Real Numbers

Case Study Based Questions

Case Study 1

A shopkeeper has 420 science stream books and 130 arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface.

Based on the above information, solve the following questions:

Q1. A number has no factor other than 1 and number itself is:

a. composite

b. prime

c. do not say anything

d. None of these

Q2. What is the maximum number of books that can be placed in each stack for this purpose?

a. 10

b. 14

c. 12

d. 15

Q3. Which mathematical concept is used to solve the problem?

a. Prime factorisation method

b. Area of triangle

c. Arithmetic progression
d. None of the above

Q4. If the shopkeeper double the quantity, then the maximum number of books that can be placed in each stack:
a. remains same
b. double
c. triple
d. None of these

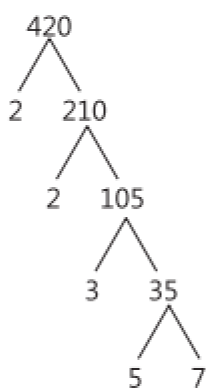
Q5. Find the LCM of the given book streams:
a. 5450
b. 5460
c. 2730
d. None of these

Solutions

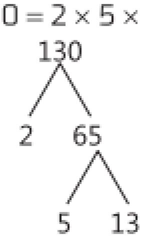
1. A number has no factor other than 1 and number itself is a prime number. So, option (b) is correct.

2. Given number of science books = 420 and number of arts books = 130

$420 = 2 \times 2 \times 3 \times 5 \times 7$



$130 = 2 \times 5 \times 13$



Maximum number of books that can be placed in each stack for the given purpose
= HCF (420, 130) = $2^1 \times 5^1$ = 10
So, option (a) is correct.

3. Prime factorsation method is used to solve the problem.
So, option (a) is correct.

4. If the shopkeeper double the quantity, then the maximum number of books that can be placed in each stack is also doubled.
So, option (b) is correct.

5. LCM of (420,130) = 2^2x3x5x7x13
= 4 x 15 x 915460
So, option (b) is correct.

Case Study 2

A seminar is being conducted by an educational organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

Based on the above information, solve the following questions:

Q1. The sum of the powers of each prime factor of 108 is:

a. 2
b. 3
C. 4
d. 5

Q2. In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number of participants that can accommodated in each room are:

a. 14

b. 12

c. 16

d. 18

Q3. What is the minimum number of rooms required during the event?

a. 11

b. 31

c. 41

d. 21

Q4. The LCM of 60, 84 and 108 is:

a. 3780

b. 3680

c. 4780

d. 4680

Q5. The product of HCF and LCM of 60, 84 and 108 is:

a. 55360

b. 35360

c. 45500

d. 45360

Solutions

1. Prime factorisation of 108

$=2\times2\times3\times3\times3$

$=2^2 \times 3^3$

.. Required sum of the powers = 2+3=5

So, option (d) is correct.

2. Using prime factorisation,

$60 = 2\times2\times3\times5=2^2\times3\times5$

$84 = 2\times2\times3\times7=2^2x3x7$

and 108 $2\times2\times3\times3\times3=22x3^3$

.. Maximum number of participants that can be accommodated in each room = HCF (60, 84, 108)

= Product of the smallest power of each common prime factor in the numbers
$=2^2 x 3=4x3=12$
So, option (b) is correct.

3. Given.
The number of participants in Hindi = 60
The number of participants in English = 84
and the number of participants in Mathematics = 108
.. Total number of participants = 60 +84 +108
= 252
Hence, minimum number of rooms required during

$$\text{event} = \frac{\text{Total number of participants}}{\text{Maximum number of participants that can be accommodated in each room}}$$

$$= \frac{252}{12} = 21$$

So, option (d) is correct.

4. LCM (60, 84, 108) = Product of the greatest power of each prime factor in the numbers
$=2^2 x 2^3 x 5 x 7$
= 4 x 27 x 5 x 7 = 3780
So, option (a) is correct.

5. HCF (60, 84, 108) = 2^2 x 3 = 12
and LCM (60, 84, 108) = 3780
HCF x LCM = 12 x 3780
= 45360
So, option (d) is correct.

Case Study 3

Old age homes mean for senior citizens who are unable to stay with their families or destitute. These old age homes have special medical facilities for senior citizens such as mobile health care systems, ambulances, nurses and provision of well balanced meals.

Himanshu, Gaurav and Gagan start preparing greeting cards for each person of an old age home on new year. In order to complete one card, they take 10, 16 and 20 min respectively. Based on the above information, solve the following questions:

Q1. Co-prime numbers are those numbers which do not have any common factor other than 1. Is this statement true?

