$ - $\vec{b}$ are perpendicular to each other .

SECTION D [5x4=20]

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q32. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$, find AB and use the product to solve the system of equation : x – y + z = 4 , x – 2y – 2z = 9 , 2x + y + 3z = 1

Q33. Using integration, find the area of the region in the first quadrant enclosed by the Y-axis, the line y = x and the circle $x^2 + y^2 = 32$.

Q34. Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

Q35 Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is of the volume of the sphere.

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

SECTION- E [4x3=12]

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks

1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Q36 Let A = {1, 2, 3, ...9} and R be the relation in A x A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A xA.

i) Write yes or no. Relation is reflexive. (1)

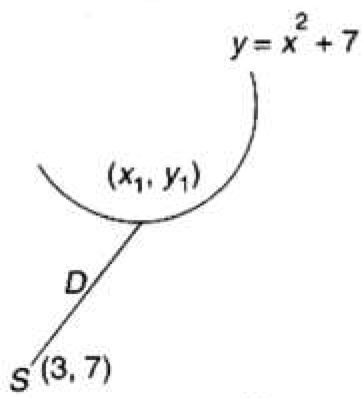
ii) Whether R symmetric ? (1)

iii) Write the elements related to (2,5). (2)

OR

show that R is an equivalent relation ?

37. Read the following text carefully and answer the questions that follow: An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$.



A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.

i. If P (x_1 , y_1) be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7) in terms of x. (1)

ii. Find the critical point such that distance is minimum. (1)

iii. . Verify by second derivative test that distance is minimum at (1, 8). (2)

OR

Find the minimum distance between soldier and helicopter? (2)

38. Nisha and Ayushi appeared for first round of an interview for two vacancies.

The probability of Nisha's selection is 1/3 and that of Ayushi's selection is ½ .

i) Find the probability that only one of them is selected. (1)

ii) The probability that none of them is selected. (1)

iii) Find the probability that at least one of them is selected. (2)

OR

Find the probability that both of them are selected. (2)

NVS RO SHILLONG
WHOLE SYLLABUS PRACTICE PAPER SET III
(2024-2025)
MARKING SCHEME
CLASS XII
MATHEMATICS(CODE-041)

Q.No	Answer	Mark
1	c) singular	1
2	c) (6,8)	1
3	d)$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$	1
4	b)	1
5	b) homogeneous differential equation	1
6	b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - { 0 }	1
7	a) 0	1
8	d) $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + \gamma(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$	1
9	a) x	1
10	c)cosx	1
11	d)	1
12	a) x=2	1
13	c)8	1
14	c) ¼	1
15	c)	1
16	d) $\frac{x}{\sqrt{1+x^2}}$	1
17	b) $\frac{\pi}{4}$	1
18	a) 4/5	1
19	c) A is true but R is false	1
20	a) Both A and R is true and R is the correct explanation of A.	1
21	Range of $\cos^{-1} x$ is $[0,\pi]$ OR, $-\pi/3$	2
22	$V=4/3\ \pi r^3$, $dv/dt = 8/3\ \pi r^2 dr/dt$, $25 = 8/3\ \pi r^2\ dr/dt$, $dr/dt = \frac{75}{8\pi r^2}$, Now, $S = 4\pi r^2$ $Ds/dt = 8\pi r\ dr/dt = 8\pi r . (\frac{75}{8\pi r^2}) = 75/r = 75/5 = 25\ cm^2/s$	½ ½ 1

23	$f(x)= \sin3x$, $f'(x) = 3\cos3x$ $f'(x)=0$, $\cos3x = 0 \Rightarrow 3x = \frac{\pi}{2},\frac{3\pi}{2}$ as $x\in [0,\frac{\pi}{2}]$ $x=\frac{\pi}{6}, x = \frac{\pi}{2}$ f(x) is increasing in $[0,\frac{\pi}{6})$, and decreasing in $(\frac{\pi}{6},\frac{\pi}{2})$ OR, $F(x)= 4x^3 - 6x^2 - 72x +30$ $f'(x) = 12x^2 - 12x - 72 = 12(x - 3)(x+2)$. $f'(x) = 0 \Rightarrow x= - 2 , 3$. Here, f(x) is increasing in $(-\infty, -2) \cup (3,\infty)$, Decreasing in (-2,3).	½ ½ ½ ½ ½ ½ 1
24	$$\int \frac{xe^x}{(1+x)^2}dx = \int \frac{(1+x-1)e^x}{(1+x)^2}dx$$ $=\int[\frac{1+x}{(1+x)^2} - \frac{1}{(1+x)^2}]e^x dx=\int[\frac{1}{(1+x)} - \frac{1}{(1+x)^2}]e^x dx$ $=e^x \frac{1}{1+x} + c$	½ 1 ½
25	$A=\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$, $A^2 = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} = \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}$ Now, $A^2 = kA - 2I \Rightarrow \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} = \begin{bmatrix}3k & -2k\\4k & -2k\end{bmatrix} - \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$ ⇨ K = 1	½ 1 ½
26	Let, $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2x}{\cos x+\sin x}dx$, ………(i) $= \int_0^{\frac{\pi}{2}} \frac{\cos^2(\frac{\pi}{2}-x)}{\cos(\frac{\pi}{2}-x)+\sin(\frac{\pi}{2}-x)}dx$, $= \int_0^{\frac{\pi}{2}} \frac{\sin^2x}{\cos x+\sin}dx$, ……….(ii) Adding (i) and (ii) : $2I = \int_0^{\frac{\pi}{2}} \frac{\cos^2x+\sin^2x}{\cos x+\sin x}dx$, $2I =\int_0^{\frac{\pi}{2}} \frac{1}{\cos x+\sin x}dx$, $= \int_0^{\frac{\pi}{2}} \frac{1}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}dx$, $=\int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{1-\tan^2\frac{x}{2}+2\tan\frac{x}{2}}dx$ Let, $\tan\frac{x}{2} = t$. Then, $d(\tan\frac{x}{2})=dt$, $(\sec^2 \frac{x}{2})1/2\, dx= dt$, Also, x=0, t = tan0=0, $x =\frac{\pi}{2}$, t= $\tan\frac{\pi}{4}=1$, $2I = \int_0^1 \frac{2dt}{2t+1-t^2} = 2\int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2)}$ $=2\frac{1}{2\sqrt{2}} \log\left\|\frac{\sqrt{2}+(t-1)}{\sqrt{2}-(t-1)}\right\| \begin{cases}1\\0\end{cases} = \frac{1}{\sqrt{2}} \log\left\|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\|$	½ ½ ½ ½ 1

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>27</td><td>Here, $\overrightarrow{a_1} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$, $\overrightarrow{b_1} = (\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$
$\overrightarrow{a_2} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k})$,, $\overrightarrow{b_2} = (2\hat{\imath} + 3\hat{\jmath} + \hat{k})$
$\overrightarrow{a_2} - \overrightarrow{a_1} = =(3\hat{\imath} + 3\hat{\jmath} + 3\hat{k})$
$\overrightarrow{b_1} X \overrightarrow{b_2} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{bmatrix} = -7\hat{\imath} + 0.\hat{\jmath} + 9\hat{k}$
Hence, SD = $\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} X \overrightarrow{b_2})}{|\overrightarrow{b_1} X \overrightarrow{b_2}|} = \frac{6}{\sqrt{112}}$</td><td>½
½
1
1</td></tr>
<tr><td>28</td><td>Drawing of graph and finding of feasible region ABCD :
<table><tr><td>Corner points</td><td>Value of Z</td></tr><tr><td>(0,8)</td><td>40</td></tr><tr><td>(1,5)</td><td>28</td></tr><tr><td>(2,4)</td><td>26(minimum)</td></tr><tr><td>(10,0)</td><td>30</td></tr><tr><td></td><td></td></tr></table></td><td>1 ½
1 ½</td></tr>
<tr><td>29</td><td>Drawing correct figure
Solving $4y = 3x^2$, $2y = 3x + 12$ gives , x = -2, 4
Area = $\int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx = 30$ units.</td><td>1
½
1 ½</td></tr>
<tr><td>30</td><td>Taking $u = x^{\sin x}$ of $y = x^{\sin x} + 4^x$,
$Y = u + 4^x$

$Dy/dx = du/dx + d/dx(4^x)$
$= du/dx + 4^x (\log 4)$ ……..(i)
Now, $u = x^{\sin x}$,
Log (u) = sinxlogx
Differentiating w.r.t x:
1/u du/dx = sinx/x + logx cosx
Du/dx = u(sinx/x + logx cosx)
$Du/dx = x^{\sin x}$ (sinx/x + logx cosx)
Thus from (i) ; $dy/dx = x^{\sin x}$ (sinx/x + logx cosx) + 4^x (log4)</td><td>½
½
½
1</td></tr>
<tr><td>31</td><td>$\vec{a} + \vec{b} = (5\hat{\imath} - \hat{\jmath} - 3\hat{k}) + (\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) = (6\hat{\imath} + 2\hat{\jmath} - 8\hat{k})$
$\vec{a} - \vec{b} = (5\hat{\imath} - \hat{\jmath} - 3\hat{k}) - (\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) = (4\hat{\imath} - 4\hat{\jmath} + 2\hat{k})$

