Series **PQ3RS/3**

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/3/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ* ***23*** *हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains* ***23*** *printed pages.*
- *Please check that this question paper contains* ***38*** *questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$ है, तो k का मान है :

(A) 2 (B) -2 (C) ± 2 (D) ∓ 2

2. 5^x का अवकलज, e^x के सापेक्ष, है :

(A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$ (B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$

(C) $\left(\frac{5}{e}\right)^x \log 5$ (D) $\left(\frac{e}{5}\right)^x \log 5$

3. यदि $|\vec{a}| = 2$ और $-3 \le k \le 2$ है, तो $|k\vec{a}| \in$:

(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

(A) 2 (B) -2 (C) ± 2 (D) ∓ 2

2. The derivative of 5^x w.r.t. e^x is :

(A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$ (B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$

(C) $\left(\frac{5}{e}\right)^x \log 5$ (D) $\left(\frac{e}{5}\right)^x \log 5$

3. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

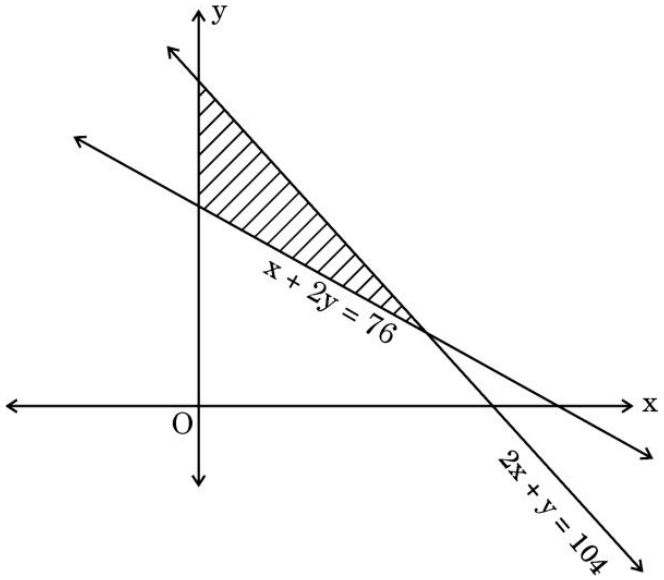
(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$

4. यदि एक रेखा दोनों x-अक्ष और z-अक्ष की धनात्मक दिशाओं में $\frac{\pi}{4}$ का कोण बनाती है, तो यह रेखा y-अक्ष की धनात्मक दिशा में जो कोण बनाती है, वह है :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

5. नीचे दिए गए सुसंगत क्षेत्र को, व्यवरोधों का निम्नलिखित में से कौन-सा समूह निरूपित करता है ?



(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

6. यदि $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ है, तो A^{-1} है :

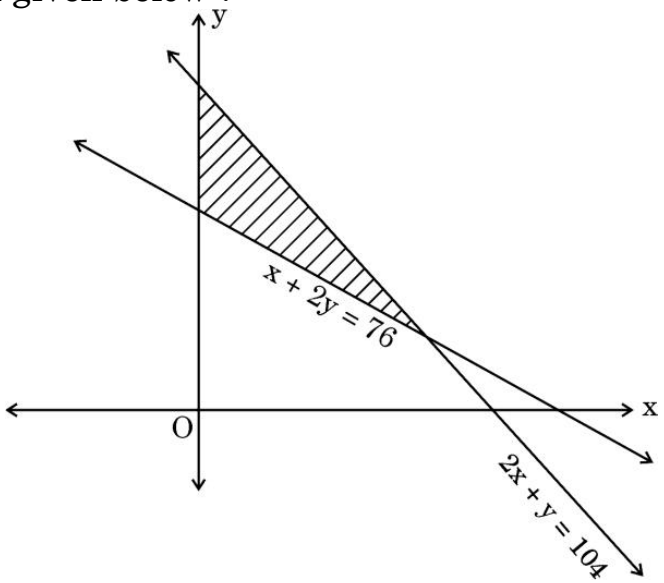
(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

5. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

7. किसी वर्ग आव्यूह A के लिए, $(A - A')$ हमेशा

(A) एक तत्समक आव्यूह होता है

(B) एक शून्य आव्यूह होता है

(C) एक विषम सममित आव्यूह होता है

(D) एक सममित आव्यूह होता है

8. एक फलन $f : R \to A$ जो $f(x) = x^2 + 1$ द्वारा परिभाषित है, आच्छादक होगा, यदि A है :

(A) $(-\infty, \infty)$ (B) $(1, \infty)$

(C) $[1, \infty)$ (D) $[-1, \infty)$

9. माना $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ एक वर्ग आव्यूह है जिसके लिए adj A = A है । तब, $(a + b + c + d)$ बराबर है :

(A) 2a (B) 2b

(C) 2c (D) 0

10. फलन $f(x) = |1 - x + |x||$:

(A) असंतत है केवल $x = 1$ पर (B) असंतत है केवल $x = 0$ पर

(C) असंतत है $x = 0, 1$ पर (D) हर बिन्दु पर संतत है

11. फलन $f(x)$ का नति परिवर्तन बिंदु वह बिंदु होता है, जिसके लिए

(A) $f'(x) = 0$ और बिंदु के बाएँ से दाएँ जाने पर $f'(x)$ का चिह्न धन से ऋण में परिवर्तित होता है ।

(B) $f'(x) = 0$ और बिंदु के बाएँ से दाएँ जाने पर $f'(x)$ का चिह्न ऋण से धन में परिवर्तित होता है ।

(C) $f'(x) = 0$ और बिंदु के बाएँ से दाएँ जाने पर $f'(x)$ का चिह्न परिवर्तित नहीं होता है ।

(D) $f'(x) \neq 0$.

12. यदि $g(x)$ एक संतत फलन है जिसके लिए $g(-x) = -g(x)$, तो $\int_0^{2a} g(x)\, dx$ बराबर है :

(A) 0 (B) $2\int_0^a g(x)\, dx$

(C) $\int_{-a}^a g(x)\, dx$ (D) $-\int_{-2a}^0 g(x)\, dx$

7. For any square matrix A, $(A - A')$ is always

(A) an identity matrix

(B) a null matrix

(C) a skew symmetric matrix

(D) a symmetric matrix

8. A function $f : R \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is :

(A) $(-\infty, \infty)$ (B) $(1, \infty)$

(C) $[1, \infty)$ (D) $[-1, \infty)$

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

(A) 2a (B) 2b

(C) 2c (D) 0

10. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at x = 1 only (B) discontinuous at x = 0 only

(C) discontinuous at x = 0, 1 (D) continuous everywhere

11. The point of inflexion of a function f(x) is the point where

(A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point.