Q2. Find the sum of the powers of all different prime factors of the numbers 10, 16 and 20.

Q3. If all of them started together, then what time will they start preparing a new card together?

OR

What is the common time to make one card?

Solutions

1. True

2. By prime factorisation,
$10=2^1\times 5^1$
$16=2x2x2x2=2^4$
$20\ 2x2x5=2^2x5^1$
.. Required sum = sum of the power of 2 + sum of the power of 5 = (1 + 4 + 2) + (1+1)=7+2=9

3. The required number of minutes after which they start preparing a new card together is the LCM of 10,

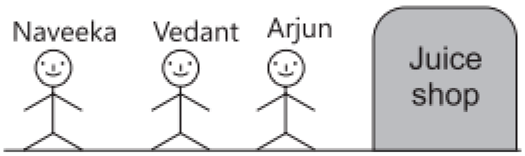
16 and 20 min.
Now, 10=2x5
16=2x2x2x2
20=2x2x5=2^2x5
.. LCM (10, 16, 20) = 2^4 x 5^1 = 16 x 5 = 80 min
So, they will start preparing a new card together after 80 min i.e., 1 h 20 min.

OR

:: The common time to make one card
= HCF of (10, 16, 20) = 2 min

Case Study 4

In a morning walk, Naveeka, Arjun and Vedant step off together, their steps measuring 240 cm, 90 cm, 120 cm respectively. They want to go for a juice shop for a health issue, which is situated near by them.



Based on the above information, solve the following questions:

Q1. Factor tree is a chain of factors, which is represented in the form of a tree. Is this statement true?

Q2. Find the sum of the powers of all common prime factors of the numbers 240, 90 and 120.

Q3. Find the minimum distance of shop from where they start to walk together, so that one can cover the distance in complete steps.

Or

Find the number of common steps covered by all of them to reach the juice shop.

Solutions

1. True

2. By prime factorisation,

$240=2x2x2x2x3x5=2^4 \times 3^1 \times 5^1$

$90=2\times3\times3\times5=2^1\times3^2x5^1$

$120=2\times2\times3\times2\times5=2^3\times3^1\times5^1$

:. Required sum = sum of the power of 2+ sum of the power of 3+ sum of the power of 5=1+1+1=3.

3. Minimum required distance to reach the juice shop

= LCM (240, 90, 120)

$240 = 2x2x2x2x3x5=2^4x3x5$

$90 = 2\times3\times3\times5=2x3^2x5$

and $120 = 2\times2\times2\times3\times5=2^3x3x5$

Now, LCM=24x32x5=16x9 × 5=720

Hence, required minimum distance is 720 cm.

OR

The number of common steps covered by all of them HCF (240, 90, 120) = 2 x 3 x 5 =30

Solutions for Questions 5 to 9 are Given Below

Case Study 5

Case Study 6

(i) Three people go for a morning walk together from the same place. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance travelled when they meet at first time after starting the walk assuming that their walking speed is same?

(a) 6120 cm (b) 12240 cm (c) 4080 cm (d) None of these

(ii) In a school Independence Day parade, a group of 594 students need to march behind a band of 189 members. The two groups have to march in the same number of columns. What is the maximum number of columns in which they can march?

(a) 9 (b) 6 (c) 27 (d) 29

(iii) Two tankers contain 768 litres and 420 litres of fuel respectively. Find the maximum capacity of the container which can measure the fuel of either tanker exactly.

(a) 4 litres (b) 7 litres (c) 12 litres (d) 18 litres

(iv) The dimensions of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm. Find the length of the largest measuring rod which can measure the dimensions of room exactly.

(a) 1 m 25 cm (b) 75 cm (c) 90 cm (d) 1 m 35 cm

(v) Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

(a) 3 and 2 (b) 2 and 5 (c) 3 and 4 (d) 4 and 5

Case Study 7

Activity on Real Numbers

In a classroom activity on real numbers, the students have to pick a number card from a pile and frame question on it if it is not a rational number for the rest of the class. The number cards picked up by first 5 students and their questions on the numbers for the rest of the class are as shown below. Answer them.

(i) Suraj picked up $\sqrt{8}$ and his question was - Which of the following is true about $\sqrt{8}$?

(a) It is a natural number (b) It is an irrational number

(c) It is a rational number (d) None of these

(ii) Shreya picked up 'BONUS' and her question was - Which of the following is not irrational?

(a) $3-4\sqrt{5}$ (b) $\sqrt{7}-6$ (c) $2+2\sqrt{9}$ (d) $4\sqrt{11}-6$

(iii) Ananya picked up $\sqrt{15}-\sqrt{10}$ and her question was - $\sqrt{15}-\sqrt{10}$ is ______ number.