$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 24-8-16 = 0$
Hence , $\overrightarrow{(\vec{a} + b)}$ and $\overrightarrow{(a - \vec{b})}$ are perpendicular to each other.</td><td>1
1 ½
½</td></tr>
</table>

32	Here, AB=$\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}=\begin{bmatrix}8 & 0 & 0\\0 & 8 & 0\\0 & 0 & 8\end{bmatrix}$ =8 I Hence, $A^{-1}=\frac{B}{8}$, Now the system can be written as , AX = D, Where, A = $\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$, D = $\begin{bmatrix}4\\9\\1\end{bmatrix}$, Hence , X = A^{-1} D , = 1/8 $\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}$ =1/8 $\begin{bmatrix}24\\-16\\-8\end{bmatrix}=\begin{bmatrix}3\\-2\\-1\end{bmatrix}$	1 ½ ½ ½ ½ 2
33	Given, equation of circle is $x^2 + y^2 = 32$ ......(i) Given ,equation of line is y = x ....(iii) Solving (i) and (ii) to get the points of intersection are (4, 4) and (-4, - 4) . So, given line and the circle intersect in the first quadrant at point A(4, 4) and The circle cut the Y-axis at point B $(0,4\sqrt{2})$. Proper sketch of the graph of given curves, Area of the required region: $\int_0^4 y dy + \int_4^{4\sqrt{2}} \sqrt{32-y^2}dy$ $=[\frac{y^2}{2}] + [\int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2-y^2 dy}$ $= ½(16\text{-}0) + \left[\frac{y\sqrt{(4\sqrt{2})^2-y^2}}{2}+\frac{(4\sqrt{2})^2}{2}sin^{-1}\frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$ $= 4\pi$ unit	1 1 1 1 1
34	Any point P on the line is (2k +1, - 3k -1,8k – 10) Let P be the foot of the perpendicular drawn from A (1,0,0) Hence D R of PA is 2k , -3 k-1 , 8k – 10. Since DR of the given line is 2 , - 3 , 8 . Hence, 2(2k) + (-3)(-3k – 1) + 8(8k – 10) = 0 77k = 77 => k = 1. Hence foot of the perpendicular is (3, - 4 , -2) The equation of the perpendicular with DR 2, - 4 , -2 is $\frac{x-3}{2}=\frac{y+4}{-4}=\frac{z+2}{-2}$	½ ½ ½ ½ ½ ½ 2
35	P(A)= 1/3 , P(A$'$) = 1 – 1/3 = 2/3 , P(B) =1/2 , P(B$'$) = ½ i) P(only one of them is selected) = P(A)P(B$'$) + P(A$'$)P(B)=1/3 .1/2 + 2/3.1/2 = 3/6 = ½ ii) P(at least one of them will be selected) = 1 – P(none of them will be selected)	1 2 1

	$= 1 - P(A' \cap B') = 1 - 2/3 \cdot ½ = 1 - 1/3 = 2/3$	1
36	i) Yes reflexive. ii) Symmetric. iii)	1 1
37	i) Distance of the point P(x,y) from (3,7) is $D = \sqrt{(x-3)^2 + (y-7)^2} = \sqrt{(x-3)^2 + (x^2+7-7)^2}$ $= \sqrt{(x-3)^2 + (x^2)^2} = \sqrt{(x-3)^2 + x^4}$ ii) Now, for extreme value of D, $d/dx\,(D^2) = 0$ $2(x-3) + 4x^3 = 0 \Rightarrow 2x^3 + x - 3 = 0 \Rightarrow x = 1$. For , x = 1 , y = 8. iii) $d^2D/dx^2 = 2 + 12x^2$, which is + ve for x = 1. Hence D is minimum at the point (1,8).	1 1 1 + 1
38	I) 1/2 ii)1/3 iii)2/3 OR 1/6	

NVS RO-SHILLONG
WHOLE SYLLABUS PRACTICE QUESTION PAPER SET-IV
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)
TIME: 3 HRS **MAX MARKS:80**
BLUE PRINT

Blue Print-2025(AS PER CBSE)

Class 12

		Section A		Section B	Section C	Section D	Section E	Total Marks(No. of Questions)
	Type of Questions	MCQ Type	Assertion and Reason Type	Very Short Answer type	Short Answer Type	Long	Case Study Type	
	Marks	1 M	1M	2M	3M	5M	4M	
Sl No.	Chapter	Number of Questions						
1	Relations and Functions	---	1	---	---	---	1	5(2)
2	Inverset Trigonometry	1	---	1	---	---	---	3(2)
3	Matrices	2	---	---	---	---	---	2(2)
4	Determinants	3	---	---	---	1	---	8(4)
5	Continuity and Differentiability	1	1	1*	---		---	4(3)
6	Applications of Derivatives	1	---	1	2	---	1	13(5)
7	Integral	2	---	---	1*	---	---	5(3)
8	Applications of Integration	1	---	---	---	1	---	6(2)
9	Differential Equations	2	---	---	---	1*	---	7(3)
10	Vector Algebra	2	---	2**	---	---	---	6(4)
11	3-D Geometry	---	---	---	1*	1*	---	8(2)
12	LPP	2	---	---	1	---	---	5(3)
13	Probability	1	---	---	1*	---	1	8(3)
Total		18	2	5	6	4	3	80(38)

* means internal option

Navodaya Vidyalaya Samiti, RO Shillong
WHOLE SYLLABUS PRACTICE PAPER SET-IV
(2024-25)
Class-XII
Subject: Mathematics (041)

Time:3 Hours **Maximum Marks:80**

General Instructions

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3**. Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5**. Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7.Use of calculators is **not** allowed.

Section A

This section contains multiple choice question (MCQ) of 1 mark each

1. If a matrix A = $\begin{pmatrix} \mathbf{0} & \boldsymbol{k} & \mathbf{-2} \\ \mathbf{3} & \mathbf{0} & \mathbf{3} \\ \mathbf{2} & \mathbf{-3} & \mathbf{0} \end{pmatrix}$ is a singular matrix. Then the value of k is
   A) -3 B) 3
   C) all real values D) None of these

2. The number of square matrices or order 3 x 3 whose every entry is either -1 or 3 is
   B) 9 B) 18
   C) 512 D) None of these

3. Function f(x) = $\frac{1}{3}\boldsymbol{x^3} + \frac{1}{2}\boldsymbol{x^2} + \boldsymbol{x} - \mathbf{102}$ is
   A) Increasing B) Strictly increasing
   C) Decreasing D) Strictly decreasing

4. If |A| = 4 where A is a square matrix of order 3, then the value of |adj A| + |A$^{/}$| is
   A) 64 B) 12
   C) 20 D) None of these

5. Integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \boldsymbol{logx}$ is
   A) logx B) x
   C) $\boldsymbol{e^x}$ D) none of these

6. The diagonal elements of a skew-symmetric matrix are
   A) 0 B) 1
   C) -1 D) None of these

7. If a matrix A = $\begin{pmatrix} \mathbf{0} & \mathbf{2b} & \mathbf{-2} \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{3a} & \mathbf{3} & \mathbf{-1} \end{pmatrix}$ is a symmetric matrix. Then the correct statement from the following is
   B) ab =1 B) ab = -1
   C) a + b = $\frac{5}{6}$ D) a + b = $-\frac{13}{6}$

8. In a single throw of a die, A = event of getting odd numbers and B = event of getting prime numbers, then
   B) A and B are independent events B) A and B are not independent events
   C) $P(A|B) = \frac{1}{3}$ D) None of these

9. Projection of the vector $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ on the vector $\vec{b} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ is
   A) 0 B) $\frac{2\sqrt{14}}{7}$
   C) 4 D) None of these

10. If $|\vec{a} \times \vec{b}| = 4$, $\vec{a} \cdot \vec{b} = 3$ and $|\vec{b}| = 5$, then $\vec{a}$ is
   A) A zero vector B) Vector with magnitude 2 units
   C) A unit vector D) None of these

11. In a linear programming problem, feasible region is the region where
   A) All possible solutions satisfying all the constraints of the problems exist.
   B) Only optimal solution exist
   C) Only non-negative solutions exist
   D) None of these

12. $\int \frac{2^x - 3^x}{5^x} dx$ is
   A) $\frac{2^x log2 - 3^x log3}{5^x log5} + C$ B) $\left(\frac{2}{5}\right)^x \log\left(\frac{2}{5}\right) - \left(\frac{3}{5}\right)^x \log\left(\frac{3}{5}\right) + C$
   C) $\left(-\frac{1}{5}\right)^x \log\left(\frac{1}{5}\right) + C$ D) None of these

13. $\int_{-3}^{3} x^7 cosx dx$ is
   A) 0 B) 1
   C) -1 D) None of these

14. General solution of the differential equation xdx + ydy = 0 is a
   A) Parabola B) Circle
   C) Hyperbola D) Ellipse

15. Domain of $\boldsymbol{y} = \mathbf{sin^{-1}}(\boldsymbol{2x-1})$ is

A) [-1,1] B) [0,2]

C) [0,1] D) None of these

16. The corner points of the feasible region of an LPP are (0,4),(0.6,1.6) and (3,0) . The minimum value of the objective function z = 4x + 6y occurs at

A) (0.6,1.6) only B) (3,0) only

C) (0.6,1.6) and (3,0) only D) at every point of the line segment joining points (3,0) and (0.6,1.6)

17. The relation described by R = {(a,b): a ≤ b, a,b are natural numbers} is

A) Equivalence relation B) Not reflexive

C) Not symmetric D) Not transitive

18. Area of the region bounded by x-axis, $x^2 = 12y$ and the line x = 3 in the first quadrant is

A) 3 sq units B) 9 sq units

C) 4.5 sq units D) None of these

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

19. Assertion(A): f(x) = [x] is not differentiable at integral points.

Reason(R) : If a function is not differentiable at a point, then it in not continuous thereat.

