MATHEMATICS – X

SAMPLE PAPER – 6 (SOLVED)

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. Let p be a prime number and k be a positive integer.

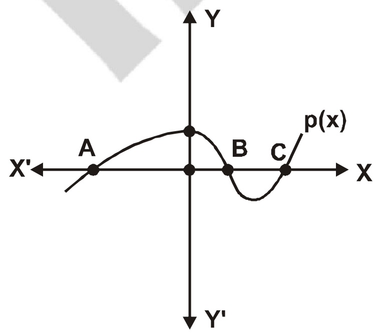
If p divides k^2, then which of these is DEFINITELY divisible by p ?

$\frac{k}{2}$	k	7k	k^3

(a) Only k (b) Only k and 7k

(c) Only k, 7k and $7k^3$ (d) all $\frac{k}{2}$, k, 7k and k^3

Q2. In figure the graph of a polynomial p(x) is shown. The number of zeroes of p(x) is



(a) 1 (b) 2 (c) 3 (d) 4

Q3. Which of these is QUADRATIC equation having one of its roots as zero

(i) $x^3 + x^2 = 0$ (ii) $x^2 - 2x = 0$ (iii) $x^2 - 9 = 0$

(a) Only (i) (b) Only (ii)

(c) Only (i) and (ii) (d) Only (ii) and (iii)

Q4. Two linear equations in variable x and y are given below :

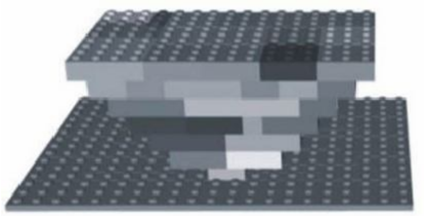
$$a_1x + b_1y + c = 0,\ a_2x + b_2y + c = 0$$

Which of the following places of information is independently sufficient to determine a solution exists or not for this pair of linear equation ?

(i) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = 1$ (ii) $\frac{a_1}{b_2} = \frac{b_1}{b_2}$ (iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq 1$ (iv) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(a) (i) (b) (i) and (iv) (c) (ii) and (iv) (d) (i) and (iii)

Q5. The cylindrical bumps on top of lego blocks are called studs. Pragun has built a solid inverted lego pyramid as shown below. The number of studs in successive floors forms an arithmetic progression Pragun figures out that the sum of the number of studs used in the first p floors is given by $(6p^2 - 2p)$. The number of studs are :



(a) 140 (b) 88 (c) 64 (d) 52

Q6. A (5, 1), B(1, 4) and C(8, 5) are the coordinates of the vertices of a triangle. Which of the following types of triangle will ΔABC be ?

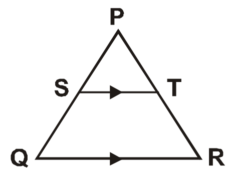
(a) Equilateral triangle (b) Scalene right-angled triangle

(c) Isosceles right-angled triangle (d) Isosceles acute-angled triangle

Q7. X-axis divides the join of (2, –3) and (5, 6) in the ratio

(a) 1 : 2 (b) 2 : 1 (c) 2 : 5 (d) 5 : 2

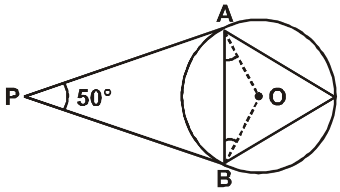
Q8. In the following figure, ST || QR, point S divides PQ in the ratio 4 : 5. If ST = 1.6 cm, what is the length of QR ?



(a) 0.71 cm (b) 2 cm

(c) 3.6 cm (d) cannot be calculated from the given data.

Q9. In the given figure, if PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is equal to



(a) 25° (b) 30° (c) 40° (d) 50°

Q10. A circle has a centre O and radii OQ and QR. Two tangents, PQ and PR, are drawn from an external point, P. In addition to the above information, which of these must also be known to conclude that the quadrilateral PQOR is a square ?

(i) OQ and OR are at angle of 90° (ii) The tangents meet at an angle of 90°

(a) Only (i) (b) only (ii) (c) either (i) or (ii) (d) both (i) and (ii)

Q11. P and Q are acute angle such that P > Q.

Which of the following is DEFINITELY true ?

(a) sin P < sin Q (b) tan P > tan Q (c) cos P > cos Q (d) cos P > sin Q

Q12. In a right-angled triangle PQR, $\angle Q = 90°$

Which is these is ALWAYS 0 ?

(a) cos P – sec R (b) tan P – cot R

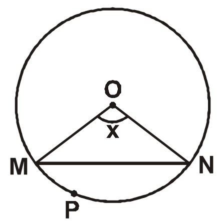
(c) sin P – cosec R (d) (cannot be known without knowing the value of P)

Q13. If the height of a vertical pole is $\sqrt{3}$ times the length of it shadow on the ground, then the angle of elevation of the Sun at that time is :

(a) 30° (b) 60° (c) 45° (d) 75°

Q14. Shown below is a circle with centre O. Chord MN subtends an angle at O.

Which of these is true for the above circle.



I. $\dfrac{x}{360°} = \dfrac{\text{length of arc MPN}}{\text{circumference of the circle}}$

II. $\dfrac{x}{360°} = \dfrac{\text{minor sector area}}{\text{area of the circle}}$

(a) only I (b) only II (c) both I and II (d) neither I nor II

Q15. The sum of circumference and the radius of a circle is 102 cm. The radius of circle is

(a) 7 cm (b) 14 cm (c) 10 cm (d) 18 cm

Q16. A number was selected at random from 1 to 100 (inclusive of both number) and it was found to be a multiple of 10.

What is the probability that the selected number is a multiple of 5 ?

(a) $\dfrac{1}{10}$ (b) $\dfrac{1}{5}$ (c) $\dfrac{1}{2}$ (d) 1

Q17. At a party, there is one last pizza slice and two people (Ananya and Pranit) who want it. To decide who gets the last slice, two fair six-sided dice are rolled, if the largest number in the roll is :

1, 3 or 6, Ananya would get the last slice, and 2, 4 or 5, Pranit would get it.

In a random roll of dice, who has higher chance of getting the last pizza slice ?

(Note : If the number on both the dice is the same, then consider that number as the larger number)

(a) Ananya

(b) Pranit

(c) Both have an equal chance

(d) (cannot be answered without knowing the exact number in a roll)

Q18. A survey was conducted on 80 gamers on how many games did they plan in a 6 day. The data is given below.

Number of games	Number of gamers
1 – 2	20
2 – 3	24
3 – 4	10
4 – 5	12
5 – 6	8
6 – 7	4
7 – 8	2

Which of the following is the modal class ?

(a) 1 - 2 (b) 2 - 3 (c) 4 - 5 (d) 7 - 8

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

(a) Both (A) and (R) are true and (R) is the correct explanation of the (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of the (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q19. **Assertion (A) :** The volume of a right circular cylinder of base radius 7 cm and height 10cm is 1540 cm^3.

Reason (R) : According to assertion, the curved surface area of cylinder is 440 cm^2.

Q20. **Assertion (A) :** If the second term of an A.P. is 13 and the fifth term is 25, then its 7th term is 33.

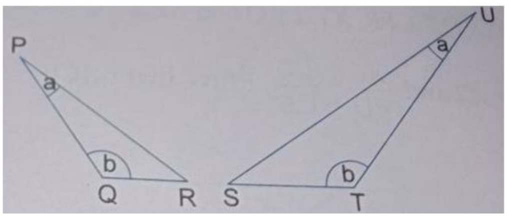
Reason (R) : If the common difference of an A.P. is 5, then $a_{18} - a_{13}$ is 25.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. M and N are positive integers such that $M = p^2q^3r$ and $N = p^3q^2$, where p, q, r are prime numbers. Find LCM (M, N) and HCF (M, N).

Q22. In the below figure, QR = 4cm, RP = 8 cm and ST = 6 cm.

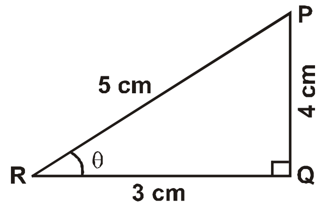


(Note : The figure is not to scale)

If the perimeter of ΔSTU is 27cm, find the length of PQ. Show your step.

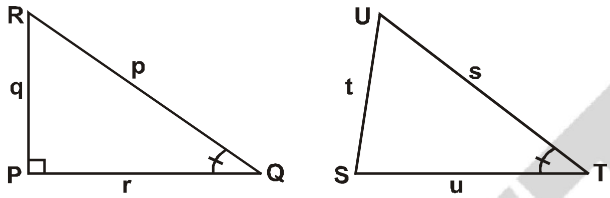
Q23. If AB is tangent drawn from a point A to a circle with centre O and BOC is a diameter of circle such that $\angle AOC = 105°$, then find $\angle OAB$.

Q24. Show that $\sin\theta = \cos(90° - \theta)$ is true using the definition of trigonometric ratios.



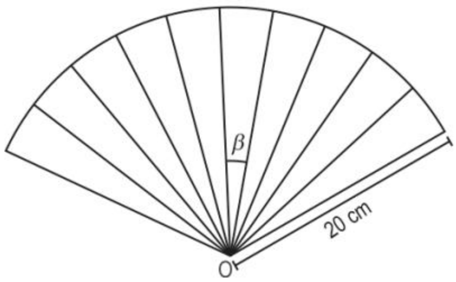
OR

In the triangles shown below, $\angle Q = \angle T$.



Write an expression each for cosQ and sin T.

Q25. The figure below is a part of a circle with centre O. Its area is $\frac{1250\pi}{9}\text{cm}^2$ and the 10 sectors are identical. Find the value of β, in degrees. Show your steps.



OR

Avikant bought a pair of glasses with wiper blades. He was curious to know the area being cleaned by each of the wiper blades. With the help of a ruler and a protractor, he found the length of each blade as 3 cm and the angle swept as 60°.

(i) Find the area that each wiper cleans in one swipe, in terms of π.

(ii) If the diameter of each circular glass is 5 cm, what percent of the area of the glass will be cleaned by the blade in one swipe ?

Show your work.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $\sqrt{5}$ is an irrational.

Q27. The difference of an integer and its reciprocal is $\frac{143}{12}$. Find the integers.

OR

Find the positive value of k, for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots.

Q28. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β.

Q29. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B \leq 90°$; $A > B$, find A and B.

Q30. The lengths of tangents drawn from an external point (point outside the circle) to a circle are equal. Prove it.

OR

ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

Q31. Radhika, a good student has ability to save her pocket money into her own piggy bank. Saving money is a skill that will be useful at all stages in person's life.

Radhika's piggy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. One day she decided to take out money from her piggy bank. If it is equally likely that one of the coins will fall out when the piggy bank is turned upside down, find the probability that the coin (i) will be 50p coin (ii) will not be a ₹5 coin (iii) will be ₹2 coin.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. In a periodic test, the sum of the marks obtained by Charvi in Mathematics and Science is 39. Had she got 3 marks less in Mathematics and 4 marks more in Science, the product of marks obtained in two subjects would have been 399. Find the marks obtained in the both subjects separately. Also, find marks obtained in Hindi if the average marks of Mathematics, Science and Hindi is 20.

OR

In a cricket match against Britain, R Ravidern, Jadeja took one wicket less that twice the number of wickets taken by Arshdeep. If the product of the number of wickets taken by these two is 15, find number of wickets taken by each.

Q33. In ΔABC and ΔPQR, AD and PM are the median respectively. If $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$, then prove that $\Delta ABC \sim \Delta PQR$.

Q34. Find the median of the following data :

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80
Frequency	0	10	25	43	65	87	96	100

Q35. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 7 cm. Find the volume of the toy. If a sphere circumscribes the toy, then find the difference of the volumes of the sphere and toy.

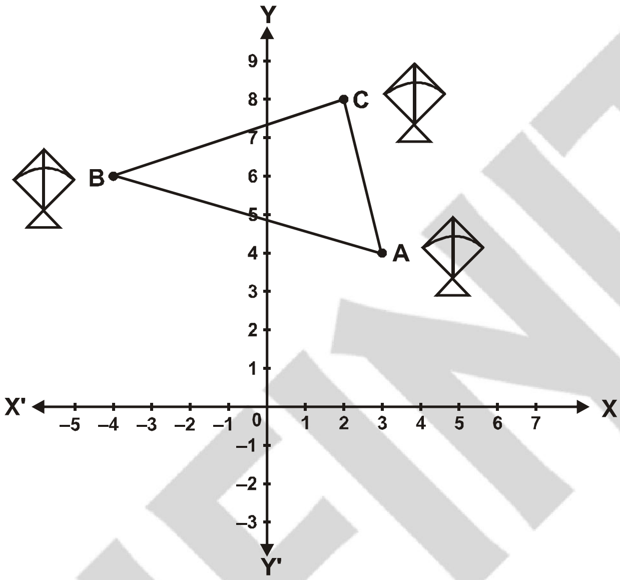
OR

14 lead shots each of radius 2 cm are packed in a cuboidal box of internal dimensions 18cm × 8 cm × 6 cm and then the box is filled with water. Find volume of water filled in the box.

SECTION - E

Case study based questions are compoulsory.

Q36. A boy standing at O, observed three kites at point A, B and C. Based on this answer the following :



(i) What is he distance between the kites at A and B ?

