BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade XII

Grade 12 Preparatory EXAMINATION-02
MATHEMATICS, SET A MARKING SCHEME

		Section- A'	
1)	d) $\boxed{R-\{-10\}}$	$2(6-0)+1(3\lambda)+1(-2\lambda-2)=0 \Rightarrow 12+\lambda-2=0$ $\Rightarrow \lambda=-10$, Singular. For Non-Singular $R-\{-10\}$	1
2)	c) $\boxed{2025\times 2024}$	$P\cdot(\text{adj } P) = -2025\, I_3 \Rightarrow \lvert P\rvert = -2025$ $\lvert \text{adj } P\rvert = \lvert P\rvert^{n-1} = \lvert P\rvert^2 \Rightarrow \lvert P\rvert + \lvert \text{adj } P\rvert = -2025+(2025)^2$ $\Rightarrow 2025(-1+2025) = 2025\times 2024$	1
3)	d) 2p = q	Z=px + qy, Z(3,0) = 3p, Z (1,2) =p + q 3p = p + q => 2p = q	1
4)	a) $\boxed{m\times n}$	$A \to m\times n$ (order). Let B be $x\times y$ (order). B' order $= y\times x$ AB' defined $m\times n \; y\times x \Rightarrow n=y$ $B'A$ defined $y\times x \; m\times n \Rightarrow m=x$ B order is $m\times n$	1
5)	b) $\boxed{\pm 1}$	$\lvert A\rvert\lvert A^T\rvert = 1 \Rightarrow \lvert A\rvert = \pm 1$ $\quad \lvert \text{adj}(\text{adj } A)\rvert = \lvert A\rvert^{(n-1)^2}$ $\Rightarrow (\pm 1)^{(n-1)^2} = \pm 1$ based on n	1
6)	c) $\boxed{3}$	$\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^4\right) = 4\left(\frac{dy}{dx}\right)^3\cdot\left(\frac{d^2y}{dx^2}\right)$ $\quad m=2, n=1, m+n=3$	1
7)	a) $\boxed{e^{2\sqrt{x}}}$	$\frac{dy}{dx}+\frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \Rightarrow P=\frac{1}{\sqrt{x}}$ $\quad IF = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$	1
8)	b) $\boxed{e^x\log(x+1)+C}$	$\int e^x\left(\frac{1}{x+1}+\log(x+1)\right)dx = e^x\cdot\log(x+1)+C$ $\int e^x\left(f'(x)+f(x)\right)dx = e^x f(x)+C$	1

9)	d) $\boxed{\frac{3}{4}}$	$1-P(\text{problem not solved}) = 1-P(\bar{A}\cap\bar{B}\cap\bar{C})$ $= 1-P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C}) = 1-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{4} = \frac{3}{4}$	1
10)	$\boxed{\text{c) } 6}$	LHL $\lim_{x\to\pi/2}\frac{k\cos x}{\pi-2x} = \lim_{h\to0}\frac{k\cos(\pi/2-h)}{\pi-\pi+2h}$ $= \lim_{h\to0}\frac{k\sin h}{2h} = \frac{k}{2}$. RHL $=3 \Rightarrow k=6$	1
11)	c) 0	$a=0,\ b=-c \Rightarrow b+c=0 \quad 2a-(b+c)=0$	1
12)	b) $\frac{\cos x}{2y-1}$	$y=\sqrt{\sin x+y} \Rightarrow y^2=\sin x+y$ $2y\frac{dy}{dx}=\cos x+\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{\cos x}{2y-1}$	1
13)	b) Max value at Q	$O(0,0) = \overset{Z}{0}$ | $P=(0,40)=120$ $R(40,0)=160$ | $Q=(30,20)=180$	1
14)	b) $x=\frac{1}{e}$	$y=x^x$ $\log y = x\log x$ | $\frac{1}{y}\frac{dy}{dx} = x\cdot\frac{1}{x}+\log x = 1+\log x$ $\frac{dy}{dx}=x^x(1+\log x)$ $1+\log x=0 \Rightarrow \log x=-1=-\log e \Rightarrow x=\frac{1}{e}$	1
15)	c) $10\sqrt{3}$	$\frac{dA}{dt}=\frac{d}{dt}\left(\frac{\sqrt{3}}{4}a^2\right)=\frac{\sqrt{3}}{4}\cdot2a\cdot\frac{da}{dt}=\frac{\sqrt{3}}{4}\cdot2\times10\times2=10\sqrt{3}$	1
16)	d) $\frac{17}{4}$	$\left\vert\int_{-2}^{0}x^3dx\right\vert+\int_{0}^{1}x^3dx=\left\vert\left[\frac{x^4}{4}\right]_{-2}^{0}\right\vert+\left[\frac{x^4}{4}\right]_{0}^{1}=\frac{16}{4}+\frac{1}{4}=\frac{17}{4}$	1
17)	a) $\frac{1}{5}\log\left\vert\frac{x^5}{x^5+1}\right\vert+C$	$\int\frac{x^4}{x^5(x^5+1)}dx \quad t=x^5,\ dt=5x^4dx \Rightarrow \int\frac{1}{5}\frac{dt}{t(t+1)}\ \frac{1}{5}\int\left(\frac{t+1-t}{t(t+1)}\right)dt$ $=\frac{1}{5}\left[\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt\right]=\frac{1}{5}\left[\log\vert t\vert-\log\vert t+1\vert\right]=\frac{1}{5}\log\left\vert\frac{x^5}{x^5+1}\right\vert+C$	1
18)	d)	$\cos\alpha+\cos\beta+\cos\gamma\neq1 \rightarrow$ not true is d)	1
19)	d)	Assertion: $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\pi/3$ A → false. Reason - True.	1
20)	c)	$\vert x-6\vert\cdot\cos x$ is not differentiable at $x=6$. So it is differentiable in $R-\{6\}$. Assertion True. Reason is false.	1

