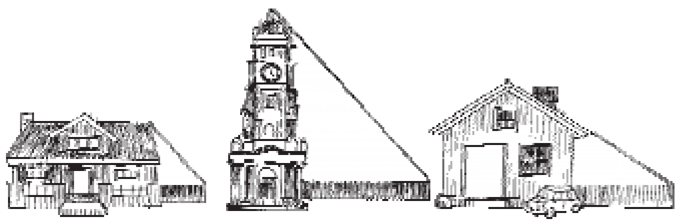
Triangles

Case Study Based Questions

Case Study 1

Digvijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Digvijay's house is 20 m when Digvijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Anshul casts 20 m shadow on the ground.



Q1. The height of the tower is:

a. 10 m

b. 20 m

c. 50 m

d. 100 m

Q2. When Digvijay's house casts a shadow of 18 cm, the length of the shadow of the tower is:

a. 18 m

b. 20 m

c. 90 m

d. 100 m

Q3. The height of Anshul's house is:

a. 20 m

b. 40 m

c. 50 m

d. 100 m

Q4. When the tower casts a shadow of 40 m, same time the length of the shadow of Anshul's house is:

a. 16 m

b. 40 m

c. 100 m

d. None of these

Q5. Which of the following similarity criterion does not exist?

a. AA

b. SAS

c. SSS

d. RHS

Solutions

1. Let CD = hm be the height of the tower.

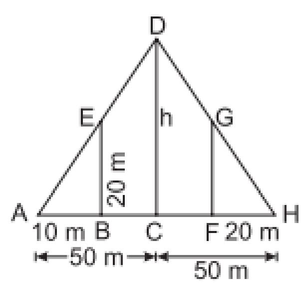
Let BE = 20 m be the height of Digvijay house and GF be the height of Anshul's house.

$\Delta ACD \sim \Delta ABE$

$\therefore \quad \frac{AC}{AB} = \frac{CD}{EB}$

$\Rightarrow \quad \frac{50}{10} = \frac{h}{20}$

$\Rightarrow \quad h = 100 \text{ m}$



So, option (d) is correct.

2. Given AB = 18 m, let AC = x

In similar ΔABE and ΔACD

$\frac{AB}{AC} = \frac{BE}{CD} \Rightarrow \frac{18}{x} = \frac{20}{100}$

$\Rightarrow \quad x = \frac{18 \times 100}{20} = 18 \times 5 = 90 \text{ m}$

3. Let height of Anshul's house be GF = h_1

Since, $\Delta HFG \sim \Delta HCD$

$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD} \Rightarrow \frac{20}{50} = \frac{h_1}{100}$

$h_1 = \frac{20 \times 100}{50} = 40 \text{ m}$

So, option (b) is correct.

4. Given, HC = 40 cm

Let length of the shadow of Anshul's house be HF = l m.

Since, $\Delta HFG \sim \Delta HCD$

$$\therefore \quad \frac{HF}{HC} = \frac{FG}{CD}$$

$$\Rightarrow \quad \frac{l}{40} = \frac{40}{100}$$

$$\Rightarrow \quad l = \frac{40 \times 40}{100} = 16 \text{ m}$$

5. RHS similarity

Criterion does not exist.

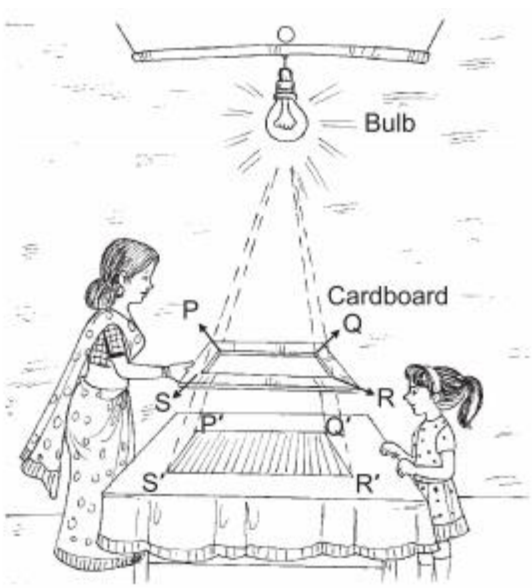
So, option (d) is correct.

Case Study 2

Gaurav placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P'Q'R'S'. Quadrilateral P'Q'R'S' is an enlargement of the quadrilateral PQRS with scale factor 1: 3. Given that PQ = 2.5 cm,

QR 3.5 cm. RS 3.4 cm and PS = 3.1 cm;

<P = 115°, <Q = 95°, <R = 65° and <S = 85°.



Based on the given information, solve the following questions:

Q1. The length of R'S' is:

a. 3.4 cm

b. 10.2 cm

c. 6.8 cm

d. 9.5 cm

Q 2. The ratio of sides P'Q' and Q'R' is:

a. 5:7

b. 7:5

c. 7:2

d. 2:7

Q3. The measurement of <Q' is:

a. 115°

b. 95°

c. 65°

d. 85°

Q4. The sum of the lengths Q'R' and P'S' is:

a. 12.3 cm

b. 6.7 cm

c. 19.8 cm

d. 9 cm

Q5. The sum of angles of quadrilateral P'Q'R'S' is:

a. 180°

b. 270°

c. 300°

d. 360°

Solutions

1. Given, scale factor is 1:3.

R'S' = 3RS

R'S' 3 x 3.4 = 10.2 cm

So, option (b) is correct.

2. Since, P'Q' 3 PQ = 3 x 2.5=7.5 cm
and Q'R' 3 QR = 3 x 3.5= 10.5 cm

$$\therefore \quad \frac{P'Q'}{Q'R'} = \frac{7.5}{10.5} = \frac{5}{7} \text{ or } 5:7$$

So, option (a) is correct.

