MARKING SCHEME
PRE-BOARD EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

GENERAL INSTRUCTIONS:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answer which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks to be awarded to them.

2. If a student has attempted an extra question, answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale (0-80) has to be used. Please do not hesitate to award full marks if the answer deserve it.

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SECTION-A

1. (b) -6 1

2. (a) $\lambda \in (0,2)$ 1

3. (d) x 1

4. (b) 36 1

5.	(c)	11	1
6.	(c)	not continuous at $x = 3$ and not differentiable at $x = 3$.	1
7.	(b)	$y = \cos^{-1} x$	1
8.	(a)	$(1, \infty)$	1
9.	(d)	$\tan x - x + c$	1
10.	(c)	6	1
11.	(a)	72	1
12.	(d)	$e^x - e^y = c$	1
13.	(a)	(2, 4)	1
14.	(b)	$\frac{3}{8}$	1
15.	(d)	10	1
16.	(c)	3	1
17.	(c)	(6, 4)	1
18.	(c)	27	1
19.	(c)	(A) is true and (R) is false.	1
20.	(a)	Both (A) and (R) are true and (R) is the correct explanation of (A).	1

SECTION-B

21. f(x) is defined if $-1 \leq 2x - 3 \leq 1$ ½

$$\Rightarrow 1 \leq x \leq 2$$ 1

Domain of f(x) = [1, 2] ½

22. $\frac{dy}{dx} = \frac{x+y}{x}$

Put $y = vx$ and $\frac{dy}{dx} = v + x.\frac{dv}{dx}$ ½

$\therefore v + x\frac{dv}{dx} = 1 + v$ ½

$\Rightarrow \int dv = \int \frac{dx}{x}$

$\Rightarrow v = \log x + c$ ½

$\Rightarrow y = x(\log x + c)$ ½

23. (a) $$\frac{d(\tan^{-1}x)}{d(\log 5x)} = \frac{\frac{d}{dx}(\tan^{-1})}{\frac{d}{dx}(\log 5x)}$$ ½

$$= \frac{\frac{1}{1+x^2}}{\frac{1}{x}}$$ 1

$$= \frac{x}{1+x^2}$$ ½

OR

(b) $y = (\tan x)^x \Rightarrow \log y = x.\log(\tan x)$ ½

On differentiating w.r.t. x,

$$\frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{\tan x}.\sec^2 x + \log(\tan x)$$ 1

$$\Rightarrow \frac{dy}{dx} = (\tan x)^x[2x\ \text{cosec}\,2x + \log(\tan x)]$$ ½

24. (a) $\because \left(\vec{a} + \lambda\vec{b}\right) \perp \vec{c} \Rightarrow \left(\vec{a} + \lambda\vec{b}\right).\vec{c} = 0$ ½

$$\Rightarrow \left[(3+\lambda)\hat{i} + (2-2\lambda)\hat{j} + (1+3\lambda)\hat{k}\right].(4\hat{i} + \hat{j}) = 0$$ ½

$$\Rightarrow 4(3+\lambda) + (2-2\lambda) = 0$$ ½

$$\Rightarrow \boxed{\lambda = -7}$$ ½

OR

24. (b) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ 1

$\alpha = \cos^{-1}\left(\frac{2}{7}\right), \beta = \cos^{-1}\left(\frac{3}{7}\right), \gamma = \cos^{-1}\left(\frac{6}{7}\right)$ 1

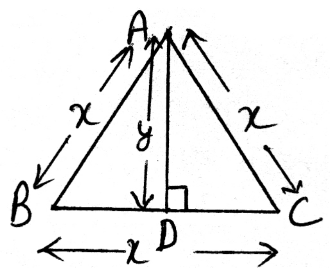
25. Required area of parallelogram = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & -2 & 3 \end{vmatrix}$ ½

$= |\hat{i} - \hat{j} - \hat{k}|$ 1

$= \sqrt{3}$ sq.units ½

SECTION-C

26.