(B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point.

(C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

(D) $f'(x) \neq 0$.

12. If g(x) is a continuous function satisfying $g(-x) = -g(x)$, then $\int_0^{2a} g(x)\,dx$ is equal to :

(A) 0 (B) $2\int_0^a g(x)\,dx$

(C) $\int_{-a}^a g(x)\,dx$ (D) $-\int_{-2a}^0 g(x)\,dx$

13. $x \log x \frac{dy}{dx} + y = 2 \log x$ एक उदाहरण है :

(A) पृथक्करणीय चर वाले अवकल समीकरण का ।

(B) समघातीय अवकल समीकरण का ।

(C) प्रथम कोटि के रैखिक अवकल समीकरण का ।

(D) ऐसे अवकल समीकरण का जिसकी घात परिभाषित नहीं है ।

14. यदि $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ और $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ हैं, तो $\vec{a}$ और $\vec{b}$ हैं :

(A) संरेख सदिश जो कि समांतर नहीं हैं

(B) समांतर सदिश

(C) परस्पर लंबवत् सदिश

(D) मात्रक सदिश

15. यदि एक रेखा x, y और z अक्षों की धनात्मक दिशा के साथ क्रमशः α, β और γ कोण बनाती है, तो निम्नलिखित में से कौन-सा सत्य ***नहीं*** है ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

16. एक रैखिक प्रोग्रामन समस्या के उद्देश्य फलन में निर्णायक चरों पर लगे प्रतिबंध कहलाते हैं :

(A) सुसंगत हल (B) व्यवरोध

(C) इष्टतम हल (D) असुसंगत हल

17. माना E और F दो ऐसी घटनाएँ हैं जिनके लिए $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$ है, तो $P(F \mid E)$ है :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

18. यदि $A = [a_{ij}]$ एक तत्समक आव्यूह है, तो निम्नलिखित में से कौन-सा सही है ?

(A) $a_{ij} = \begin{cases} 0, & \text{यदि } i = j \\ 1, & \text{यदि } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{यदि } i \neq j \\ 1, & \text{यदि } i = j \end{cases}$

13. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

15. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

(C) optimal solutions (D) infeasible solutions

17. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F \mid E)$ is :

(A) $0{\cdot}6$ (B) $0{\cdot}4$ (C) $0{\cdot}5$ (D) 0

18. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए ।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* सदिश $\vec{a}$ का, सदिश $\vec{b}$ पर प्रक्षेप उतना ही होता है जितना सदिश $\vec{b}$ का, सदिश $\vec{a}$ पर ।

तर्क (R) : संख्यात्मक दृष्टि से $\vec{a}$ और $\vec{b}$ के बीच का कोण उतना ही होता है जितना $\vec{b}$ और $\vec{a}$ के बीच में ।

20. *अभिकथन (A) :* प्रत्येक अदिश आव्यूह एक विकर्ण आव्यूह होता है ।

तर्क (R) : एक विकर्ण आव्यूह में, विकर्ण के सभी अवयव शून्य होते हैं ।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. (क) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

अथवा

(ख) दिया गया है $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ और $F(1) = 0$, $F(x)$ ज्ञात कीजिए ।

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

20. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

22. बिंदु A और B, जिनके स्थिति सदिश क्रमशः $\hat{i} + 2\hat{j} - \hat{k}$ और $-\hat{i} + \hat{j} + \hat{k}$ हैं, को मिलाने वाले रेखा-खंड को 4 : 1 के अनुपात से बाह्य विभाजित करने वाले बिंदु C का स्थिति सदिश ज्ञात कीजिए । $|\overrightarrow{AB}| : |\overrightarrow{BC}|$ भी ज्ञात कीजिए ।

23. यदि $\vec{a}$, $\vec{b}$ और $\vec{c}$ ऐसे तीन मात्रक सदिश हैं जिनके लिए $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, तो सदिश $\vec{a}$ और $\vec{c}$ के बीच का कोण ज्ञात कीजिए ।

24. $\left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\} + \tan^2\left\{\sec^{-1}(3)\right\}\right]$ का मान ज्ञात कीजिए ।

25. (क) यदि $x = e^{x/y}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

अथवा

(ख) $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ के $x = 1$ पर अवकलनीय होने की जाँच कीजिए ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. ज्ञात कीजिए :

$$\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4}\, dx$$

27. (क) ज्ञात कीजिए :

$$\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x\, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\, dx$$

22. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

23. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.

24. Find the value of $\left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\} + \tan^2\left\{\sec^{-1}(3)\right\}\right]$.

25. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4}\, dx$$

27. (a) Find :

$$\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\, dx$$

28. निम्नलिखित रैखिक प्रोग्रामन समस्या को आलेख विधि से हल कीजिए :
निम्न व्यवरोधों के अंतर्गत

$$x + y \geq 8$$
$$x + 2y \leq 16$$
$$4x + y \leq 29$$
$$x, y \geq 0$$

$z = 600x + 400y$ का न्यूनतमीकरण कीजिए ।

29. P, Q और R के किसी कंपनी के CEO के रूप में चुने जाने की संभावनाएँ क्रमश: 4 : 1 : 2 के अनुपात में हैं । नए CEO, P, Q या R के तहत कंपनी के पिछले वर्ष की तुलना में लाभ बढ़ाने की प्रायिकताएँ क्रमश: 0·3, 0·8 और 0·5 हैं । यदि कंपनी पिछले वर्ष से लाभ बढ़ाती है, तो प्रायिकता ज्ञात कीजिए कि यह R की CEO के पद पर नियुक्ति के कारण हुई है ।

30. (क) यदि $x \cos (p + y) + \cos p \sin (p + y) = 0$ है, तो सिद्ध कीजिए कि

$$\cos p \frac{dy}{dx} = -\cos^2 (p + y), \text{ जहाँ } p \text{ स्थिरांक है ।}$$

अथवा

(ख) a और b के वे मान ज्ञात कीजिए जिनके लिए फलन f, जो परिभाषित है :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{यदि } x < 2 \\ a + b, & \text{यदि } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{यदि } x > 2 \end{cases}$$

संतत फलन है ।

31. (क) अंतराल ज्ञात कीजिए जिनमें फलन $f(x) = \frac{\log x}{x}$ निरंतर वर्धमान या निरंतर ह्रासमान है ।

अथवा

(ख) अंतराल [1, 2] में $f(x) = \frac{x}{2} + \frac{2}{x}$ द्वारा प्रदत्त फलन f के निरपेक्ष उच्चतम और निरपेक्ष निम्नतम मानों को ज्ञात कीजिए ।

28. Solve the following linear programming problem graphically :

Minimize $z = 600x + 400y$,

subject to the constraints

$$x + y \geq 8$$

$$x + 2y \leq 16$$

$$4x + y \leq 29$$

$$x, y \geq 0.$$

29. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

30. (a) If $x \cos (p + y) + \cos p \sin (p + y) = 0$, prove that $\cos p \frac{dy}{dx} = -\cos^2 (p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

31. (a) Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval [1, 2].