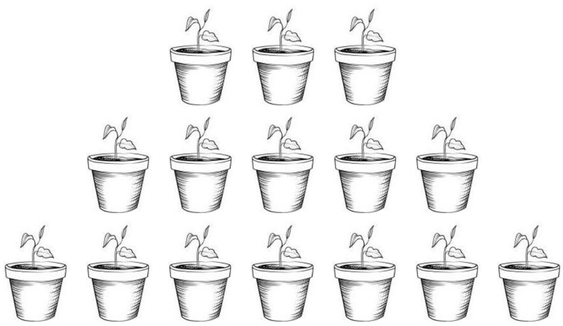
(ii) What is the ratio by which y-axis divided the line segment AB ?

(iii) What are the coordinates of point on the y-axis equidistant from B and C ?

OR

What is the length of median of triangle through B ?

Q37. Reema being a plant lover decides to open a nursery and she bought few plants with pots. She wants to place pots in such a way that the number of pots in first row is 3, in second row is 5 and in third row is 7 and so on....



Based on the above answer the following :

(i) How many pots are there in 15th row ?

(ii) If there are total 120 pots. How many rows are there ?

(iii) If there are 21 rows then how many pots will be there in middle row ?

OR

If she increases two pots in each row. How many plants will be there in 10 rows ?

Q38. The status of unity is the world's tallest statue, located in Gujarat. A man from some distance from the foot of the statue in the same plane observe that the angle of elevation of top of statue is 45°, after covering a distance of 253.73 feet towards the statue the angle of elevation of the to of the statue become 60°. (use $\sqrt{3}$ = 1.73).

On the basis of above information answer the following :

(i) Draw a neat labelled diagram to show the above situation.

(ii) Find the relation between the height of statue and initial distance of man from statue.

(iii) Find the height of statue.

OR

If the man starts moving always from the statue. At what distance from the statue the angle of elevation will be 30° ?

MATHEMATICS – X
SOLUTIONS : SAMPLE PAPER – 6

A-1. (c)

Explanation : k^2 is divisible by p.

$\therefore$ k will be divisible by p.

From the given condition we can check the divisibility by k.

$$\frac{k}{2}\sqrt{k} = \frac{1}{2}$$

$$k + k = 1$$

$$7k + k = 7$$

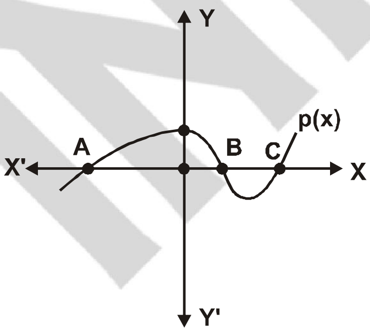
$$k^3 + k = k^2$$

From above we can see that the conditions (ii), (iii) and (iv) are completely divisible by k, so these will also be divisible by p but the condition (i) is not completely divisible by k, so it will also not be divisible by p.

A-2. (c)

Explanation : According to the property of the polynomials,

Number of zeroes = Number of points at which graph intersects the X-axis.



From the figure it is clear that the graph intersects X-axis at three different points. Therefore, the polynomial has 3 zeroes.

A-3. (b)

Explanation :

(i) $x^3 + x^2 = 0$

It is cubic so it is not possible.

(ii) $x^2 - 2x = 0$

$\Rightarrow$ $x(x-2) = 0$

$\Rightarrow$ $x = 0$ and $x = 2$

Thus, it has one of the roots as zero.

(iii) $x^2 - 9 = 0$

$\Rightarrow$ $x + 3 = 0$ and $x - 3 = 0$

$\Rightarrow$ $x = -3$ and $x = 3$

It does not have any of its roots as zero.

Thus only (ii) has one of the roots as zero.

A-4. (b)

Explanation : According to given linear equations

(i) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c}{c}$

$\Rightarrow$ $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = 1$

$\Rightarrow$ $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = 1$

Hence (i) is sufficient to determine the solution

(ii) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ is not sufficient.

(iii) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq 1$ is not any condition.

(iv) $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ is condition of unique solution so it is sufficient.

Thus, equation (i) and (iv) are sufficient.

A-5. (d)

Explanation :

$S_p = 6p^2 - 2p$

$\therefore \quad S_1 = 6 \times 1^2 - 2 \times 1$

$= 6 - 2 = 4$

$S_2 = 6 \times 2^2 - 2 \times 2$

$= 24 - 4 = 20$

Now, $S_2 = T_2 + T_1$

$20 = T_2 + 4 \quad (\text{As } S_1 = T_1)$

$T_2 = 16$

Common difference (d) $= T_2 - T_1$

$= 16 - 4 = 12$

Thus, $a = 4$, $d = 12$ and $n = 5$

$\therefore \quad T_n = a + (n-1)d$

$= 4 + (5-1)\,12$

$= 4 + 48 = 52$

A-6. (c)

Explanation :

In ΔABC,

$AB = \sqrt{(1-5)^2 + (4-1)^2}$

$= \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16+9} = \sqrt{25}$

$= 5$ units

$BC = \sqrt{(8-1)^2 + (5-4)^2}$

$= \sqrt{(7)^2 + (1)^2}$

$= \sqrt{49+1} = \sqrt{50}$

$= 5\sqrt{2}$ units

$CA = \sqrt{(8-5)^2 + (5-1)^2}$

$= \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25}$

$= 5$ units

$\Rightarrow \quad AB = AC$

Thus, ΔABC is isosceles ...(i)

By Applying Pythagoras theorem, we can see that

$BC^2 = AB^2 + CA^2$

$(5\sqrt{2})^2 = (5)^2 + (5)^2$

$50 = 50$

So, ΔABC is right angled triangle.

(By converse of Pythagoras theorem) ...(ii)

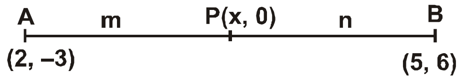
From (i) and (ii)

ΔABC is Right Angled Isosceles Triangle.

A-7. (a)

Explanation :

Let P(x, 0) be a point on X-axis which divides the join of A(2, –3) and B(5, 6) in the ratio m : n, then using section formula.



$Y = \dfrac{my_2 + ny_1}{m+n}$

$\Rightarrow \quad 0 = \dfrac{m \times 6 + n \times (-3)}{m+n}$

$\Rightarrow \quad 6m - 3n = 0$

$\Rightarrow \quad 2m - n = 0$

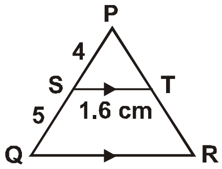
$\Rightarrow \quad 2m = n$

$\Rightarrow \quad \dfrac{m}{n} = \dfrac{1}{2}$

i.e., $m : n = 1 : 2$

A-8. (c)

Explanation :



$ST \parallel QR$ (Given)

$$\frac{PS}{PQ} = \frac{ST}{QR} \quad \text{[Conc. B.P.T]}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{1.6}{QR}$$

$$\therefore \quad QR = \frac{9}{4} \times 1.6 = 3.6 \text{ cm}$$

A-9. (a)

Explanation :

In ΔOAB, we have

(Radii of same circle)

(Angles opposite to equal sides are equal)

As OA and PA are radius and tangent respectively at point of contact A.

So, $\angle OAP = 90°$

$$\angle P + \angle A + \angle O + \angle B = 360°$$

$$\Rightarrow 50° + 90° + \angle O + 90° = 360°$$

$$\Rightarrow \angle O = 360° - 90° - 90° - 50°$$

$$\Rightarrow \angle O = 130°$$

Again, in ΔOAB,

$$\angle O + \angle OAB + \angle OBA = 180°$$

$$\Rightarrow 130° + \angle OAB + \angle OAB = 180°$$

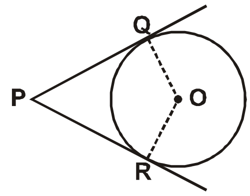
$(\because \angle OBA = \angle OAB)$

$$\Rightarrow 2\angle OAB = 180° - 130° = 50°$$

$$\Rightarrow \angle OAB = 25°$$

A-10. (c)

Explanation :



(i) $OQ \perp PQ$

And, $QR \perp PR$

($\because$ Radius is perpendicular to the tangent through the point of contact)

$\therefore \quad \angle OQP = \angle ORP = 90°$

Hence, OQ and OR are at angle of 90°

(ii) In square all angles are right angles

Hence, both conditions either (i) or (ii) should be known to conclude that PQOR is a square.

A-11. (b)

Explanation :

$P > Q$ (given)

Let $P = 60°$ and $Q = 0°$

(i) $\sin P < \sin Q$ = False

(As, $\sin P > \sin Q$)

(ii) $\tan P > \tan Q$ = True

(iii) $\cos P > \cos Q$ = False

(As, $\sin P < \cos Q$)

(iv) $\cos P > \sin Q$ = false

(As $\cos P < \sin Q$)

A-12. (b)

Explanation :

In Right Angle Triangle PQR,

$\angle Q = 90°$

(i) $\cos P - \sec R$

$$\frac{b}{c} = \frac{c}{a} \neq 0 \qquad \text{False}$$

(ii) $\tan P - \cot R$

$$\frac{a}{b} - \frac{a}{b} = 0 \qquad \text{True}$$

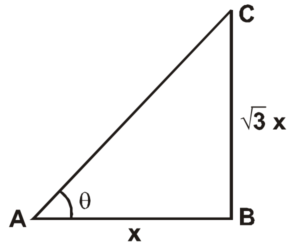
(iii) sin P – cosec R

$\frac{a}{c} - \frac{c}{b} \neq 0$ False

A-13. (b)

Explanation :

Let the length of shadow is x



Then height of pole = $\sqrt{3}x$

$\tan\theta = \frac{CB}{AB}$

$\tan\theta = \frac{\sqrt{3}x}{x}$

$\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60°$

$\theta = 60°$

A-14. (c)

Explanation : According to formula

$$\frac{x}{360°} \times 2\pi r$$

(ii) Area of Minor sector = $\frac{\pi r^2\theta}{360°}$

A-15. (b)

Explanation : Let radius of circle be r.

∴ Circumference of circle = 2πr

According to question

$2\pi r + r = 102$

or $4\left(2\times\frac{22}{7}+1\right)=102$

or $r = \frac{102\times 7}{51}$ = 51 cm

A-16. (d)

Explanation : Total marks = 100

Numbers selected as multiple of 10 = 10

∴ Total outcomes = 10

Favorable outcomes (Multiple of 5)=10

P (number is a multiple of 5) = $\frac{10}{10} = 1$

A-17. (b)

Explanation : Total outcome = 6×6 = 36

Favourable outcomes for Ananya = (1, 2), (2, 1), (1,5), (2, 4), (3, 3), (4, 2),(5, 1) =

∴ P (Ananya getting last slice) = $\frac{7}{36}$ ...(i)

Favourable outcome for Pranit = (1, 1), (1, 3), (2, 2), (3, 1), (1, 4), (2, 3), (3, 2), (4, 1) = 8 outcomes

∴ P (Pranit getting last slice) = $\frac{8}{36}$ ...(ii)

From (i) and (ii)

Pranit has a higher chance of getting the last prize slice.

A-18. (b)

Explanation : Mode is of the number that occurs the highest number of times.

So, 24 is Mode.

∴ Modal Class is 2 – 3

A-19. (a)

Explanation : In case of assertion :

Here r = 7 cm, h = 10 cm

Volume of Cylinder = $\pi r^2 h$

$= \frac{22}{7}\times(7)^2\times 10$

$= 1540 \text{ cm}^3$

$\therefore$ Assertion is true.

In case of reason :

Curved Surface area of cylinder

$= 2\pi rh$

$= 2\times\frac{22}{7}\times 7\times 10$

$= 440\ cm^2$

$\therefore$ Reason is true.

Both (A) and (R) are true and (R) is the correct explanation of A.

A-20. (b)

Explanation : In case of assertion

In the given A.P., $a_2 = 13$ and $a_5 = 25$

$a + d = 13$

$a + 4d = 25$

Solving these equations, we get a = 9 and d = 4.

Thus, $a_n = a + (n - 1)d$

$\Rightarrow a_7 = 9 + (7 - 1)4 = 33$

$\therefore$ Assertion is true.

In case of reason :

In the given A.P., d = 5

Thus,

$a_{18} - a_{13} = a + 17d - a - 12d = 5d = 25$

$\therefore$ Reason is true.

Hence both assertion and reason are true but reason is not the correct explanation for assertion.

A-21. $M = p^3q^3r$

$N = p^3q^2$

$\therefore$ LCM (M, N) = p^3q^3r

HCF (M, N) = p^2q^2

A-22. In ΔPQR and ΔSTU

$\angle P = \angle U = a$ (Given)

$\angle Q = \angle T = b$ (Given)

$\therefore$ $\Delta PQR = \Delta STU$ (By AA similarity)

$$\frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta STU} = \frac{QR}{ST}$$

($\because$ Ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides)

$$\Rightarrow \frac{\text{Perimeter of } \Delta PQR}{27} = \frac{4}{6}$$

$$= \frac{4\times 27}{6} = 18\ cm$$

Now, in ΔPQR

Perimeter = PQ + QR + PR

18 = PQ + 4 + 8

PQ = 18 – 12

PQ = 6 cm

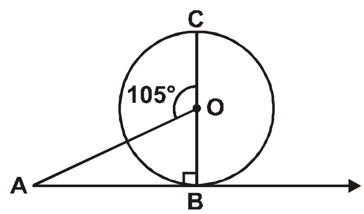
A-23. Given, AB and BOC are the tangent and diameter of the circle with centre O, respectively.