20. Assertion(A): f(x) = x^4, where x is any prime number is one-one function.

Reason(R):A function is one – one if $\boldsymbol{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$ for all $\boldsymbol{x_1, x_2} \in \boldsymbol{domain}$

Section B

<u>This section contains 5 very short answer type (VSA) of 2 marks each</u>

21. Simplify: $\mathbf{tan^{-1}}\frac{\boldsymbol{1-sin\theta}}{\boldsymbol{cos\theta}}$

22. Find the rate of change in the area of a circle with respect to its radius when the radius is 10 cm.

23. Find the derivative of $\tan^{-1} x$ with respect to $\sin^{-1} x$, x$\in [-1,1]$

OR,

Find $\frac{d^2y}{dx^2}$ if $x = a(1 + cos\theta)$ and $y = a(\theta + sin\theta)$

24. Let, α, β and γ be the angles made by a vector with the three co-ordinate axes. Find the value of $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$.

OR,

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the angle between the vectors $\vec{a}$ and $\vec{b}$.

25. Find the unit vector which is perpendicular to the vectors $3\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$.

SECTION C

This section contains 6 short answer type (SA) of 3 marks each

26. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

27. Show that $y = \frac{4sin\theta}{2+cos\theta} - \theta$ is an increasing function of θin $\left[0, \frac{\pi}{2}\right]$.

28. Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

OR,

Find the value of k if the points (k,-10,3),(1,-1,3) and (3,5,3) are collinear.

29. Evaluate the integral: $\int \frac{x^2}{(x^2+1)(x^2+5)} dx$

OR,

Evaluate the integral: $\int_0^2 (2 - x)^m x dx$

30. Determine the maximum value of z =11x+7y subject to the constraints

$2x + y \leq 6,\ x \leq 2,\ x \geq 0,\ y \geq 0$

31. Two dice are thrown together and the total score is noted. The events E,F and G are 'a total score of 4', 'a total score of 9 or more' and 'a total score divisible by 5' respectively.

Calculate P(E), P(F) and P(G) and decide which pairs of events are independent.

OR,

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits the target ,is $\frac{2}{5}$.If both try to hit the target independently ,find the probability that the target is hit.

Section D

This section contains 4long answer type (LA) of 5 marks each

32. Find the area of the region bounded by the curves $x^2=y, y=x+2$ and x-axis, using integration.

33. If $A=\begin{vmatrix} \mathbf{2} & \mathbf{-3} & \mathbf{5} \\ \mathbf{3} & \mathbf{2} & \mathbf{-4} \\ \mathbf{1} & \mathbf{1} & \mathbf{-2} \end{vmatrix}$ find $\boldsymbol{A^{-1}}$.Use it to solve the system of equations $2x -3y+5z=11; 3x+2y-4z=-5; x+y-2z=-3$

34.Show that the differential equation $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right)=0$ is homogeneous.Find the particular solution of this differential equation,given that x=1 when $y=\frac{\pi}{2}$

OR,

Classify the differential equation $\frac{dy}{dx}$ + 2tanx. y =sinx on the basis of its order degree . Find the particular solution of this differential equation given that y=0 when $x=\frac{\pi}{3}$

35.Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it and passing through the point (4,0, -5).

OR,

Find the co-ordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5,4,2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(\widehat{2\imath} + 3\hat{\jmath} - \hat{k})$, where $\boldsymbol{\lambda}$ is a scalar. Also, find the image of P in this line .

Section E

This section contains 3 case study based questions of 4 marks each

36. Case study – 1

A potter made a mud vessel where the shape of the pot is based on f(x) = |x – 3| + |x – 2|, where f(x) represents the height of the pot.

(A) When x > 4, what will be the height of the pot in terms of x? 1 mark

(B) Will the slope of the pot vary with the value of x? 1 mark

(C) What is $\frac{dy}{dx}$ at x = 3? 2 marks

OR,

(C) Will the potter be able to make a pot using the function f(x) =[x]? 2 mark

37. Case study – 2

Students of Grade 12, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the sapling along the line $y = x - 4$. Let, L be the set of all lines which are parallel on the ground and R be a relation on L.

Based on the given information, answer the following questions:

(A) Let, f: R → R be defined by $f(x) = x - 4$, then find the range of f(x). 1 mark

(B) Is f one-one? 1 mark

(C) Let, $R=\{(L_1,L_2): L_1 \parallel L_2 \text{ where } L_1,L_2 \in L\}$, then, show that R is an equivalence relation. 2 marks

OR,

(C) Write the equivalence class of the line $3x - 4y = 5$. 2 marks

38. Case study - 3

Jyoti CNC is the largest CNC (Computer Numerical Control) machine manufacturing company of India. Their unit in Bhubaneswar, Odisha has three machine operators A,B and C. The operators supervise the machines while they execute the task and make any necessary adjustments to produce a better result. Their main focus is to minimize defects as it increased the cost of operations.

The first operator a produces 1% defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively.

Machine operators	% of the time on the job

A	50%
B	30%
C	20%

Based on the given information, answer the following questions:

(A) What is the conditional probability that the defective item is produced by the operator A? 2 marks

(B) The factory in charge wants to do a quality check. During inspection he picks on item from the stockpile at random. If the chosen item is defective, then what is the probability that it is not produced by the operator C? 2 marks

NVS RO SHILLONG
WHOLE SYLLABUS PRACTICE PAPER SET IV
(2024-2025)
MARKING SCHEME CLASS XII
MATHEMATICS(CODE-041)

SECTION:A

(Solution of MCQs of 1Mark each)

QUESTION NUMBER	ANSWER	MARKS
1	A	
2	C	
3	B	
4	C	
5	B	
6	A	
7	B	
8	B	
9	B	
10	C	
11	A	
12	B	
13	A	
14	B	
15	C	
16	D	
17	C	
18	A	
19	C	
20	A	
21	$\tan^{-1}\frac{1-sin\theta}{cos\theta} = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)}\right)$ $= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right)$ $= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right) = \frac{\pi}{4}-\frac{\theta}{2}$	1 1
22	**Answer: let, r be the radius of the circle** **Then, A = area of the circle = πr^2**	1

	Then, $\frac{dA}{dr} = 2\pi r$ At, r = 10 cm, $\frac{dA}{dr} = 20\pi\ cm^2/cm$	1
23	: $\frac{d(\tan^{-1}x)}{d(\sin^{-1}x)} = \frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\sin^{-1}x)}$ $= \frac{\frac{1}{1+x^2}}{\frac{1}{\sqrt{1-x^2}}} = \frac{\sqrt{1-x^2}}{1+x^2}$	1 1
23(OR)	$\frac{dy}{dx} = \frac{1+cos\theta}{-sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = -\cot\frac{\theta}{2}$ So, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\cot\frac{\theta}{2}\right) = \frac{1}{2}cosec^2\frac{\theta}{2}\frac{d\theta}{dx} =$ $\frac{\frac{1}{2}cosec^2\frac{\theta}{2}}{-sin\theta} = -\frac{1}{4}cosec^3\frac{\theta}{2}\sec\frac{\theta}{2}$	1 1
24	let the vector be $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{\jmath}$ A/Q, $cos\alpha = \frac{\vec{r}\cdot\hat{\imath}}{\lvert\vec{r}\rvert} = \frac{a}{\sqrt{a^2+b^2+c^2}}$ Similarly, $cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}$ and $cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$ So, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$	1 1
24(OR)	we have, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c} \Rightarrow \lvert\vec{a} + \vec{b}\rvert^2 = \lvert-\vec{c}\rvert^2$ $\Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}\cdot\vec{b} = \lvert\vec{c}\rvert^2$ $\Rightarrow 1 + 1 + 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert cos\theta = 1 \Rightarrow cos\theta = -\frac{1}{2}$ $\theta = \frac{2\pi}{3}$	1 1
25	: let, $\vec{a} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 5\hat{k}$. Here $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ 1 & 2 & -5 \end{vmatrix}$	1