(a) a natural (b) an irrational (c) a whole (d) a rational

(iv) Suman picked up $\frac{1}{\sqrt{5}}$ and her question was - $\frac{1}{\sqrt{5}}$ is ______ number.

(a) a whole (b) a rational (c) an irrational (d) a natural

(v) Preethi picked up $\sqrt{6}$ and her question was - Which of the following is not irrational?

(a) $15+3\sqrt{6}$ (b) $\sqrt{24}-9$ (c) $5\sqrt{150}$ (d) None of these

Case Study 8

Decimal Expansion

Decimal form of rational numbers can be classified into two types.

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form

$\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n \cdot 5^m$, where n, m are non-negative integers and vice-versa.

- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n\, 5^m$, where n and m are non-negative integers. Then x has a non-terminating repeating decimal expansion.

(i) Which of the following rational numbers have a terminating decimal expansion?

(a) 125/441 (b) 77/210 (c) 15/1600 (d) $129/(2^2 \times 5^2 \times 7^2)$

(ii) $23/(2^3 \times 5^2) =$

(a) 0.575 (b) 0.115 (c) 0.92 (d) 1.15

(iii) $441/(2^2 \times 5^7 \times 7^2)$ is a __________ decimal.

(a) terminating (b) recurring
(c) non-terminating and non-recurring (d) None of these

(iv) For which of the following value(s) of p, $251/(2^3 \times p^2)$ is a non-terminating recurring decimal?

(a) 3 (b) 7 (c) 15 (d) All of these

(v) $241/(2^5 \times 5^3)$ is a ______ decimal.

(a) terminating (b) recurring
(c) non-terminating and non-recurring (d) None of these

Case Study 9

Divisibility Rules

HCF and LCM are widely used in number system especially in real numbers in finding relationship between different numbers and their general forms. Also, product of two positive integers is equal to the product of their HCF and LCM.

Based on the above information answer the following questions.

(i) If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, then which of the following is true?

(a) HCF $= pq^2 \times$ LCM (b) LCM $= pq^2 \times$ HCF
(c) LCM $= p^2q \times$ HCF (d) HCF $= p^2q \times$ LCM

(ii) A boy with collection of marbles realizes that if he makes a group of 5 or 6 marbles, there are always two marbles left, then which of the following is correct if the number of marbles is p?

(a) p is odd (b) p is even (c) p is not prime (d) both (b) and (c)

(iii) Find the largest possible positive integer that will divide 398, 436 and 542 leaving remainder 7, 11, 15 respectively.

(a) 3 (b) 1 (c) 34 (d) 17

(iv) Find the least positive integer which on adding 1 is exactly divisible by 126 and 600.

(a) 12600 (b) 12599 (c) 12601 (d) 12500

(v) If A, B and C are three rational numbers such that $85C - 340A = 109$, $425A + 85B = 146$, then the sum of A, B and C is divisible by

(a) 3 (b) 6 (c) 7 (d) 9

5. **(i) (d):** For a number to end in zero it must be divisible by 5, but $4^n = 2^{2n}$ is never divisible by 5.
So, 4^n never ends in zero for any value of n.

(ii) (c): We know that product of two rational numbers is also a rational number.
So, $a^2 = a \times a$ = rational number
$a^3 = a^2 \times a$ = rational number
$a^4 = a^3 \times a$ = rational number
....
....
$a^n = a^{n-1} \times a$ = rational number.

(iii) (d): Let $x = 2m + 1$ and $y = 2k + 1$
Then $x^2 + y^2 = (2m + 1)^2 + (2k + 1)^2$
$= 4m^2 + 4m + 1 + 4k^2 + 4k + 1 = 4(m^2 + k^2 + m + k) + 2$
So, it is even but not divisible by 4.

(iv) (a): Let three consecutive positive integers be n, $n + 1$ and $n + 2$.
We know that when a number is divided by 3, the remainder obtained is either 0 or 1 or 2.
So, $n = 3p$ or $3p + 1$ or $3p + 2$, where p is some integer.
If $n = 3p$, then n is divisible by 3.
If $n = 3p + 1$, then $n + 2 = 3p + 1 + 2 = 3p + 3 = 3(p + 1)$ is divisible by 3.
If $n = 3p + 2$, then $n + 1 = 3p + 2 + 1 = 3p + 3 = 3(p + 1)$ is divisible by 3.
So, we can say that one of the numbers among n, $n + 1$ and $n + 2$ is always divisible by 3.