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. (क) दिया गया है कि फलन $f(x) = x^4 - 62x^2 + ax + 9$ स्थानीय उच्चतम मान $x = 1$ पर प्राप्त करता है । 'a' का मान ज्ञात कीजिए, और सभी अन्य बिंदु ज्ञात कीजिए जिन पर इस फलन $f(x)$ का स्थानीय उच्चतम या स्थानीय निम्नतम मान प्राप्त होता है ।

अथवा

(ख) एक आयताकार धातु की चादर का परिमाप 300 cm है । एक सिलेंडर बनाने के लिए इसको एक किनारे से लपेटा जाता है । आयताकार चादर की विमाएँ ज्ञात कीजिए जिससे बनाए गए सिलेंडर का आयतन अधिकतम हो ।

33. समाकलन का प्रयोग करके, रेखाओं $x - 2y = 4$, $x = -1$, $x = 6$ तथा x-अक्ष के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

34. (क) रेखाओं $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ और $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ के प्रतिच्छेदन बिंदु से गुज़रने वाली उस रेखा का समीकरण ज्ञात कीजिए जो इन दी गई रेखाओं के लम्बवत् है ।

अथवा

(ख) एक समांतर चतुर्भुज ABCD के दो शीर्ष A(–1, 2, 1) और B(1, – 2, 5) हैं । यदि C और D से गुज़रने वाली रेखा का समीकरण $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$ है, तो भुजाओं AB और CD के बीच की दूरी ज्ञात कीजिए । अतः, समांतर चतुर्भुज ABCD का क्षेत्रफल ज्ञात कीजिए ।

35. समुच्चय $A = \{x : -10 \leq x \leq 10, x \in Z\}$ पर एक संबंध R परिभाषित है

$R = \{(x, y) : (x - y), 5$ से विभाजित है$\}$. दर्शाइए कि R एक तुल्यता संबंध है । तुल्यता-वर्ग [5] भी लिखिए ।

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

33. Find the area of the region bounded by the lines $x - 2y = 4$, $x = -1$, $x = 6$ and x-axis, using integration.

34. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

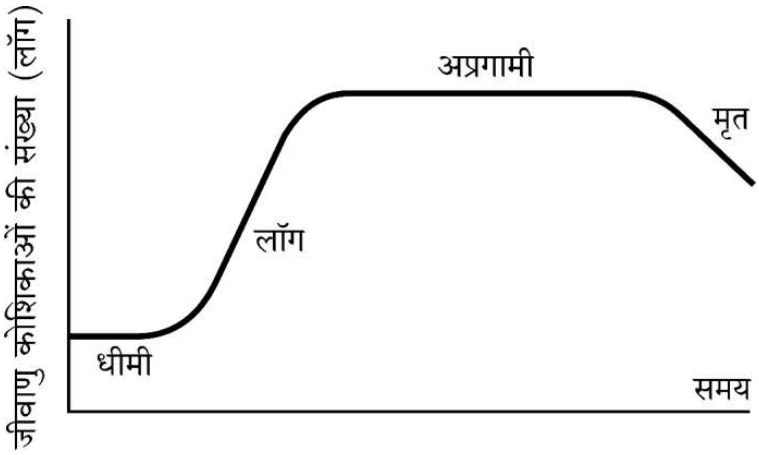
35. A relation R on set $A = \{x : -10 \leq x \leq 10, x \in Z\}$ is defined as $R = \{(x, y) : (x - y) \text{ is divisible by } 5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5].

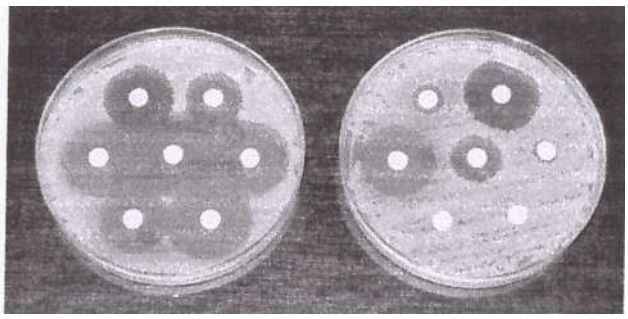
खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. एक निश्चित संख्या में जीवाणुओं का एक जीवाणु नमूना एक निश्चित समय में चरघातांकी रूप से बढ़ता हुआ देखा गया है । चरघातांकी वृद्धि मॉडल का उपयोग करके, जीवाणु के इस नमूने की वृद्धि दर की गणना की जाती है ।





जीवाणुओं की वृद्धि को दर्शाने वाला अवकल समीकरण इस प्रकार दिया गया है : $\frac{dP}{dt} = kP$, जहाँ P किसी भी समय 't' पर जीवाणुओं की जनसंख्या है ।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) दिए गए अवकल समीकरण का व्यापक हल ज्ञात कीजिए और इसको 't' के चरघातांकी फलन के रूप में व्यक्त कीजिए । *2*

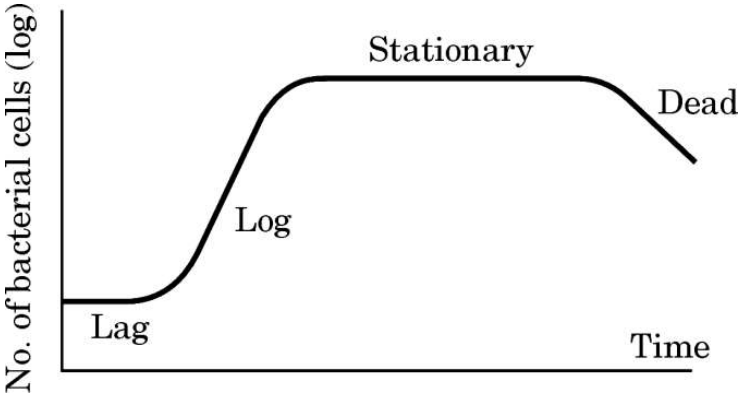
(ii) यदि जीवाणुओं की जनसंख्या $t = 0$ पर 1000 और $t = 1$ पर 2000 है, तो k का मान ज्ञात कीजिए । *2*