Section- B

(21) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$ — (1/2)

$= \sin^{-1}\left(\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x\right)$ — (1/2)

$= \sin^{-1}\left(\sin(x+\pi/4)\right)$ — (1/2)

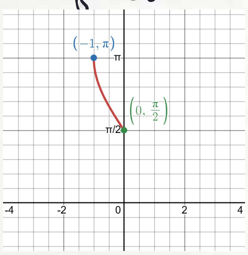
$= x + \pi/4$

as $-\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} - \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$

$\Rightarrow 0 < x + \frac{\pi}{4} < \frac{\pi}{2} \in PVB$ — (1/2)

(OR)

Graph of $\cos^{-1}x$ — (1)



Range of $\cos^{-1}x = [0, \pi]$ — (1/2)

Range of $\tan^{-1}x = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ — (1/2)

(22) $f(1) = 3 - 1 = 2$ — (1/2)

LHD $\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{(1-h)^2+1-2}{-h} =$
$= \lim_{h\to 0} \frac{h(h-2)}{-h} = 2$ — (1/2)

RHD $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{3-(1+h)-2}{h} = -1$ — (1/2)

LHD $\neq$ RHD. So $f(x)$ is NOT differentiable @ $x = 1$ — (1/2)

(23) $f'(x) = x^2(-e^{-x}) + e^{-x}(2x) = e^{-x}(2x - x^2)$ — (1/2)

$f'(x) > 0$ (strictly increasing) as $e^{-x} > 0$ — (1/2)

$(2x - x^2) > 0 \Rightarrow x(x-2) < 0 \Rightarrow 0 < x < 2$ — (1/2)

Hence strictly increasing in $(0, 2)$ — (1/2)

OR

23) $f(x) = \dfrac{4\sin x}{2+\cos x} - x$

$f'(x) = \dfrac{(2+\cos x)\,4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$ — $\frac{1}{2}$

$= \dfrac{8\cos x + 4\cos^2 x + 4\sin^2 x}{(2+\cos x)^2} - 1 = \dfrac{8\cos x + 4}{(2+\cos x)^2} - 1$

$= \dfrac{8\cos x + 4 - (2+\cos x)^2}{(2+\cos x)^2} = \dfrac{\cos x\,(4-\cos x)}{(2+\cos x)^2}$ $\Big\}$ — $\frac{1}{2}$

as $x \in [0, \pi/2]$, $\cos x \geq 0$, $4 - \cos x \geq 0$ & $(2+\cos x)^2 \geq 0$ — $\frac{1}{2}$

$\Rightarrow f'(x) \geq 0 \Rightarrow f$ is increasing in $[0, \frac{\pi}{2}]$ — $\frac{1}{2}$

24) $\vec{AD} = \vec{AB} - \vec{DB} = \hat{i} - \hat{j} + \hat{k}$ — $\frac{1}{2}$

Area $= |\vec{AB} \times \vec{AD}| = \sqrt{25+1+16} = \sqrt{42}$ sq units — $\frac{1}{2}$

$\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} - 4\hat{k}$ — 1

$\therefore$ Area $= \sqrt{42}$ sq units.