We know that, tangent at any point on a circle is perpendicular to the radius through the point of contact.

$\therefore$ $OB \perp AB$

$\Rightarrow \angle OBA = 90°$

Given $\angle AOC = 105°$



In ΔABO,

$\angle AOC = \angle OAB + \angle OBA$

[$\because$ External angle = Sum of opposite internal angles]

$\Rightarrow 105° = \angle OAB + 90°$

$\Rightarrow \angle OAB = 105° - 90°$

$\Rightarrow \angle OAB = 15°$

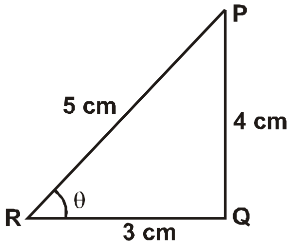
A-24. In ΔPQR

$$\angle P + \angle Q + \angle R = 180^\circ$$

(Angle sum property)

$$\angle P = 180^\circ - (90^\circ + \theta)$$

$$\angle P = 90^\circ - \theta$$



According to trigonometry ratio

$$\cos(90 - \theta)^\circ = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{4}{5}$$

From (i) and (ii)

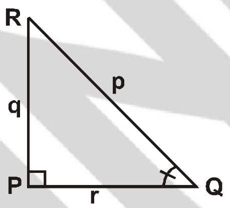
$$\sin\theta = \cos(90^\circ - \theta)$$

Hence Proved.

OR

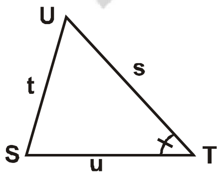
In right ΔPQR

$$\cos Q = \frac{r}{p},\ \sin Q = \frac{q}{p}$$



As $\angle Q = \angle T$ (given)

$\therefore$ $\sin Q = \sin T$



So, $\sin T = \frac{q}{p}$

Thus $\cos Q = \frac{r}{p}$ and $\sin T = \frac{q}{p}$

A-25. Area of sector $= \frac{1250\pi}{9}\text{cm}^2$ (Given)

$$\Rightarrow \frac{\theta}{360^\circ} \times \pi \times (20)^2 = \frac{1250\pi}{9}$$

($\because$ radius = 20 cm)

$$\Rightarrow \theta = \frac{1250\pi}{9} \times \frac{360^\circ}{\pi \times 400}$$

$$\theta = 125^\circ$$

θ = value of 10 sectors

$$\beta = \frac{\theta}{10} = \frac{125^\circ}{10} = 12.5^\circ$$

Thus, value of $\beta = 12.5^\circ$

OR

(i) Radius = Length of wiper

= 3 cm

Angle (q)= 60°

Area that wiper cleans = Area of sector

$$= \frac{\theta}{360^\circ}\pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \pi \times (3)^2$$

$$= \frac{3}{2}\pi$$

(ii) Area of glass $= \pi r^2$

$$= \pi \times \frac{5}{2} \times \frac{5}{2}$$

$$= \frac{25}{4}\pi\ \text{cm}^2$$

Percentage of area cleaned

$$= \frac{1.5\pi}{\frac{25}{4}\pi} \times 100$$

$$= 1.5 \times \frac{4}{25\pi} \times 100 = 24\%$$

A-26. Let $\sqrt{5}$ is a rational number and $\sqrt{5} = \frac{a}{b}$, where a and b are co prime and $b \neq 0$.

Now, $(\sqrt{5})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow \quad 5b^2 = a^2 \quad$ ...(i)

$\Rightarrow$ 5 is a factor of a^2

$\therefore$ a is also divisible by 5.

Let a = 5c, where c is some integer.

Substituting a = 5c in (i), we get

$5b^2 = (5c)^2$

$\Rightarrow \quad 5b^2 = 25c^2$

$\Rightarrow \quad b^2 = 5c^2$

$\Rightarrow$ 5 is a factor of b^2

$\therefore$ 5 is a factor of b.

$\therefore$ 5 is a common factor of a and b

This contradicts the fact that a and b are co prime so, our assumption is wrong.

Here, $\sqrt{5}$ is irrational.

A-27. Let the integer be x and its reciprocal be $\frac{1}{x}$.

According to question

$$x - \frac{1}{x} = \frac{143}{12}$$

$\Rightarrow \quad \frac{x^2 - 1}{x} = \frac{143}{12}$

$\Rightarrow \quad 12x^2 - 12 = 143x$

$\Rightarrow \quad 12x^2 - 143x - 12 = 0$

$\Rightarrow \quad 12x^2 - 144x + x - 12 = 0$

$\Rightarrow \quad 12x(x - 12) + 1(x - 12) = 0$

$\Rightarrow \quad (x - 12)(12x + 1) = 0$

$\Rightarrow \quad x = 12$ or $x = -\frac{1}{22}$

Rejecting $x = -\frac{1}{22}$, because x is an integer.

$\therefore \quad x = 12$

$\therefore$ The required integer is 12.

OR

If the equation $x^2 + kx + 64 = 0$ has real roots, then $D \geq 0$.

$\Rightarrow \quad k^2 - 4 \times 1 \times 64 \geq 0$

$\Rightarrow \quad k^2 \geq 256$

$\Rightarrow \quad k^2 \geq (16)^2$

$\Rightarrow \quad k \geq 16 \quad [\because k > 0] \quad$ ...(i)

If the equation $x^2 - 8x + k = 0$ has real roots then $D \geq 0$

$\Rightarrow \quad 64 - 4k \geq 0$

$\Rightarrow \quad 4k \leq 64$

$\Rightarrow \quad k \leq 16 \quad$ ...(ii)

From (i) and (ii), we get

$k = 16$

A-28. Let $\quad p(x) = 4x^2 + 4x + 1$

$\because \quad \alpha, \beta$ are zeroes of p(x)

$\therefore \quad \alpha + \beta = \text{sum of zeroes} = \frac{-b}{a}$

$\Rightarrow \quad \alpha + \beta = \frac{-4}{4} = -1 \quad$ ...(i)

Also $\quad \alpha\beta = \text{Product of zeroes} = \frac{c}{a}$

$\Rightarrow \quad \alpha\beta = \frac{1}{4} \quad$ ...(ii)

Now a quadratic polynomial whose zeroes are 2α and 2β.

$x^2 -$(sum of zeroes)x + Product of zeroes

$$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$$

$$= x^2 - 2(\alpha + \beta)x + 4\alpha\beta$$

$$= x^2 - 2 \times (-1)\, x + 4 \times \frac{1}{4}$$

[Using (i) and (ii)]

$$= x^2 + 2x + 1$$

A-29. $\tan(A + B) = \sqrt{3}$

$\Rightarrow$ $\tan(A + B) = \tan 60°$

$\Rightarrow$ $A + B = 60°$ ...(i)

$\tan(A - B) = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan(A - B) = \tan 30°$

$\Rightarrow$ $A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$2A = 90°$

$\Rightarrow$ $A = \dfrac{90°}{2} = 45°$

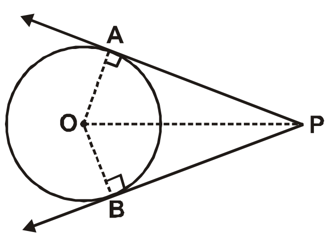
From (i),

$45° + B = 60°$

$\Rightarrow$ $B = 60° - 45° = 15°$

Hence, $\angle A = 45°$, $\angle B = 15°$

A-30. Given : A circle C(O, r). P is a point outside the circle 3 and PA and PB are tangents to a circle.



To prove : PA = PB

Construction : Join OA, OB and OP.

Proof : In ΔOAP and ΔOBP.

$\angle OAP = \angle OBP = 90°$

(Radius is perpendicular to the tangent at the point of contact).

OA = OB

(Radii of the same circle)

OP = OP (common)

$\therefore$ $\Delta OAP \cong \Delta OBP$

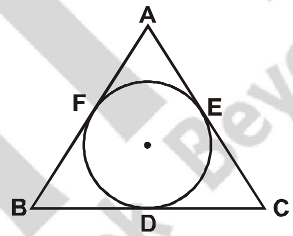
(RHS congruence rule)

$\Rightarrow$ PA = PB [CPCT]

Hence Proved

OR

Given : In an isosceles ΔABC, AB = AC, circumscribed a circle.



To Prove : BD = DC

Proof : AB = AC (Given) ...(i)

AF = AE

(Tangents from an external point A to a circle are equal) ...(ii)

Subtracting (ii) from (i), we get

AB – AF = AC – AE

$\Rightarrow$ BF = CE ...(iii)

Now, BF = BD

(Tangents from an external point B to a circle are equal)

Also, CE = CD

(Tangents from an external point C to a circle are equal)

$\Rightarrow$ BD = CD

$\therefore$ BC is bisected at the point of contact.

Hence Proved

A-31. Total number of coins

$= 100 + 50 + 20 + 10 = 180$

(i) Number of 50p coins = 100

$\therefore$ Probability of getting a 50p coin

$$= \frac{100}{180} = \frac{5}{9}$$

(ii) Number of ₹5 coins = 10

Number of coins other than ₹5 coins

$= 180 - 10 = 170$

$\therefore$ Probability of not getting ₹5 coin

$$= \frac{170}{180} = \frac{17}{18}$$

(iii) Number of ₹2 coins = 20

$\therefore$ Probability of getting ₹2 coin

$$= \frac{20}{180} = \frac{1}{9}$$

Probability of getting ₹2 coin

A-32. Let marks in Maths be x and marks in Science be y.

According to the first condition

$x + y = 39 \Rightarrow y = 39 - x$ ...(i)

$(x - 3)(x + 4) = 399$

$\Rightarrow xy + 4x - 3y - 12 = 399$

$\Rightarrow xy + 4x - 3y = 411$ [From (i)]

$\Rightarrow x(39 - x) + 4x - 3(39 - x) = 411$

$\Rightarrow 39x - x^2 + 4x - 117 + 3x = 411$

$\Rightarrow x^2 - 46x + 528 = 0$

$\Rightarrow x^2 - 24x - 22x + 528 = 0$

$\Rightarrow (x - 24)(x - 22) = 0$

Then $x = 24$

$y = 39 - 24 = 15$

and $x = 22$

$y = 39 - 22 = 17$

$\therefore$ Marks in Maths 24 and marks in Science 15 or marks in Maths 22 and marks in Science 17.

Let marks in Hindi be z

$$\frac{39 + z}{3} = 20$$

$39 + z = 60 \Rightarrow z = 21$

Hence, marks obtained in Hindi are 21.

OR

Let number of wickets taken by Arshdeep be x then number of wickets taken by Jadeja be (2x – 1).

According to the question

$x(2x - 1) = 15$

$\Rightarrow 2x^2 - x - 15 = 0$

$\Rightarrow 2x^2 - 6x + 5x - 15 = 0$

$\Rightarrow 2x(x - 3) + 5(x - 3) = 0$

$\Rightarrow (x - 3)(2x + 3) = 0$

$\Rightarrow x = 3$ or $x = \frac{-5}{3}$ (not possible)

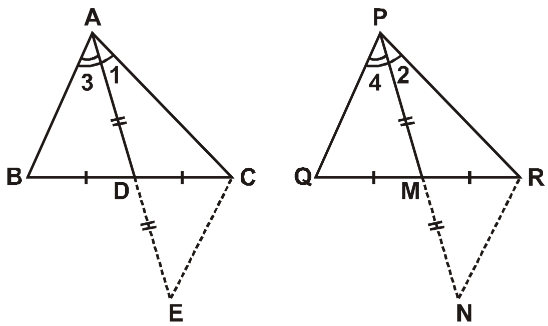
$\therefore$ Number of wickets taken by Arshdeep = 3.

Number of wickets taken by Jadeja

$= 2x - 1 = 2 \times 3 - 1 = 5$

A-33. **Given :** ΔABC and ΔPQR in which

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ where AD and PM are medians.



To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E and PM to N such that AD = DE and PM = MN and join EC and NR.

Proof : In ΔADB and ΔEDC

AD = ED [By construction]

$\angle ADB = \angle EDC$

[Vertically opposite angles]

BD = CD [As AD is median]

$\Delta ADB \cong \Delta EDC$

[SAS congruency rule]

$\therefore$ AB = EC [CPCT]

Similarly PQ = NR

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \text{[Given]}$$

$$\frac{EC}{NR} = \frac{AC}{PR} = \frac{\frac{1}{2}AE}{\frac{1}{2}PN}$$

$$\frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$$

$\Delta ACE \sim \Delta PRN$ [SSS similarlity]

Similarly $\angle 1 = \angle 2$

[Corresponding $\angle$s of similar Δ's are equal]

Similarly $\angle 3 = \angle 4$

$\therefore$ $\angle BAC = \angle QPR$

In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR}$$

and $\angle BAC = \angle QPR$

$\therefore$ $\Delta ABC = \Delta PQR$

[SAS similarity rule]

A-34.