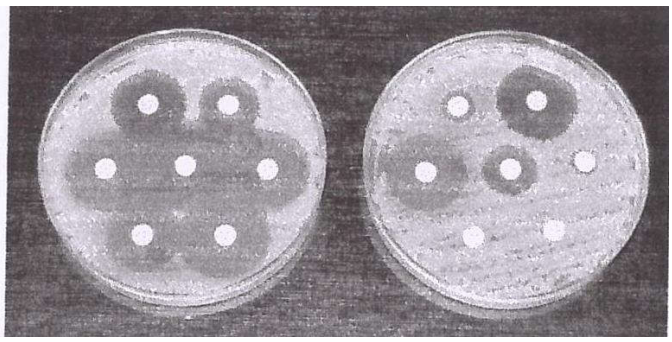
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2*

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*

प्रकरण अध्ययन – 2

37. छात्रवृत्ति एक छात्र को शिक्षा के लिए भुगतान करने में मदद करने के लिए प्रदान की जाने वाली धनराशि है । कुछ छात्रों को उनकी शैक्षणिक उपलब्धियों के आधार पर छात्रवृत्ति दी जाती है, जबकि अन्य को उनकी वित्तीय आवश्यकताओं के आधार पर पुरस्कृत किया जाता है ।

हर वर्ष एक स्कूल कुछ मानदंडों के आधार पर बालिकाओं और मेधावी उपलब्धि हासिल करने वालों को छात्रवृत्ति प्रदान करता है । सत्र 2022 – 23 में, स्कूल ने कुछ छात्राओं को ₹ 3,000 प्रत्येक की मासिक छात्रवृत्ति और शैक्षणिक और साथ ही खेल में मेधावी उपलब्धि हासिल करने वालों को ₹4,000 प्रत्येक की मासिक छात्रवृत्ति की पेशकश की ।

कुल मिलाकर, 50 छात्रों को छात्रवृत्ति प्रदान की गई और स्कूल द्वारा छात्रवृत्तियों पर ₹ 1,80,000 मासिक खर्च किया गया ।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) दी गई सूचना को आव्यूहों का उपयोग करते हुए, बीजगणितीय रूप में व्यक्त कीजिए । *1*

(ii) जाँच कीजिए कि क्या प्राप्त हुआ आव्यूह समीकरण निकाय संगत है या नहीं । *1*

(iii) (क) आव्यूहों का प्रयोग करके, स्कूल द्वारा प्रत्येक प्रकार की दी गई छात्रवृत्तियों की संख्या ज्ञात कीजिए । *2*

अथवा

(iii) (ख) यदि छात्रवृत्ति की धनराशियाँ, जो एक बालिका को और एक मेधावी छात्र को प्रदान की गई हैं, को परस्पर बदल दिया जाए, तो स्कूल का मासिक खर्च क्या होगा ? *2*

Case Study – 2

37. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

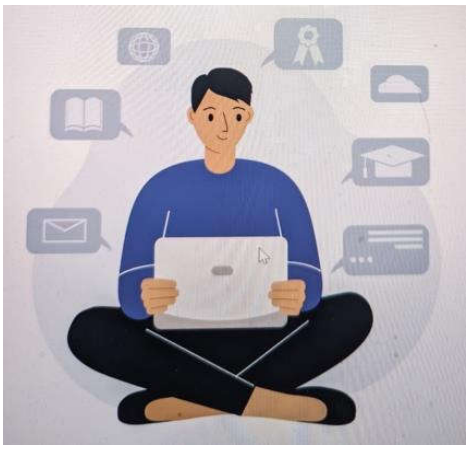
(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

प्रकरण अध्ययन – 3

38. स्व-अध्ययन छात्रों को सीखने में आत्मविश्वास पैदा करने में मदद करता है । इससे छात्रों का आत्म-सम्मान बढ़ता है । हाल के सर्वेक्षणों से पता चला है कि लगभग 50% छात्रों ने इंटरनेट संसाधनों का उपयोग करके स्वयं शिक्षा प्राप्त की और खुद को कुशल बनाया ।



एक छात्र स्वयं के कौशल को बढ़ाने में एक दिन में 1 घंटे से 6 घंटे तक का समय व्यतीत कर सकता है । एक छात्र द्वारा व्यतीत किए गए घंटों की संख्या का प्रायिकता बंटन नीचे दिया गया है :

$$P(X = x) = \begin{cases} kx^2, & x = 1, 2, 3 \text{ के लिए} \\ 2kx, & x = 4, 5, 6 \text{ के लिए} \\ 0, & \text{अन्यथा} \end{cases}$$

जहाँ x घंटों की संख्या को प्रदत्त करता है ।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) ऊपर दिए गए प्रायिकता बंटन को प्रायिकता बंटन तालिका के रूप में व्यक्त कीजिए । *1*

(ii) k का मान ज्ञात कीजिए । *1*

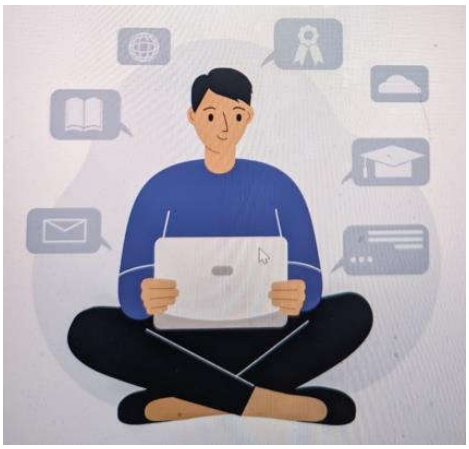
(iii) (क) छात्र द्वारा व्यतीत किए गए घंटों की संख्या का माध्य ज्ञात कीजिए । *2*

अथवा

(iii) (ख) $P(1 < X < 6)$ ज्ञात कीजिए । *2*

Case Study – 3

38. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

(iii) (a) Find the mean number of hours spent by the student. *2*

OR

(iii) (b) Find $P(1 < X < 6)$. *2*

Marking Scheme

Strictly Confidential

(For Internal and Restricted use only)