	$=5\hat{\imath}+15\hat{\jmath}+7\hat{k}$ So, unit vector along the perpendicular to the vectors = $\frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}$ $= \pm\frac{5\hat{\imath}+15\hat{\jmath}+7\hat{k}}{\sqrt{299}}$	1
26	: let, l be the length of the edge of the cube and V be the volume So, V = l^3 A/Q, $\frac{dV}{dt} = k \Rightarrow \frac{d}{dt}(l^3) = k, k$ being a constant So, $\frac{dl}{dt} = \frac{k}{3l^2}$ Then, S = surface are of the cube = $6l^2$ So, $\frac{dS}{dt} = 12l\frac{dl}{dt} = 12\,l\frac{k}{3l^2} = \frac{4k}{l} \Rightarrow \frac{dS}{dt} = \frac{constant}{l}$ Thus, change of S is inversely proportional to l. Hence proved	1 1 1
27	Here, $y = \frac{4sin\theta}{2+cos\theta} - \theta$ So, $\frac{dy}{d\theta} = \frac{8cos\theta+4\cos^2\theta+4\sin^2\theta-(4+4cos\theta+\cos^2\theta)}{(2+cos\theta)^2}$ $= \frac{cos\theta(4-cos\theta)}{(2+cos\theta)^2}$ For θin $\left[0,\frac{\pi}{2}\right]$, $0 \le cos\theta \le 1$ $4-cos\theta > 0$and$(2+cos\theta)^2 > 0$ Hence, $\frac{dy}{d\theta} = \frac{(non-negative\ quantity)(positive\ quantity)}{positive\ quantity} = non-negative\ quantity$ Thus, $\frac{dy}{d\theta} \ge 0 \Rightarrow y$ is increasing in $\left[0,\frac{\pi}{2}\right]$	1 1 1

28	**Given lines are** $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}\Rightarrow\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$ **---(i)** $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}\Rightarrow\frac{x-1}{-\frac{3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$ **---(ii)** ***T*□*e d*.rs of the lines (i) and (ii) are** a_1=-3 , $b_1=\frac{2p}{7}$, $c_1=2$ a_2=$\frac{-3p}{7}$, $b_2=$ 1 , c_2 = -5	1 1
	Two lines (i) and (ii)are at right angle if $a_1a_2+b_1b+cc_2=0$ $=(-3)\left(-\frac{3p}{7}\right)+\left(\frac{2p}{7}\right)(1)+(2)(-5)=0$ $\Rightarrow 9p+2p=70\Rightarrow p=\frac{70}{11}$	1
28(OR)	**let, the given points be A(k,-10,3),B(1,-1,3) and C(3,5,3) respectively.** **Since, A,B and C are collinear,** **The d.rs of the lines AB and BC are in proportion**	1 1
	The d.rs of the line AB are 1-k,9 and 0 **The d.rs of the line BC are 2 ,6 and 0**	1
	According to the question $\Rightarrow\frac{1-k}{2}=\frac{9}{6}=\frac{0}{0}$ $\Rightarrow k=-2$	

29	$\int \frac{x^2}{(x^2+1)(x^2+5)} dx$ $= \frac{1}{4}\int \frac{5(x^2+1)-(x^2+5)}{(x^2+1)(x^2+5)} dx$ $= \frac{1}{4}\int \left(\frac{5}{x^2+5} - \frac{1}{x^2+1}\right) dx$ $= \frac{1}{4}\left(\sqrt{5}\tan^{-1}\frac{x}{\sqrt{5}} - tan^{-1}x\right) + c$	1 1 1
29(OR)	$: \; I = \int_0^2 (2-x)^m x dx$ $= \int_0^2 x^m(2-x)\, dx \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ $= \frac{2x^{m+1}}{m+1} - \frac{x^{m+2}}{m+2}\Big\|_0^2$ $= \frac{2^{m+2}}{m+1} - \frac{2^{m+2}}{m+2} - 0 + 0$ $= \frac{2^{m+2}}{(m+1)(m+2)}$	1 1 1
30		For graph 2

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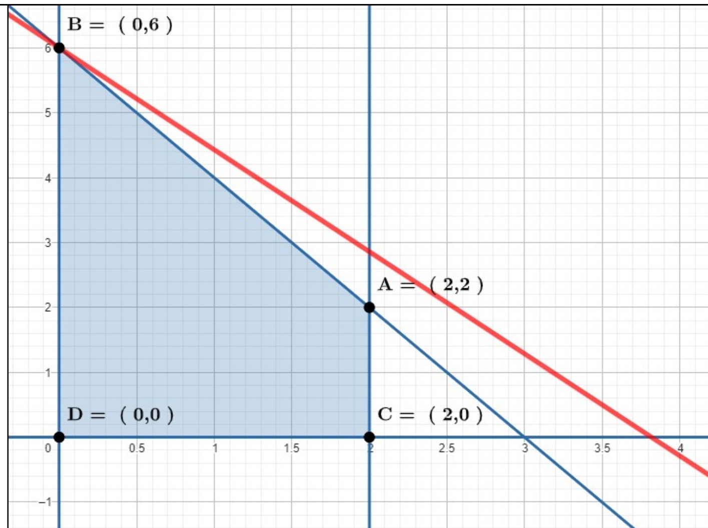
<td>

The corner points areO(0,0),C(2,0),a(2,2) and B(0,6)
The maximum value of Z is 42 and which attained at B(0,6)</td>
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<td>31</td>
<td>When two dice are thrown together, the number of possible outcomes=36
Given
E:event of outcomes whose total score is 4
={(1,3)(2,2)(3,1)}
F: event of outcomes whose total score is 9 or more
{(3,6)(4,5),(4,6)(5,4),(5,5),(5,6),(6,3),(6,4),(6,5)}
G: event of outcomes whose total score is divisible by 5
{(1,4)(2,3),(3,2),(4,1),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)}

: P(E) $=\frac{1}{12}$, P(F) $=\frac{1}{4}$, P(G) $=\frac{5}{18}$
E∩ F={}, E∩ G={},F∩G={(4,6),(5,5),(6,4)}

P(E∩ F) = 0
P(E)XP(F)=$\frac{1}{48}$
P(E∩ F) ≠P(E)XP(F)

E and F are not independent

P(E∩ G) = 0
P(E)×P(G)=$\frac{1}{12} \times \frac{5}{18} = \frac{5}{216}$
∴ P(E∩ G) ≠P(E)×P(G)
E and G are not independent</td>
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	P(F∩ G)=$\frac{1}{12}$ **P(F)×P(G)=**$\frac{1}{4}\times\frac{5}{18}=\frac{5}{72}$ **∴ P(F∩ G) ≠P(F)×P(G)** **F and G are not independent** **No pairs are independent**	1
31(or)	**Probability that A hits the target,P(A)=** $\frac{1}{3}$ **Probability that B hits the target,P(B)=** $\frac{2}{5}$ **Probability that A does not hit the target,P(**$\bar{A}$**)=** $1-\frac{1}{3}$ $=\frac{2}{3}$ **Probability that B does not hit the target,P(**$\bar{B}$**)=** $1-\frac{2}{5}$ $=\frac{3}{5}$ **Probability that the target is hit=At least one of them hit the target** **=1 - P(**$\bar{A}$**) P(**$\bar{B}$**)** **=1-**$\frac{2}{3}\times\frac{3}{5}$ $=\frac{3}{5}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1
32	**Given curves** $x^2=y$**………………(i)** $y=x+2$**……………(ii)** **and x-axis** **The points of intersection of the curves (i) and (ii) is given by** $x^2=x+2 \Rightarrow x^2-x-2=0 \Rightarrow x=-1,2$ **Area of the shaded region=**$\int_{-2}^{-1}(x+2)dx + \int_{-1}^{0} x^2 dx$ $= \left\|\frac{x^2}{2}+2\right\|_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$	Figure 1 1 1 1 1

	$=\frac{5}{6}$ **square units**	
33	A=$\begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} \Rightarrow |A|=-1$ $\therefore A^{-1}$ **exist** **Calculation of co-factor ofA** $A_{11}=0$ $A_{12}=2$ $A_{13}=1$ $A_{21}=-1$ $A_{22}=-9$ $A_{23}=-5$ $A_{31}=2$ $A_{32}=23$ $A_{33}=13$ AdjA=$\begin{vmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{vmatrix}$ $A^{-1}=\frac{1}{|A|}$adjA=$\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ **The given system of equations can be written in matrix form as** $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ ⇒AX=B ⇒X=A^{-1}B ⇒X=$\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ ⇒ $\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ∴ x=1, y=2 , z=3	1 $1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$

34	**Given differential equation is $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right)=0$** $\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$**.......(i)** **Hence the given differential equation is homogeneous.** **Let y=vx** $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ **The equation (i) will reduces to** $v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\left(\frac{vx}{x}\right) = v - \operatorname{cosec}v$ $\Rightarrow x\frac{dv}{dx} = -\operatorname{cosec}v$ $\Rightarrow -\sin v\,dv = \frac{dx}{x}$ **Integrating** $-\int \sin v\, dv = \int \frac{dx}{x}$ $\Rightarrow \cos v = \log x + C$ $\Rightarrow \cos\frac{y}{x} = \log x + C$**...........(II)** **Putting** $y=\frac{\pi}{2}, x=1$ **in (ii),we get** $\cos\left(\frac{\pi}{2}\right) = \log 1 + C$ $\Rightarrow C = 0$ **Hence the particular solution is** $\cos\frac{y}{x} = \log x$	1 1 1 1 1
34(OR)	$\frac{dy}{dx} + 2\tan x.y = \sin x$ **is a linear equation of degree 1.** **The given differential equation is** $\frac{dy}{dx} + 2\tan x.y$	1