Class Intervals	Cumulative frequency (c.f)	No. of Students frequency (f)
0 – 10	0	0
10 – 20	10	10
20 – 30	25	15
30 – 40	43	18
40 – 50	65	22
50 – 60	87	22
60 – 70	96	9
70 – 80	100	4
Total		100

$$\frac{N}{2} = \frac{100}{2} = 50$$

Here $l = 40$, f = 22, d = 43, h = 10

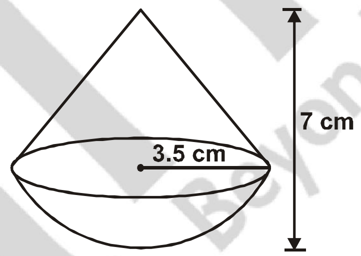
$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 40 + \left(\frac{50-43}{22}\right) \times 10$$

$$= 40 + \frac{7}{22} \times 10 = 40 + \frac{70}{22}$$

$= 40 + 3.18 = 43.18$ marks

A-35. Let radius of hemisphere be r, then radius of cone be r.



Radius of sphere, r = 3.5 cm

Height of cone, h = 7 cm – 3.5 cm

= 3.5 cm

Volume of toy

$$= \frac{2}{3}\pi r^2 + \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (2r + h)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5\ (2 \times 3.5 + 3.5)$$

$= 134.75\ cm^2$.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^2$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$

$= 179.67\ cm^3$

Difference of volume

$= 179.67\ cm^3 - 134.75\ cm^3$

= 44.92 cm3

OR

Radius of a lead shot, r = 2 cm

Dimensions of cuboid

= 18 cm × 8 cm × 6 cm

Volume of water = Volume of cuboid –

14 × volume of one lead shot

$= l \times b \times h - 14 \times \frac{4}{3}\pi r^3$

$= 18 \times 8 \times 6 - 14 \times \frac{4}{3}\pi r^3$

$= 18 \times 8 \times 6 - 14 \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$

$= 394.67\ cm^3$.

A-36. (i) Coordinates of A, B and C are A(3, 4), B(–4, 6) and C(2, 8)

Distance between A and B

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4-3)^2 + (6-4)^2}$

$= \sqrt{49+4} = \sqrt{53}$ units

(ii) Let y-axis be divided by the line AB in the ratio k : 1 at point (0, a)

$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$0 = \frac{k \times 3 + 1 \times (-4)}{(k+1)}$

$\Rightarrow 3k - 4 = 0$

k = 4/3

Hence, y-axis is divided by the line segment in the ratio 4 : 3.

(iii) Let point on y-axis equidistant from B and C be (0, y)

BP = CP

$\sqrt{(-4-0)^2 + (6-y)^2} = \sqrt{(2-0)^2 + (8-y)^2}$

On squaring both sides, we get

$16 + 36 + y^2 - 12y = 4 + 64 + y^2 - 16y$

$\Rightarrow$ 52 – 12y = 68 – 16y

$\Rightarrow$ y = 4

Point on y-axis which is equidistant form B and C is (0, 4).

OR

Mid point of AC $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{5}{2}, \frac{12}{2}\right) = \left(\frac{5}{2}, 6\right)$

Length of median through B

$= \sqrt{\left(\frac{5}{2} + 4\right)^2 + (6-6)^2}$

$= \frac{13}{2}$ units

A-37. Given series is 3, 5, 7

(i) Number of pots in each row from an AP, where a = 3, and = 2

Number of pots in 15th row will be given by

a_{15} = a + (15 – 1)d

= a + 14d

= 3 + 14 × 2 = 31

(ii) If $S_n = 120$, n = ?

$S_n = \frac{n}{2}[2 \times 3 + (n-1)d]$

$\Rightarrow 120 = \frac{n}{2}[6 + (n-1)2]$

$\Rightarrow \quad 120 = \frac{n}{2}(2n+4)$

$\Rightarrow \quad 120 = n^2 + 2n$

$\Rightarrow \quad n^2 + 2n - 120 = 0$

$\Rightarrow \quad n^2 + 12n - 10n - 120 = 0$

$\Rightarrow \quad (n + 12)(n - 10) = 0$

$\Rightarrow \quad n + 12 = 0$ or $n - 10 = 0$

$\Rightarrow \quad n = -12$ (rejected) or $n = 10$

If 120 pots are there then number of rows = 10.

(iii) If there are 21 rows, then 11th row will be middle row.

Number of pots in middle row

$a_{11} = a + 10d$

$= 3 + 10 \times 2 = 23$

OR

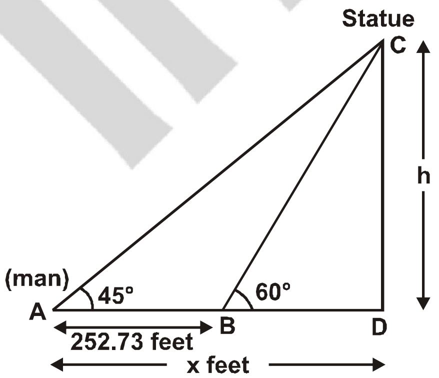
If two pots are increased in each row then number of pots in each row 5, 7, 9...

No. of plants in 10 rows

$= \frac{10}{2} \times [2 \times 5 + (10 - 1) \times 2]$

$= 5 \times [10 + 18] = 5 \times 28 = 140$

A-38. (i)



(ii) Let initial distance of man from statue be x feet and height of statue be h feet.

In ΔADC,

$\tan 45° = \frac{h}{x} \Rightarrow h \;\; x$

(iii) In ΔBDC,

$$\tan 60° = \frac{h}{x - 252.73}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{h - 252.73}$$

$$\Rightarrow \quad \sqrt{3}h = 252.73\sqrt{3} - h$$

$$\Rightarrow \quad (\sqrt{3} - 1)h = 252.73 \times \sqrt{3}$$

$$h = \frac{252.73 \times \sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{252.73(3 + \sqrt{3})}{2}$$

$$= \frac{252.73 \times 4.73}{2}$$

$= 597.70$ feet

Initial distance of man from statue = 597.70 feet.

OR

Let the distance from statue be y feet the angle elevation be 30°

$$\tan 30° = \frac{597.70}{y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{597.70}{y}$$

$$\Rightarrow \quad y = 597.70 \times \sqrt{3}$$

$= 1034.02$ feet

So, at distance 1034.02 feet from statue angle of elevation will be 30°.

MATHEMATICS – X

SAMPLE PAPER – 7 (SOLVED)

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. The sum of two irrational numbers

(a) is always irrational (b) is always rational

(c) can be rational or irrational (d) None of these

Q2. A quadratic equation is such that its roots are the HCF and LCM of the smallest prime number and the smallest composite number, then quadratic equation is

(a) $x^2 - 2x + 4 = 0$ (b) $x^2 - 6x + 8 = 0$ (c) $x^2 - 4x + 2 = 0$ (d) $x^2 - 8x + 6 = 0$

Q3. If (-3) is one of the zeroes of the quadratic polynomial, $(k-1)x^2 + kx - 3$, then the sum of the zeroes of the quadratic polynomial is

(a) 2 (b) 3 (c) 1 (d) –2

Q4. For what value of p will the following pair of linear equations are parallel ?

$3x - y - 5 = 0,\ 6x - 2y - p = 0$

(a) all real numbers except 10 (b) 10

(c) 5/2 (d) 1/2

Q5. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2, -5)$ is the mid point of PQ, then the coordinates of P and Q are respectively.

(a) $(0, -5)$ and $(2, 0)$ (b) $(0, 10)$ and $(-4, 0)$

(c) $(0, 4)$ and $(-10, 0)$ (d) $(0, -10)$ and $(4, 0)$

Q6. If $\Delta PQR \sim \Delta ABC$, PQ = 6 cm, AB = 8 cm and perimeter of ΔABC is 36 cm then perimeter of ΔPQR is

(a) 20.25 cm (b) 27 cm (c) 48 cm (d) 64 cm

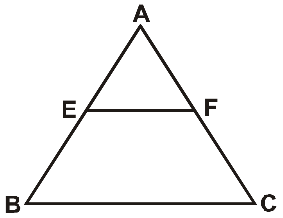
Q7. If cosec A – cot A – k, then the value of cosec A + cot A is

(a) 1 + k (b) $\frac{1}{k}$ (c) 1 – k (d) $1-\frac{1}{k}$

Q8. $\frac{2\tan 30°}{1-\tan^2 30°}$ is equal to

(a) cos 60° (b) sin 60° (c) tan 60° (d) sin 30°

Q9. In ΔABC, EF || BC, AB = 4 cm, AE = 1.8 cm, ∠C = ∠A, then the value of EF is

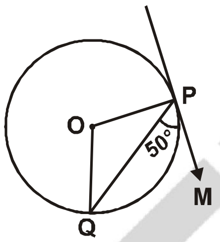


(a) 2 cm (b) 2.1 cm (c) 1.8 cm (d) 2.2 cm

Q10. XY is drawn parallel to the base BC of ΔABC cutting AB at X and AC at Y. If AB = 4BX and YC = 2cm, then the value of AY is

(a) 2 cm (b) 4 cm (c) 6 cm (d) 8 cm

Q11. In the given figure, O is the centre of a circle and PQ is the chord. If the tangent PR at P makes and angle of 50° with PQ. Then ∠POQ is



(a) 100° (b) 80° (c) 90° (d) 75°

Q12. A wire, is in the form of a circle of radius 14cm, bent to a square then the side of square into which it can be sent is

(a) 22 cm (b) 2π cm (c) (π + 14) cm (d) 11 cm

Q13. If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5 cm and length 6 cm, then the volume of water which flows out of cylindrical cup is

(a) 38.808 cm^3 (b) 55.4 cm^3 (c) 19.4 cm^3 (d) 471.4 cm^3

Q14. For the following distribution

Class Interval	0 – 10	10 – 20	20 – 30	30 – 40
Frequency	15	18	11	16

the sum of lower limit of median class and lower limit of modal class is

(a) 30 (b) 10 (c) 20 (d) 40

Q15. If the perimeter and the area of a circle are numerically equal, then the radius of circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

Q16. Which one of the following is the correct relationship between Mean, Median and Mode ?

(a) Mode = 2 Median – 3 Mean (b) Mode = Median – 2 Mean

(c) Mode = 2 Median – Mean (d) Mode = 3 Median – 2 Mean

Q17. A card is drawn from a deck of 52 cards. The probability of getting a king or spade is

(a) $\frac{1}{52}$ (b) $\frac{1}{26}$ (c) $\frac{15}{52}$ (d) $\frac{4}{13}$

Q18. The value of $\frac{\tan 60°}{\cot 30°} + \frac{\sec 30°}{\operatorname{cosec} 60°}$ is

(a) 0 (b) 1 (c) 2 (d) 3

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** If HCF of two numbers is 'p' and LCM is q, then $\frac{q}{p}$ is always a natural number.

Statement R (Reason) : The HCF is always factor of LCM.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is the false but reason (R) is true.

Q20. **Statement A (Assertion) :** The points (1, –3), (2, 3) and (–4, 6) are collinear.

Statement R (Reason) : Three points A, B and C are collinear if AB + BC = AC.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

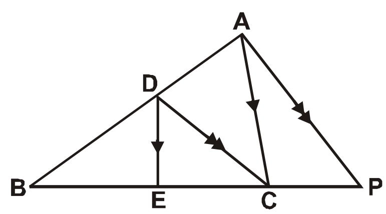
(d) Assertion (A) is the false but reason (R) is true.

SECTION - B

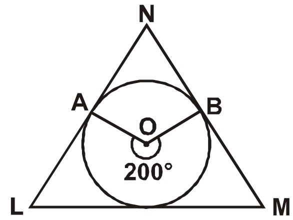
Section B consists of 5 questions of 2 marks each.

Q21. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

Q22. In the adjoining figure, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.



Q23. In the figure below, a circle with centre O is inscribed inside ΔLMN. A and B are the points of tangency. Find $\angle ANB$. Show your steps.



Q24. If $\cos(A + 2B) = 0,\ 0° \le (A + 2B) \le 90°$ and $\cos(B - A) = \frac{\sqrt{3}}{2},\ 0° \le (B - A) \le 90°$, then find $\text{cosec}\,(2A + B)$. Show your work.

OR

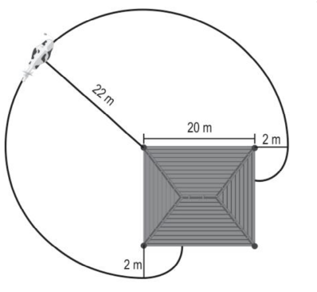
State whether the following statements are true or false. Give reasons.

(i) At the value of $\sin\theta$ increases, the value of $\tan\theta$ decreases.

(ii) When the value of $\sin\theta$ is maximum, the value of $\text{cosec}\,\theta$ is also maximum.

(Note $0° < \theta < 90°$).

Q25. A cow is tied at one of the corners of a square shed. The length of the rope is 22 m. The cow can only eat the grass outside the shed as shown below.



What is the area that the cow can graze on ? Show your steps.

OR

The perimeter of sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

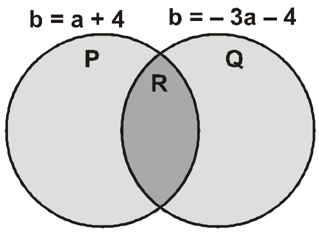
SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Find all pairs of positive integers whose sum is 91 and HCF is 13. Show your work.

Q27. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation.

Q28. The two circles represent the ordered pairs, (a, b), which are solutions of the respective equations. The circles are divided into 3 regions P, Q and R. Show your work.



OR

Shown below is a pair of linear equations.