Senior School Certificate Examination, 2024

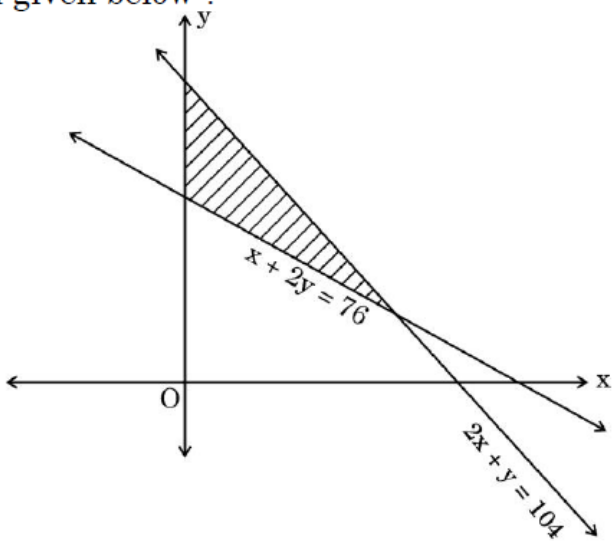
MATHEMATICS PAPER CODE 65/3/2

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (**✓**) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. NO.	EXPECTED ANSWER / VALUE POINT	MARKS
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) of 1 mark each.	
Q1	If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is : (A) 2 (B) -2 (C) ± 2 (D) ∓ 2	
Ans	(D) ∓ 2	1
Q2	The derivative of 5^x w.r.t. e^x is : (A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$ (B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$ (C) $\left(\frac{5}{e}\right)^x \log 5$ (D) $\left(\frac{e}{5}\right)^x \log 5$	
Ans	(C) $\left(\frac{5}{e}\right)^x \log 5$	1
Q3	If $\lvert \vec{a} \rvert = 2$ and $-3 \le k \le 2$, then $\lvert k\vec{a} \rvert \in$: (A) $[-6, 4]$ (B) $[0, 4]$ (C) $[4, 6]$ (D) $[0, 6]$	
Ans	(D) $[0, 6]$	1
Q4	If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is : (A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π	
Ans	(C) $\frac{\pi}{2}$	1

Q5	Of the following, which group of constraints represents the feasible region given below ?  (A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$ (B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$ (C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$ (D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$	
Ans	(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$	1
Q6	If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is : (A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	
Ans	(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	1

Q7	For any square matrix A, $(A - A')$ is always (A) an identity matrix (B) a null matrix (C) a skew symmetric matrix (D) a symmetric matrix	
Ans	(C) a skew symmetric matrix	1
Q8	A function $f : R \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is : (A) $(-\infty, \infty)$ (B) $(1, \infty)$ (C) $[1, \infty)$ (D) $[-1, \infty)$	
Ans	(C) $[1, \infty)$	1
Q9	Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to : (A) 2a (B) 2b (C) 2c (D) 0	
Ans	(A) 2a	1
Q10	A function $f(x) = \lvert 1 - x + \lvert x \rvert \rvert$ is : (A) discontinuous at x = 1 only (B) discontinuous at x = 0 only (C) discontinuous at x = 0, 1 (D) continuous everywhere	
Ans	(D) continuous everywhere	1
Q11	The point of inflexion of a function f(x) is the point where (A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point. (B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point. (C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point. (D) $f'(x) \neq 0$.	
Ans	(C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.	1

Q12	If g(x) is a continuous function satisfying g(– x) = – g(x), then $\int_0^{2a} g(x)\,dx$ is equal to : (A) 0 (B) $2\int_0^{a} g(x)\,dx$ (C) $\int_{-a}^{a} g(x)\,dx$ (D) $-\int_{-2a}^{0} g(x)\,dx$	
Ans	D) $-\int_{-2a}^{0} g(x)\,dx$	1
Q13	$x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a : (A) variable separable differential equation. (B) homogeneous differential equation. (C) first order linear differential equation. (D) differential equation whose degree is not defined.	
Ans	(C) first order linear differential equation.	1
Q14	If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are : (A) collinear vectors which are not parallel (B) parallel vectors (C) perpendicular vectors (D) unit vectors	
Ans	(C) perpendicular vectors	1
Q15	If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ? (A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ (C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ (D) $\cos\alpha + \cos\beta + \cos\gamma = 1$	
Ans	(D) $\cos\alpha + \cos\beta + \cos\gamma = 1$	1

Q16	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called : (A) feasible solutions (B) constraints (C) optimal solutions (D) infeasible solutions	
Ans	(B) constraints	1
Q17	Let E and F be two events such that P(E) = 0·1, P(F) = 0·3, P(E ∪ F) = 0·4, then P(F | E) is : (A) 0·6 (B) 0·4 (C) 0·5 (D) 0	
Ans	(D) 0	1
Q18	If A = $[a_{ij}]$ is an identity matrix, then which of the following is true ? (A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\ i, j$ (C) $a_{ij} = 0, \forall\ i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$	
Ans	(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19	*Assertion (A) :* Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$. *Reason (R) :* Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1

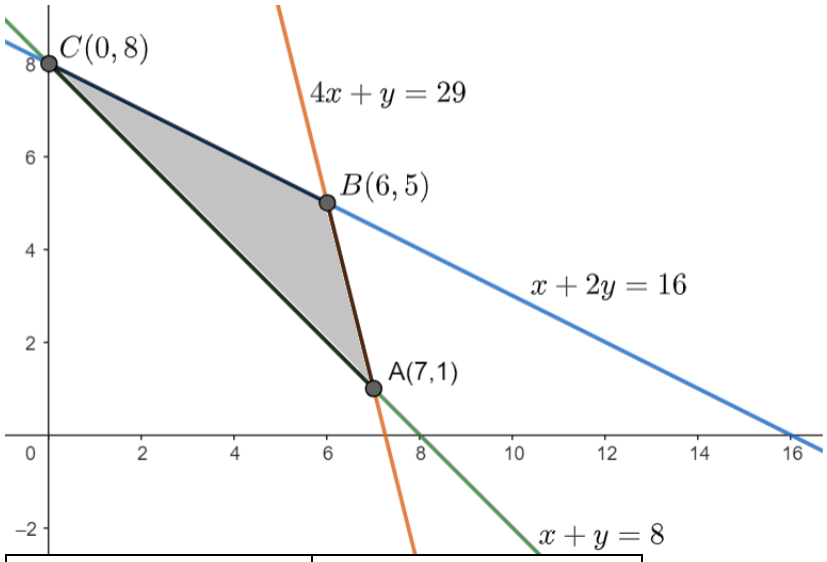
Q20	*Assertion (A) :* Every scalar matrix is a diagonal matrix. *Reason (R) :* In a diagonal matrix, all the diagonal elements are 0.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
SECTION B Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.		
Q21(a)	Evaluate : $\int_0^{\pi/2} \sin 2x \cos 3x \, dx$	
Ans	$I = \int_0^{\frac{\pi}{2}} \sin 2x \cos 3x \, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 5x - \sin x)\, dx$ $= \frac{1}{2}\left[-\frac{1}{5}\cos 5x + \cos x\right]_0^{\frac{\pi}{2}}$ $= -\frac{2}{5}$	1 ½ ½
Q21(b)	Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.	
Ans	$F(x) = \int \frac{1}{\sqrt{2x - x^2}} dx$ $= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx$ $= \sin^{-1}(x-1) + c$ when $x = 1, y = 0$ gives $c = 0$ $\therefore F(x) = \sin^{-1}(x-1)$	½ ½ ½ ½
Q22	Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $\lvert \overrightarrow{AB} \rvert : \lvert \overrightarrow{BC} \rvert$.	