25) Gen Pt on line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$ is

$P(3\lambda-2, 2\lambda-1, 2\lambda+3)$ — $\frac{1}{2}$

Point $Q(1, 3, 3)$.

Distance $PQ = \sqrt{(3\lambda-2-1)^2 + (2\lambda-1-3)^2 + (2\lambda+3-3)^2}$

$\Rightarrow PQ^2 = 25 \Rightarrow (3\lambda-3)^2 + (2\lambda-4)^2 + (2\lambda)^2 = 25$

$9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$

$17\lambda^2 - 34\lambda = 0 \Rightarrow \lambda = 0, 2$ $\Big\}$ — 1

$\therefore$ Points are $(-2, -1, 3)$ and $(4, 3, 7)$ — $\frac{1}{2}$

SECTION - C

26) $x = a\cos\theta + b\sin\theta$

$\frac{dx}{d\theta} = a(-\sin\theta) + b\cos\theta$ — ½

$y = a\sin\theta - b\cos\theta \qquad \frac{dy}{d\theta} = a\cos\theta - b(-\sin\theta)$ — ½

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta} = \frac{x}{-y} = -\frac{x}{y}$ — ½

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{y}\right) \Rightarrow \frac{y(-1) + x\left(\frac{dy}{dx}\right)}{y^2} = \frac{d^2y}{dx^2}$ — 1

$\Rightarrow y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx} \Rightarrow y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ — ½

27) $p(\text{getting a six}) = \frac{1}{6} = p(S)$ — ½

$p(\text{not getting a six}) = \frac{5}{6} = p(F)$ — ½

$p(A \text{ wins}) = S, FFS, FFFFS, \ldots$ — ½

$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{1}{6} + \left(\frac{5}{6}\right)^2\frac{1}{6} + \left(\frac{5}{6}\right)^4\cdot\frac{1}{6} + \cdots$

$\Rightarrow$ Infinite GP with $a = \frac{1}{6}$, $r = \left(\frac{5}{6}\right)^2$

$S_\infty = \frac{a}{1-r} = \frac{1/6}{1 - \frac{25}{36}} = \frac{1}{6}\times\frac{36}{11} = \frac{6}{11}$ $\therefore p(A \text{ wins}) = \frac{6}{11}$ — 1

$p(B \text{ wins}) = 1 - \frac{6}{11} = \frac{5}{11}$ — ½

(OR)

$S = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)\}$ $n(S) = 12$ — ½

$p(X=3)$ getting sum as 3 $= \frac{2}{12} = \frac{1}{6}$

$p(X=4)$ " " " 4 $= \frac{2}{12} = \frac{1}{6}$

$p(X=5)$ getting sum as 5 $= \frac{4}{12} = \frac{2}{6}$ — 1

$p(X=6)$ getting sum as 6 $= \frac{2}{12} = \frac{1}{6}$

$p(x=7) = \frac{2}{12} = \frac{1}{6}$

prob. distribution table

X	3	4	5	6	7
p_i	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

— (1/2)

$\text{Mean} = \sum x_i p_i = \frac{3}{6} + \frac{4}{6} + \frac{10}{6} + \frac{6}{6} + \frac{7}{6} = \frac{30}{6} = 5$ — (1)

28) $\int \frac{2\cos x \, dx}{(1-\sin x)(1+\sin^2 x)}$ Let $t = \sin x$, $dt = \cos x \, dx$ — (1/2)

$= \int \frac{2\, dt}{(1-t)(1+t^2)}$ Using Partial fractions,

$\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2} \Rightarrow$

$2 = A(1+t^2) + (Bt+C)(1-t)$ — (1/2)

$2 = (A+C) + (A-B)t^2 + (B-C)t$

Comparing co-efficients, $A-B=0$, $B-C=0 \Rightarrow A=B=C$

$A+C=2 \Rightarrow 2A=1 \Rightarrow A=B=C=1$ — (1/2)

$\Rightarrow \int \frac{2\,dt}{(1-t)(1+t^2)} = \int\left(\frac{1}{1-t} + \frac{t+1}{1+t^2}\right)dt = \int \frac{1}{1-t}dt + \int \frac{t}{1+t^2}dt + \int \frac{1}{1+t^2}dt$