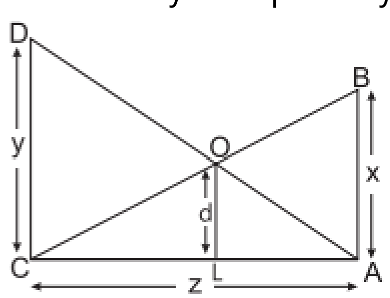
3. Quadrilateral P'Q'R'S' is similar to PQRS
<Q'=<Q=95°
So, option (b) is correct.

4. Q'R' = 3 QR = 3 x 3.5= 10.5 cm
and P'S' 3 PS = 3 x 3.1 9.3 cm
Q'R' + P'S' 10.5 + 9.3 = 19.8 cm
So, option (c) is correct.

5. Since, PQRS P'Q'R'S'
<P' = <P = 115°
<Q'=ZQ=95°
<R' = <R = 65°
and <S' <5 = 85°
<P' + <Q'+<R' + <S′ = 115° + 95° + 65° +85°
= 360°
i.e., the sum of angles of quadrilateral P'Q'R'S' is 360°.
So, option (d) is correct.

Case Study 3

Anika is studying in class X. She observe two poles DC and BA. The heights of these poles are x m and y m respectively as shown in figure:



These poles are z m apart and O is the point of intersection of the lines joining the top of each pole to the foot of opposite pole and the distance between point O and L is d. Few questions came to his mind while observing the poles.

Based on the above information, solve the following questions:

Q1. Which similarity criteria is applicable in ΔACAB and CLO?

Q2. If x=y, prove that BC: DA = 1 : 1.

Q3. If CL = a, then find a in terms of x, y and d.

OR

If AL = b, then find b in terms of x, y and d.

Solutions

1. In ΔCAB and ΔCLO, we have
<CAB = <CLO 90°
<C = <C (common)
.. By AA similarity criterion,
ΔCAB ~ ΔCLO

2. In ΔDCA and ABAC,
DC = BA [:: x = y (Given)]
<DCA = <BAC [Each 90°)
CA = AC [Common]
By SAS similarity criterian,
ΔDCA - ΔBAC

$$\therefore \quad \frac{DA}{BC} = \frac{DC}{BA} = \frac{y}{x}$$

$$\Rightarrow \quad \frac{BC}{DA} = \frac{x}{y} = \frac{x}{x} = \frac{1}{1}$$

$\therefore$ BC : DA = 1 : 1 **proved.**

3. ΔCAB ~ ΔCLO

$$\therefore \quad \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{z}{a} = \frac{x}{d} \Rightarrow a = \frac{zd}{x}$$

OR

In ΔALO and ΔACD ,

We have

$\angle ALO = \angle ACD = 90°$

$\angle A = \angle A$ (common)

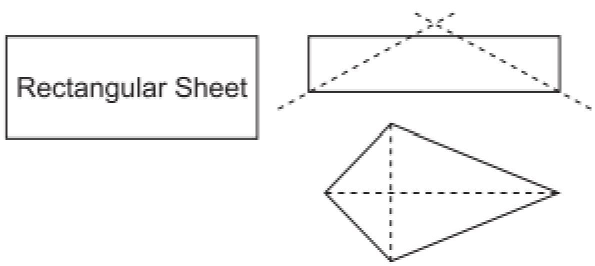
∴ By AA similarity criterion,

ΔALO ~ ΔACD

$$\therefore \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{b}{z} = \frac{d}{y} \Rightarrow b = \frac{zd}{y}$$

Case Study 4

Before Basant Panchami, Samarth is trying to make kites at home. So, he take a rectangular sheet and fold it horizontally, then vertically and fold it transversally. After cutting transversally, he gets a kite shaped figure as shown below:



Based on the above information, solve the following questions:

Q1. What is the angle between diagonals of a rectangle?

Q2. Prove that two triangles divided by a diagonal in rectangle are similar as well as congruent.

Q3. Prove that the longest diagonal of a kite bisect a pair of opposite angle.

OR

By which similarity criterion the triangles formed by longest diagonal in a kite are similar?

Solutions

1. Diagonals of a rectangle can bisect each other at any angle.

2. In ΔABC and ΔCDA

AB = CD

<B = <D

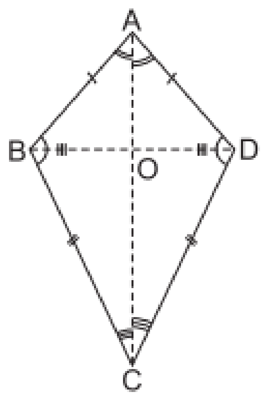
BC= DA

ΔABC = ΔCDA

(By SAS)

When two triangles are congruent, then they are similar also.

3. In ΔAOB and ΔAOD,



AB = AD

OA = OA (common)

BO = DO

(diagonal AC bisect the other diagonal BD)

∴ ΔAOB ~ ΔAOD

(by SSS similarity)

⇒ ∠BAO = ∠DAO ...(1)

In ΔBOC and ΔDOC,

BC = DC

OC = OC (common)

BO = OD

[Diagonal AC bisect the other diagonal BD]

∴ ΔBOC ~ ΔDOC [by SSS similarity]

⇒ ∠BCO = ∠DCO ...(2)

From (1) and (2), It is clear that, the longest diagonal of a kite bisect a pair of opposite angle.

OR

In ΔABC and ΔADC,

AB = AD

BC = DC

AC = AC (common)

ΔABC ~ ΔADC

(by SSS criterion)

In ΔABC and ΔADC,

$$AB = AD$$
$$\angle ABC = \angle ADC$$
$$BC = DC$$

$\therefore \quad \Delta ABC \sim \Delta ADC$

(by SAS criterion)

In ΔABC and ΔADC,

$$\angle B = \angle D$$
$$\angle BAC = \angle DAC$$

($\because$ $\angle BAO = \angle BAC$, $\angle DAO = \angle DAC$, proved above)

$$\angle BCA = \angle DCA$$

($\because$ $\angle BCO = \angle BCA$, $\angle DCO = \angle DCA$, proved above)

$\therefore \quad \Delta ABC \sim \Delta ADC$ (by AAA similarity)

So, required similarity criterions are SSS, SAS and AAA.