	=sinx……..(i) I.F=$e^{\int pdx}$ =$e^{\int 2tanxdx}$ =$\sec^2$x Multiplying both sides of equation (i) by I.F we get y.(I.F)=$\int sinx.sec^2x\,dx$ y.$\sec^2$x = secx + C……………….(II) When y=0 , x= $\frac{\pi}{3}$ (ii) $\Rightarrow$ C=-2 Putting the value of C in (ii),we get y.$\sec^2$x = secx -2 $\Rightarrow$y=cosx-2cos^2x Which the required particular solution.	1 1 1 1
35	Given line is $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}\Rightarrow\frac{x-0}{2}=\frac{y-3}{2}=\frac{z-1}{1}$ ---(i) A line parallel to (i) and passing through (4,0,-5) is $\frac{x-4}{2}=\frac{y-0}{2}=\frac{z+5}{1}$ ---(ii) $\therefore \overrightarrow{a_1}$=$3\hat{j}+\hat{k}$, $\overrightarrow{a_2}$=$4\hat{i}-5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ $\overrightarrow{a_2}-\overrightarrow{a_1}$=$(4\hat{i}-5\hat{k})-(3\hat{j}+\hat{k})$=$4\hat{i}-3\hat{j}-6\hat{k}$ $(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}$=$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -3 & -6\\ 2 & 2 & 1\end{vmatrix}$=9$\hat{i}$ -16$\hat{j}$ +14$\hat{k}$ $\lvert\vec{b}\rvert$=$\sqrt{2^2+2^2+1^2}$=3	1 1 1

	So, required distance between the parallel lines (i) and (ii) is given by $$\frac{\left|(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\right|}{\lvert\vec{b}\rvert}$$ $=\dfrac{\lvert 9\hat{\imath}-16\hat{\jmath}+14\hat{k}\rvert}{3}$ $=\dfrac{\sqrt{81+256+196}}{3}$ $=\frac{\sqrt{533}}{3}$units	1 1	
35(OR)	Given line is $$\vec{r}=-\hat{\imath}+3\hat{\jmath}+\hat{k}+\lambda\left(\widehat{2\imath}+3\hat{\jmath}-\hat{k}\right)$$ It can be written in Cartesian form as $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$............(i) Let Q$(\alpha,\beta,\gamma)$ be the foot of perpendicular drawn from P(5,4,2) to the line (i) and $P^{/}(x_1,y_1,z_1)$ be the image of P on the line (i) $\because$ Q(α,β,γ) lie on line (i) $\frac{\alpha+1}{2}=\frac{\beta-3}{3}=\frac{\gamma-1}{-1}=\lambda$ $\Rightarrow \alpha=2\lambda-1;\ \beta=3\lambda+3\ ;\ \gamma=-\lambda+1$ $Now, \overrightarrow{PQ}=(\alpha-5)\hat{\imath}+(\beta-4)\hat{\jmath}+(\gamma-2)\hat{k}$ Obviously $\overrightarrow{PQ}\perp\vec{b}$ $\Rightarrow \overrightarrow{PQ}\,.\,\vec{b}=0$ $\Rightarrow (\alpha-5)2+(\beta-4)3+(\gamma-2)(-1)=0$ $\Rightarrow 2\alpha+3\beta-\gamma-20=0$ $\Rightarrow 2(2\lambda-1)+3(3\lambda+3)-(-\lambda+1)-20=0$ $\Rightarrow \lambda=1$ Hence the co-ordinates of the foot of perpendicular Q are (1,6,0)		1 1 1 1

	∴ **Length of perpendicular** = $\sqrt{(5-1)^2+(4-6)^2+(2-0)^2}$ **=2$\sqrt{6}$ units** **Also ,Since Q is the mid point of PP/** $\therefore 1=\frac{x_1+5}{2} \Rightarrow x_1 = -3$ $6=\frac{y_1+4}{2} \Rightarrow y_1 = 8$ $0=\frac{z_1+2}{2} \Rightarrow z_1 = -2$ **Therefore required image is (-3,8,-2)**	1
36	**A potter made a mud vessel where the shape of the pot is based on f(x) = |x – 3| + |x – 2|, where f(x) represents the height of the pot.** (D) **When x > 4, what will be the height of the pot in terms of x?** **Answer: when x > 4, then x – 3 > 0 and x – 2 > 0 so, f(x) = x – 3 + x – 2 =2x – 5** (E) **Will the slope of the pot vary with the value of x?** **Answer:** **we can redefine f(x) as** $f(x) = \begin{cases} -2x+5, & x \le 2 \\ 1, & 2 < x < 3 \\ 2x-5, & x \ge 3 \end{cases}$ **We can see that, for slope of the pot is – 2 when $x \le 2$** **0 when $2 < x < 3$** **2 when $x \ge 3$** **So, slopes vary for value of x** (F) **What is $\frac{dy}{dx}$ at x = 3?** **Answer:** **LHD** $= \lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0^-} \frac{1-1}{h} = 0$ **And ,**	1 1 1+1=2

	RHD = = $\lim_{h\to 0^+}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0^+}\frac{2h}{h} = 2$ **So, derivative does not exist at x = 3** **OR,** **(C) Will the potter be able to make a pot using the function f(x) =[x]?** **Answer: As the function f(x) =[x] is discontinuous at every integral points, so he can only construct pots of height always less than 1 units.**	2
37	**(A)Let, f: R → R be defined by f(x) = x – 4, then find the range of f(x).** **Answer: for x ∈ R,** $-\infty < x < \infty \Rightarrow -\infty < x - 4 < \infty$ **So, range of f(x) = R** **(B)Is f one-one?** **Answer: let,** x_1,x_2 **be two arbitrary entries in R such that** $f(x_1) = f(x_2)$ $\Rightarrow x_1 - 4 = x_2 - 4$ $\Rightarrow x_1 = x_2$ **Hence, f is one – one** **(C)Let,** $R=\{(L_1,L_2): L_1 \parallel L_2 \text{ where } L_1,L_2 \in L\}$**, then, show that R is an equivalence relation.** **Answer: R is symmetric as for any** $L_1\in L$, $L_1 \parallel L_1$ **So,** $(L_1,L_1)\in R$ **for any** $L_1\in L$ **Again,** $(L_1,L_2)\in R \Rightarrow L_1 \parallel L_2 \Rightarrow L_2 \parallel L_1 \Rightarrow (L_2,L_1)\in R$ **So, R is symmetric** **Moreover,** $(L_1,L_2)\in R$, $(L_2,L_3)\in R \Rightarrow L_1 \parallel L_2 \text{ and } L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3 \Rightarrow (L_1,L_3)\in R$ **So, R is transitive** **And, hence R is equivalence relation** **OR,** **(C) Write the equivalence class of the line 3x – 4y = 5.**	1 1 2 2

	Answer: [3x – 4y = 5]={3x – 4y = k: k∈ R}	
38	**(C) What is the conditional probability that the defective item is produced by the operator A?** **Answer:** **Let, D be the event that product is defective and E_1 be the event that it produced by operator A** **So, $P(E_1\|D) = \frac{P(E_1)P(D\|E_1)}{P(A)}$** $= \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}} = \frac{5}{34}$ **(B)The factory in charge wants to do a quality check. During inspection he picks on item from the stockpile at random. If the chosen item is defective, then what is the probability that it is not produced by the operator C?** **Answer: P(defective items not produce by C)** **= 1 – P(defective item produced by C)** **$= 1-P(E_3\|D) = 1-\frac{P(E_3)P(D\|E_3)}{P(A)}$** $= 1 - \frac{\frac{20}{100}\times\frac{7}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}} = 1 - \frac{14}{34}$ $= \frac{20}{34} = \frac{10}{17}$	1 1 1 1

NVS RO-SHILLONG
WHOLE SYLLABUS PRACTICE QUESTION PAPER SET-V
(2024-25)
CLASS: XII
SUBJECT: MATHEMATICS (041)
TIME: 3 HRS **MAX MARKS:80**
BLUE PRINT

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6**. Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Sl. No.	Typology of question	Total marks	Weightage
1	Remembering Understanding	44	55
2	Application	20	25
3	Analysing Evaluating Creating	16	20
4	Total	80	100

Sl. No.	Type of Question	No. of Questions	Total mark	Weightage
1	MCQ	20	20	25
2	VSA	05	10	12.5
3	SA	06	18	22.5
4	LA	04	20	25
5	Case Base	3	12	15
Total		38	80	100

Unit wise Weightage

Sl. No.	Unit	Mark
1	Relation Function	8
2	Algebra	10
3	Calculus	35
4	Vector & Geometry	14
5	LPP	5
6	Probability	8
		80

Navodaya Vidyalaya Samiti, RO Shillong
WHOLE SYLLABUS PRACTICE PAPER SET-V
(2024-25)
Class-XII
Subject: Mathematics (041)

Time:3 Hours **Maximum Marks:80**

General Instructions:

i. This Question paper contains 38 questions. All questions are compulsory.
ii. This Question paper is divided into five Sections - A, B, C, D and E.
iii. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
iv. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
v. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
vi. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
vii. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
viii. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
ix. Use of calculator is not allowed.