$= \frac{\log|1-t|}{-1} + \frac{1}{2}\int \frac{2t}{1+t^2}dt + \tan^{-1}t + C$ $u = 1+t^2$, $du = 2t\,dt$ — (1 1/2)

$= -\log|1-t| + \frac{1}{2}\int \frac{du}{u} + \tan^{-1}t + C$

$= -\log|1-\sin x| + \frac{1}{2}\log|1+\sin^2 x| + \tan^{-1}(\sin x) + C$

29) DE: $\left(x^2y + yx\sqrt{y^2-x^2}\right)dx - x^3\,dy = 0$ $\div$ by x^3/transpose

$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x^2}\sqrt{y^2-x^2} = \frac{y}{x} + \frac{y}{x}\sqrt{\left(\frac{y}{x}\right)^2 - 1}$ $\therefore$ Homogenous DE — (1)

Let $y = vx$

$\frac{dy}{dx} = v(1) + x\frac{dv}{dx}$ — (1/2)

29) continued...

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + v\sqrt{v^2-1} \Rightarrow x\frac{dv}{dx} = v\sqrt{v^2-1}$$ — 1/2

$$\Rightarrow \int \frac{dv}{v\sqrt{v^2-1}} = \int \frac{dx}{x}$$ — 1/2

$$\sec^{-1} v = \log|x| + \log c = \log(cx)$$

$$\sec^{-1}\left(\frac{y}{x}\right) = \log cx \text{ (or) } \log|x| + C$$ — 1/2

(OR)

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2 .$$ Divide by $(x+1)$

$$\frac{dy}{dx} - \frac{1}{(x+1)}y = e^{3x}(x+1).$$ This is LDE $\frac{dy}{dx} + Py = Q$ — 1/2

$$P = \frac{-1}{x+1}, \quad Q = e^{3x}(x+1)$$ — 1/2

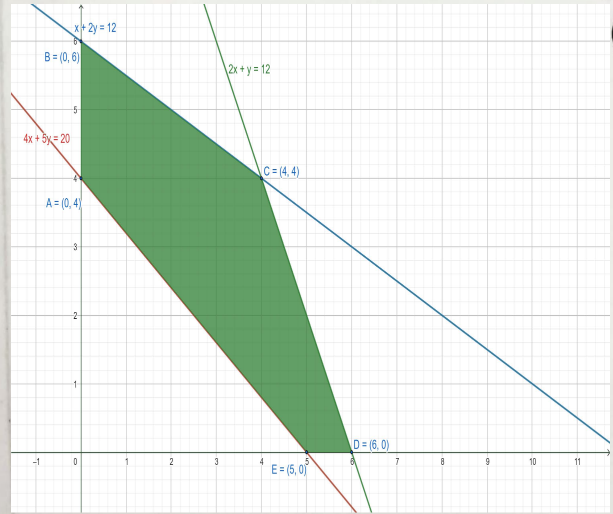
$$IF = e^{\int P\,dx} = e^{\int \frac{-1}{x+1}dx} = e^{-\log|x+1|} = \frac{1}{x+1}$$ — 1/2

Solution : $y(IF) = \int Q(IF)\,dx$ — 1/2

$$y\left(\frac{1}{x+1}\right) = \int e^{3x}\cdot(x+1)\cdot\frac{1}{(x+1)}\,dx = \int e^{3x}dx = \frac{e^{3x}}{3} + C$$ — 1

$$y = \frac{e^{3x}(x+1)}{3} + C(x+1)$$

30)



Correct Graph (1 1/2)

① $x + 2y \le 12$

$\frac{x}{12} + \frac{y}{6} \le 1$

origin included

② $2x + y \le 12$

$\frac{x}{6} + \frac{y}{12} \le 1$

Origin included

③ $4x + 5y \ge 20$

$\frac{x}{5} + \frac{y}{4} \ge 1$

origin not included

Bounded region ABCDEA.

Maximum occurs at Corner Point.

