KENDRIYA VIDYALAYA SANGATHAN, KOLKATA REGION

PRE-BOARD EXAMINATION – 2024-25

<u>CLASS</u> – XII **<u>MAX.MARKS</u> – 80**

<u>SUB</u>. – MATHEMATICS (041) **<u>TIME</u> – 03 HOURS**

General Instructions:

1. This Question paper contains **- five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's and 02 Assertion-Reason** based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has **4 Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.
7. Use of calculator is not allowed.

SECTION A

(Multiple Choice Questions of 1 mark each)

1. The domain of $\cos^{-1}(3x-2)$is

(A) $\left(\frac{1}{3},2\right)$ (B) $\left[\frac{1}{3},1\right]$ (C) [-1,1] (D) $\left[\frac{-1}{3},\frac{1}{3}\right]$

2. Let $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}$ be a square matrix such that $adj\ A=A$. Then $(p+q+r+s)$ *is equal to*

(A) 2p (B)2q (C) 2r (D) 0

3. If A and B are symmetric matrix of the same order , then $(AB'-BA')$is a

(A) Skew Symmetric matrix (B)Null matrix (C) Symmetric matrix (D)None of these

4. If the area of triangle is 40 sq units with vertices (1,-6), (5,4) and (k,4). then k is

(A) 13 (B) -3 (C) -13,-2 (D) 13,-3

5. Given that A is a square matrix of order 3 and |A|= -2, then | adj(2A)| is equal to

(A) -128 (B) +4 (C) 64 (D)256

6. If $A.(\text{adj } A)=\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$, then the value of $|A|+|\text{adj } A|$ is equal to :

(A)12 (B)9 (C) 3 (D)27

7. The value of k for which the function $f(x)=\begin{cases}x^2, x\geq 0\\ kx, x<0\end{cases}$ is differentiable at $x=0$ is :

(A)1 (B)2 (C)Any real number (D)0

8. The function f (x) = [x], where [x] denotes the greatest integer function, is continuous at

(A) 4 (B) – 2 (C) 1 (D) 1.5

9. The function $f(x) = tanx - x$

(A) Always increases (B)Always decreases

(C)Never increases (D)Sometimes increases and sometime decreases

10. The anti derivative of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is

(A) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$ (B) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$ (D) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

11. The area of the region bounded by the lines $y = x, x = 0, x = 3$ and x-axis is:

(A) $\frac{1}{5}$ sq. units (B) $\frac{9}{4}$ sq. units (C) $\frac{9}{2}$ sq. units (D) $\frac{4}{5}$ sq. units

12. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is –

(A) $4e^{3x} + 3e^{-4y} = C$ (B) $e^{3x} + 3e^{-4y} = C$ (C) $3e^{3x} + 4e^{-4y} = C$ (D) $4e^{3x} - 3e^{-4y} = C$

13. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of

(A) Straight lines (B)Circles (C)Parabolas (D)Ellipses

14. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$ then $\vec{a} . \vec{b}$ is

(A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D)None of these

15. If a line makes equal acute angles with coordinate axes , then direction cosines of the line is

(A)1,1,1 (B) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (D) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

16. The corner point of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15) and (0,20) . let Z = px+ qy , where p,q >0.

Condition on p and q so that the maximum of z occurs at both the points (15,15) and (0,20) is

(A)p = q (B)p = 2q (C)q = 2p (D)q = 3p

17. Let A and B be two events such that P(A) = 0.6 , P(B) = 0.2 and $P\left(\frac{A}{B}\right) = 0.5$ then $P\left(\frac{A'}{B'}\right)$ equals

(A)1/10 (B)3/10 (C)3/8 (D)6/7

18. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).

Let F = 4x + 6y be the objective function. The Minimum value of F occurs at

(A) (0, 2) only (B) (3, 0) only (C) the mid point of the line segment joining the points (0, 2) and (3, 0) only (D) any point on the line segment joining the points (0, 2) and (3, 0).

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true

19. **Assertion(A):** $\int_{-2}^{2} log\left(\frac{1 + x}{1 - x}\right) dx = 0$

Reason(R): If f is an odd function, then $\int_{-a}^{a} f(x)\, dx = 0$

20. **Assertion(A):** Let A= {2,4,6} , B={3,5,7,9} and defined a function f = { (2,3),(4,5), (6,7)} from A to B ,then f is not onto.

Reason(R): A function f: A→B is said to be onto, if every element of B is the image of some element of A under f.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the value of tan^{-1}(-1) + $sin^{-1}\left(-\frac{1}{2}\right) + cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

22. (a)Find the derivative of $(cosx)^x$ with respect to x.