SECTION A

(*This section comprises of Multiple Choice Question of 1 mark each*)

Q1. If for a square matrix A, $\boldsymbol{A}.(\boldsymbol{adjA}) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then the value of $|A| + |adjA|$ is

a. 20 b. 30 c. 45 d. None of these.

Q2. Assume X, Y, Z ,W and P are matrices of order 2 x n, 3 x k, 2 x p, n x 3 and p x k respectively. Then the restriction on n ,k and p so that PY + WY will be defined are:

a. k = 3, p = n b. k is arbitrary, p = 2 c. p is arbitrary, k=3 d. k=2, p=3

Q3. The interval in which the function f defined by $f(x) = e^x$ is strictly increasing, is

a. $[1,\infty)$ b. $(-\infty, 0)$ c. $(-\infty,\infty)$ d. $(0, \infty)$

Q4. If A B and are non-singular matrices of same order with $\boldsymbol{det}(\boldsymbol{A}) = \mathbf{5}$, then $\boldsymbol{det}(\,B^{-1}AB)^2$ is equal to

a. 5 b. 5^2 c. 5^4 d. 5^5

Q5. The value of n such that the differential equation $x^n \frac{dy}{dx} = \boldsymbol{y}(\boldsymbol{logy} - \boldsymbol{logx} + \mathbf{1})$, where x & y are positive real number is homogeneous, is

a. 0 b . 1 c. 2 d. 3

Q6. If the points (x_1, y_1), (x_2, y_2) and $(x_1 + x_2, y_1 + y_2)$ are collinear, then $x_1 y_2$ is equal to

a. $x_2\, y_1$ b. $x_1\, y_1$ c. $x_2\, y_2$ d. None of these.

Q7. If A = $\begin{pmatrix} 0 & 2 & c \\ -2 & a & -b \\ 5 & 7 & 0 \end{pmatrix}$ is a skew symmetric matrix, then value of a+b+c is

a. 0 b. 2 c. 5 d. None of these

Q.8. For any two events A and B , if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ & $P(A \cap B) = \frac{1}{4}$, then $P(\frac{\bar{A}}{\bar{B}})$ is

a. $\frac{3}{8}$ b. $\frac{8}{9}$ c. $\frac{5}{8}$ d. $\frac{1}{4}$

Q9. For what value of 'a' the vectors 2i-3j+4k and ai+6j-8k are collinear

a. 5 b. 4 c. 7 d. None

Q10. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then value of $|\vec{a} - \vec{b}|$ is

a. 3 b. 4 c. 5 d. 8

Q11. Of all the points of the feasible region, for maximum or minimum of objective function, the point lies

a. Inside the feasible region b. At the boundary line of the feasible region.
c . Vertex point of the boundary of the feasible region d. None of these

Q12. $\int \frac{dx}{x\cos^2(1+\log x)}$ is

a. $\tan|1 + \log x|$ + c b. 1+logx + c c. Sec(1+logx) + c
d. None of these

Q13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Sin^5 x\, dx$ is

a. 0 b. 1 c. 2 d. None of these

Q14. Find the value of m+n , where m & n are order and degree of differential equation

$$\frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$$

a. -1 b. 4 c. 5 d. 6

Q15. $\text{Tan}^{-1}\{\sin(-\frac{\pi}{2})\}$ is

a. π b. $\frac{\pi}{2}$ c . $-\frac{\pi}{2}$ d. $-\frac{\pi}{4}$

Q16. $\int_{0.5}^{1.5}[x]dx$ is

a. 0.5 b. 1 c. 2.5 d. None of these

Q17. The function $f: R \to Z$ defined by f(x) = $[x]$, where $[.]$ denotes the greatest integer function, is

a. Continuous at x=2.5 but not differentiable at x = 2.5
b. Not Continuous at x = 2.5 but differentiable at x = 2.5
c. Not Continuous at x = 2.5 and not differentiable at x=2.5
d. Continuous as well as differentiable at x = 2.5

Q18. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = y$. Then the area (in sq units) of the region bounded by parabola , $x^2 = y$ and the line , y= 4 is

a. $\frac{64}{3}$ b. $\frac{32}{3}$ c. $\frac{128}{3}$ d. $\frac{56}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

Q19. Assertion (A): Given a relation,R={(x,y) : x,y $\in Z$; $x^2 + y^2 \leq 9$}, the domain of R = { -3,-2,-1,0,1,2,3}

Reason (R) : For domain of R , put y = 0, then $x^2 \leq 9$

Q20. Assertion (A): Consider the function defined as $f(x) = |x| + |x - 1|$, $x \in R$. Then ,f(x) is not differentiable at x=0 and x = 1 .

Reason (R):Suppose f be defined and continuous on (a,b) and c $\in$ (a ,b) ,the f(x) is not differentiable at

$$x = c \quad \text{if} \lim_{h\to 0^-} \frac{f(c+h)- f(c)}{h} \neq \lim_{h\to 0^+} \frac{f(c+h)- f(c)}{h}$$

SECTION B

(*This section comprises of 5 very short answer (VSA) type questions of 2 marks each*.)

Q21. Find the Principal value of $\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$.

Q22. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 5x^2 -2x+1$. Find the marginal revenue when x = 5.

Q23. Find derivative of $\sin^{-1} x$ with respect to e^x .

Or

Find derivative of $(\sin x)^x$ with respect to x

Q24. If vectors $\vec{a} = 2\hat{i}+ 2\hat{j}+ 3\hat{k}$, $\vec{b} = -\hat{i}+ 2\hat{j}+ \hat{k}$ and $\vec{c} = 3\hat{i}+ \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.

Or

Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector

Q25. The two co-initial adjacent sides of a parallelogram are $2\hat{i}- 4\hat{j}- 5\hat{k}$ and $2\hat{i}+ 2\hat{j}+ 3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram.

SECTION C

(*This section comprises of 6 short answer (SA) type questions of 3 marks each.*)

Q26. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

Q27. Find the intervals in which the function f given by f(x) = sinx +cos x, $0\leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Q28. Find the value of τ if the lines : $\frac{1-x}{3} = \frac{7y-14}{2\tau} = \frac{5z-10}{11}$ & $\frac{7-7x}{3\tau} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

Or

Find the vector and the cartesian equation of the line that passes through (−1, 2, 7) and is perpendicular to the lines $\vec{r}$ = 2î+ ĵ− 3k̂+ λ(î+ 2ĵ+ 5k̂) and

$\vec{r}$ = 3î+ 3ĵ− 7k̂+ μ(3î− 2ĵ+ 5k̂)

Q29. Evaluate: $\int \{ \frac{1}{logx} - \frac{1}{(logx)^2} \} \boldsymbol{dx}$; (where $\boldsymbol{x} > \mathbf{1}$).

Or

Evaluate : $\int_0^1 x(1-x)^n dx$, **where $\boldsymbol{n} \in \boldsymbol{N}$**

Q30. Consider the following Linear Programming Problem:

Minimise z = x+2y Subject to

2x + y ≥ 3 , x+2y ≥ 6, x,y ≥ 0

Show graphically that the minimum of Z occurs at more than two points.

Q31. The probability that it rains today is 0.4. If it rains today, the probability that it will rain tomorrow is

0.8. If it does not rain today, the probability that it will rain tomorrow is 0.7. If

$P1$: denotes the probability that it does not rain today.

$P2$: denotes the probability that it will not rain tomorrow, if it rains today.

$P3$: denotes the probability that it will rain tomorrow, if it does not rain today.

$P4$: denotes the probability that it will not rain tomorrow, if it does not rain today.

(i) Find the value of $P_1 x P_4 - P_2 x P_3$. [2Marks].

(ii) Calculate the probability of raining tomorrow. [1Mark]

Or

A random variable X can take all non – negative integral values and the probability that X takes the value r is proportional to 5^{-r} . Find P(X<3)

SECTION D

(This section comprises of 4 long answer (LA) type questions of 5 marks each.)

Q32. Find the area enclosed by the ellipse, $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Q33. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the following equations:

x-2y+2z =1 , 2y-3z = 1 & 3x-2y +4z = 2

Q34. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1+logx)^2}{logy}$

Q.35. Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(7\hat{i} - 6\hat{j} + \hat{k})$ and $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \mu(\hat{i} - 2\hat{j} + \hat{k})$ where λ and μ are parameters.