Corner point table

Corner point	$Z = 50x + 30y$
$A(0,4)$	120
$B(0,6)$	180
$C(4,4)$	320
$D(6,0)$	300
$E(5,0)$	250

— 1

Max value at $C(4,4)$

Value is 320. — ½

31) $\int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{2} |x(x-1)(x-2)|\, dx \Rightarrow x = 0, 1, 2.$

$x^3 - x = 0$ — ½

$\Rightarrow x = 0, 1, 2$

$x^3 - x$ +ve $-1 < x < 0,\ 1 < x < 2$

$x^3 - x$ -ve $0 < x < 1$ — ½

$$\int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} -(x^3 - x)\,dx + \int_{1}^{2} (x^3 - x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[-\frac{x^4}{4} + \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$ — 1½

$$= 0 - \left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{-1}{4} + \frac{1}{2}\right) + \left(\frac{16}{4} - \frac{4}{2} - \left(\frac{1}{4} - \frac{1}{2}\right)\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = 2 + \frac{3}{4} = \frac{11}{4}$$ — ½

$\boxed{\text{Section - D}}$

32) Given $(a,b)\, R\, (c,d) \iff ad(b+c) = bc(a+d)$

(i) Reflexive: $ab(a+b) = ab(a+b)$

$ab(b+a) = ba(a+b)$ — commutative property of addition/multiplication on N — 1

$\Rightarrow (a,b)\, R\, (a,b),\ \forall (a,b) \in$

$\therefore$ Reflexive.

32) continued...

Symmetric : Let $(a,b), (c,d)$ be an arbitrary element of $N \times N$ such that $(a,b) R (c,d)$

$ad(b+c) = bc(a+d)$

$bc(a+d) = ad(b+c) \Rightarrow cb(d+a) = da(c+b)$ — 1

{Commutative property of multiplication/ addition on N}

$\Rightarrow cb(d+a) = da(c+b)$

$\Rightarrow (c,d) R (a,b)$

$\therefore (a,b) R (c,d) \Rightarrow (c,d) R (a,b)$ for all $(a,b), (c,d) \in N \times N$

<u>Transitive:</u> $(a,b), (c,d), (e,f)$ be arbitrary elements of $N \times N$

Such that $(a,b) R (c,d), \ (c,d) R (e,f)$

$ad(b+c) = bc(a+d)$

$\frac{b+c}{bc} = \frac{a+d}{ad}$ — 1/2

$\frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$ — ①

$cf(d+e) = de(c+f)$

$\frac{d+e}{de} = \frac{c+f}{cf}$ — 1/2

$\frac{1}{d} + \frac{1}{e} = \frac{1}{f} + \frac{1}{c}$ — ②

Adding ① & ② $\cancel{\frac{1}{c}} + \frac{1}{b} + \cancel{\frac{1}{d}} + \frac{1}{e} = \frac{1}{a} + \cancel{\frac{1}{d}} + \cancel{\frac{1}{c}} + \frac{1}{f}$

$\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$ — 1

$\Rightarrow af(b+e) = be(a+f)$

$\Rightarrow (a,b) R (e,f)$ — 1/2

$\Rightarrow (a,b) R (c,d) \ \& \ (c,d) R (e,f) \Rightarrow (a,b) R (e,f)$

for all $(a,b), (c,d), (e,f) \in N \times N$. $\therefore$ R is transitive

As it is reflexive, symmetric & transitive

$\therefore$ It is an equivalence relation. — 1/2

(OR)

32) One-one:

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ — $\frac{1}{2}$

$\Rightarrow \frac{\cancel{2}x_1}{1+x_1^2} = \frac{\cancel{2}x_2}{1+x_2^2} \Rightarrow x_1 + x_1x_2^2 = x_2 + x_2x_1^2$

$\Rightarrow (x_1-x_2) - (x_1x_2^2 - x_2x_1^2) = 0$ — 1

$\Rightarrow (x_1-x_2) - x_1x_2(x_1-x_2) = 0$

$\Rightarrow (x_1-x_2)(1-x_1x_2) = 0 \Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

So if $x_1x_2 = 1$, $x_1 \neq x_2$. Hence f is not one-one — $\frac{1}{2}$

Onto

Let $y = f(x)$ where $x \in R$, $y = 0$ $x = 0$. — $\frac{1}{2}$

If $y \neq 0$, $y = \frac{2x}{1+x^2} \Rightarrow yx^2 + y - 2x = 0$

$\Rightarrow yx^2 - 2x + y = 0$. Solve for $x = \frac{2 \pm \sqrt{4-4y^2}}{2y}$ — 1

$\Rightarrow x = \frac{1 \pm \sqrt{1-y^2}}{y}$

For x to be real, $1-y^2 \geq 0 \Rightarrow y^2 \leq 1 \Rightarrow -1 \leq y \leq 1$ — $\frac{1}{2}$

Range $= [-1, 1] \neq$ codomain. $\therefore$ f is not onto — $\frac{1}{2}$

For function to become onto, $A = [-1, 1]$ — $\frac{1}{2}$

33) $|A| = 3(3-6) - 2(-12-14) + 1(12+7) = 62 \neq 0$, A^{-1} exists. — $\frac{1}{2}$

