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No. of pages - 24 **(E)**

PRE-BOARD EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

समय : 3 घंटे **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच-पाँच अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQ's) of 1 mark each.

1. If matrix $A=\begin{bmatrix} a & d & e \\ 1 & b & f \\ 2 & 3 & c \end{bmatrix}$ is a skew-symmetric matrix, then the value of a+b+c+d+e+f is : 1

(a) 6 (b) − 6

(c) 0 (d) 4

2. If the angle between $\vec{p}=\lambda^2\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{q}=-\hat{i}+5\hat{j}+\lambda\hat{k}$ is acute, then : 1

(a) $\lambda \in (0,2)$ (b) $\lambda \in R-(0,2)$

(c) $\lambda \in R-\{0\}$ (d) $\lambda \in R-\{2\}$

3. Integrating factor for linear differential equation $x.\frac{dy}{dx}+y=x^2$ is : 1

(a) log x (b) x^2

(c) 1 (d) x

4. If $A.(\text{adj } A)=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$, then the value of $|\text{adj } A|$ is : 1

(a) 6 (b) 36

(c) 216 (d) 1296

5. If points (a, 4), (–1, 2) and (5, 3) are collinear, then the value of a is : 1

(a) 9 (b) 10

(c) 11 (d) 12

6. The function $f : R \to Z$ defined by

$$f(x) = [x]$$

where [.] denotes the greatest integer function, is : 1

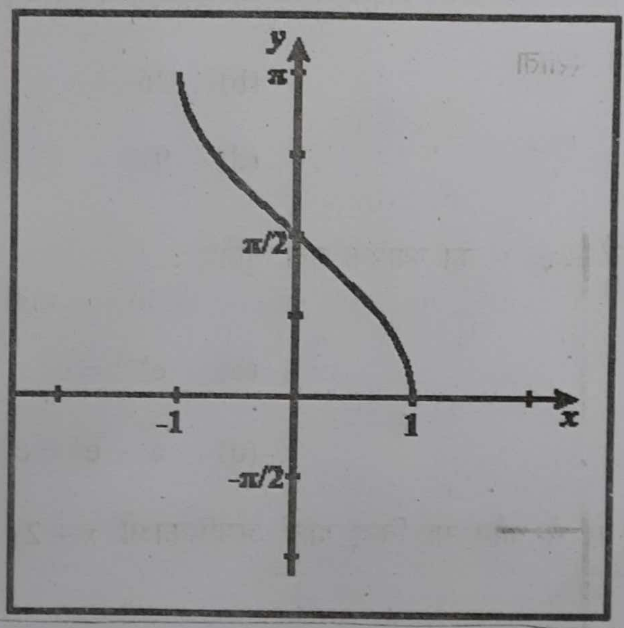
(a) continuous at $x = 3$, but not differentiable at $x = 3$.

(b) not continuous at $x = 3$, but differentiable at $x = 3$

(c) not continuous at $x = 3$ and not differentiable at $x = 3$.

(d) continuous as well as differentiable at $x = 3$

7. The graph drawn below depicts: 1



(a) $y = \sin^{-1} x$

(b) $y = \cos^{-1} x$

(c) $y = \text{cosec}^{-1} x$

(d) $y = \cot^{-1} x$

8. The interval, in which the function f defined by $f(x) = x^2 - 2x + 5$ is strictly increasing, is : 1

(a) $(1, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, 0)$

(d) $(0, -\infty)$

9. $\int \tan^2 x \, dx$ equals: 1

(a) $-\tan x - x + c$ (b) $x - \tan x + c$

(c) $\tan x + x + c$ (d) $\tan x - x + c$

10. $\int_{-4}^{-2} |x| \, dx$ equals: 1

(a) 2 (b) 4

(c) 6 (d) 8

11. The area (in square units) of the region bounded by the curve $x^2 = 4y$ and the line $y = 9$ is : 1

(a) 72 (b) 36

(c) 18 (d) 9

12. Choose the general solution of differential equation $\frac{dy}{dx} = e^{x-y}$: 1

(a) $e^{x-y} = c$ (b) $e^{x+y} = c$

(c) $e^x + e^y = c$ (d) $e^x - e^y = c$

13. Which of the following points satisfies both the inequalities $x + 2y \geq 8$ and $2x + y \leq 10$? 1

(a) (2, 4) (b) (3, 2)