Solutions for Questions 5 to 14 are Given Below

Case Study 5

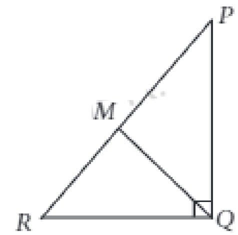
(v) If ΔPQR is right triangle with $QM \perp PR$, then which of the following is not correct?

(a) $\Delta PMQ \sim \Delta PQR$

(b) $QR^2 = PR^2 - PQ^2$

(c) $PR^2 = PQ + QR$

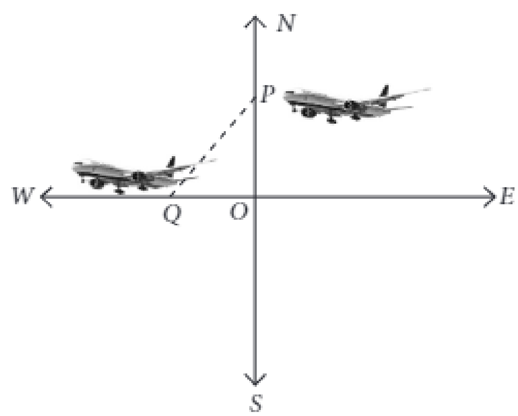
(d) $\Delta PMQ \sim \Delta QMR$



Case Study 6

Application of Pythagoras Theorem

An aeroplane leaves an airport and flies due north at a speed of 1200 km /hr. At the same time, another aeroplane leaves the same station and flies due west at the speed of 1500 km/hr as shown below. After $1\frac{1}{2}$ hr both the aeroplanes reaches at point P and Q respectively.



Based on the above information, answer the following questions.

(i) Distance travelled by aeroplane towards north after $1\frac{1}{2}$ hr is

(a) 1800 km (b) 1500 km (c) 1400 km (d) 1350 km

(ii) Distance travelled by aeroplane towards west after $1\frac{1}{2}$ hr is

(a) 1600 km (b) 1800 km (c) 2250 km (d) 2400 km

(iii) In the given figure, $\angle POQ$ is

(a) 70° (b) 90° (c) 80° (d) 100°

(iv) Distance between aeroplanes after $1\frac{1}{2}$ hr , is

(a) $450\sqrt{41}$ km (b) $350\sqrt{31}$ km

(c) $125\sqrt{12}$ km (d) $472\sqrt{41}$ km

(v) Area of ΔPOQ is

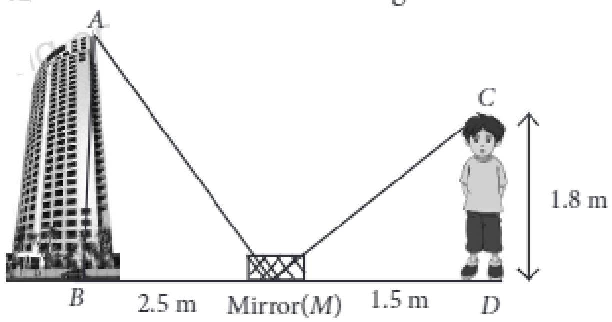
(a) 185000 km^2 (b) 179000 km^2

(c) 186000 km^2 (d) 2025000 km^2

Case Study 7

Measurement of Height

Rohit's father is a mathematician. One day he gave Rohit an activity to measure the height of building. Rohit accepted the challenge and placed a mirror on ground level to determine the height of building. He is standing at a certain distance so that he can see the top of the building reflected from mirror. Rohit eye level is at 1.8 m above ground. The distance of Rohit from mirror and that of building from mirror are 1.5 m and 2.5 m respectively.



Based on the above information, answer the following questions.

(i) Two similar triangles formed in the above figure is

(a) ΔABM and ΔCMD (b) ΔAMB and ΔCDM (c) ΔABM and ΔCDM (d) None of these

(ii) Which criterion of similarity is applied here?

(a) AA similarity criterion (b) SSS similarity criterion

(c) SAS similarity criterion (d) ASA similarity criterion

(iii) Height of the building is

(a) 1 m (b) 2 m (c) 3 m (d) 4 m

(iv) In ΔABM, if $\angle BAM = 30°$, then $\angle MCD$ is equal to

(a) 40° (b) 30° (c) 65° (d) 90°

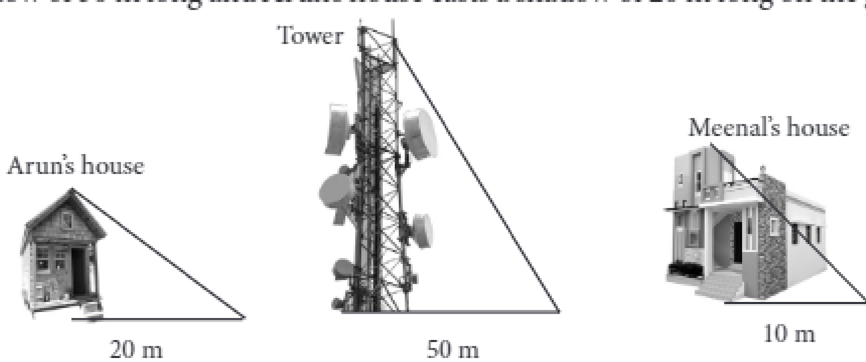
(v) If ΔABM and ΔCDM are similar where $CD = 6$ cm, $MD = 8$ cm and $BM = 24$ cm, then AB is equal to

(a) 16 cm (b) 18 cm (c) 12 cm (d) 14 cm

Case Study 8

Application of Similar Triangles

Meenal was trying to find the height of tower near his house. She is using the properties of similar triangles. The height of Meenal's house is 20 m. When Meenal's house casts a shadow of 10 m long on the ground, at the same time, tower casts a shadow of 50 m long and Arun's house casts a shadow of 20 m long on the ground as shown below.