Ans	Position vector of $C = \vec{r} = \dfrac{4\vec{b} - \vec{a}}{3}$ i.e. $\vec{r} = \dfrac{1}{3}\left(-5\hat{i} + 2\hat{j} + 5\hat{k}\right)$ Now, $\overrightarrow{AB} = -2\hat{i} - \hat{j} + 2\hat{k} \Rightarrow \left\lvert\overrightarrow{AB}\right\rvert = 3$ $\overrightarrow{BC} = -\dfrac{1}{3}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) \Rightarrow \left\lvert\overrightarrow{BC}\right\rvert = 1$ $\left\lvert\overrightarrow{AB}\right\rvert : \left\lvert\overrightarrow{BC}\right\rvert = 3:1$	1 1
Q23	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.	
Ans	Given $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 1$ Now $\vec{a} - \vec{c} = -\vec{b}$ $(\vec{a} - \vec{c}).(\vec{a} - \vec{c}) = (-\vec{b}).(-\vec{b})$ $\Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{c}\rvert^2 - 2\vec{a}.\vec{c} = \lvert\vec{b}\rvert^2$ $\Rightarrow 1 + 1 - 2\lvert\vec{a}\rvert\lvert\vec{c}\rvert\cos\theta = 1$ $\Rightarrow 2 - 2(1)(1)\cos\theta = 1$ $\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$	½ ½ ½ ½
Q24	Find the value of $\left[\sin^2\left\{\cos^{-1}\left(\dfrac{3}{5}\right)\right\} + \tan^2\left\{\sec^{-1}(3)\right\}\right]$.	
Ans	Required value $= \left[1 - \cos^2\left(\cos^{-1}\dfrac{3}{5}\right)\right] + \left[\sec^2\left(\sec^{-1}3\right) - 1\right]$ $= \left(1 - \dfrac{9}{25}\right) + (9 - 1)$ $= \dfrac{216}{25}$	1 ½ ½
Q25(a)	If $x = e^{x/y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x - 1}{(\log x)^2}$	
Ans	$x = e^{\frac{x}{y}} \Rightarrow \log x = \dfrac{x}{y} \Rightarrow y = \dfrac{x}{\log x}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{(\log x)(1) - x\left(\dfrac{1}{x}\right)}{(\log x)^2} = \dfrac{\log x - 1}{(\log x)^2}$	1 1

Q25(b)	Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.	
Ans	LHD at $x=1$ $= \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{\left[(1-h)^2+1\right]-2}{-h} = 2$ RHD at $x=1$ $= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{\left[3-(1+h)\right]-2}{h} = -1$ as LHD $\neq$ RHD, so $f(x)$ is not differentiable at $x=1$	1 ½ ½
	SECTION C Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.	
Q26	Find : $\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4}\, dx$	
Ans	Let $\log x = t$; $\frac{1}{x}dx = dt$ Given integral $= \int \frac{1}{t^2 - 5t + 4}\, dt$ $= \int \frac{1}{\left(t-\frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\, dt$ $= \frac{1}{3}\log\left\lvert\frac{t-4}{t-1}\right\rvert + C$ $= \frac{1}{3}\log\left\lvert\frac{\log x - 4}{\log x - 1}\right\rvert + C$	½ 1 1 ½
Q27(a)	Find : $\int \frac{2+\sin 2x}{1+\cos 2x} e^x\, dx$	

Ans	$I=\int \frac{2+\sin 2x}{1+\cos 2x}e^x\,dx$ $=\int \frac{2+2\sin x\cos x}{2\cos^2 x}e^x\,dx$ $=\int \left(\sec^2 x+\tan x\right)e^x dx$ $=e^x.\tan x+c$	1 1 1
Q27(b)	Evaluate : $\int_0^{\pi/4} \frac{1}{\sin x+\cos x}\,dx$	
Ans	$I=\int_0^{\frac{\pi}{4}} \frac{1}{\sin x+\cos x}\,dx$ $=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} \frac{1}{\cos\frac{\pi}{4}\sin x+\sin\frac{\pi}{4}\cos x}\,dx$ $=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} \frac{1}{\sin\left(x+\frac{\pi}{4}\right)}dx==\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} \operatorname{cosec}\left(x+\frac{\pi}{4}\right)dx$ $=\frac{1}{\sqrt{2}}\left[\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$ or $-\frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$	1 1 1
Q28	Solve the following linear programming problem graphically : Minimize z = 600x + 400y, subject to the constraints $x + y \geq 8$ $x + 2y \leq 16$ $4x + y \leq 29$ $x, y \geq 0.$	

Ans



Corner Point	Value of $z = 600x + 400y$
$A(7,1)$	4600
$B(6,5)$	5600
$C(0,8)$	3200

$z_{min} = 3200$ when $x = 0, y = 8$

For correct graph 1½ marks

For correct table 1 mark

½

Q29

The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

Ans

Let E_1 : P is appointed as CEO,
E_2 : Q is appointed as CEO,
E_3 : R is appointed as CEO
A : company increase profits from previous year

½

here, $P(E_1) = \frac{4}{7}, P(E_3) = \frac{1}{7}, P(E_1) = \frac{2}{7}$

$P(A|E_1) = 0.3, P(A|E_2) = 0.8, P(A|E_3) = 0.5$

1

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$= \frac{\frac{2}{7} \times 0.5}{\frac{4}{7} \times 0.3 + \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5}$$