Let, length of each side = x cm and length of median = y cm ½

$DC = \frac{BC}{2} = \frac{x}{2}$ ½

In ΔADC, using pathagorus theorem,

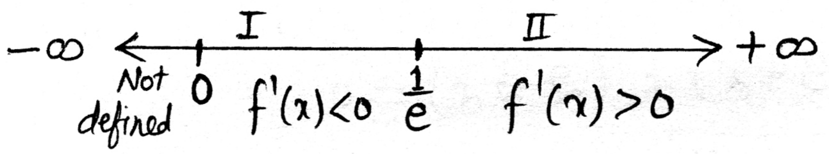
$$x^2 = y^2 + \left(\frac{x}{2}\right)^2$$

$$\Rightarrow x = \frac{2y}{\sqrt{3}}$$ 1

$$\frac{dx}{dt} = \frac{2}{\sqrt{3}}.\frac{dy}{dt} = 4 \text{ cm/s}$$ 1

27. $f(x) = x.\log x \Rightarrow f'(x) = 1 + \log x$ ½

Put $f'(x) = 0 \Rightarrow \boxed{x = \frac{1}{e}}$ ½



1

$\therefore f(x)$ is strictly decreasing on $\left(0, \frac{1}{e}\right)$ and f(x) is strictly increasing on $\left(\frac{1}{e}, \infty\right)$. 1

28. (a) $|\vec{a}| = 3\sqrt{6}, |\vec{b}| = 5\sqrt{2}, \vec{a}.\vec{b} = 45$ 1

$$Q = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right) = \frac{\pi}{6}$$ 1

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{9\sqrt{2}}{2}$ units 1

OR

(b) Parallel vector to the required line $(\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 3 & -2 & 1 \end{vmatrix}$ 1½

$= 10\hat{i} + 11\hat{j} - 8\hat{k}$ ½

Equation of required line is given by :

$\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + k(10\hat{i} + 11\hat{j} - 8\hat{k})$ ½

and

$\frac{x-2}{10} = \frac{y+3}{11} = \frac{z-4}{-8}$ ½

29. (a) Let $I = \int_0^{\pi/4} \log(1 + \tan x)\, dx$ ...(1)

Applying property $\int_0^a f(x)\, dx = \int_0^a f(a-x) dx$

$I = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$ 1

$\Rightarrow I = \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$ ...(2) 1

On adding equations (1) and (2), we get

$$2I = \log 2 \int_{0}^{\pi/4} dx = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2 \qquad 1$$

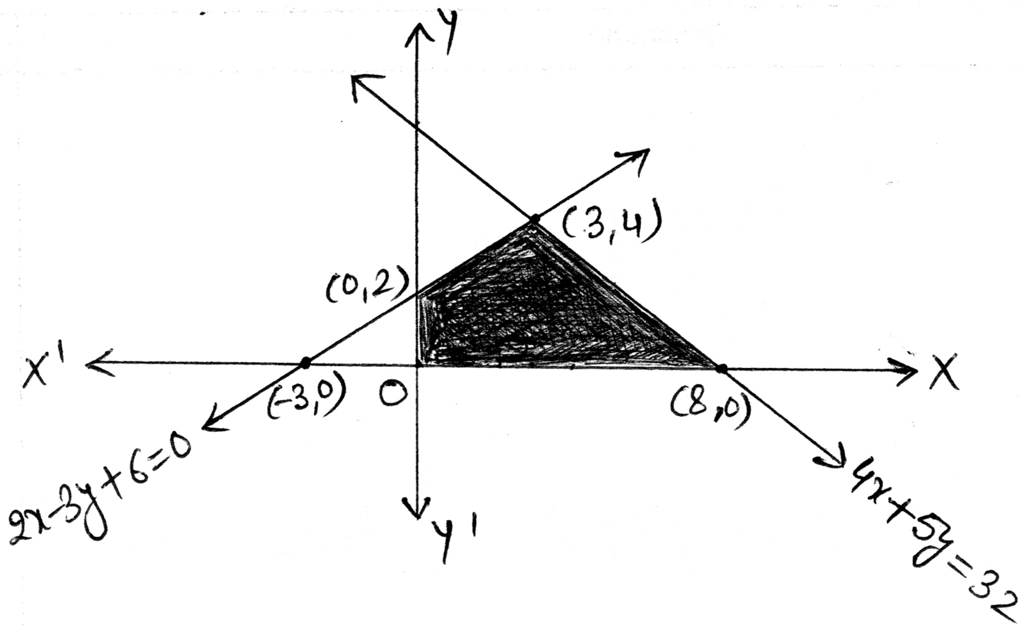
OR

(b) $$\int e\left(\frac{1+\sin x)}{1+\cos x}\right)dx = \int e^{x}\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^{2}\frac{x}{2}}\right)dx \qquad 1$$

$$= \int e\left(\frac{1}{2}\sec^{2}\frac{x}{2} + \tan\frac{x}{2}\right)dx \qquad 1$$

$$= e^{x}.\tan\frac{x}{2} + c \qquad 1$$

30.