Based on the above information, answer the following questions.

(i) What is the height of tower?

(a) 100 m (b) 50 m (c) 15 m (d) 45 m

(ii) What will be the length of shadow of tower when Meenal's house casts a shadow of 15 m?

(a) 45 m (b) 70 m (c) 75 m (d) 72 m

(iii) Height of Arun's house is

(a) 80 m (b) 75 m (c) 60 m (d) 40 m

(iv) If tower casts a shadow of 40 m, then find the length of shadow of Arun's house.

(a) 18 m (b) 16 m (c) 17 m (d) 14 m

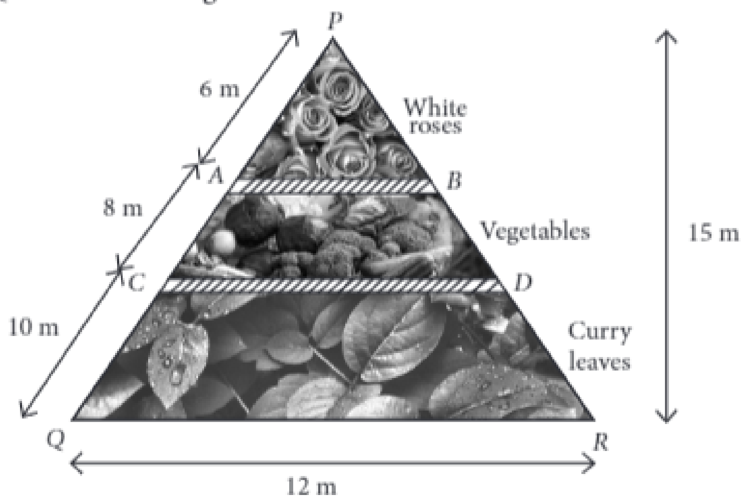
(v) If tower casts a shadow of 40 m, then what will be the length of shadow of Meenal's house?

(a) 7 m (b) 9 m (c) 4 m (d) 8 m

Case Study 9

Gardening in the Backyard

In the backyard of house, Shikha has some empty space in the shape of a ΔPQR. She decided to make it a garden. She divided the whole space into three parts by making boundaries AB and CD using bricks to grow flowers and vegetables where $AB || CD || QR$ as shown in figure.



Based on the above information, answer the following questions.

(i) The length of AB is

(a) 3 m (b) 4 m (c) 5 m (d) 6 m

(ii) The length of CD is

(a) 4 m (b) 5 m (c) 6 m (d) 7 m

(iii) Area of whole empty land is

(a) 90 m^2 (b) 60 m^2 (c) 32 m^2 (d) 72 m^2

(iv) Area of ΔPAB is

(a) $\frac{45}{4} m^2$ (b) $\frac{45}{8} m^2$ (c) $\frac{8}{45} m^2$ (d) $\frac{4}{45} m^2$

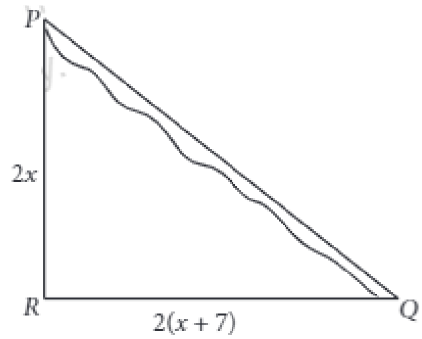
(v) Area of ΔPCD is

(a) $\frac{12}{245} m^2$ (b) $\frac{245}{12} m^2$ (c) $\frac{243}{8} m^2$ (d) $\frac{245}{8} m^2$

Case Study 10

Inspection of Road

Minister of a state went to city Q from city P. There is a route via city R such that $PR \perp RQ$. $PR = 2x$ km and $RQ = 2(x+7)$ km. He noticed that there is a proposal to construct a 26 km highway which directly connects the two cities P and Q.



Based on the above information, answer the following questions.

(i) Which concept can be used to get the value of x?

(a) Thales theorem (b) Pythagoras theorem

(c) Converse of thales theorem (d) Converse of Pythagoras theorem

(ii) The value of x is

(a) 4 (b) 6 (c) 5 (d) 8

(iii) The value of PR is

(a) 10 km (b) 20 km (c) 15 km (d) 25 km

(iv) The value of RQ is

(a) 12 km (b) 24 km (c) 16 km (d) 20 km

(v) How much distance will be saved in reaching city Q after the construction of highway?

(a) 10 km (b) 9 km (c) 4 km (d) 8 km

Case Study 11

Class teacher draw the shape of quadrilateral on board. Ankit observed the shape and explored on his notebook in different ways as shown below.

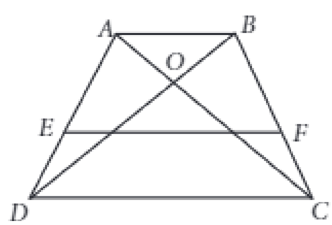


Fig. 1

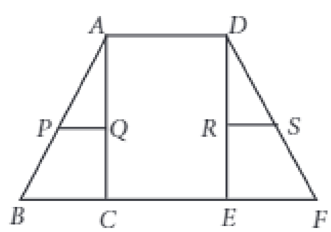


Fig. 2

Based on the above information, answer the following questions.