SECTION- E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Q.36. Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension 25 by 40 cm to make container packets without top. Let x cm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps. Based on the above information, answer the following questions:

(i) Express the volume (V) of each container as function of x only. [1Mark]

(ii) Find $\frac{dv}{dx}$ [1Mark]

(iii) For what value of x , the volume of each container is maximum? [2Marks]

OR

Check whether V has a point of inflection at x = $\frac{65}{6}$. [2mark]

Q37. An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = { 1, 2, 3 } . G = {g_1, g_2} where B represents the set of boys selected and G the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions.

On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relations possible from B to G . How many such relations are possible? [1Mark]

(ii) Write the smallest equivalence relation on G. [1mark]

(iii) Ravi defines a relation from B to B as $\boldsymbol{R}_1 = \{(\boldsymbol{b}_1, \boldsymbol{b}_2), (\boldsymbol{b}_2, \boldsymbol{b}1)\}$. Write the minimum ordered pairs to be added in $\boldsymbol{R1}$ so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive

[2mark]

OR

(iii). If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; (where $0 \leq x \leq 20\sqrt{2}$ & $0 \leq y \leq 200$), then state whether the track represents a one-one and onto function or not. (Justify). [2Marks]

Q.38. Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage –II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I(simultaneously). Assume that all the birds have equal chances of flying. On the basis of the above information, answer the following questions:

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I.

[2Marks]

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II?

[2Marks]

NVS RO SHILLONG
WHOLE SYLLABUS PRACTICE PAPER SET V
(2024-2025)
MARKING SCHEME CLASS XII
MATHEMATICS(CODE-041)

SECTION:A

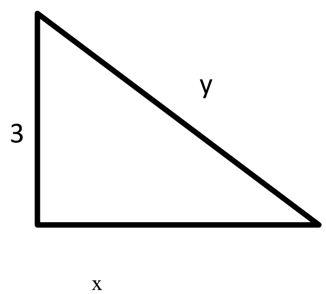
(Solution of MCQs of 1 Mark each)

Q.No	ANS	HINTS/SOLUTION
1.	(B)	$\|A\|=5$, $\|A\| + \|adjA\|=5+25=30$
2.	(A)	$P \xrightarrow{\text{Order}} p \times k$; $Y \xrightarrow{\text{Order}} 3 \times k$; $W \xrightarrow{\text{Order}} n \times 3$; $Y \xrightarrow{\text{Order}} 3 \times k$ For PY to exist $k = 3$ Order of $PY = p \times k$ Order of $WY = n \times k$ For $PY + WY$ to exist order(PY) = order(WY) $\therefore p = n$
3.	(C)	
4.	(B)	25
5.	(B)	
6.	(A)	
7.	(B)	
8.	(C)	
9.	(D)	
10.	(C)	
11.	(C)	
12.	(A)	
13.	(A)	
14.	(C)	m=3, n=2 & m+n+5
15.	(D)	$\text{Tan}^{-1}(-1) = -\frac{\pi}{4}$
16.	(A)	0.5
17.	(D)	
18.	(B)	
19.	(A)	
20.	(C)	

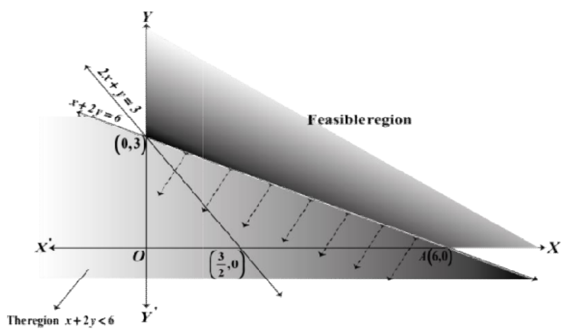
Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$Tan^{-1}(2Sin\frac{\pi}{3})$ $= Tan^{-1}(\sqrt{3})$ $=\frac{\pi}{3}$	1 1
22.	The marginal revenue = $R'(x=5)$ = 48	1 1
23.	let $y= Sin^{-1}x$ $dy/dx = \frac{1}{\sqrt{1-x^2}}$ & $dz/dx = e^x$ So $dy/dz = \frac{1}{e^x\sqrt{1-x^2}}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR 23.	$y=(sinx)^x$ take log on both sides logy=xlog sinx $dy/dx = (sinx)^x(xcotx+logsinx)$	$\frac{1}{2}$ $\frac{1}{2}$ 1
24.	We have $\vec{b} + \lambda\vec{c} = (-1 + 3\lambda)\hat{i} + (2 + \lambda)\hat{j} + \hat{k}$ $(\vec{b} + \lambda\vec{c}) . \vec{a} = 0 \Rightarrow 2(-1 + 3\lambda) + 2(2 + \lambda) + 3 = 0$ $\lambda = -\frac{5}{8}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
OR 24.	$\lvert\vec{x}\rvert^2-1 = 12$ $\lvert\vec{x}\rvert = \sqrt{13}$	$\frac{1}{2}$
25.	$\vec{d_1} = \vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{d_2} = \vec{b} - \vec{a} = 6\hat{j} + 8k$	$\frac{1}{2}$ 1 $\frac{1}{2}$

	Area of the parallelogram = $\frac{1}{2}\left\|\overrightarrow{d_1 x d_2}\right\| = \frac{1}{2}\begin{vmatrix} i & j & k \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix}$ $= \frac{1}{2}\|-4i - 32j + 24k\|$ Area of the parallelogram = $\frac{1}{2}\sqrt{1616}$= $2\sqrt{101}$ sq unit	.
Section –C		
[This section comprises of solution short answer type questions (SA) of 3 marks each]		
26.	 $x^2 + 3^2 = y^2$ when x=5 then x=4 & $2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ so, $\frac{dy}{dt}$ = 160cm/sec	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
27.	$F'(x) = 0$ $X=\frac{\pi}{4}$ or $\frac{5\pi}{4}$ $f'(x) > 0$ on $x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ $f'(x) < 0$ at $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ strictly increasing on $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ strictly decreasing on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	1 1 $\frac{1}{2}$ $\frac{1}{2}$
28	Ans: $\tau = 7$ OR, Line perpendicular to the lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 7\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$. has a vector parallel it is given by $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} i & j & k \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = = 20\hat{i} + 10\hat{j} - 8\hat{k}$ 3 −2 5 $\therefore$ equation of line in vector form is $\vec{r} = -\hat{i} + 2\hat{j} + 7\hat{k} + a(10\hat{i} + 5\hat{j} -$	1 1 1

<table>
<tr><td></td><td>$4\hat{k}$)
Eqn in Cartesian form is $\frac{x+1}{10} = \frac{y-2}{5} = \frac{z-7}{-4}$</td><td></td></tr>
<tr><td>29.</td><td>I= $\int \left\{\frac{1}{logx} - \frac{1}{(logx)^2}\right\} dx$
$=\int \frac{1}{logx} dx - \int \left\{\frac{1}{(logx)^2}\right\} dx$
Use By part in 1st integration
I= $\frac{x}{logx}$ +C</td><td>1
1

1</td></tr>
<tr><td>OR
29</td><td>Let I $=\int_0^1 x(1-x)^n dx$
$=\int_0^1 (1-x)\{1-(1-x)^n$ dx , (using prop)
$=\int_0^1 x^n dx - \int_0^1 x^{n+1}$ dx
$= \frac{1}{(n+1)(n+2)}$</td><td>1
1
1</td></tr>
<tr><td>30.</td><td>The feasible region determined by given constraints, is as shown.


The corner points of the unbounded feasible region are $A(6,0)$ and $B(0,3)$.

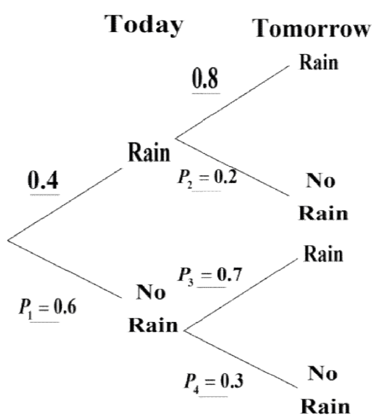
The values of Z at these corner points are as follows:
<table><tr><td>Corner point</td><td>Value of the objective function
$Z = x+2y$</td></tr><tr><td>$A(6,0)$</td><td>6</td></tr><tr><td>$B(0,3)$</td><td>6</td></tr></table>We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z = 6$.