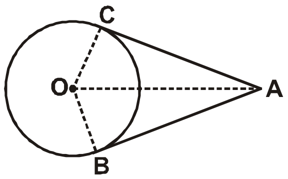
$x + 0.999y = 2.999$

$0.999x + y = 2.998$

(i) Without finding the values of x and y, prove that $x - y = 1$.

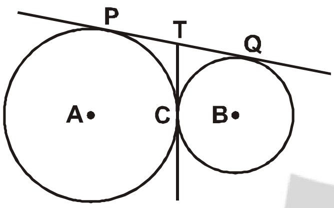
(ii) Find the values of x and y. Show your work.

Q29. Given below is the diagram of pair of pulleys. The length of AC is 12 cm. In the given figure, $\angle CAB = 20°$. What is the measure of $\angle AOC$?



OR

In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Q30. Prove that : $\dfrac{\operatorname{cosec}^2 x - \sin^2 x \cot^2 x - \cot^2 x}{\sin^2 x} = 1$

Q31. Arti owns a manufacturing company. She hires 5 supervisors and 20 operators of a 6-months project. The table given below shows their salary backup.

Position	Salary for the two months	Salary for the remaining four months
Supervisor	Between ₹18,000 to ₹20,000	Between ₹22,000 to ₹25,000
Operator	Between ₹8,000 to ₹10,000	Between ₹13,000 to ₹15,000

The mean salary of five supervisors for the first two months is ₹19,000.

The salary of three supervisors are ₹18,000; ₹18,500 and ₹20,000 respectively. Find the sum of other two supervisor's salary for first two months.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

The difference of two numbers is 5 and the difference of their reciprocal is $\frac{1}{10}$. Find the numbers.

Q33. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarly criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Q34. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. How many litres of water is left in the cylinder, if the radius of the cylinder is 60 cm and its length is 180 cm.

OR

The cost of fencing a circular field at the rate of ₹24 per m is ₹5280. The field is to be ploughed at the rate of ₹0.50 per m^2. Find the cost of ploughing the field.

Q35. The students of Class X of a school decided to donate their pocket money to purchase mineral water bottles for the people using contaminated water in a nearby village. They packed the mineral water bottles in different boxes. These boxes contained varying number of mineral water bottles. The following table shows the distribution of mineral water bottles according to the number of boxes :

No. of mineral water bottles	No. of boxes
50 – 52	20
53 – 55	120
56 – 58	105
59 – 61	125
62 – 64	30

Find the mean number of mineral water bottles kept in a packing box.

SECTION - E

Case study based questions are compoulsory.

Q36. A leading LED TV manufacturing company manufactures 18000 LED TVs in the second year and 19800 LED Tvs in tenth year. Assuming that the company increases the manufacturing of LED TV uniformly every year by fixed numbers.

Based on the above answer the following :

(i) How much, the manufacturing of LED TV is increased every year ?

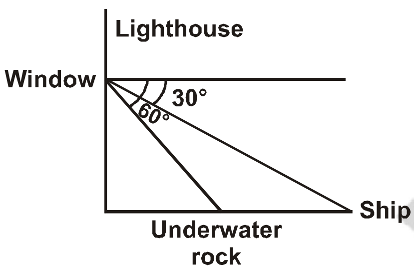
(ii) How many LED TVs were manufactured in the seventh year ?

(iii) How many LED TVs were manufactured in ten years ?

OR

If company is 12 year old, find number of LED TVs produced in last 3 years.

Q37. A lighthouse 1000 feet high is situated at the edge of the sea. From the window at the middle of the lighthouse, the guard can see an underwater rock and a ship making the angles of depression 60° and 30° respectively. The ship is behind the underwater rock exactly and come towards it in a straight line.



Based on the above, answer the following :

(i) Find the distance between underwater rock and base of lighthouse.

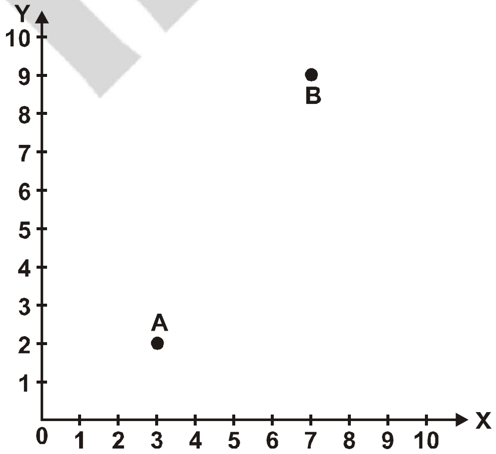
(ii) Find the distance between ship and underwater rock.

(iii) If the speed of ship is 3 feet/s, then find the time taken by ship to collide with underwater rock.

OR

Find the initial between guard and ship.

Q38. On the occasion of children's day in a school, sports are organised. Kavita and Pooja are standing at points A and B whose coordinates are (3, 2) and (7, 9) respectively. Jitender fixes are country flag at the mid point (M) of the line joining the points A and B.



Based on the above, answer the following :

(i) Find distance between Kavita and Pooja.

(ii) Find the coordinates of flag (M).

(iii) If M divides AB internally in the ratio of 2 : 1 i.e., $\frac{AM}{MB} = 2$, then where is the flag ported?

OR

Find the distance of both Kavita and Pooja from the origin.

MATHEMATICS – X

SOLUTIONS : SAMPLE PAPER – 7

A-1. (c) The sum of two irrational numbers can be rational or irrational.

A-2. (b) Smallest prime number HCF = 2

Smallest composite number LCM = 4

Quadratic equation

x^2 – (Sum of zeroes)x + Product of zeroes = 0

$x^2 - 6x + 8 = 0$

A-3. (d) The polynomial is $(k-1)x^2 + kx - 3$

Since x = –3 is the zero of given polynomial.

So $(k-1)(-3)^2 + k(-3) - 3 = 0$

$\Rightarrow 9k - 9 - 3k - 3 = 0$

$\Rightarrow 6k = 12$

$\Rightarrow k = 2$

So polynomial is $x^2 + 2x - 3$

Sum of zeroes = $-\frac{b}{a} = -2$

A-4. (a) Given lines are

$3x - y - 5 = 0$

and $6x - 2y - p = 0$

Since lines are parallel

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p}$

$\Rightarrow \frac{1}{2} \neq \frac{5}{p}$

$\Rightarrow p \neq 10$

p = all real numbers except 10.

A-5. (d) Let the piont where line intersect the y-axis be P(0, b) and the point where the line intersect the x-axis be Q(a, 0) and point of PQ = (2, 5)

By mid-point formula

$\Rightarrow \left(\frac{0+a}{2}, \frac{b+0}{2}\right) = (2, -5)$

On comparing, $\frac{0+a}{2} = 2 \Rightarrow a = 4$

and $\frac{b+0}{2} = 5 \Rightarrow b = -10$

So, coordinates of points P and Q are (0, –10) and (4, 0) respectively.

A-6. (b) Given, $\Delta PQR \sim \Delta ABC$

$$\frac{PQ}{AB} = \frac{PQ+QR+PR}{AB+BC+AC}$$

$$\frac{6}{8} = \frac{\text{perimeter of } \Delta PQR}{36}$$

Perimeter of $\Delta PQR = \frac{6 \times 36}{8} = 27$cm

A-7. (b) Given, cosec A – cot A = k

We know $\text{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow (\text{cosec}A - \cot A)(\text{cosec}A + \cot A) = 1$

$\Rightarrow k(\text{cosec}A + \cot A) = 1$

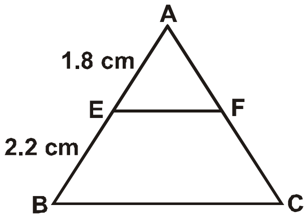
$\Rightarrow (\text{cosec}A + \cot A) = \frac{1}{k}$

A-8. (c) $\frac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

A-9. (c)



In ΔAEF and ΔABC, EF || BC

$\angle AEF = \angle ABC$ (Corresponding angles)

$\angle A = \angle A$ (common)

$\angle AFE = \angle ACB$ (Corresponding angles)

So $\Delta AEF \sim \Delta ABC$ (AAA criteria)

$$\Rightarrow \frac{AE}{EF} = \frac{AB}{BC} \quad (\because \text{ CPST})$$

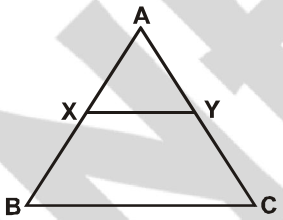
$$\frac{1.8}{EF} = \frac{BC}{BC}$$

(since $\angle A = \angle C$, $\therefore$ AB = BC)

$$\Rightarrow \frac{1.8}{EF} = 1$$

EF = 1.8 cm

A-10. (c)



Given, AB = 4BX

YC = 2 cm

Now, XY || BC (Given)

$$\frac{AX}{BX} = \frac{AY}{YC} \quad (\because \text{ BPT})$$

Adding 1 on both side

$$\frac{AX}{BX} + 1 = \frac{AY}{YC} + 1$$

$$\Rightarrow \frac{AX + BX}{BX} = \frac{AY + YC}{YC}$$

$$\Rightarrow \frac{AB}{BX} = \frac{AC}{YC}$$

$$4 = \frac{AC}{YC} \quad \text{(As AB = 4BX)}$$

AC = 8 cm

AY = AC – YC

= 8 cm – 2 cm = 6 cm

A-11. (a) $\angle OPR = 90°$

(Tangent is perpendicular to radius at the point of contact)

$\angle QPR = 50°$ (Given)

$\angle OPQ = \angle OPR - \angle QPR$

$= 90° - 50° = 40°$

In ΔOPQ, OP = OQ (Radii of a circle)

$\therefore \angle OPQ = \angle OQP = 40°$

$\angle OPQ + \angle OQP + \angle POQ = 180°$

(Angle sum property of a triangle)

$\therefore$ $40° + 40° + \angle POQ = 180°$

$\Rightarrow$ $\angle POQ = 180° - 80° = 100°$

A-12. (a) Perimeter of square = circumference of circle

$4 \times \text{side} = 2\pi r$

$$4 \times \text{side} = 2 \times \frac{22}{7} \times 14$$

side = 22 cm

A-13. (a) Volume of water flows out = Volume of the marble

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$= 8.8 \times 4.41$

$= 38.808 \text{ cm}^3$.

A-14. (c)

Class Interval	Frequency	c.f.
0 – 10	15	15
10 – 20	18	33
20 – 30	11	44
30 – 40	16	60

$\frac{n}{2} = \frac{60}{2} = 30$

Median class = 10 – 20

Modal class = 10 – 20

Sum of lower limits of median class and modal class = 10 + 10 = 20

A-15. (a) According to the question,

$2\pi r = \pi r^2$

r = 2 units

A-16. (d) Correct relationship is

Mode = 3 Median – 2 Mean

A-17. (d) Number of king or spade

= 4 + 22 (as king of spade is counted with kings)

= 16

P(getting a king or spade)

$= \frac{16}{52} = \frac{4}{13}$

A-18. (c) $\frac{6an60°}{\cot 30°} + \frac{\sec 30°}{\cos ec60°}$

$= \frac{\sqrt{3}}{\sqrt{3}} + \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 1 + 1 = 2$

A-19. (a) Since we know that HCF is always the factor of LCM. So here q is LCM and p is HCF $\Rightarrow \frac{q}{p}$ is a natural number. So, Assertion (A) is true and Reason (R) is true and correct explanation of A.

A-20. (d) A(1, –3), B(2, 3) and C(– 4, 6)

$AB = \sqrt{(2-1)^2 + (3+3)^2}$

$= \sqrt{37}$ Units

$BC = \sqrt{(-4-2)^2 + (6-3)^2}$

$= \sqrt{36+9} = \sqrt{45}$ units

$AC = \sqrt{(-4-1)^2 + (6+3)^2}$

$= \sqrt{25+81} = \sqrt{106}$ units

$AB^2 \neq AB^2 + BC^2$

So, Assertion (A) is false but reason (R) is true.

A-21. Let the assume to the .... that $5+2\sqrt{7}$ is rational then $5+2\sqrt{7}$ and eqn form $\frac{p}{q}$ where p and q are is .... and $q \neq p$.

$\Rightarrow \frac{p}{q} = 5+2\sqrt{7}$

$\Rightarrow \frac{p}{q} - 5 = 5\sqrt{7}$

$\Rightarrow \frac{p-5q}{2q} = \sqrt{7}$

$\frac{p-5q}{2q}$ is rational as p and q are undergoes this coordinate the given fact that $\sqrt{7}$ is irrational.,

∴ Our assumption is wrong.

$5+5\sqrt{7}$ is irrational. Proved.

A-22. In ΔABP,

DC || AP [Given]

$\therefore \frac{BD}{DA} = \frac{BC}{CP}$ (From BPT) ...(i)

In ΔABC,

DE || AC [Given]

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (From BPT) ..(ii)

From equation (i) and (ii), we have

$\frac{BE}{EC} = \frac{BC}{CP}$ Hence Proved.