OR

22. (b) Find$\frac{dy}{dx}$ $if\ x = a(\theta - sin\theta)\ and\ y = a(1 + cos\theta)$

23. It is given that $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a '.

24. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $then\ find\ the\ value\ of\ \vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a}$.

25. (a) Find the area of the parallelogram whose diagonals are $4\hat{\imath}$ -$\hat{\jmath}$ - $3\hat{k}$ and $-2\hat{\imath}$ +$\hat{\jmath}$ - $2\hat{k}$.

OR

25. (b) If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0), (0,1,2) respectively, find ∠ABC

SECTION C

This section comprises of short answer type-questions (SA) of 3 marks each.

26. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?
27. Find the intervals of increasing and decreasing nature of the function

    $f(x)=x^3+6x^2+9x-8$.

28. (a)Evaluate $\int \frac{1}{9x^2+6x+5}\, dx$.

OR

28. (b) Evaluate $\int_0^{\frac{\pi}{4}} \log(1+tanx)\ dx$

29. (a) Find the Cartesian equation of the line which passes through the point (– 2, 4, – 5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

OR

29. (b) If $\vec{a}$ *is a unit vector such that* $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=80$ *then find* $|\vec{x}|$.
30. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

OR

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

31. Solve the Linear Programming Problem Graphically:
    Minimize and Maximize Z = 3x + 9y
    Subject to the constraints: x + 3y ≤ 60, x + y ≥ 10, x + y ≥ 10, x ≤ y, x ≥ 0, y ≥ 0

SECTION D

This section comprises of long answer type-questions (LA) of 5 marks each.

32. Find the area of the region lying in the first quadrant enclosed by x-axis , the line $x=y$ and the circle $x^2+\ y^2=32$
33. If $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 1 & -2 & 1\end{pmatrix}$, then find A^{-1} and hence solve the system of system of linear equations:$x+y+z=6,\ \ y+3z=7\ \text{ and }\ x-2y+z=0.$
34. (a) Evaluate: $\int \frac{1\ -\ x^2}{x\,(1\ -\ 2\,x)}dx$

OR

34. (b) Evaluate: $\int_1^4 |x-1| + |x-2| + |x-3|$ dx.
35. (a) Find the shortest distance between the lines whose equations are

$$\vec{r}=(1-t)\hat{\imath}+(t-2)\hat{\jmath}+(3-2t)\hat{k}$$
$$\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$$

OR

35.(b) Find the image of the point (1, 6, 3) in the line $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{4}$.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:**

An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = { b_1,,b_2,b_3} and G = {g_1,g_2} where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions. On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relations possible from B to G . How many such relations are possible

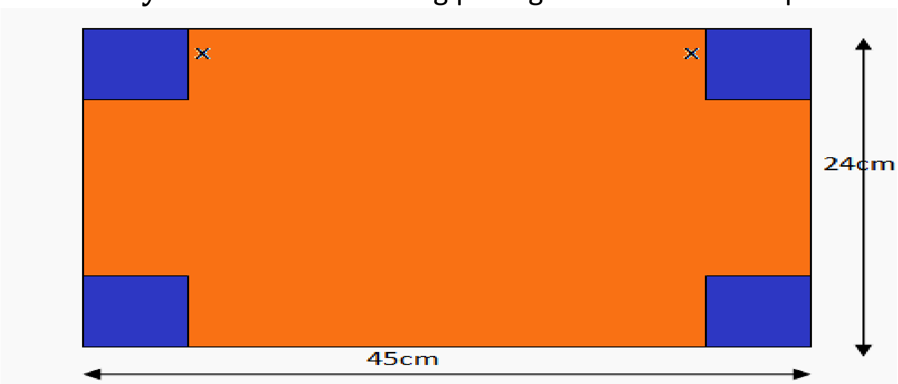
(ii) Write the smallest equivalence relation on G.

(iii) (a) Ravi defines a relation from B to B as R_1 = {(b_1,b_2),(b_2,b_1)}. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive.

OR

(iii) (b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; (where$0 \le x \le 20\sqrt{2}$ & $0 \le y \le 200$), then state whether the track represents a one-one and onto function or not. (Justify).

a. **Case-Study 2:** Read the following passage and answer the questions given below.



A rectangular piece of length 45 cm by 24 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. If ' x ' is the side of the square to be cut off from each corner, then

(i) What should be the volume (V) of the box formed in the term of x only ?

(ii) Find $\frac{dV}{dx}$

(iii) (a) What should be the side of the square(x) so that the volume of the box is maximum?

(iii) (b) What will be the maximum volume of the box?

37. **Case-Study 3:** Read the following passage and answer the questions given below.
A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

Based on the above information, answer the following questions.

(i) What is the probability that the doctor arrives late?

(ii) When the doctor arrives late, what is the probability that he comes by cab?