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Corner Points	Max : Z = 8x + 5y
(0, 0)	0
(8, 0)	64 (Max.)
(3, 4)	44
(0, 2)	10

½

Hence, $Z_{max} = 64$ at point (8, 0) ½

31. (a) (i) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A).P(B)$

$= 0.3 + 0.6 - 0.3 \times 0.6$

$= 0.72$ 1

(ii) $P(A) \text{ and not } B) = P(A \cap \overline{B})$

$= P(A) - P(A \cap B)$

$= 0.3 - 0.18$

$= 0.12$ 1

(iii) P (neither A nor B) $= P(\overline{A} \cap \overline{B}) = P(A \cup B)'$

$= 1 - P(A \cup B) = 0.28$ 1

OR

(b)

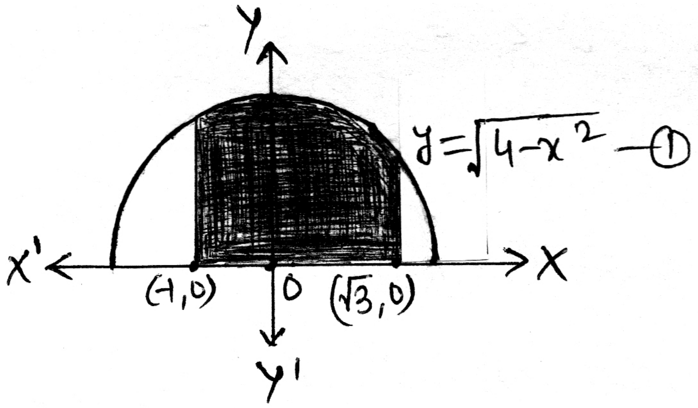
X	P(X)	X.P(X)
0	$\frac{1}{8}$	0
1	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$

2

Mean = $\Sigma X.\ P(X) = \frac{12}{8} = \frac{3}{2}$ or 1.5 1

SECTION-D

32.



1

Required shaded region = $\int_{-1}^{\sqrt{3}} y_1 \, dx$ 1

$= \int_{-1}^{\sqrt{3}} \sqrt{4-x^2} \; dx$ ½

$= \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) \right]_{-1}^{\sqrt{3}}$ 1

$= (\sqrt{3} + \pi)$ sq.units 1½

33. $|A| = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & 1 \\ 4 & 1 & 3 \end{vmatrix} = 7$

$\because |A| \neq 0 \Rightarrow A^{-1}$ exists 1

$\text{adj } A = \begin{bmatrix} 5 & 4 & -3 \\ 1 & 5 & -2 \\ -7 & -7 & 7 \end{bmatrix}$ 1½

$$A^{-1} = \frac{\text{adj A}}{|A|} = \frac{1}{7}\begin{bmatrix} 5 & 4 & -3 \\ 1 & 5 & -2 \\ -7 & -7 & 7 \end{bmatrix}$$ ½

Given equations in matrix form, can be written as

$$\underset{A}{\underset{\downarrow}{\begin{bmatrix} 3 & -1 & 1 \\ 1 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}}} \underset{X}{\underset{\downarrow}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}} = \underset{B}{\underset{\downarrow}{\begin{bmatrix} 5 \\ 5 \\ 8 \end{bmatrix}}}$$

$\Rightarrow A.X = B$

$\Rightarrow (A^{-1}.A).X = A^{-1}.B$ (Pre-multiplying by A^{-1}]

$\Rightarrow I.X = A^{-1}.B$

$\Rightarrow X = A^{-1}.B$ ½

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 5 & 4 & -3 \\ 1 & 5 & -2 \\ -7 & -7 & 7 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \\ 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$ 1

On comparing, x = 3, y = 2, z = –2 ½

34. (a) $f(x) = |x-2|$ at $x = 2$

CASE-I : CONTINUITY

LHL = 0 ½

RLH = 0 ½

f(2) = 0 ½

$\because \text{LHL} = f(2) = \text{RHL}$

$\therefore f(x)$ is continuous at x = 2 ½

CASE-II : DIFFERENTIABILITY

LHD = –1 1

RHD = 1 1

$\because \text{LHD} \neq \text{RHD at } x = 2$ ½

$\therefore f(x)$ is not differentiable at x = 2 ½

OR

(b) Let $y = \tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{\pi}{2} - \frac{x}{2}$ or $\frac{x}{2}$ 2

$\Rightarrow \frac{dy}{dx} = \pm\frac{1}{2}$ ½

Let $t = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\pi}{2} - 2\tan^{-1} x$ 1

$\Rightarrow \frac{dt}{dx} = -\frac{2}{1+x^2}$ ½

$\frac{dy}{dt} = \pm\frac{1+x^2}{4}$ 1

35. (a) $\overrightarrow{a_1} = 6\hat{i} + 2\hat{j} + 2\hat{k};\ \overrightarrow{b_1} = \hat{i} - 2\hat{j} + 2\hat{k}$ ½