(i) In fig. 1, if $ABCD$ is a trapezium with $AB \parallel CD$, E and F are points on non-parallel sides AD and BC respectively such that $EF \parallel AB$, then $\dfrac{AE}{ED} =$

(a) $\frac{BE}{CD}$ (b) $\frac{AB}{CD}$ (c) $\frac{BF}{FC}$ (d) None of these

(ii) In fig. 1, if $AB \parallel CD$, and $DO = 3x - 19$, $OB = x - 5$, $OC = x - 3$ and $AO = 3$, then the value of x can be

(a) 5 or 8 (b) 8 or 9 (c) 10 or 12 (d) 13 or 14

(iii) In fig. 1, if $OD = 3x - 1$, $OB = 5x - 3$, $OC = 2x + 1$ and $AO = 6x - 5$, then the value of x is

(a) 0 (b) 1 (c) 2 (d) 3

(iv) In fig. 2, in ΔABC, if $PQ \parallel BC$ and $AP = 2.4$ cm, $AQ = 2$ cm, $QC = 3$ cm and $BC = 6$ cm, then $AB + PQ$ is equal to

(a) 7.2 cm (b) 5.9 cm (c) 2.6 cm (d) 8.4 cm

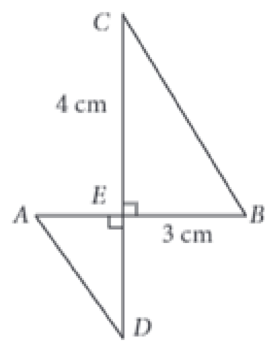
(v) In fig. 2, in ΔDEF, if $RS \parallel EF$, $DR = 4x - 3$, $DS = 8x - 7$, $ER = 3x - 1$ and $FS = 5x - 3$, then the value of x is

(a) 1 (b) 2 (c) 8 (d) 10

Case Study 12

Diwali Decoration

Ankita wants to make a toran for Diwali using some pieces of cardboard. She cut some cardboard pieces as shown below. If perimeter of ΔADE and ΔBCE are in the ratio 2 : 3, then answer the following questions.



(i) If the two triangles here are similar by SAS similarity rule, then their corresponding proportional sides are

(a) $\frac{AE}{CE} = \frac{DE}{BE}$ (b) $\frac{BE}{AE} = \frac{CE}{DE}$

(c) $\frac{AD}{CE} = \frac{BE}{DE}$ (d) None of these

(ii) Length of BC =

(a) 2 cm (b) 4 cm (c) 5 cm (d) None of these

(iii) Length of AD =

(a) 10/3 cm (b) 9/4 cm (c) 5/3 cm (d) 4/3 cm

(iv) Length of ED =

(a) 4/3 cm (b) 8/3 cm (c) 7/3 cm (d) Can't be determined

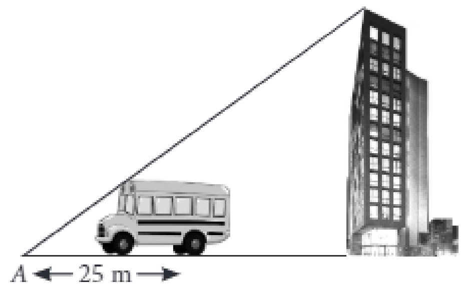
(v) Length of AE =

(a) $\frac{2}{3} \times BE$ (b) $\sqrt{AD^2 - DE^2}$ (c) $\frac{2}{3} \times \sqrt{BC^2 - CE^2}$ (d) All of these

Case Study 13

Aruna visited to her uncle's house. From a point A, where Aruna was standing, a bus and building come in a straight line as shown in the figure.

Based on the above information, answer the following questions.



(i) Which similarity criteria can be seen in this case, if bus and building are considered in a straight line?

(a) AA (b) SAS (c) SSS (d) ASA

(ii) If the distance between Aruna and the bus is twice as much as the height of the bus, then the height of the bus is

(a) 40 m (b) 12.5 m (c) 15 m (d) 25 m

(iii) If the distance of Aruna from the building is twelve times the height of the bus, then the ratio of the heights of bus and building is

(a) 3 : 1 (b) 1 : 4 (c) 1 : 6 (d) 2 : 3

(iv) What is the ratio of the distance between Aruna and top of bus to the distance between the tops of bus and building?

(a) 1 : 5 (b) 1 : 6 (c) 2 : 5 (d) Can't be determined

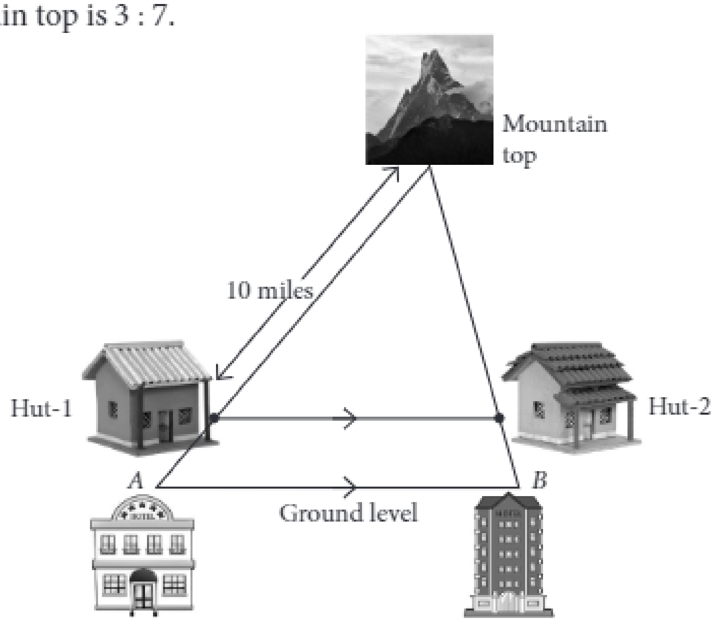
(v) What is the height of the building?

(a) 50 m (b) 75 m (c) 120 m (d) 30 m

Case Study 14

Mountain Trekking

Two hotels are at the ground level on either side of a mountain. On moving a certain distance towards the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to hut-2 and that of hut-2 to mountain top is 3 : 7.



Based on the above information, answer the following questions.