Cofactors		
$C_{11} = -3$	$C_{21} = 9$	$C_{31} = 5$
$C_{12} = 26$	$C_{22} = -16$	$C_{32} = -2$
$C_{13} = 19$	$C_{23} = 5$	$C_{33} = -11$

— 2

$\text{adj } A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ — $\frac{1}{2}$

$3x + 4y + 7z = 14$

$2x - y + 3z = 4$

$x + 2y - 3z = 0$

$$\underbrace{\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}}_{A^T} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}}_{B}$$

$\Rightarrow (A^T) X = B$

Premultiply by $(A^T)^{-1}$

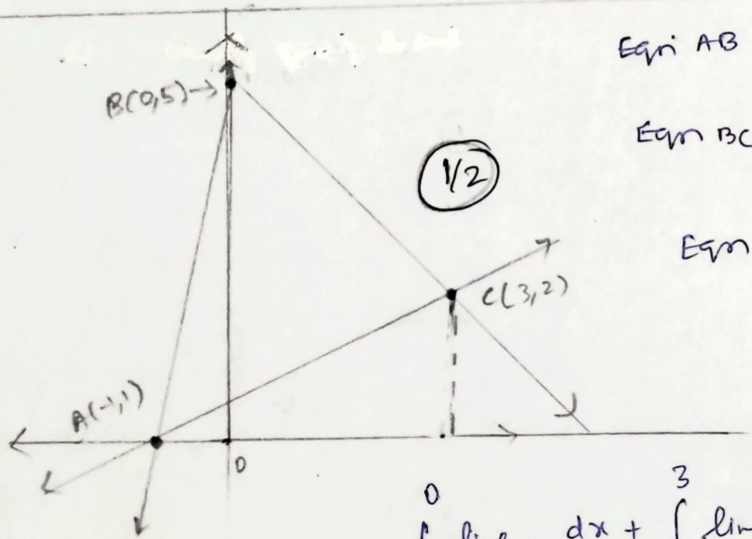
$(A^T)^{-1} \cdot A^T \cdot X = (A^T)^{-1} \cdot B \Rightarrow IX = (A^T)^{-1} \cdot B \Rightarrow X = (A^T)^{-1} B$

$X = (A^{-1})^T B \quad \{ (A^{-1})^T = (A^T)^{-1} \}$ — 1

$$X = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}^T \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$ — 1

$$X = \frac{1}{62} \begin{bmatrix} -42 + 104 \\ 126 - 64 \\ 70 - 8 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow x = 1, y = 1, z = 1$$

34)



Eqn AB $\Rightarrow \frac{x-0}{+1} = \frac{y-5}{+4} \Rightarrow y = 4x + 5$ — 1/2

Eqn BC $\Rightarrow \frac{x-0}{3} = \frac{y-5}{-3} \Rightarrow y = -x + 5$ — 1/2

Eqn AC $\Rightarrow \frac{x+1}{4} = \frac{y-1}{1} \Rightarrow y = \frac{x+5}{4}$ — 1/2

Required Area $= \int_{-1}^{0} \text{line}_{AB}\, dx + \int_{0}^{3} \text{line}_{BC}\, dx - \int_{-1}^{3} \text{line}_{AC}\, dx$ — 1

$= \int_{-1}^{0} 4x + 5\, dx + \int_{0}^{3} (-x+5)\, dx - \frac{1}{4} \int_{-1}^{3} (x+5)\, dx$

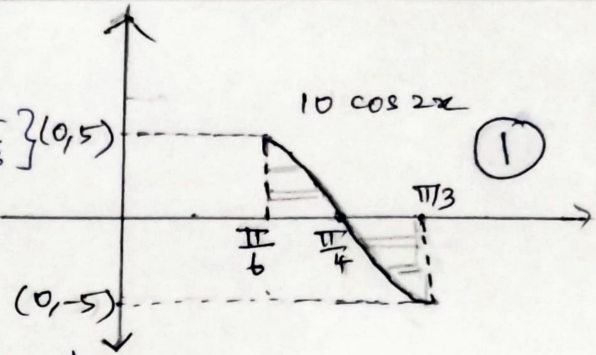
$= \left. \frac{4x^2}{2} + 5x \right]_{-1}^{0} + \left[\frac{-x^2}{2} + 5x \right]_{0}^{3} - \frac{1}{4} \left[\frac{x^2}{2} + 5x \right]_{-1}^{3}$ — 1 1/2