It can be seen that the value of Z at points A and B is same. If we take any other point on the line $x + 2y = 6$ such as (2,2) on line,</td><td>1

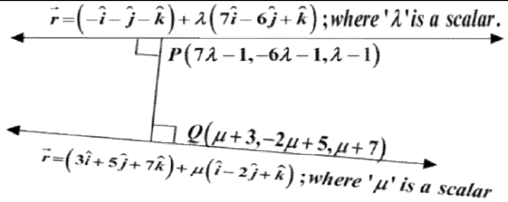
1

$\frac{1}{2}$

$\frac{1}{2}$</td></tr>
</table>

	$x + 2y = 6$, then $Z = 6$. Thus, the minimum value of Z occurs for more than 2 points, and is equal to 6.	
31	Since the event of raining today and not raining today are complementary events so if the probability that it rains today is 0.4 then the probability that it does not rain today is 1- 0.4 = 0.6 , so, $P_1 = 0.6$ If it rains today, the probability that it will rain tomorrow is 0.8 then the probability that it will not rain tomorrow is 1- 0.8 = 0.2 . If it does not rain today, the probability that it will rain tomorrow is 0.7 then the probability that it will not rain tomorrow is 1-0.7 = 0.3  *(i)* $P_1 P_4 - P_2 P_3 = 0.6 \times 0.3 - 0.2 \times 0.7 = 0.04$. *(ii)* Let E_1 and E_2 be the events that it will rain today and it will not rain today respectively. $P(E_1) = 0.4$ & $P(E_2) = 0.6$ A be the event that it will rain tomorrow. $P\left(\frac{A}{E_1}\right) = 0.8$ & $P\left(\frac{A}{E_2}\right) = 0.7$ We have, $P(A) = P(E_1)P\,({}_{E}{}^{A}1) + P(E_2)P\,({}_{E}{}^{A}2) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74$. The probability of rain tomorrow is 0.74.	1 1 $\frac{1}{2}$ $\frac{1}{2}$
31 (OR)	Given $P(X = r)\alpha_5 \frac{1}{5^r}$ $P(X = r) = k.\frac{1}{5^r}$, where k is a non-zero constant)	$\frac{1}{2}$

	$P(r=0)=k.\frac{1}{5^0}$ $P(r=1)=k.\frac{1}{5^1}$ $P(r=2)=k.\frac{1}{5^2}$ $P(r=3)=k.\frac{1}{5^3}$ ………………………… ………………………… We have, $P(X=0)+P(X=1)+P(X=2)+\ldots\ldots\ldots\ldots\ldots=1$ K=$\frac{4}{5}$ So, P(X<3) = $P(X=0)+P(X=1)+P(X=2)$ $=\frac{124}{125}$	$\frac{1}{2}$ $\frac{1}{2}$
Section –D [This section comprises of solution of long answer type questions (LA) of 5 marks each]		
32.	Equation is $\frac{x^2}{9}+\frac{y^2}{4}=1$, so y= $\frac{2}{3}\sqrt{9-x^2}$ Area of Ellipse =4x area of shaded region= 4 $\int_0^3 y\,dx$ $=\frac{8}{3}\int\sqrt{9-x^2}$ dx $=6\pi\ sq\ unit$	1 1+1 1 1
33	Let A=$\begin{pmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{pmatrix}$, X=$\begin{pmatrix}x\\ y\\ z\end{pmatrix}$, B = $\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$, C = $\begin{pmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{pmatrix}$ AC= I So, A^{-1} = C Using matrix method, X= A^{-1}B So, x= 0, y= 5 & z= 3	1 1 2 1
34.	Here, $y^x = e^{y-x}$ Taking log on both sides Xlogy = y-x X=$\frac{y}{1+logy}$ Find $\frac{dx}{dy}$ So, $\frac{dy}{dx}$ =	1 2 1

		1
35	 Given that equation of lines are $\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$................ (i) and $\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$.................. (ii) $\vec{PQ} = \vec{OQ} - \vec{OP} = (\mu + 3 - 7\lambda + 1)\hat{\imath} + (-2\mu + 5 + 6\lambda + 1)\hat{\jmath} + (\mu + 7 - \lambda + 1)\hat{k}$ $i.e.,$ $\vec{PQ} = (\mu - 7\lambda + 4)\hat{\imath} + (-2\mu + 6\lambda + 6)\hat{\jmath} + (\mu - \lambda + 8)\hat{k}$; &$(\mu - 7\lambda + 4).1 + (-2\mu + 6\lambda + 6).(-2) + (\mu - \lambda + 8).1 = 0$ $\mathbf{20\mu - 86\lambda = 0} \Rightarrow \mathbf{10\mu - 43\lambda = 0}$ & $6\mu - 20\lambda = 0 \Rightarrow 3\mu - 10\lambda = 0$ On solving the above equations, we get $\boldsymbol{\lambda} = \mu = 0$ So, the position vector of the points P and Q are $-\hat{\imath} - \hat{\jmath} - \hat{k}$ and $3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ respectively. $\vec{PQ} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ and $\lvert\vec{PQ}\rvert = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116} = 2\sqrt{29}$ $units.$	1 2 1 1
	Section –E [This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)	
36.	(i) $V = (40 - 2x)(25 - 2x)x\,cm^3$ (ii) dV/dx = $4(3x - 50)(x - 5)$ (iii) (a) For extreme values , dv/dx = $4(3x - 50)(x - 5) = 0$ $\Rightarrow x = \frac{50}{3}$ or $x = 5$	1 1 **1⁄2** **1⁄2**

	$\frac{d^2V}{dx^2} = 24x - 260$ ∴ $\frac{d^2V}{dx^2}$ at $x = 5$ is $-140 < 0$ ∴ V is maximum $when x = 5$ (iii) OR (b) For extreme values , dv/dx= $4(3x^2 - 65x + 250)$ ∴ $\frac{d^2V}{dx^2} = 4(6x - 65)$ ∴ Dv/dx at x = 65/6 exists and $\frac{d^2V}{dx^2}$ $at\ x = 65/6$ is 0 ∴∴ $\frac{d^2V}{dx^2}$ at x=65/6 is negative & $\frac{d^2V}{dx^2}$ at x= 65/6 is positive $\therefore x = \frac{65}{6}$ is a point of inflection.	**1/2** **1/2** **1/2** **1/2** **1/2** **1/2**
37	i)Number of relations is equal to the number of subsets of the set $B\ xG = 2^{n(B\ xG)}$ $= 2^6$ ($\boldsymbol{Where n(A)}$ **denotes the number of the elements in the finite set** A) ii) Smallest Equivalence relation on G is $\{(\boldsymbol{g_1}, \boldsymbol{g_1}), (\boldsymbol{g_2}, \boldsymbol{g_2})\}$ iii) (a) (A) reflexive but not symmetric = $\{(\boldsymbol{b_1}, \boldsymbol{b_2}), (\boldsymbol{b_2}, \boldsymbol{b_1}), (\boldsymbol{b_1}, \boldsymbol{b_1}), (\boldsymbol{b_2}, \boldsymbol{b_2}), (\boldsymbol{b_3}, \boldsymbol{b_3}), (\boldsymbol{b_2}, \boldsymbol{b_3})\}$. So the minimum number of elements to be added are $(\boldsymbol{b_1}, \boldsymbol{b_1}), (\boldsymbol{b_2}, \boldsymbol{b_2}), (\boldsymbol{b_3}, \boldsymbol{b_3}), (\boldsymbol{b_2}, \boldsymbol{b_3})$ {Note : it can be any one of the pair from, $(\boldsymbol{b_3}, \boldsymbol{b_2}), (\boldsymbol{b_1}, \boldsymbol{b_3}), (\boldsymbol{b_3}, \boldsymbol{b_1})$in place of $(\boldsymbol{b_2}, \boldsymbol{b_3})$ ***also***} (B) reflexive and symmetric but not transitive = $\{(\boldsymbol{b_1}, \boldsymbol{b_2}), (\boldsymbol{b_2}, \boldsymbol{b_1}), (\boldsymbol{b_1}, \boldsymbol{b_1}), (\boldsymbol{b_2}, \boldsymbol{b_2}), (\boldsymbol{b_3}, \boldsymbol{b_3}), (\boldsymbol{b_2}, \boldsymbol{b_3}), (\boldsymbol{b_3}, \boldsymbol{b_2})\}$.	1 1 1

	So the minimum number of elements to be added are $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$ OR (iii) (b) One-one and onto function $^2= 4y$. let$y = f(x) = \frac{x^2}{4}$ Let $x_1, x_2 \in [0, 20\sqrt{2}]$ such that $f(x_1)=f(x_2)$ $\Rightarrow x_1^2=x_2^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1=x_2$ as $x_1, x_2 \in [0, 20\sqrt{2}]$ $\therefore f$ is one-one Now,$0 \leq y \leq 200$ hence the value of y is non-negative and $f(2\sqrt{y}) = y$ $\therefore$ for any arbitrary $y \in [0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$ hence f is onto function.	1 1 1
38.	Let E_1be the event that one parrot and one owl flew from cage –I E_2 be the event that two parrots flew from Cage-I A be the event that the owl is still in cage-I (i) Total ways for A to happen From cage I 1 parrot and 1 owl flew and then from Cage-II 1 parrot and 1 owl flew back + From cage I 1 parrot and 1 owl flew and then from Cage-II 2 parrots flew back + From cage I 2 parrots flew and then from Cage-II 2 parrots came back. $= (5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})$ Probability that the owl is still in cage –I = $= \frac{35 + 280}{35 + 105 + 280} = \frac{315}{420} = \frac{3}{4}$ (ii) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in cage-I is $P(E^1/A)$ $P\left(E_1/A\right) = \frac{P(E_1 \cap A)}{P(\ \cap\ )\ \ \cap}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1

	(by Baye's Theorem) $E1A$ +P($E2A$) $= \frac{\frac{35}{420}}{\frac{315}{420}} = \frac{1}{9}$	