(i) What is the ratio of the perimeters of the triangle formed by both hotels and mountain top to the triangle formed by both huts and mountain top?

(a) 5 : 2 (b) 10 : 7 (c) 7 : 3 (d) 3 : 10

(ii) The distance between the hotel *A* and hut-1 is

(a) 2.5 miles (b) 29 miles (c) 4.29 miles (d) 1.5 miles

(iii) If the horizontal distance between the hut-1 and hut-2 is 8 miles, then the distance between the two hotels is

(a) 2.4 miles (b) 11.43 miles (c) 9 miles (d) 7 miles

(iv) If the distance from mountain top to hut-1 is 5 miles more than that of distance from hotel *B* to mountain top, then what is the distance between hut-2 and mountain top?

(a) 3.5 miles (b) 6 miles (c) 5.5 miles (d) 4 miles

(v) What is the ratio of areas of two parts formed in the complete figure?

(a) 53 : 21 (b) 10 : 41 (c) 51 : 33 (d) 49 : 51

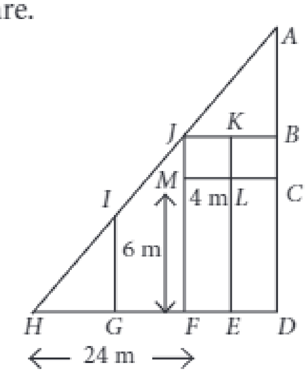
HINTS & EXPLANATIONS

5. **(i) (b):** As *JKLM* is a square.

$\therefore$ $ML = JM = 4$ m

So, $JF = 6 + 4 = 10$ m

Required distance between initial and final position of insect = HJ



$$= \sqrt{(HF)^2 + (JF)^2}$$

$$= \sqrt{(24)^2 + (10)^2}$$

$$= \sqrt{676} = 26 \text{ m}$$

(ii) (a): By Pythagoras, $n^2 + m^2 = r^2$

(iii) (a): In ΔABJ and ΔADH

$\angle B = \angle D = 90°$

$\angle A = \angle A$ (common)

$\therefore$ By AA similarity criterion, $\Delta ABJ \sim \Delta ADH$.

(iv) (d): Since, $\Delta ABJ \sim \Delta ADH$ [By AA similarity criterion]

$$\therefore \quad \frac{AB}{AD} = \frac{AJ}{AH}$$

(v) (c): Since, $PR^2 = PQ^2 + QR^2$ [By Pythagoras theorem]

6. **(i) (a):** Speed = 1200 km/hr

$$\text{Time} = 1\frac{1}{2} \text{ hr} = \frac{3}{2} \text{ hr}$$

$\therefore$ Required distance = Speed × Time

$$= 1200 \times \frac{3}{2} = 1800 \text{ km}$$

(ii) (c): Speed = 1500 km/hr

$$\text{Time} = \frac{3}{2} \text{ hr}$$

$\therefore$ Required distance = Speed × Time

$$= 1500 \times \frac{3}{2} = 2250 \text{ km}$$

(iii) (b): Clearly, directions are always perpendicular to each other.

$\therefore$ $\angle POQ = 90°$

(iv) (a): Distance between aeroplanes after $1\frac{1}{2}$ hour

$$= \sqrt{(1800)^2 + (2250)^2} = \sqrt{3240000 + 5062500}$$

$$= \sqrt{8302500} = 450\sqrt{41} \text{ km}$$

(v) (d): Area of $\Delta POQ = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 2250 \times 1800 = 2250 \times 900 = 2025000 \text{ km}^2$$

7. **(i) (c):** Since, $\angle B = \angle D = 90°$, $\angle AMB = \angle CMD$

($\because$ Angle of incident = Angle of reflection)

$\therefore$ By similarity criterion, $\Delta ABM \sim \Delta CDM$

(ii) (a)

(iii) **(c)**: $\because \Delta ABM \sim \Delta CDM$

$$\therefore \frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{1.8} = \frac{2.5}{1.5}$$

$$\Rightarrow AB = \frac{2.5 \times 1.8}{1.5} = 3 \text{ m}$$

(iv) **(b)**: Since, $\Delta ABM \sim \Delta CDM$

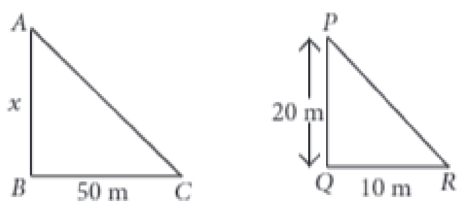
$\therefore \angle A = \angle C = 30°$

[$\because$ Corresponding angles of similar triangles are also equal]

(v) **(b)**: Since, $\Delta ABM \sim \Delta CDM$

$$\therefore \frac{AB}{CD} = \frac{BM}{MD} \Rightarrow \frac{AB}{6} = \frac{24}{8} \Rightarrow AB = 18 \text{ cm}$$

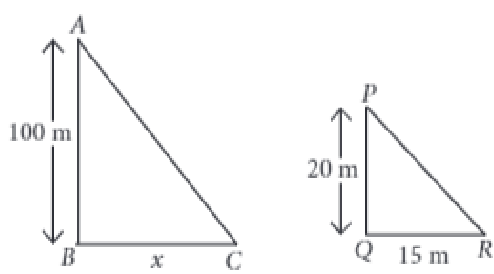
8. **(i)** **(a)**: Since, $\Delta ABC \sim \Delta PQR$



$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{x}{20} = \frac{50}{10} \Rightarrow x = 100$$

Thus, height of tower is 100 m.