$= -(2-5) + \left(\frac{-9}{2} + 15\right) - \frac{1}{4}\left[\frac{9}{2} + 15 - \left(\frac{1}{2} - 5\right)\right]$

$= 3 + \frac{21}{2} - \frac{1}{\cancel{4}} \cdot \cancel{24}^{6} = \frac{15}{2}$ sq. units — 1/2

(OR)

34) $y = 10\cos 2x \quad \left\{ \frac{\pi}{6} \le x \le \frac{\pi}{3} \right\}$



(1)

Required area =

$$= \int_{\pi/6}^{\pi/4} 10\cos 2x \, dx + \left| \int_{\pi/4}^{\pi/3} 10\cos 2x \, dx \right| \quad \text{— (1)}$$

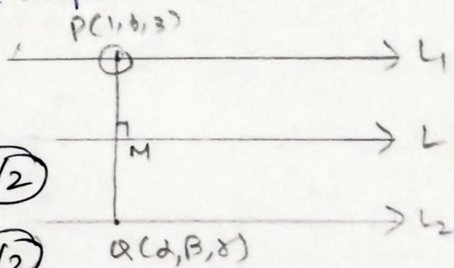
$$= 10\,\frac{\sin 2x}{2}\Big]_{\pi/6}^{\pi/4} + \left| 10\,\frac{\sin 2x}{2}\Big]_{\pi/4}^{\pi/3} \right| = 5\sin 2x\Big]_{\pi/6}^{\pi/4} + \left| 5\sin 2x\Big]_{\pi/4}^{\pi/3} \right| \quad \text{— (1)}$$

$$= 5\left(\sin \pi/2 - \sin \pi/3\right) + \left| 5\left(\sin 2\pi/3 - \sin \pi/2\right) \right| \quad \text{— (1)}$$

$$= 5\left(1 - \frac{\sqrt{3}}{2}\right) + 5\left| \frac{\sqrt{3}}{2} - 1 \right| = 5\left(1 - \frac{\sqrt{3}}{2}\right) + 5\left(1 - \frac{\sqrt{3}}{2}\right) \quad \text{— (1/2)}$$

$$= 10\left(1 - \frac{\sqrt{3}}{2}\right) \text{ sq units} \quad \text{— (1/2)}$$

35) Let M be the foot of perpendicular from P(1, 6, 3) and Q be the image of point P with respect to line L



Line L : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

Gen Point on L : $\lambda,\ 2\lambda+1,\ 3\lambda+2$ — (1/2)

DR (Direction Ratios) of L $\langle 1, 2, 3 \rangle$ — (1/2)

DR PM are $\langle \lambda-1,\ 2\lambda-5,\ 3\lambda-1 \rangle$ — (1/2)

PM ⊥ L ⇒ dot product = 0 ⇒ $(\lambda-1)1 + (2\lambda-5)2 + (3\lambda-1)3 = 0$

$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0 \Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$ (1)

Coordinates of M (1, 3, 5). M is the midpoint of PQ. (1/2)

Let Q (α, β, γ) ⇒ $\frac{\alpha+1}{2} = 1,\ \frac{\beta+6}{2} = 3,\ \frac{\gamma+3}{2} = 5 \Rightarrow \alpha = 1,\ \beta = 0,\ \gamma = 7$ (1)

∴ Q is (1, 0, 7)

Equation of L2; L2 passes through Q(1, 0, 7). DR $\langle 1, 2, 3 \rangle$ (parallel to L) (1/2)

$$\Rightarrow \frac{x-1}{1} = \frac{y-0}{2} = \frac{z-7}{3} \quad (1/2)$$

Case-study - Section-E

36) (i) First square length = 'x' m

Side = $\frac{x}{4}$ m — (1/2)

Second Square length = $(a-x)$ m, Side = $\left(\frac{a-x}{4}\right)$ m. — (1/2)

Combined area $A = \frac{x^2}{16} + \frac{(a-x)^2}{16} = \frac{x^2}{16} + \frac{a^2+x^2-2ax}{16}$

$= \frac{2x^2+a^2-2ax}{16}$ — (1)

(ii) $\frac{dA}{dx} = \frac{4x+0-2a}{16} = \frac{4x-2a}{16}$ — (1/2)

$\frac{dA}{dx} = 0 \Rightarrow 4x = 2a \Rightarrow x = a/2$ (critical point) — (1/2)