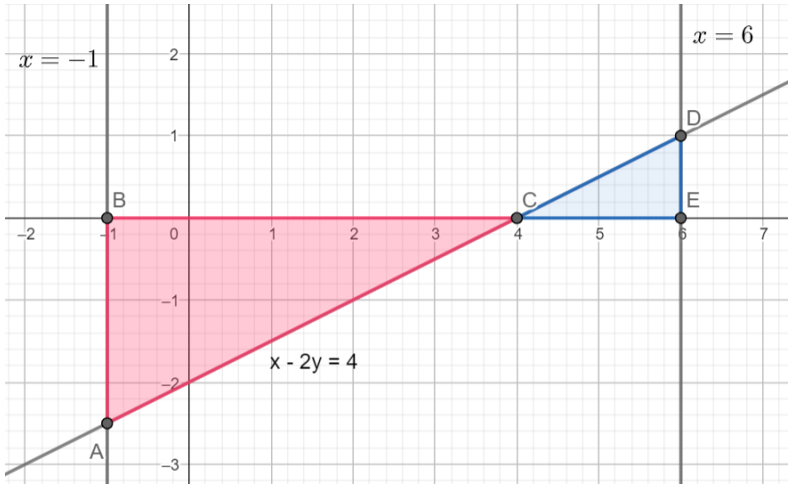
1

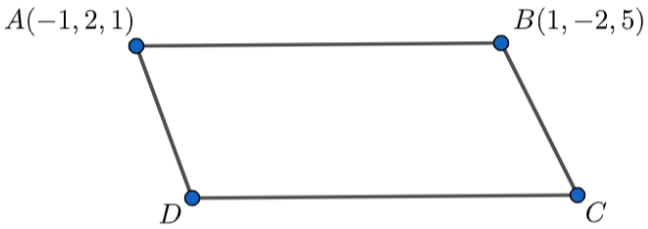
$$= \frac{1}{3}$$

½

Q30(a)	If x cos (p + y) + cos p sin (p + y) = 0, prove that $\cos p \frac{dy}{dx} = -\cos^2(p+y)$, where p is a constant.	
Ans	$x\cos(p+y)+\cos p\sin(p+y)=0$ $\Rightarrow x=\frac{-\cos p\sin(p+y)}{\cos(p+y)} \Rightarrow x=-\cos p.\tan(p+y)$ $\Rightarrow \frac{dx}{dy}=-\cos p.\sec^2(p+y)$ $\Rightarrow \frac{dy}{dx}=\frac{-1}{\cos p.\sec^2(p+y)}$ $\Rightarrow \cos p\frac{dy}{dx}=-\cos^2(p+y)$	1 1 ½ ½
Q30(b)	Find the value of a and b so that function f defined as : $f(x)=\begin{cases}\frac{x-2}{\lvert x-2\rvert}+a, & \text{if } x<2\\ a+b, & \text{if } x=2\\ \frac{x-2}{\lvert x-2\rvert}+b, & \text{if } x>2\end{cases}$ is a continuous function.	
Ans	$f(x)=\begin{cases}\frac{x-2}{-(x-2)}+a & ;x<2\\ a+b & ;x=2\\ \frac{x-2}{(x-2)}+b & ;x>2\end{cases} \Rightarrow f(x)=\begin{cases}-1+a & ;x<2\\ a+b & ;x=2\\ 1+b & ;x>2\end{cases}$ $\lim_{x\to 2^-} f(x)=-1+a, \lim_{x\to 2^+} f(x)=1+b$ and $f(2)=a+b$ as f is continous at $x=2 \therefore -1+a=1+b=a+b$ $\Rightarrow a=1, b=-1$	1 1 ½+½
Q31(a)	Find the intervals in which the function f(x) = $\frac{\log x}{x}$ is strictly increasing or strictly decreasing.	
Ans	$f(x)=\frac{\log x}{x} \Rightarrow f'(x)=\frac{1-\log x}{x^2}; x>0$ for strictly increasing/decreasing, put $f'(x)=0 \Rightarrow x=e$ for strictly increasing, $x\in(0,e)$ and for strictly decreasing $x\in(e,\infty)$	1 1 ½+½

Q31(b)	Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval [1, 2].	
Ans	$f(x) = \frac{x}{2} + \frac{2}{x}$; $x \in [1, 2]$ $\Rightarrow f'(x) = \frac{1}{2} - \frac{2}{x^2}$ for absolute maximum / minimum, put $f'(x) = 0$ $\Rightarrow x^2 = 4 \Rightarrow x = 2$ Now, $f(1) = \frac{5}{2}$ and $f(2) = 2$ $\therefore$ absolute maximum value $= \frac{5}{2}$ and absolute minimum value $= 2$	1 ½ ½+½ ½
SECTION D Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.		
Q32(a)	It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.	
Ans	$f(x) = x^4 - 62x^2 + ax + 9 \Rightarrow f'(x) = 4x^3 - 124x + a$ as at $x = 1$, f attains local maximum value, $f'(1) = 0 \Rightarrow a = 120$ now, $f'(x) = 4x^3 - 124x + 120 = 4(x-1)(x^2 + x - 30) = 4(x-1)(x-5)(x+6)$ Critical points are $x = -6, 1, 5$ $f''(x) = 12x^2 - 124$ $f''(-6) > 0$, $f''(1) < 0$, $f''(5) > 0$ so f attains local maximum value at $x = 1$ and local minimum value at $x = -6, 5$	½ 1 1 1 ½ 1
Q32(b)	The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.	

Ans	Let length of rectangle be x cm and breadth be $(150-x)$ cm. Let r be the radius of cylinder $\Rightarrow 2\pi r = x \Rightarrow r = \frac{x}{2\pi}$ $V = \pi r^2 h = \pi\left(\frac{x^2}{4\pi^2}\right)(150-x) = \frac{75x^2}{2\pi} - \frac{x^3}{4\pi}$ $\frac{dV}{dx} = \frac{150x}{2\pi} - \frac{3x^2}{4\pi}$ $\frac{dV}{dx} = 0 \Rightarrow x = 100$ cm $\left.\frac{d^2V}{dx^2}\right\|_{x=100\text{ cm}} = -\frac{75}{\pi} < 0 \Rightarrow V$ is maximum when $x = 100$ cm. Length of rectangle is 100 cm and breadth of rectangle is 50 cm.	1 1 1 1 ½ ½
Q33	Find the area of the region bounded by the lines x – 2y = 4, x = –1, x = 6 and x-axis, using integration.	
Ans	 Required area = $\left\| \int_{-1}^{4} \left(\frac{x-4}{2}\right) dx \right\| + \int_{4}^{6} \left(\frac{x-4}{2}\right) dx$ $= \left\| \frac{(x-4)^2}{4} \right\|_{-1}^{4} + \frac{(x-4)^2}{4} \Big]_{4}^{6}$ $= \frac{25}{4} + 1 = \frac{29}{4}$	For correct figure 1 mark 2 1 1
Q34(a)	Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.	