(ii) **(c)**: Since, $\Delta ABC \sim \Delta PQR$

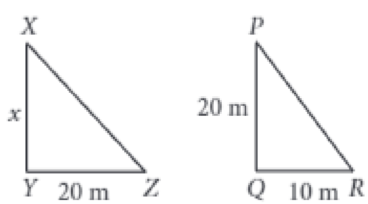


$$\therefore \frac{100}{20} = \frac{x}{15} \Rightarrow x = \frac{1500}{20} = 75 \text{ m}$$

(iii) **(d)**: Since, the shapes are similar

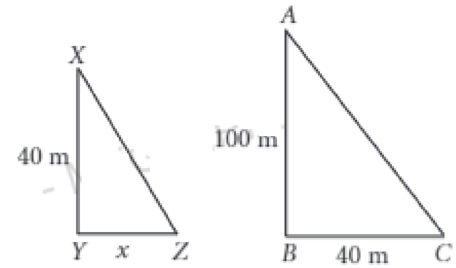
$$\therefore \frac{x}{20} = \frac{20}{10}$$

$$\Rightarrow x = \frac{20 \times 20}{10} = 40 \text{ m}$$



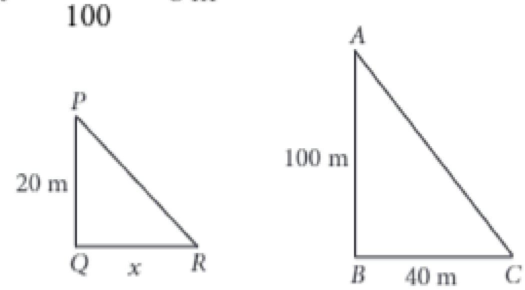
(iv) **(b)**: Since, the shapes are similar, so, $\frac{40}{100} = \frac{x}{40}$

$\Rightarrow x = 16$ m



(v) **(d)**: Since, the shapes are similar, so, $\frac{20}{100} = \frac{x}{40}$

$$\Rightarrow x = \frac{20 \times 40}{100} = 8 \text{ m}$$



9. **(i)** **(a)**: In ΔPAB and ΔPQR,

$\angle P = \angle P$ (Common)

$\angle A = \angle Q$ (Corresponding angles)

By AA similarity criterion, $\Delta PAB \sim \Delta PQR$

$$\therefore \frac{AB}{QR} = \frac{PA}{PQ} \Rightarrow \frac{AB}{12} = \frac{6}{24} \Rightarrow AB = 3 \text{ m}$$

(ii) **(d)**: Similarly, ΔPCD and ΔPQR are similar.

$$\therefore \frac{PC}{PQ} = \frac{CD}{QR} \Rightarrow \frac{14}{24} = \frac{CD}{12} \Rightarrow CD = 7 \text{ m}$$

(iii) **(a)**: Area of whole empty land

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 15 = 90 \text{ m}^2$$

(iv) **(b)**: Since, $\Delta PAB \sim \Delta PQR$.

$$\therefore \frac{ar(\Delta PAB)}{ar(\Delta PQR)} = \left(\frac{PA}{PQ}\right)^2 = \left(\frac{6}{24}\right)^2 = \frac{1}{16}$$

$$\Rightarrow ar(\Delta PAB) = \frac{1}{16} \times 90 = \frac{45}{8} \text{ m}^2$$

[$\because ar(\Delta PQR) = 90 \text{ m}^2$]

(v) **(d)**: Since, $\Delta PCD \sim \Delta PQR$.

$$\therefore \frac{ar(\Delta PCD)}{ar(\Delta PQR)} = \left(\frac{PC}{PQ}\right)^2 = \left(\frac{14}{24}\right)^2 = \left(\frac{7}{12}\right)^2$$

$$\Rightarrow ar(\Delta PCD) = \frac{90 \times 49}{144} = \frac{245}{8} \text{ m}^2$$

10. **(i)** (b)

(ii) **(c)**: Using Pythagoras theorem, we have

$PQ^2 = PR^2 + RQ^2$

$\Rightarrow \quad (26)^2 = (2x)^2 + (2(x+7))^2 \Rightarrow 676 = 4x^2 + 4(x+7)^2$

$\Rightarrow \quad 169 = x^2 + x^2 + 49 + 14x \Rightarrow x^2 + 7x - 60 = 0$

$\Rightarrow \quad x^2 + 12x - 5x - 60 = 0$

$\Rightarrow \quad x(x+12) - 5(x+12) = 0 \Rightarrow (x-5)(x+12) = 0$

$\Rightarrow \quad x = 5, x = -12$

$\therefore \quad x = 5$ [Since length can't be negative]

(iii) (a): $PR = 2x = 2 \times 5 = 10$ km

(iv) (b): $RQ = 2(x+7) = 2(5+7) = 24$ km

(v) (d): Since, $PR + RQ = 10 + 24 = 34$ km

Saved distance = 34 – 26 = 8 km

11. **(i) (c)**

(ii) (b): Since, $\Delta AOB \sim \Delta COD$ [By AA similarity criterion]

$\therefore \frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{3}{x-3} = \frac{x-5}{3x-19}$

$\Rightarrow \quad 3(3x-19) = (x-5)(x-3)$

$\Rightarrow \quad 9x - 57 = x^2 - 3x - 5x + 15 \Rightarrow x^2 - 17x + 72 = 0$

$\Rightarrow \quad (x-8)(x-9) = 0 \Rightarrow x = 8$ or 9

(iii) (c): Since, $\Delta AOB \sim \Delta COD$ [By AA similarity criterion]

$\therefore \frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{6x-5}{2x+1} = \frac{5x-3}{3x-1}$

$\Rightarrow \quad (6x-5)(3x-1) = (5x-3)(2x+1)$

$\Rightarrow \quad 18x^2 - 6x - 15x + 5 = 10x^2 + 5x - 6x - 3$

$\Rightarrow \quad 8x^2 - 20x + 8 = 0 \Rightarrow 2x^2 - 5x + 2 = 0$

From options, $x = 2$ is the only value that satisfies this equation.