A-23. Reflex $\angle AOB$ + minor $\angle AOB = 360°$ (Complete Angle)

So, minor $\angle AOB = 360° - 260°$

$= 100°$

$OA \perp LN$ (point of tangency)

Similarly,

$OB \perp NM$

$\therefore \quad \angle OAN = 90°$

and $\quad \angle OBN = 90°$

Now, in quadrilateral NAOB

$$\angle NAO + \angle AOB + \angle OBN + \angle BNA = 360°$$

$$90° + 100° + 90° + \angle BNA = 360°$$

$$\angle ANB = 360° - 280° = 80°$$

Hence, $\angle ANB = 80°$

A-24. $\cos(A + 2B) = 0$ (given)

$\Rightarrow \cos(A + 2B) = \cos 90°$ $(\because \cos 90° = 0)$

$\Rightarrow \quad A + 2B = 90° \quad$ ...(i)

Now, $\cos(B - A) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \cos(B - A) = \cos 30°$

$B - A = 30° \quad$ ...(ii)

Add both the equations

$A + 2B + B - A = 90° + 30°$

$\Rightarrow \quad 3B = 120°$

$\Rightarrow \quad B = 40°$

Substitute value of B in equation (i)

$A + 2 \times 40° = 90°$

$A = 10°$

Thus, $\text{cosec}(2A + B) = \text{cosec}(2 \times 10 + 40)$

$= \text{cosec } 60°$

$= \dfrac{2}{\sqrt{3}}$

Hence, $\text{cosec}(2A + B) = \dfrac{2}{\sqrt{3}}$

OR

(i) False

We know that

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

So, $\tan\theta$ is directly proportional to $\sin\theta$.

Hence, if $\sin\theta$ increases the value of $\tan\theta$ also increases.

(ii) False

As $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$

So, $\sin\theta$ is inversely proportional to $\text{cosec}\theta$.

Hence if $\sin\theta$ is maximum value the value of $\text{cosec}\theta$ is not maximum.

A-25. Total area cow can grazed = (3 quarters sector with radius 22m) + (2 quarters sector with radius 2m)

$$\therefore \text{ Total area} = \left[\frac{3}{2}\pi \times 484 + \frac{1}{3}\pi \times 4\right]$$

$$= \left(\frac{3}{4}\pi \times 484 + \frac{1}{2}\pi \times 4\right)$$

$$= 363\pi + 2\pi$$

$$= 365\pi \text{ m}^2$$

OR

Given, radius of circle (r) = 5.2 cm

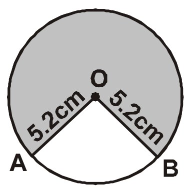
i.e, $\quad OA = OB = r = 5.2$ cm

and the perimeter of a sector = 16.4 cm

As we know that perimeter of the sector

$$= 2r + \frac{2\pi r\theta}{360°}$$

$$\Rightarrow \quad 16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360°}$$



$\Rightarrow \quad \frac{2\pi \times 5.2 \times \theta}{360°} = 6$

$\Rightarrow \quad \theta = \frac{6 \times 360°}{2\pi \times 5.2}$

Now, area of sector

$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{6 \times 360°}{2\pi \times 5.2 \times 360°} \times \pi \times (5.2)^2$

= 15.6 sq. units

A-26. Let pair of numbers be 'a' and 'b'

$\therefore$ HCF (a, b) = 13 (Given)

So, a and b be will be of the form

a = 13 x

b = 13y

(where x and y are co-primes)

Now a + b = 91 (Given)

So, 13x + 13y = 91

x + y = 7

Now all possible values of x and y are :

1 and 6

2 and 5

3 and 4

So, all values of a and b are

13 and 78

26 and 65

39 and 52

A-27. $3x^2 + px + 4 = 0$

$\because \frac{2}{3}$ is a root so it must satisfy the given equation

$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$

$\frac{4}{3} + \frac{2p}{3} + 4 = 0$

On solving, we get

p = – 8

$3x^2 - 8x + 4 = 0$

$3x^2 - 6x - 2x + 4 = 0$

$3x(x - 2) - 2(x - 2) = 0$

$x = \frac{2}{3}$ or x = 2

Hence, x = 2

So, the other root is 2.

A-28. From figure given

P $\Rightarrow$ (b = a + 4)

If a = 0, then b = 4

$\therefore$ Ordered pair of P(0, 4)

Q $\Rightarrow$ (b = – 3a – 4)

If a = 0, then b = – 4

$\therefore$ Ordered pair of P (0, 4)

R $\Rightarrow$ Equate both P and Q equation as LHS is equal

b = a + 4

b = – 3a – 4

a + 4 = – 3a – 4

a + 3a = – 4 – 4

4a = – 8

a = – 2

Now, if a = – 2

b = – 2 + 4 = 2

Thus ordered pair of R = (–2, 2)

OR

(i) x + 0.999y = 2.999

0.999x + y = 2.998

(–) (–) (–)

0.001x – 0.001y = 0.001

0.001(x – y) = 0.001

Thus x – y = 1 ...(i)

(ii) Add both the equation

x + 0.999y = 2.999

0.999x + y = 2.998

1.999x + 1.999y = 5.997

1.999(x + y) = 5.997

$\Rightarrow \quad x + y = 3 \qquad ...(ii)$

By solving (i) and (ii) simultaneously, we get

$$\begin{aligned} x - y &= 1 \\ x + y &= 3 \\ \hline 2x &= 4 \\ x &= 2 \end{aligned}$$

Equating value of x, we get

$$\begin{aligned} 2 - y &= 1 \\ -y &= 1 - 2 \\ -y &= -1 \\ y &= 1 \end{aligned}$$

Thus x = 2 and y = 1

A-29. Angle between two tangents = 20°

As tangents are equally inclined to each other.

$\therefore \quad \angle CAO = \angle BAO = 10°$

Now, in ΔAOC

$$\angle CAO + \angle AOC + \angle ACO = 180°$$

Here $\angle ACO = 80°$

(Tangent at any point of a circle is perpendicular to the radius through the point of contact)

So, $10° + 90° + \angle AOC = 180°$

$\therefore \quad \angle AOC = 180° - 100°$

$= 80°$

Here $\angle AOC = 80°$.

OR

Since, PT = TC

and QT = TC

[Tangent of circle from external point]

So, PT = QT

Now PQ = PT + TQ

$\Rightarrow$ PQ = PT + PT

$\Rightarrow$ PQ = 2PT

$\Rightarrow \quad \frac{1}{2}PQ = PT$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

A-30. $\frac{\operatorname{cosec}^2 x - \sin^2 x \cot^2 x - \cot^2 x}{\sin^2 x} = 1$

$$\text{LHS} = \frac{1 - \sin^2 x \cot^2 x}{\sin^2 x}$$

$(\because \operatorname{cosec}^2 x - \cot^2 x = 1)$

$$= \frac{1}{\sin^2 x} - \cot^2 x$$

$$= \operatorname{cosec}^2 x - \cot^2 x$$

$$= 1$$

Thus, LHS = RHS Hence Proved

A-31. Mean salary of 5 supervisors for first two months = ₹19,000

$$\Rightarrow \frac{\text{Sum of the salaries of 5 supervisors}}{\text{Total no. of supervisiors}} = 19,000$$

$$\Rightarrow \frac{18,000 + 18,500 + 20,000 + 2 \text{ supervisor's salary}}{5} = 19,000$$

$\Rightarrow$ 56,500 + 2 supervisor's salary = 95,000

Hence, 2 supervisor's salary

= 95,000 – 56,500 = ₹38,500

A-32. Let the speed of the train be x km/h

Distance travelled = 360 km

$\therefore$ Time taken $= \frac{360}{x}$ hours

The speed of the train becomes (x + 5) km/h, if the speed had been 5 km/h more.

Distance = 360 km

$\therefore$ Time taken $= \frac{360}{x+5}$ hours

According to question,

$$\frac{360}{x} = \frac{360}{x+5} + 1$$

$$\Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1$$

$\Rightarrow$ $1800 = x^2 + 5x$

$\Rightarrow$ $x^2 + 5x - 1800 = 0$

$\Rightarrow$ $x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow$ $(x + 45)(x - 40) = 0$

$\Rightarrow$ $x = -45$ or $x = 40$

Rejecting $x = -45$

$\therefore$ Speed of the train = 40 km/h

OR

Let the two numbers be x and x – 5

According to question,

$$\frac{1}{x-5} - \frac{1}{x} = \frac{1}{10} \quad \left(\text{since } \frac{1}{x-5} \geq \frac{1}{x}\right)$$

$$\Rightarrow \frac{x - x + 5}{(x-5)x} = \frac{1}{10}$$

$\Rightarrow$ $(x - 5)x = 50$

$\Rightarrow$ $x^2 - 5x - 50 = 0$

$\Rightarrow$ $(x - 10)(x + 5) = 0$

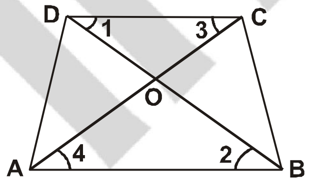
$\Rightarrow$ $x = 10$ or $x = -5$

when $x = 10$, then $x - 5 = 10 - 5 = 5$

When $x = -5$, then $x - 5 = -5 - 5 = 10$

Thus, the required numbers are either 10 and 5 or – 5 and – 10

A-33. **Given :** Diagonals AC and BD intersect at O.



AB || DC

To Prove : $\frac{OA}{OC} = \frac{OB}{OD}$

Proof : In ΔAOB and ΔCOD

$\angle 1 = \angle 2$

$\angle 3 = \angle 4$ [Alternate angles]

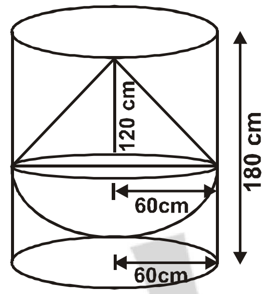
$\therefore$ $\Delta AOB = \Delta COD$ [Alternate angles]

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

[Corresponding sides of similar triangles]

A-34. Radius of cone = 60 cm

Height of cone = 120 cm



$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (60)^2 \times 120$

$= 144000\pi \text{ cm}^3$

Radius of hemisphere = 60 cm

$\therefore$ Volume of hemisphere

$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (60)^3$

$= 144000\pi \text{ cm}^3$

$\therefore$ Volume of solid = Volume of cone + Volume of hemisphere

$= 144000\pi \text{ cm}^3 + 144000\pi \text{ cm}^3$

$= 288000\pi \text{ cm}^3$

Now, volume of cylinder = $\pi r^2 h$

$= \pi \times (60)^2 \times 180 = 648000\pi \text{ cm}^3$.

Volume of water left in the cylinder = Volume of cylinder – Volume of solid

$= 648000\pi \text{ cm}^3 - 288000\pi \text{ cm}^3$

$= 360000\pi \text{ cm}^3$

$= 360000 \times \frac{22}{7} \text{cm}^3$

$= 1131428.57 \text{ cm}^3$

$= \frac{1131428.57}{1000} l = 1131.42\ l$

OR

₹24 is the cost for fencing 1m of circular field.

₹5280 is the cost for fencing $= \frac{1}{24} \times 5280$

= 220 m of circular field

Circumference of the field = 220 m

$\Rightarrow \quad 2\pi r = 220$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{44} = 35$ m

$\therefore$ Area of the field $= \pi r^2 = \pi(35)^2$

$= 1225\pi \text{ m}^2$

Cost of ploughing the field

= ₹$1225\pi \times 0.50$ = ₹1925

A-35. Let A = 57, h = 3

Number of mineral water bottles	Number of boxes (f_i)	Class marks (x_i)	$n_i = \frac{x_i - A}{h}$	$f_i u_i$
49.5 – 52.5	20	51	–2	–40
52.5 – 55.5	120	54	–1	–120
55.5 – 58.5	105	57 = A	0	0
58.5 – 61.5	125	60	1	125
61.5 – 64.5	30	63	2	60
Total	n = 400			$\Sigma f_i u_i = 25$

Here A = 57, h = 3, n = 400 and $\Sigma f_i u_i = 25$

By Step-deviation method,

Mean, $\bar{x} = A + h \times \frac{1}{n} \times \Sigma f_i u_i$

$= 57 + 3 \times \frac{1}{400} \times 25$

$= 57 + \frac{75}{400} = 57 + 0.1875$

= 57.1875=~~57.19 (approx)~~

A-36. (i) $a_2 = 18000 \Rightarrow a + d = 18000$

$a_{10} = 19800 \Rightarrow a + 9d = 19800$

$8d = 1800$

$d = 225$

$a = 17775$

Manufacturing increases every year = 225

(ii) $a_7 = a + 6d$

$= 17775 + 6 \times 225$

$= 17775 + 1350$

$= 19125$

(iii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 17775 + 9 \times 225]$

$= 187875$

OR

Number of LED produced in last 3 years $= S_{12} - S_9$

$= \frac{12}{2}(2a + 11d) - \frac{9}{2}(2a + 8d)$

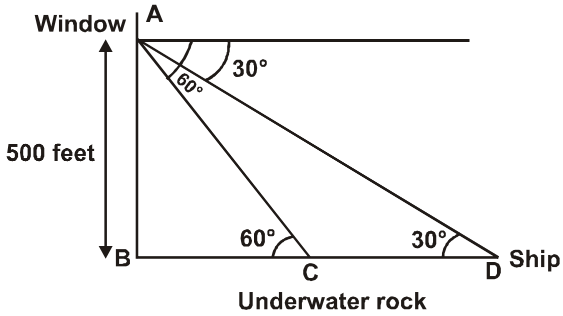
$= \frac{1}{2}(24a + 132d - 18a - 72d)$

$= \frac{1}{2}(6a + 60d) = 3a + 30d$

$= 3 \times 17775 + 30 \times 225$

$= 60075$

A-37. (i) In ΔABC,



$$\frac{500}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{500}{BC} = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{500}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$$

$$\Rightarrow \quad BC = \frac{500 \times 1.73}{3} = 288.33 \text{ feet}$$

Hence, distance between understand rock and base of lighthouse is 288.33 feet (approx).