(c) (–2, 4) (d) (–5, 6)

14. If for any two events A and B, $P(A) = 0.8$, $P(B) = 0.4$ and $P(A \cup B) = 0.9$, then $P\left(\frac{B}{A}\right)$ is : 1

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

15. If $|\vec{a}|=6, |\vec{b}|=8$ and $|\vec{a}+\vec{b}|=10$, then $|\vec{a}-\vec{b}|=$ 1

(a) 4
(b) 6
(c) 8
(d) 10

16. Let $A=\begin{bmatrix} x & 1 \\ 2 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 11 & 6 \\ 12 & 11 \end{bmatrix}$. If $A^2 = B$ the value of x is: 1

(a) 1
(b) 2
(c) 3
(d) 4

17. If the objective function for a LPP is $Z = 5x + 9y$ and the corner points of the bounded feasible region are (2, 3), (8, 0), (6, 4) and (0, 6), then the maximum value of Z occurs at : 1

(a) (2, 3)
(b) (8, 0)
(c) (6, 4)
(d) (0, 6)

18. If $|A| = 27$, where A is 3×3 matrix, then $|9A^{-1}|$ equals: 1

(a) 3
(b) 9
(c) 27
(d) 81

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. Assertion (A): $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x \cos x + \sin^5 x + 1)dx = \pi$ 1

Reason (R): $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if} \quad f(x) = \text{odd function} \\ 0 & \text{if} \quad f(x) = \text{even function} \end{cases}$

20. Assertion (A): The function f: N→N defined by $f(x) = x^2$ is not onto function. 1

Reason (R): The function f(x) is onto if Range of f = Co-domain.

SECTION-B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \sin^{-1}(2x - 3)$. 2

22. Find the general solution of differential equation $x.dy = (x+y).dx$. 2

23. (a) Find the derivative of $\tan^{-1}x$ with respect to $\log(5x)$. 2

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$ 2

24. (a) If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} + \hat{j}$ are such that $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then find the value of λ. 2

OR

(b) The coordinates of points A and B are (3, -1, 3) and (5, 2, 9) respectively. Find the angles which $\overrightarrow{AB}$ makes with X, Y and Z-axis. 2

25. Two co-initial adjacent sides of a parallelogram are $(2\hat{i}-3\hat{j}+5\hat{k})$ and $(\hat{i}-2\hat{j}+3\hat{k})$. Find the area of parallelogram. 2

SECTION-C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing. 3

27. Find the intervals in which function f(x) = x.log(x) is strictly increasing and strictly decreasing, where $x > 0$. 3

28. (a) An ant is moving along the vector $\vec{a}=5\hat{i}+5\hat{j}+2\hat{k}$. Few crystals are kept along the vector $\vec{b}=3\hat{i}+4\hat{j}+5\hat{k}$, which is inclined at angle θ with $\vec{a}$, then find the angle θ. Also, find the length of projection of $\vec{a}$ on $\vec{b}$. 3

OR

(b) Find the vector and cartesian equation. of the line thet passes through the point (2, -3, 4) and is perpendicular to the lines $\vec{r}=(3\hat{i}-\hat{j}+7\hat{k})+\lambda(\hat{i}+2\hat{j}+4\hat{k})$ and $\vec{r}=(7\hat{j}-9\hat{k})+\mu(3\hat{i}-2\hat{j}+\hat{k})$. 3

29. (a) Evaluate: $\int_0^{\pi/4} \log(1+\tan x)\, dx$ 3

OR

(b) Evaluate: $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ 3

30. Solve the following Linear Programming problem graphically: 3

Maximise: $z = 8x + 5y$

Subject : $4x + 5y \leq 32$

$2x - 3y + 6 \geq 0$

$x, y \geq 0$

31. (a) Given two independent events A and B such that P(A) = 0.3 and P(B) = 0, then find : 3