$\left.\frac{d^2A}{dx^2}\right|_{x=a/2} = \frac{1}{4} > 0 \Rightarrow$ point of local minima — (1/2)

Side length square 1 $= \frac{a/2}{4} = \frac{a}{8}$ m

Side length Square 2 $= \frac{a-a/2}{4} = \frac{a}{8}$ m — (1/2)

(OR)

$\frac{dA}{dx} = \frac{4x-2a}{16}$. $\frac{dA}{dx} = 0 \Rightarrow 4x = 2a \Rightarrow x = a/2$ — (1)

$\left.\frac{d^2A}{dx^2}\right|_{x=a/2} = \frac{1}{4} > 0$. Minima — (1/2)

Area $= \frac{x^2}{16} + \frac{(a-x)^2}{16} = \frac{1}{16}\left(\frac{a^2}{4}+\frac{a^2}{4}\right) = \frac{1}{16}\cdot\frac{a^2}{2} = \frac{a^2}{32}$ sq m — (1/2)

37) $\vec{PQ} = (3\hat{i}+4\hat{j}-\hat{k}) - (-2\hat{i}+\hat{j}+3\hat{k}) = 5\hat{i}+3\hat{j}-4\hat{k}$ — (1/2)

Angle between $\vec{PQ}$ and $\vec{PR}$. $\cos\theta = \frac{\vec{PQ}\cdot\vec{PR}}{|\vec{PQ}|\,|\vec{PR}|}$ — (1/2)

$|\vec{PQ}| = \sqrt{25+9+16} = \sqrt{50}$

$|\vec{PR}| = \sqrt{25+1+4} = \sqrt{30}$

$\vec{PR} = 5\hat{i}+\hat{j}-2\hat{k}$

$\cos\theta = \frac{25+3+8}{\sqrt{50}\sqrt{30}} = \frac{36}{10\sqrt{15}}$

$\theta = \cos^{-1}\left(\frac{18}{5\sqrt{15}}\right)$ — (1)

37) (ii) $\vec{RQ} = \vec{PQ} - \vec{PR} = 2\hat{j} - 2\hat{k}$ — $\frac{1}{2}$

Let S be the point on PQ (one third fuel is used)

$1 : 2$, P — S — Q — $\frac{1}{2}$

$$\vec{OS} = \frac{1(\vec{OQ}) + 2(\vec{OP})}{3}$$

$$\vec{OS} = \frac{(3\hat{i} + 4\hat{j} - \hat{k}) + 2(-2\hat{i} + \hat{j} + 3\hat{k})}{3} = \frac{-\hat{i} + 6\hat{j} + 5\hat{k}}{3}$$ — $\frac{1}{2}$

p.v of $S = -\frac{\hat{i}}{3} + 2\hat{j} + \frac{5}{3}\hat{k}$ — $\frac{1}{2}$

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38) Let $I+$ be the event when a person is infected and $I-$ be the event when person is not infected.

Let $T+$ be the event a person is tested positive and $T-$ denote where test is negative.

(i) Given $P(I+) = \frac{5}{100}$, $P(I-) = \frac{95}{100}$, $P(T+/I-) = \frac{4}{100}$ (false +ve)

$P(T-/I+) = \frac{3}{100}$ (false negative)

$P(T+/I+) = 1 - P(T-/I+) = 1 - \frac{3}{100} = \frac{97}{100}$ — $\frac{1}{2}$

$P(T+) = P(I+) \cdot P(T+/I+) + P(I-) \cdot P(T+/I-)$ — $\frac{1}{2}$

$= \frac{5}{100} \cdot \frac{97}{100} + \frac{95}{100} \cdot \frac{4}{100} = \frac{865}{10000} = 0.0865$ — 1

(ii) $P(I+/T+) = \frac{P(I+) \cdot P(T+/I+)}{P(I+)} = \frac{(5 \times 97)/10^4}{865/10^4} = \frac{97}{173}$ — $1\frac{1}{2}$

$P(I-/T+) = 1 - \frac{97}{173} = \frac{76}{173}$ — $\frac{1}{2}$

(OR)

$P(T-) = 1 - P(T+) = 1 - \frac{865}{10000} = 0.9135$ — $\frac{1}{2}$

$P(I+/T-) = \frac{P(I+) \cdot P(T-/I+)}{P(T-)} = \frac{15/10^4}{9135/10^4} = \frac{1}{609}$ — $1\frac{1}{2}$