(ii) In ΔABD,

$$\frac{500}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = 500\sqrt{3} = 500 \times 1.73$$

$$= 865 \text{ feet}$$

$\therefore$ Distance between ship and rock

$$= 865 \text{ feet} - 288.33 \text{ feet}$$

$$= 376.67 \text{ feet}$$

(iii) $$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{DC}{\text{Speed}}$$

$$= \frac{576.67}{3}$$

$$= 192.22 \text{ seconds}$$

OR

In ΔABD,

$$\frac{500}{AD} = \frac{1}{2}$$

$$\Rightarrow \quad AD = 1000 \text{ feet}$$

Distance between guard and ship = 1000 feet.

A-38. (i) The given points are : A(3, 2) and B(7, 9), then

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7-3)^2 + (9-2)^2}$$

$$= \sqrt{4^2 - 7^2} = \sqrt{16+49}$$

$$= \sqrt{65} \text{ units}$$

(ii) Coordinates of flag

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(5, \frac{11}{2}\right)$$

(iii) AM = BM = 2 : 1

Coordinates of point M

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{14+3}{3}, \frac{18+2}{3}\right) = \left(\frac{17}{3}, \frac{20}{3}\right)$$

OR

$$AO = \sqrt{(3-0)^2 + (2-0)^2}$$

$$= \sqrt{9+4} = \sqrt{13} \text{ units}$$

Distance of Kavita from origin

$$= \sqrt{13} \text{ units}$$

$$BO = \sqrt{(7-0)^2 + (9-0)^2}$$

$$= \sqrt{49+81} = \sqrt{130} \text{ units}$$

Distance of Pooja from origin

$$= \sqrt{130} \text{ units}$$

X – MATHEMATICS

SAMPLE PAPER – 8 (SOLVED)

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. 4 bells toll together at 9.00 am. They toll after 7, 9, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours ?

(a) 3 (b) 4 (c) 5 (d) 6

Q2. A quadratic polynomial whose zeroes are $\frac{3}{5}$ and $-\frac{1}{2}$ are ________.

(a) $10x^2 - x - 3$ (b) $10x^2 + x - 3$ (c) $10^2 - x + 3$ (d) none of these

Q3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = 1$ (c) $a = 2, b = -6$ (d) $a = 0, b = -6$

Q4. Three runners running around a circular track, can complete one revolution in 2, 3 and 4 hrs respectively. They will meet again at the starting point after

(a) 8 hrs (b) 6 hrs (c) 12 hrs (d) 18 hrs

Q5. If A and B are the points (–3, 4) and (2, 1) respectively, then the coordinates of the point on AB produced such that AC = 2BC are

(a) (2, 4) (b) (3, 7) (c) (7, – 2) (d) none of these

Q6. What is the largest number that divides each one of 1152 and 1664 exactly ?

(a) 32 (b) 64 (c) 128 (d) 256

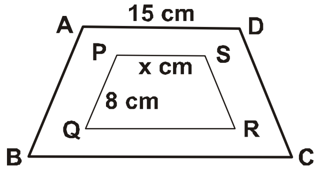
Q7. In right triangle, $\angle B = 90°$, AB = 24 cm, BC = 7 cm, then cos C =

(a) $\frac{7}{24}$ (b) $\frac{24}{25}$ (c) $\frac{25}{24}$ (d) $\frac{7}{25}$

Q8. In ΔABC, $\angle C = 90°$, then $\tan A + \tan B =$

(a) $\frac{b^2}{ac}$ (b) $a + b$ (c) $\frac{a^2}{bc}$ (d) $\frac{c^2}{ab}$

Q9. If quadrilateral ABCD and PQRS are similar, then x =



(a) 4 cm (b) 5 cm (c) 6 cm (d) 7 cm

Q10. Distance between two parallel tangents is 14cm, then the radius of circle is

(a) 6 cm (b) 7 cm (c) 12 cm (d) 14 cm

Q11. $\sin 45° - \cos 45°$ is equal to

(a) $2 \cos \theta$ (b) 0 (c) $2 \sin \theta$ (d) 1

Q12. The length of the minute hand a wall clock is 7 cm, then how much area does it sweep in 20 minutes ?

(a) 51 cm^2 (b) 49.33 cm^2 (c) 51.33 cm^2 (d) 52 cm^2

Q13. The curved surface area of a cylinder of height 14 cm is 88 cm^2, then diameter of the cylinder is

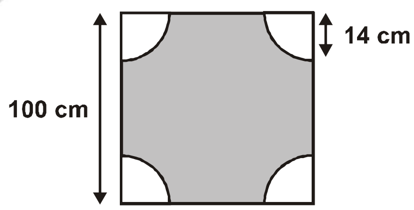
(a) 8.5 cm (b) 1 cm (c) 1.5 cm (d) 2 cm

Q14. The relationship between mean, median and mode for a moderately skewed distribution is

(a) mean = median – 2 mean (b) mode = 3 median – 2 mean

(c) mode = 2 median – 3 mean (d) mode = median – mean

Q15. In figure, at each corner of square side 100 cm, a quadrant of radius 14 cm is formed, then area of shaded region is



(a) 9834 cm^2 (b) 9348 cm^2 (c) 9384 cm^2 (d) 9884 cm^2

Q16. The mean age of combined group of men and women is 30 years. If the mean of the age of men and women are respectively 32 and 27, then the percentage of women in the group is

(a) 30 (b) 20 (c) 50 (d) 40

Q17. Radius of circumcircle of a triangle ABC is $5\sqrt{10}$ units. If point P is equidistant from A(1, 3), B(–3, 5) and C(5, –1), then AP =

(a) 5 units (b) $5\sqrt{5}$ units (c) 25 units (d) $5\sqrt{10}$ units

Q18. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** Pair of linear equations : $9x + 3y + 12 = 0$, $8x + 6y + 24 = 0$ have infinitely many solutions.

Statement R (Reason) : Pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** PA and PB are two tangents to a circle with centre O, such that $\angle AOB = 110°$, then $\angle APB = 90°$.

Statement R (Reason) : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

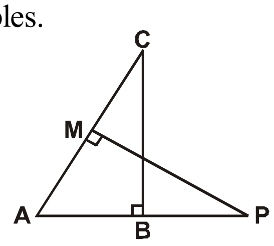
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. 5 books and 7 pens together cost Rs. 79, whereas 7 books and 5 pens together cost Rs. 77. Represent this situation in the form of linear equation in two variables.

Q22. Amandeep draws two right-angled triangle ABC and AMP right-angled at B and M respectively, as shown in figure.

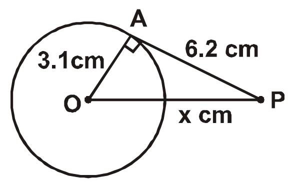


Prove that :

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Q23. In the given figure, O is the centre of the circle. The radius of the circle is 3.1 cm and PA is a tangent drawn to the circle from point P. If OP = x cm and AP = 6.2 cm, then find the value of x.



OR

AB is a tangent drawn from a point A to a circle with centre O and BOC is a diameter of the circle such that $\angle AOC = 110°$. Find $\angle OAB$.

Q24. Find the area of the quadrant of a circle whose circumference is 44 cm.

Q25. If $\tan\theta = \frac{1}{\sqrt{3}}$, then evaluate $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.

OR

If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, find A and B.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Manju and Manish participate in a cycle race, organised for National integration. Manju takes 18 minutes to complete one round, while Manish takes 12 minutes for the same. Suppose they both start at the same time and go in the same direction. After how many minutes, will they meet again at the starting point ?

Q27. Solve for x : $\frac{x-2}{x-4} + \frac{x-6}{x-8} = 6\frac{2}{3}$, $(x \neq 4, 8)$

Q28. Abhishek is planning a journey by ship to Andaman. Andaman trip in itself is an advaenture. There are three port in India from where you can sail to Andaman : Kolkata, Chennai and Vishakhapatnam. Abhishek did not know the length of journey so he took the help of an expert who helped him by solving a simple mathematical situation related to ships.

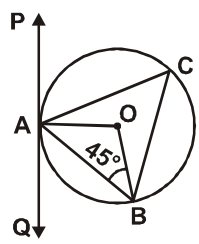
The ship covered a certain distance at a uniform speed. If the speed would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And if the speed of ship were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

OR

Two pipes running together can fill a cistern in 6 minutes. If one pipe takes 5 minutes more than the other to fil the cistern, find the time in which eace pipe would fill the cistern.

Q29. From a window (120 metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60° and 45° respectively. Show that the height of the opposite house is $120(1+\sqrt{3})$ metres.

Q30. In the given figure, PAQ is a tangent to the circle with centre O at a point A. If $\angle OBA = 45°$, find the value of $\angle BAQ$ and $\angle ACB$.



OR

The incircle of ΔABC touches the sides BC, CA and AB at D, E and F respectively. Show that AF + BD + CE + AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC).

Q31. Cards marked with numbers 4 to 99 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is :

(i) a perfect square

(ii) a multiple of 7

(iii) a prime mumber less than 30

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. Determine graphically the coordinates o the vertices of triangle formed by the equation $2x - 3y + 6 = 0$ and $2x + 3y - 18 = 0$; and the y-axis.Also, find the area of this triangle.

OR

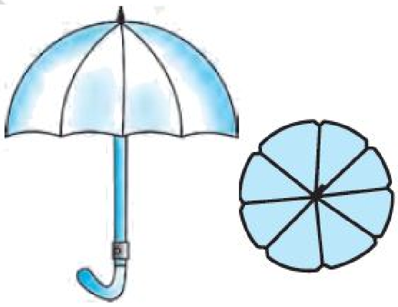
Eight times a two-digit number is equal to three times the number obtained by reversing the order of the digits. If the difference between the digits of the number is 5, find the number.

Q33. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Q34. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

OR

An umbrella has 8 ribs which are equally speed (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Q35. Find mean, median and mode of the following data :

Classes	Frequency
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

SECTION - E

Case study based questions are compoulsory.

Q36. The houses of four friends are located by point A, B, P and Q shown in figure.

If coordinates of A and B with respect to coordinate axes are known and P and Q trisect the AB. Then answer the following questions based on it

(i) Find the coordinates of P.

(ii) Find the coordinates of Q.

(iii) Find the distance PQ.

OR

Find the distance AB.

Q37. Deepak and Sanju works together in a bank in Delhi. Hometown of both of them is Rampur in Uttar Pradesh which is at a distance of 300 km from Delhi. To reach Rampur from Delhi they travel partly by train and partly by bus. This Diwali they travelled separately to Rampur. Deepak travels 60 km by train and remaining by bus and taken 4 hrs. Sanju travels 100 km by train and remaining by bus and takes 4 hrs. 10 minuts.

(i) If speed of train is x km/h and speed of bus is y km/h then write algebraic representation of the situation.

(ii) Find the speed of the bus.

(iii) If speed of the train 90 km/h and speed of the bus is 60 km/h then find time taken by Deepak to travel 60 km by train and 240 km by bus.

OR

If speed of the train is 120 km/h and speed of bus is 60 km/h then find time taken by Sanju to travel 120 km by train and 180 km by bus.

Q38. Akshat appears for a multiple choice questions test with four choices one of which is right. He either guesses or copies or known the answer to a question. Total number of questions in the test is 50.

He knows the answer to 50% of the questions, he guesses the answer of 15 questions and copies the answer of remaining questions.

(i) What is the probability that he knows the answer of a question ?

(ii) What is probability that Akshat guesses the answer of a question ?

(iii) What is the probability that Akshat copies the answer of a question ?

OR

What is the probability that Akshat does not copy the answer of a question ?

X – MATHEMATICS

SOLUTIONS : SAMPLE PAPER – 8

A-1. (c) LCM of 7, 8, 11, 12 = 1848

$\therefore$ Bells will toll together after every 1848 sec.

$\therefore$ In next 3 hrs, number of times the bells will toll number

$$= \frac{3 \times 3600}{1848} = 5.84$$

$\Rightarrow$ 5 times

A-2. (a) Zeroes of quadraticpolynomial are $\frac{3}{5}$ and $-\frac{1}{2}$

$\therefore$ quadratic polynomial

= k[x^2 – (Sum of zeroes) product of zeroes]

$$= k\left[x^2 - \left[\frac{3}{5} + \left(\frac{-1}{2}\right)\right]x + \frac{3}{5} \times \left(-\frac{1}{2}\right)\right]$$

$$= k\left(x^2 - \frac{x}{10} - \frac{3}{10}\right)$$

$$= \frac{k}{10}[10x^2 - x - 3]$$

where k is any constant.