(i) P (A or B)

(ii) P (A and not B)

(iii) P (neither A nor B)

OR

(b) Calculate the mean for a random variable X defined as the number of tails in three tosses of a coin. 3

SECTION-D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -1$ and $x = \sqrt{3}$ and x-axis. 5

33. Let $A = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$, then find A^{-1} and using it, solve the following system of equations: 5

$$3x - y + z = 5$$
$$x + 2y + z = 5$$
$$4x + y + 3z = 8$$

(a) Show that the function $f(x)=|x-2|, x\in R$ is continuous, but not differentiable at $x=2$. 5

OR

(b) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ with respect to $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. 5

(a) Find the shortest distance between the lines $\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2}$ and $\vec{r}=(-4\hat{i}-\hat{k})+\lambda(3\hat{i}-2\hat{j}-2\hat{k})$. 5

OR

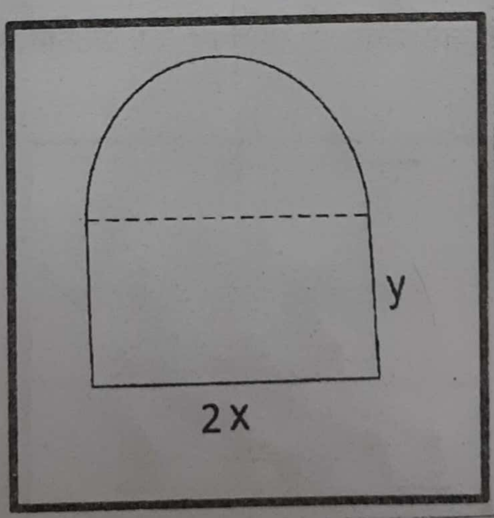
(b) Find the distance of point (7, 5, 5) from the line $\frac{x-1}{2}=\frac{y}{3}=z-4$. Also find the image of point (7, 5, 5) with respect to the given line. 5

SECTION-E

This section comprises of 3 case study/ passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (I), (II), (II) of marks 1, 1, 2 respectively. The third Case study question has two sub-parts (I) and (II) of 2 marks each.

CASE STUDY-I

5. Prashant is a carpenter, who wants to make a window in the form of rectangle surmounted by a semi-circular opening. The total perimeter of the window should be Jo meter, let diameter of semi-circular opening be 2x meter and height of rectangular part be y meter, as shown in figure.



Based on the above information, answer the following questions:

(I) Find y in terms of x only. 1

(II) Find the area of the window in terms of x only. 1

(III) (a) Using first derivative test, find the value of x, at which the area of the window is maximum. 2

OR

(b) Using second derivative test, find the value of x, at which the area of the window is maximum. 2

CASE STUDY-II

37. On the sports day, a school organized 100m race competition under two different categories – Boys and Girls. There were 15 participants initially, but finally 3 from category I and 4 from category II were selected for the final race. Vandita forms two sets P and Q with these participants for her school project.

Let P = {a, b, c} and Q = {d, e, f, g}; where P represents the set of Boys selected and Q the set of girls selected for the final race.

Based on the above information, answer the following questions :

(I) How many reflexive relations are possible from P to P? 1

(II) Among all the possible relations from Q to P, how many functions can be formed from Q to P? 1

(III) (a) Let relation R: $Q \rightarrow Q$ be defined by R = {(x, y): the number of steps taken by x and y is the same in 100 m race}.

Check whether R is reflexive, symmetric and transitive. Justify your answer. 2

OR

(b) A function f: $P \rightarrow Q$ be defined by f = {(a, d), (b, g), (c,e)}. 2

Check whether f is injective and subjective. Justify your answer.

CASE STUDY-III

38. During examination, Vyom has to reach examination centre and he has three options to reach examination centre by scooter, bus or car. The probability of his going by scooter, bus or car are 0.3, 0.1 and 0.6 respectively. The probability that he will be late is 0.2, 0.5 and 0.3 if he travels by scooter, bus or car respectively.