(v) (d): Any odd number is of the form of $(2k + 1)$, where k is any integer.
So, $n^2 - 1 = (2k + 1)^2 - 1 = 4k^2 + 4k$
For $k = 1$, $4k^2 + 4k = 8$, which is divisible by 8.
Similarly, for $k = 2$, $4k^2 + 4k = 24$, which is divisible by 8.
And for $k = 3$, $4k^2 + 4k = 48$, which is also divisible by 8.
So, $4k^2 + 4k$ is divisible by 8 for all integers k, *i.e.*, $n^2 - 1$ is divisible by 8 for all odd values of n.

6. **(i) (b):** Here $80 = 2^4 \times 5$, $85 = 17 \times 5$ and $90 = 2 \times 3^2 \times 5$
L.C.M of 80, 85 and 90 $= 2^4 \times 3 \times 3 \times 5 \times 17 = 12240$
Hence, the minimum distance each should walk when they at first time is 12240 cm.

(ii) (c): Here $594 = 2 \times 3^3 \times 11$ and $189 = 3^3 \times 7$
HCF of 594 and 189 $= 3^3 = 27$
Hence, the maximum number of columns in which they can march is 27.

(iii) (c): Here $768 = 2^8 \times 3$ and $420 = 2^2 \times 3 \times 5 \times 7$
HCF of 768 and 420 $= 2^2 \times 3 = 12$
So, the container which can measure fuel of either tanker exactly must be of 12 litres.

(iv) (b): Here, Length = 825 cm, Breadth = 675 cm and Height = 450 cm
Also, $825 = 5 \times 5 \times 3 \times 11$, $675 = 5 \times 5 \times 3 \times 3 \times 3$ and $450 = 2 \times 3 \times 3 \times 5 \times 5$
HCF $= 5 \times 5 \times 3 = 75$
Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

(v) (a): LCM of 8 and 12 is 24.
∴ The least number of pack of pens = 24/8 = 3
∴ The least number of pack of note pads = 24/12 = 2

7. **(i) (b):** Here $\sqrt{8} = 2\sqrt{2}$ = product of rational and irrational numbers = irrational number

(ii) (c): Here, $\sqrt{9} = 3$
So, $2 + 2\sqrt{9} = 2 + 6 = 8$, which is not irrational.

(iii) (b): Here $\sqrt{15}$ and $\sqrt{10}$ are both irrational and difference of two irrational numbers is also irrational.

(iv) (c): As $\sqrt{5}$ is irrational, so its reciprocal is also irrational.

(v) (d): We know that $\sqrt{6}$ is irrational.
So, $15 + 3\sqrt{6}$ is irrational.
Similarly, $\sqrt{24} - 9 = 2\sqrt{6} - 9$ is irrational.
And $5\sqrt{150} = 5 \times 5\sqrt{6} = 25\sqrt{6}$ is irrational.

8. **(i) (c):** Here, the simplest form of given options are
$125/441 = 5^3/(3^2 \times 7^2)$, $77/210 = 11/(2 \times 3 \times 5)$, $15/1600 = 3/(2^6 \times 5)$
Out of all the given options, the denominator of option (c) alone has only 2 and 5 as factors. So, it is a terminating decimal.

(ii) (b): $23/(2^3 \times 5^2) = 23/200 = 0.115$

(iii) (a) : $441/(2^2 \times 5^7 \times 7^2) = 9/(2^2 \times 5^7)$, which is a terminating decimal.

(iv) (d) : The fraction form of a non-terminating recurring decimal will have at least one prime number other than 2 and 5 as its factors in denominator.
So, p can take either of 3, 7 or 15.

(v) (a) : Here denominator has only two prime factors *i.e.*, 2 and 5 and hence it is a terminating decimal.

9. **(i) (b) :** LCM of x and $y = p^3q^3$ and HCF of x and $y = p^2q$
Also, LCM $= pq^2 \times$ HCF.

(ii) (d) : Number of marbles $= 5m + 2$ or $6n + 2$.
Thus, number of marbles, p = (multiple of 5×6) + 2
$= 30k + 2 = 2(15k + 1)$
= which is an even number but not prime

(iii) (d) : Here, required numbers
= HCF (398 – 7, 436 – 11, 542 –15)
= HCF (391, 425, 527) = 17

(iv) (b) : LCM of 126 and 600 $= 2 \times 3 \times 21 \times 100 = 12600$
The least positive integer which on adding 1 is exactly divisible by 126 and 600 = 12600 – 1 = 12599

(v) (a) : Here $85C - 340A = 109$ and $425A + 85B = 146$
On adding them, we get
$85A + 85B + 85C = 255 \Rightarrow A + B + C = 3$, which is divisible by 3.