CHENNAI SAHODAYA SCHOOLS COMPLEX

(General Instructions)

- This question paper contains 9 printed pages.
- This question paper contains 38 questions.
- Write down the question number before attempting.
- An additional reading time of 15 minutes.
- This question paper contains 5 sections A, B, C, D and E.
- Each section is compulsory. However there are internal choices in some questions.
- Section A has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each.
- Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- Section E has 3 case based questions of 4 marks each.

COMMON EXAMINATION Class –12

(MATHEMATICS- 041/SET – 1)

Time Allowed: 3 hours **Maximum Marks: 80**

Roll No: **Date:**

SECTION – A

(In this section there are 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each)

1. Given a matrix $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ then the value of $|4AA^{-1}|$ is

(a) 4 (b) $4(xt - yz)$ (c) 0 (d) 16

2. If A and B are two matrices such that AB=A and BA= B then A^2 is equal to

(a) A (b) B (c) O (d) I

3. The value of the determinant $\begin{vmatrix} x & x^2 & x^3 \\ x^2 & x^3 & x^4 \\ x^3 & x^4 & x^5 \end{vmatrix}$ is

(a) x^8 (b) 0 (c) x^7 (d) 1

4. The cofactor of the element a_{23} in $A = \begin{pmatrix} 1 & 3 & -4 \\ 2 & -6 & 9 \\ 0 & 4 & -3 \end{pmatrix}$

(a) 4 (b) -4 (c) 36 (d) -36

5. If $A = [a_{ij}]$ is a skew symmetric matrix of order 3 then the value of $5a_{12} + 6a_{22} - 5a_{21}$ is

(a) 1 (b) 0 (c) $-10a_{12}$ (d) $-10a_{21}$

6. Given $f(x) = \begin{cases} mx + 1 & if\ x \leq \frac{\pi}{2} \\ \sin x + n & if\ x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then

(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$

(c) $n = \frac{m\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

7. If $y = \sqrt{\sin x + y}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$

8. The area of an equilateral triangle increases at the rate of $10\sqrt{3}\ cm^2/sec$. The rate at which its perimeter increases when side is $10cm$ is

(a) $0.6\ cm/sec$ (b) $2\ cm/sec$

(c) $10\ cm/sec$ (d) $6\ cm/sec$

9. The order and degree of the differential equation

$\frac{d^2y}{dx^2} = \sqrt{\frac{dy}{dx} + x}$ is

(a) O - 2 D – 2 (b) O – 2 D – not defined

(c) O – 1 D – not defined (d) O – 1 D – $\frac{1}{2}$

10. The value of $\int \frac{e^x(x-1)}{x^2} dx$ is

(a) $\frac{e^x}{x} + c$ (b) $xe^x + c$ (c) $\frac{x}{e^x} + c$ (d) $\frac{1}{xe^x} + c$

11. If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2 = 400\ and\ |\vec{a}| = 5$ then $\left|\vec{b}\right|$=

(a) 16 (b) 20 (c) 4 (d) 5

12. The value of $\hat{\imath} \times (\hat{\jmath} + \hat{k}) + \hat{\jmath} \times (\hat{k} + \hat{\imath}) + \hat{k} \times (\hat{\imath} + \hat{\jmath})$ is

(a) 0 (b) $\hat{\imath}$ (c) $\hat{\jmath}$ (d) $\vec{0}$

13. If the direction cosines of a line are k, k, k then k =

(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\pm \frac{1}{\sqrt{3}}$ (d) $\pm \frac{1}{3}$

14. Equation of a line passing through (1,1,1) and parallel to z axis

(a) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$ (b) $\frac{x+1}{0} = \frac{y+1}{0} = \frac{z+1}{1}$

(c) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (d) $\frac{x-1}{1} = \frac{y-1}{0} = \frac{z-1}{0}$.

15. The angle between the vectors $(\hat{\imath} - \hat{\jmath}) and \left(\hat{\jmath} - \hat{k}\right)$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

16. The value of $\cos\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\}$

(a) $-\frac{\pi}{3}$ (b)-1 (c) 0 (d) 1

17. If A and B are any two events having $P(A \cup B) = \frac{2}{3}$ and

$P(\bar{B}) = \frac{1}{2}$ then $P(A \cap \bar{B})$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

18. E and F are mutually exclusive and exhaustive events. The odds against the event E is 2:3 then odds in favour of F is

(a) 2:3 (b) 1:3 (c) 3:1 (d) 3:2

Assertion – Reason based questions

These type of questions consist of two statements.

Statement I is called Assertion (A) and statement II is called Reason (R) Choose the correct answer out of the following choices

(a) Both **A** and **R** are true and R is the correct explanation of **A**

(b) Both **A** and **R** are true and R is not correct explanation of **A** (c) **A** is true but **R** is false

(d) **A** is false but **R** is true.

19. **Assertion (A):** The corner points of the feasible region for an L.P.P. are $(0,0), (0,40), (30,20)$ and $(60,0)$.The objective function $Z = x + y$ has maximum value at $(60,0)$.

Reason (R) : If the feasible region for an L.P.P. is bounded then the objective function $Z = ax + by$ has both minimum and maximum values.

20. **Assertion (A):** The modulus function $f(x) = |x|$ is differentiable for all $x \in R$

Reason (R): A function f(x) is said to be differentiable if it is differentiable at every point on its domain.

SECTION – B

(This section comprises of VSA questions of 2 marks each.)

21. Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ in its simplest form. **(OR)**

Given set $A = \{1,2,3\}$. $R_1$and R_2 are relations on A given

by $R_1 = \{(1,1), (1,2), (1,3), (2,2), (3,3)\}$ and
$R_2 = \{(1,1), (2,2), (1,3), (3,1)\}$ Determine whether R_1

and R_2 are reflexive, symmetric and transitive.

22. Prove that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is

increasing on R

23. Solve the differential equation $y \log y\, dx - x\, dy=0$ **(OR)**

Solve the differential equation $y - \frac{dy}{dx} + \frac{y}{x} = 0$

24. Find the shortest distance between the lines

$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and

$\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

25. Find the value of λ when scalar projection of

$\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

SECTION – C

(This section comprises of SA questions of 3 marks each.)

26. A circular blot of ink in a blotting paper increases in area in such a way that the radius r cm at time 't' seconds is given by $r = 2t^2 - \frac{t^3}{4}$. Find the rate of increase of area when t=2

27. Bag I contains 4 white and 2 black balls. Bag II contains 3 white and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from bag II. The ball so drawn is found to be black in colour. Find the probability that the transferred ball is black.

28. Two cards are drawn successively without replacement from a well shuffled pack of 52. Find the probability distribution of the number of kings. Also find its mean.

29. Evaluate $\int \frac{x^2}{x^4 - x^2 - 12} dx$ **(OR)** Evaluate: $\int \frac{x+7}{3x^2 + 25x + 28} dx$

30. Evaluate: $\int_2^5 (|x-1| + |x-3|) dx$ **(OR)** $\int_2^3 \frac{dx}{x(x^3-1)}$

31. Solve the differential equation $(3x^2 + y)\frac{dx}{dy} = x$, $x > 0$

when $x = 1, y = 1$

SECTION – D

(This section comprises of LA questions of 5 marks each.)

32. On the set of integers Z consider the relation

$R = \{(a, b): (a - b)$ is divisible by 5$\}$ .Show that R is an equivalence relation on Z .Write the equivalence class of 4.

(OR)

The function $f: N \rightarrow N$, defined by

$$f(x) = \begin{cases} x + 1 & if\ x\ is\ odd \\ x - 1 & if\ x\ is\ even \end{cases}$$

Show that the function f is both one-one and onto.

33. A diet is to contain 30 units of vitamin A, 40 units of vitamin B and 20 units of vitamin C. Three types of foods F1, F2 and F3 are available. One unit of Food F1 contain 3 units of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. One unit of food F2 contains 1 unit of vitamin A, 2 units of vitamin B and 1 unit of vitamin C. One unit of food F3 contains 5 units of vitamin A, 3 units of vitamin B and 2 units of vitamin C.

If the diet contains x units of food F1, y units of food F2 and z units of Food F3 ,find the values of x, y and z using matrix method.

34. Using integration find the area bounded by the curve

$y^2 = x$ and $x = 2y + 3$ in the second quadrant and x axis.

(OR)

Using integration find the area of the region

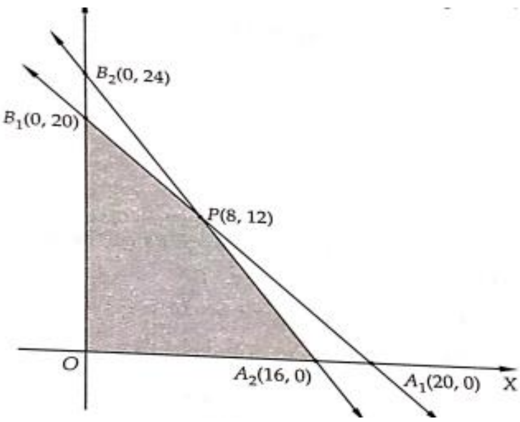
$\{(x, y): x^2 + y^2 \leq 4, x \geq -1, x + y \leq 2, y \geq 0\}$

35. Find the foot of the perpendicular and image of the point $P(1,6,3)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also find the equation of the perpendicular to the line from P.

SECTION – E

(This section contains 3 case study questions 4 marks each)

36. The feasible region for an L.P.P is shown in the figure;



(i) Write all the constraints representing the feasible region.in the above L.P.P. **(2 marks)**

(ii) Find the point at which the maximum value of the objective function $z = 10x + 5y$ occurs. **(2 marks)**

37. A cable network provider in a small town has 500 subscribers and he used to collect ₹300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from the past experience that for every increase of ₹1 one subscriber will discontinue the service. If ₹x is the monthly increase in subscription amount then find the

(i) the number of subscribers **(1 mark)**

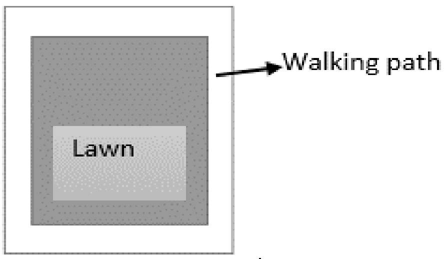
(ii) revenue function. **(1 mark)**

(iii) maximum revenue using first derivative test **(OR)**

(iii) maximum revenue using second derivative test. **(2 marks)**

38. A gardener wants to develop a rectangular lawn with an area $800m^2$. He also wants to construct a walking path of width $1m$ surrounding the lawn.

(Refer figure)



If the length of the lawn is 'x' metres then

(i) find the area of the walking path **(1 mark)**

(ii) If the cost of developing lawn is ₹50 per sq.m. and cost of constructing walking path is ₹100 per sq.m. then find the total cost function. **(1 mark)**

(iii) Find 'x' at the least construction cost. **(2 marks)**

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