(iv) (d): Since $\Delta APQ \sim \Delta ABC$ [By AA similarity criterion]

$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} \Rightarrow \frac{2.4}{AB} = \frac{2}{5} = \frac{PQ}{6}$

$\therefore \quad AB = \frac{2.4 \times 5}{2} = 6$ cm and $PQ = \frac{2 \times 6}{5} = 2.4$ cm

$\therefore \quad AB + PQ = 6 + 2.4 = 8.4$ cm

(v) (a): Since, $\Delta DRS \sim \Delta DEF$ (By AA similarity criterion)

$\therefore \quad \frac{DE}{DR} = \frac{DF}{DS} \Rightarrow \frac{DE}{DR} - 1 = \frac{DF}{DS} - 1$

$\Rightarrow \frac{DE - DR}{DR} = \frac{DF - DS}{DS} \Rightarrow \frac{ER}{DR} = \frac{FS}{DS}$

$\Rightarrow \frac{DR}{ER} = \frac{DS}{FS} \Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow \quad 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$\Rightarrow \quad 4x^2 - 2x - 2 = 0 \Rightarrow 2x^2 - x - 1 = 0$

Only option (a) *i.e.*, $x = 1$ satisfies this equation.

12. **(i) (b):** If ΔAED and ΔBEC, are similar by SAS similarity rule, then their corresponding proportional sides are $\frac{BE}{AE} = \frac{CE}{DE}$

(ii) (c): By Pythagoras theorem, we have

$BC = \sqrt{CE^2 + EB^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ cm

(iii) (a): Since ΔADE and ΔBCE are similar.

$\therefore \quad \frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \frac{AD}{BC}$

$\Rightarrow \quad \frac{2}{3} = \frac{AD}{5} \Rightarrow AD = \frac{5 \times 2}{3} = \frac{10}{3}$ cm

(iv) (b): $\frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \frac{ED}{CE}$

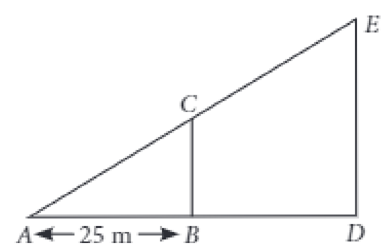
$\Rightarrow \quad \frac{2}{3} = \frac{ED}{4} \Rightarrow ED = \frac{4 \times 2}{3} = \frac{8}{3}$ cm

(v) (d): $\frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \frac{AE}{BE} \Rightarrow \frac{2}{3} BE = AE$

$\Rightarrow \quad AE = \frac{2}{3}\sqrt{BC^2 - CE^2}$

Also, in ΔAED, $AE = \sqrt{AD^2 - DE^2}$

13. Let BC represents the height of bus and DE represents the height of building.



(i) (a): In ΔABC and ΔADE,

$\angle A = \angle A$ (Common)

$\angle B = \angle D$ (Corresponding angles)

$\therefore \quad \Delta ABC \sim \Delta ADE$ (By AA similarity criteria)

(ii) (b): We have, $AB = 2BC$

$\Rightarrow \quad BC = \frac{25}{2} = 12.5$ m

So, height of bus = 12.5 m

(iii) (c): We have, $AD = 12\ BC$

$\Rightarrow \quad AD = 12 \times 12.5 = 150$ m

$\because \quad \Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE} \Rightarrow \frac{BC}{DE} = \frac{25}{150} = \frac{1}{6}$

So, ratio of heights of bus and building is 1 : 6.

(iv) **(a)**: Since, $\Delta ABC \sim \Delta ADE$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{AC}{AE} = \frac{1}{6}$$

$$\Rightarrow \frac{AC}{AE - AC} = \frac{1}{6-1} \Rightarrow \frac{AC}{EC} = \frac{1}{5}$$

$\therefore$ Required ratio = 1 : 5

(v) **(b)**: Height of the building = DE

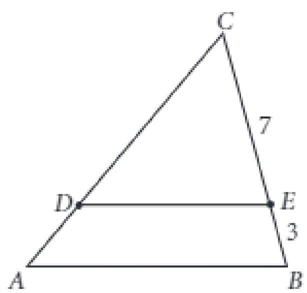
Now, $\frac{BC}{DE} = \frac{1}{6}$

$\Rightarrow DE = 6BC = 6 \times 12.5 = 75$ m

14. **(i)** **(b)**: Let ΔABC is the triangle formed by both hotels and mountain top. ΔCDE is the triangle formed by both huts and mountain top.

Clearly, $DE \parallel AB$ and so

$\Delta ABC \sim \Delta DEC$ [By AA-similarity criterion]



Now, required ratio = Ratio of their corresponding sides $= \frac{BC}{EC} = \frac{10}{7}$ *i.e.*, 10 : 7.

(ii) **(c)**: Since, $DE \parallel AB$, therefore

$$\frac{CD}{AD} = \frac{CE}{EB} \Rightarrow \frac{10}{AD} = \frac{7}{3} \Rightarrow AD = \frac{10 \times 3}{7} = 4.29 \text{ miles}$$

(iii) **(b)**: Since, $\Delta ABC \sim \Delta DEC$

$\therefore \frac{BC}{EC} = \frac{AB}{DE}$ [$\because$ Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{10}{7} = \frac{AB}{8} \Rightarrow AB = \frac{80}{7} = 11.43 \text{ miles}$$

(iv) **(a)**: Given, $DC = 5 + BC$.

Clearly, $BC = 10 - 5 = 5$ miles

Now, $CE = \frac{7}{10} \times BC = \frac{7}{10} \times 5 = 3.5$ miles

(v) **(d)**: Clearly, the ratio of areas of two triangles (*i.e.*, ΔABC to ΔDEC)

$$= \left(\frac{BC}{EC}\right)^2 = \left(\frac{10}{7}\right)^2 = \frac{100}{49}$$

$$\therefore \text{ Required ratio} = \frac{ar(\Delta CDE)}{ar(EBAD)} = \frac{49}{100-49} = \frac{49}{51}$$