$\overrightarrow{a_2} = -4\hat{i} - \hat{k};\ \overrightarrow{b_2} = 3\hat{i} - 2\hat{j} - 2\hat{k}$ ½

$\overrightarrow{a_2} - \overrightarrow{a_1} = -10\hat{i} - 2\hat{j} - 3\hat{k}$ ½

$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$ 1

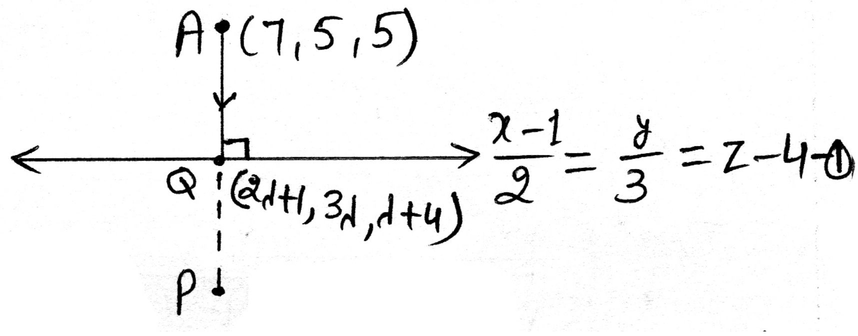
$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = 4 \times 3 = 12$ 1

Shortest distance = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

= 9 units 1½

OR

(b)



Let foot of perpendicular drawn from pt. (7, 5, 5) = Q

Then = $Q \equiv (2\lambda+1, 3\lambda, \lambda+4)$ 1

$\overrightarrow{AB} = \overrightarrow{OQ} - OA = (2\lambda-6)\hat{i} + (3\lambda-5)\hat{j} + (\lambda-1)\hat{k}$ 1

$\because \overrightarrow{AQ} \perp \text{Line}(1)$

$\therefore 2(2\lambda-6) + 3(3\lambda-5) + i.(\lambda-1) = 0$

$\Rightarrow \boxed{\lambda = 2}$ 1

$Q \equiv (5, 6, 6)$

Required distance = $AQ = \sqrt{6}$ units 1

Image of pt. (7, 5, 5) = (3, 7, 7) 1

SECTION-E

36. (I) ATQ, $2x + 2y + \pi x = 10$

$\Rightarrow y = 5 - \left(\frac{\pi + 2}{2}\right)x$ 1

(II) Area of window (A) = $2xy + \frac{1}{2}\pi x^2$

$= 10x - \left(\frac{\pi}{2} + 2\right)x^2$ 1

(III) (a) $\frac{dA}{dx} = 10 - (\pi + 4)x$ ½

Put $\frac{dA}{dx} = 0 \Rightarrow \boxed{x = \frac{10}{\pi + 4}}$ ½

If $x < \frac{10}{\pi + 4}$, then $\frac{dA}{dx} > 0$ and

If $x > \frac{10}{\pi + 4}$, then $\frac{dA}{dx} < 0$

$\therefore$ A is maximum at $x = \frac{10}{\pi + 4}$ met 1

OR

(iii) (b) $\frac{dA}{dx} = 10 - (\pi + 4)x$ ½

Put $\frac{dA}{dx} = 0 \Rightarrow \boxed{x = \frac{10}{\pi + 4}}$ ½

$\frac{d^2A}{dx^2} = -(\pi + 4) < 0$

$\therefore$ A is maximum at $x = \frac{10}{\pi + 4}$ met. 1

37. (I) Number of reflexive relations = 64 1

(II) Number of functions from Q to P = 81 1

(III) (a) Reflexive relation ½

Symmetric relation ½

Transitive relation 1

OR

(III) (b) Injective 1

Surjective 1

38. Let E_1 : Event that vyom choose scooter for travel.

E_2 : Event that vyom choose bus for travel.

E_3 : Event that vyom choose car for travel.

A : Even that vyom reaches late.

ATQ, $P(E_1) = 0.3, P(E_2) = 0.1 \; P(E_3) = 0.6$

$$P\left(\frac{A}{E_1}\right) = 0.2, \; P\left(\frac{A}{E_2}\right) = 0.5, \; P\left(\frac{A}{E_3}\right) = 0.3$$

(I) $P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)$ 1

$= 0.29$ 1

(II) By Baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(A)}$$ 1

$$= \frac{0.06}{0.29} = \frac{6}{29}$$ 1