Ans	$l_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$; $l_2 : \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu$ any point on l_1 is $(\lambda, 2\lambda+1, 3\lambda+2)$ & any point on l_2 is $(1, -3\mu, 2\mu+7)$ If l_1 and l_2 intersect, $\lambda = 1, 2\lambda+1 = -3\mu$ and $3\lambda+2 = 2\mu+7 \Rightarrow \lambda = 1$ and $\mu = -1$ Point of intersection of l_1 and l_2 is $(1,3,5)$. Let d.r.'s of required line be $< a, b, c >$. Then, $a + 2b + 3c = 0$ and $-3b + 2c = 0 \Rightarrow \frac{a}{13} = \frac{b}{-2} = \frac{c}{-3}$ Required equation of line is $\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}$	1 1 1 1 1
Q34(b)	Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.	
Ans	 d.r's of CD are $< 1, -2, 2 >$ $\therefore$ d.r's of AB are $< 1, -2, 2 >$ $\therefore$ Equation of AB is $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$ $\therefore$ Equation of CD is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$ Let $\vec{a}_1 = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = 4\hat{i} - 7\hat{j} + 8\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ Now, $\vec{a}_2 - \vec{a}_1 = 5\hat{i} - 9\hat{j} + 7\hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -9 & 7 \\ 1 & -2 & 2 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k}$	½ ½ 1 1

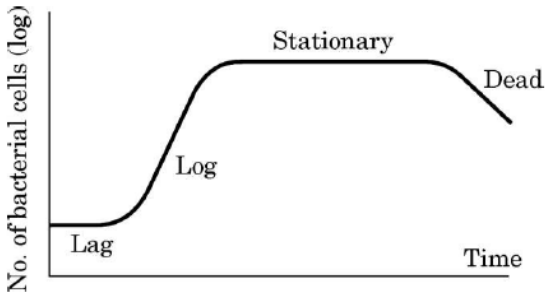
	Distance between AB and CD is given by $d = \frac{\left\|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right\|}{\left\|\vec{b}\right\|}$	½
	$d = \frac{\sqrt{16 + 9 + 1}}{\sqrt{1 + 4 + 4}} = \frac{\sqrt{26}}{3}$	½
	$CD = \sqrt{2^2 + (-4)^2 + (4)^2} = 6$	1
	Area of parallelogram ABCD = b × h = $6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26}$	
Q35	A relation R on set A = {x : – 10 ≤ x ≤ 10, x ∈ Z} is defined as R = {(x, y) : (x – y) is divisible by 5}. Show that R is an equivalence relation. Also, write the equivalence class [5].	
Ans	For reflexive relation To prove (x, x) ∈ R, x – x = 0 which is divisible by 5 ∴ (x, x) ∈ R ⇒ R is reflexive	1
	For symmetric relation Let (x, y) ∈ R ⇒ x – y is divisible by 5 ⇒ x – y = 5m ⇒ y – x = 5(– m) ⇒ y – x is divisible by 5 ⇒ (y, x) ∈ R ∴ R is symmetric	1
	For transitive relation Let (x, y) ∈ R and (y, z) ∈ R x – y is divisible by 5 ⇒ x – y = 5 m y – z is divisible by 5 ⇒ y – z = 5 n ⇒ x – y + y – z = 5(m – n) ⇒ x – z = 5(m – n) ∴ x – z is divisible by 5 ⇒ (x, z) ∈ R ∴ R is transitive. R is an equivalence relation.	2
	[5] = {– 10, – 5, 0, 5, 10}	1

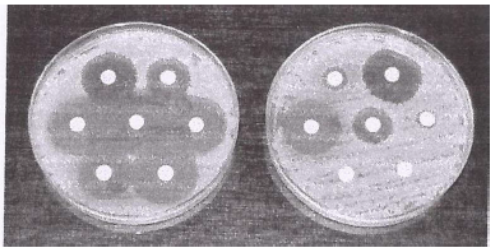
SECTION E

Questions no. 36 to 38 are case study based questions carrying 4 marks each.

Q36

A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2*

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*

Ans(i)

$\frac{dP}{dt} = kP \Rightarrow \int \frac{dP}{P} = \int k\,dt$ — 1

$\Rightarrow \log P = kt + C$ or $P = e^{kt+C}$ — 1

Ans(ii)

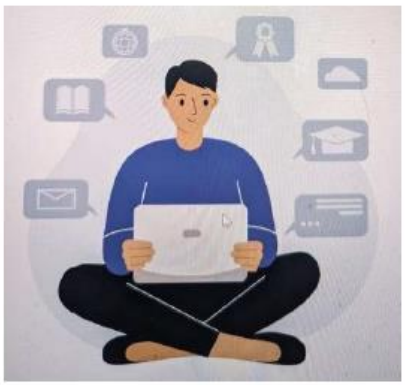
$\log P = kt + C$

when $t = 0, P = 1000 \Rightarrow C = \log 1000$ — ½

when $t = 1, P = 2000 \Rightarrow \log 2000 = k + \log 1000$ — ½

$\Rightarrow k = \log 2$ — 1

Q37	A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.   Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports. In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000. Based on the above information, answer the following questions : (i) Express the given information algebraically using matrices. *1* (ii) Check whether the system of matrix equations so obtained is consistent or not. *1* (iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2* **OR** (iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*	
Ans(i)	Let No. of girl child scholarships = x No. of meritorious achievers = y $x + y = 50$ $3000x + 4000y = 180000$ or $3x + 4y = 180$ $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$	1
Ans(ii)	$\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \neq 0$ $\therefore$ system is consistent.	1

Ans (iii)(a)	Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$ $AX = B \Rightarrow X = A^{-1}B$ $X = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$ $\Rightarrow x = 20, y = 30$	½ 1 ½
	OR	
Ans (iii)(b)	Required expenditure = ₹ [30(3000) + 20(4000)] = ₹ 1,70,000	1 1
Q38	Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.   A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below : $P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$ where x denotes the number of hours. Based on the above information, answer the following questions : (i) Express the probability distribution given above in the form of a probability distribution table. *1* (ii) Find the value of k. *1* (iii) (a) Find the mean number of hours spent by the student. *2* **OR** (iii) (b) Find $P(1 < X < 6)$. *2*	

<table>
<tr><td>Ans(i)</td><td><table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>P(X)</td><td>k</td><td>4k</td><td>9k</td><td>8k</td><td>10k</td><td>12k</td></tr></table></td><td>1</td></tr>
<tr><td>Ans(ii)</td><td>$k + 4k + 9k + 8k + 10k + 12k = 1$
$\Rightarrow k = \frac{1}{44}$</td><td>1</td></tr>
<tr><td>Ans (iii) (a)</td><td>$\text{Mean} = \sum x_i p_i = k + 8k + 27k + 32k + 50k + 72k$
$= 190k$
$= \frac{190}{44}$ or $\frac{95}{22}$</td><td>1
1</td></tr>
<tr><td colspan="3">OR</td></tr>
<tr><td>Ans (iii)(b)</td><td>$P(1 < X < 6) = 4k + 9k + 8k + 10k$
$= 31k$
$= \frac{31}{44}$</td><td>1
1</td></tr>
</table>