A-3. (d) $x^2 + (a + 1)x + b$

$\because$ x = 2 is a zero

and x = – 3 is another zero

$\therefore$ $(2)^2 + (a + 1)2 + b = 0$

and $(-3)^2 + (a + 1)2 + b = 0$

$\Rightarrow$ 4 + 2a + 2 + b = 0

and 9 – 3a – 3 + b = 0

$\Rightarrow$ 2a + b = – 6 ...(i)

and –3a + b = – 6 ...(ii)

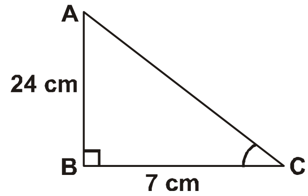
Solving (i) and (ii), we get a = 0 and b = – 6

A-4. (c)

A-5. (c)

A-6. (c

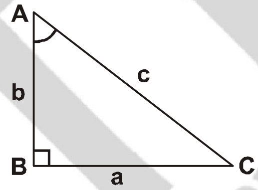
A-7. (d) In ΔABC, ∠B = 90°



$$\cos C = \frac{BC}{AC} = \frac{7}{\sqrt{(24)^2 + (7)^2}}$$

$$= \frac{7}{25}$$

A-8. (d) In ΔABC, ∠C = 90°



$$\tan A = \frac{BC}{AC}$$

$$\tan B = \frac{AC}{BC}$$

$$\therefore \quad \tan A + \tan B = \frac{BC}{AC} + \frac{AC}{BC}$$

$$= \frac{BC^2 + AC^2}{AC \cdot BC}$$

$$\tan A + \tan B = \frac{AB^2}{AC \cdot BC}$$

$$= \frac{c^2}{b \cdot a} = \frac{c^2}{ab}$$

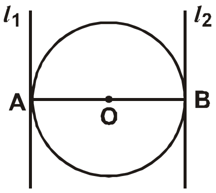
A-9. (c) Quadrilateral ABCD and quadrilateral PQRS are similar

$$\therefore \quad \frac{PS}{AD} = \frac{PQ}{AB} \text{ (Similarly criteria)}$$

$$\frac{x}{15} = \frac{8}{20}$$

$$x = \frac{8}{20} \times 15 = 6cm$$

A-10. (b) Distance between two parallel tangents is always equal to the diameter of circle.



Here tangents l_1 and l_2 are parallel and AB is diameter of circle.

Hence, AB = 14 cm

$\therefore$ Radius = $\frac{1}{2}$AB = 7 cm

A-11. (b)

A-12. (c) Angle subtended by minute hand in 20 minutes

$= \frac{360°}{60} \times 20 = 120°$

$\therefore$ Area swept in 20 minutes

$= \frac{22}{7} \times \frac{7 \times 7 \times 120°}{360°}$

= 51.33 cm^2

A-13. (d) Height of cylinder = 14 cm

Radius of cylinder - r

$\therefore$ Curved surface area = $2\pi rh$

$88 = 2 \times \frac{22}{7} \times r \times 14$

Diameter = 2r

$= \frac{88 \times 7}{22 \times 14}$ = 2 cm

A-14. (b) Mode = 3 median – 2 mean

A-15. (c) Radius of quadrant = 14 cm

Area of quadrant = $\frac{\pi(14)^2 \times 90°}{360°}$

$= \frac{22}{7} \times \frac{14 \times 14 \times 90°}{360°}$

= 154 cm^2

Area of four quadrants = 4(154)

= 616 cm^2

Angle of shaded region

= area of square – (ara of four quadrants)

= 10000 cm^2 – 616 cm^2

= 9384 cm^2

A-16. (d) Let no. of men be x, and women be y.

Total age of the group = 30(x + y)

Total age of men = 32x years

Total age of women = 27y years

$\Rightarrow$ 30(x + y) = 32x + 27y

$\Rightarrow$ 30x + 30y = 32x + 27y

$\Rightarrow x = \frac{3}{2}y$

$\therefore$ % of women = $\frac{y}{x+y} \times 100$

$\Rightarrow \frac{y}{\frac{3}{2}y + y} \times 100 = 40\%$

A-17. (d)

A-18. (c) $\sin\theta - \cos\theta = 0$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 0$

$\Rightarrow (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) = 0$

$\Rightarrow -2\sin\theta\cos\theta = -1$

$\Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{4}$

$\sin^4\theta + \cos^4\theta = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta$

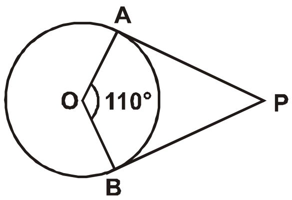
$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= (1)^2 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

A-19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-20. (d) Assertion (A) is false but reason (R) is true.

As per information given in question we have figure given below :



Radius is perpendicular to the tangent at point of conctant.

Thus, OA ⊥ AL and OB ⊥ PB

In quadrilateral, OAPB, we have

$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$

$90° + \angle APB + 90° + 110° = 360°$

$\angle APB = 70°$

Assertion (A) is false but reason (R) is true.

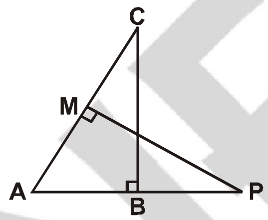
A-21. Let the cost of 1 book be Rs. x and the cost of 1 pen be Rs. y.

According to question,

$5x + 7y = 79$ ...(i)

and $7x + 5y = 77$ ...(ii)

A-22. **Given :** In ΔABC, ∠B = 90° and in ΔAMP, ∠M = 90°



To Prove : (i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Proof :

(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle BAC = \angle MAP$ (Common)

∴ ΔABC ~ ΔAMP (AA similarlity)

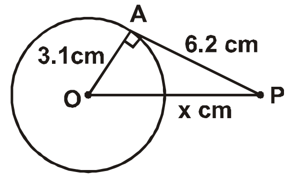
(ii) As ΔABC ~ ΔAMP,

$\therefore \frac{AC}{AP} = \frac{BC}{MP}$

(Ratios of the corresponding sides of similar triangles

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ Hence Proved.

A-23. In right-angeld ΔOAP,



$OP^2 = OA^2 + AP^2$

(Using pythagoras theorem)

$x^2 = (3.1)^2 + (6.2)^2$

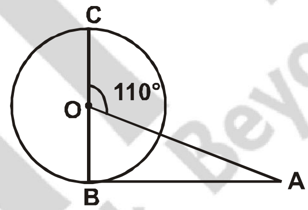
$x^2 = 9.61 + 38.44$

$x^2 = 48.05$

$x = 6.93$ cm

OR

$\angle AOB + \angle AOC = 180°$ (linear pair)



$\therefore \angle AOB = 180° - \angle AOC$

$= 180° - 110° = 70°$

In ΔAOB,

$\angle OBA + \angle OAB + \angle AOB = 180°$

$\therefore 90° + \angle OAB + 70° = 180°$

$\angle OAB = 180° - 160° = 20°$

A-24. Circumference of the circle = 44 cm

$\Rightarrow 2\pi r = 44$ cm

$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7$ cm

∴ Area of the quadrant of a circle

$= \frac{1}{4}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} cm^2$

A-25. Given $\tan\theta = \frac{1}{\sqrt{3}}$

As we know that $\tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

Putting θ = 30° in $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$, we get

$$\frac{\operatorname{cosec}^2 30° - \sec^2 30°}{\operatorname{cosec}^2 30° + \sec^2 30°} = \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{8}{16} = \frac{1}{2}$$

OR

$$\sin(A - B) = \frac{1}{2} = \sin 30°$$

$\Rightarrow$ $A - B = 30°$ ...(i)

$$\cos(A + B) = \frac{1}{2} = \cos 60°$$

$\Rightarrow$ $A + B = 60°$ ...(ii)

Adding (i) and (ii), we get

$2A = 90° \Rightarrow A = 45°$

Putting the value in (i), we get

$45° - B = 30°$

$\Rightarrow$ $B = 15°$

A-26. Factors of $12 = 2 \times 2 \times 3 = 2^2 \times 3$

Factor of $18 = 2 \times 3 \times 3 = 2 \times 3^2$

LCM $(12, 18) = 2^2 \times 3^2 = 36$

$\therefore$ After 36 mintues, they will meet again at the staring point.

A-27. $\frac{x-2}{x-4} + \frac{x-6}{x-8} = 6\frac{2}{3}$

$\Rightarrow$ $\frac{(x-2)(x-8)+(x-6)(x-4)}{(x-4)(x-8)} = \frac{20}{3}$

$\Rightarrow$ $14x^2 - 180x + 520 = 0$

$\Rightarrow$ $7x^2 - 90x + 260 = 0$

Here, $a = 7$, $b = -90$, $c = 260$

$\therefore$ Discriminate,

$D = b^2 - 4ac$

$= (-90)^2 - 4 \times 7 \times 260$

$= 820$

Using quadratic formula, we have

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{90 \pm \sqrt{820}}{2 \times 7}$$

$$= \frac{90 \pm \sqrt{820}}{14} = \frac{45 \pm \sqrt{205}}{7}$$

Hence, $x = \frac{45 + \sqrt{205}}{7}, \frac{45 - \sqrt{205}}{7}$

A-28. Let the usual speed of ship be x km/h and the usual time be y hours.

$\therefore$ Distance covered = xy km

Case I :

When speed = (x + 6) km

then time taken = (y – 4) hour

Now, distance covere = xy

$\Rightarrow$ $(x + 6)(y - 4) = xy$

$\Rightarrow$ $2x - 3y = -12$ ...(i)

Case II :

When speed = (x – 6) km/h

then time taken = (y + 6) hour

Now distance covered = xy

$\Rightarrow$ $(x - 6)(y + 6) = xy$

$x - y = 6$ ...(ii)

Solving (i) and (ii), we get

$x = 30$, $y = 24$

$\therefore$ Distance covered = $xy = 30 \times 24$

= 720 km

$\therefore$ The length of the journey = 720 km

OR

Let the time taken by first pipe to fill the cistern be x minutes

$\therefore$ In 1 minute, it can fill $\frac{1}{x}$ of cistern.

Time taken by second pipe to fill the cister

= (x + 5) minutes

$\therefore$ In 1 minute, it fill $\frac{1}{x+5}$ of cistern.

According to question

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$\Rightarrow$ $x^2 - 7x - 30 = 0$

$\Rightarrow$ $(x - 10)(x + 3) = 0$

$\Rightarrow$ $x = 10, -3$

$\Rightarrow$ $x = 10$ [$x = -3$ is rejected]

$\therefore$ Time taken by first pipe = 10 minute

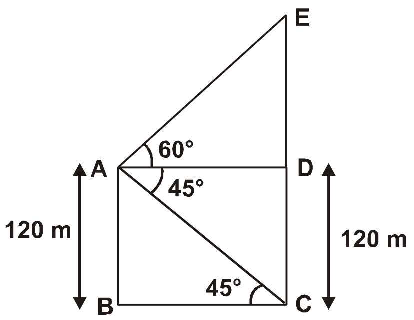
Time taken by second pipe = 15 minutes

A-29. Let bet he window and CE be the opposite house.

Now, CD = AB = 120 m ...(i)

(Opposite sides of a rectangle)

In right-angled ΔABC, $\tan 45° = \frac{AB}{BC}$



$\Rightarrow \quad 1 = \frac{120}{BC}$

$\Rightarrow \quad BC = 120$ m ...(ii)

Now, AD = BC

(Opposite sides of a rectangle)

ADS = 120 m [From (ii)]...(iii)

In right-angled ΔADE,

$\tan 60° = \frac{DE}{AD}$

$\Rightarrow \quad \sqrt{3} = \frac{DE}{120}$ [From (iii)]

$\Rightarrow \quad DE = 120\sqrt{3}$ m

$\therefore$ Height of the opposite house

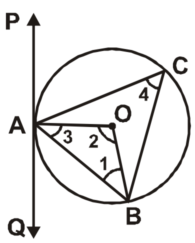
CE = CD + DE

= 120 m + $120\sqrt{3}$ m

= $120(1+\sqrt{3})$m

A-30. Given : PAQ is a tangent to th circle with centre O at a piont A and $\angle OBA = 45°$.

To find : $\angle BAQ$ and $\angle ACB$



We have OA = OB

(Radii of the same circle)

$\Rightarrow \quad \angle 3 = \angle 1$

(Angle opposite to equal sides of a triangle are equal)

$\Rightarrow \quad \angle 3 = 45°$ ($\because \angle 1 = 45°$, given)

Also, in ΔOAB

$\angle 1 + \angle 2 + \angle 3 = 180°$

(Angle sum property of a triangle)

$\Rightarrow \quad 45° + \angle 2 + 45° = 180°$

$\Rightarrow \quad \angle 2 = 180° - 90° = 90°$

Now $\angle 4 = \frac{1}{2}\angle 2 = 45°$

(Degree measure theorem)

$\Rightarrow \quad \angle ACB = 45°$

Now, $\angle BAQ = \angle OAQ - \angle 3$

$= 90° - 45° = 45°$

[$OA \perp AQ$]

OR

Given : Sides AB, BC and CA of ΔABC, touches the incircle at D, E and F respectively.

To prove :

AF + BD + CD = AE + BF + CD

$= \frac{1}{2}$ (perimeter of ΔABC)

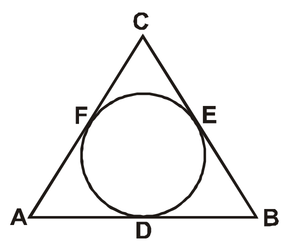
Proof : Since lengths of the tangents drawn from an external point to a circle are equal

Therefore,

AF = AE ...(i)

BD = BF ...(ii)

CE = CD ...(iii)



Adding (i), (ii) and (iii), we get

AF + BD + CE = AE + BF + CD

Now,

perimeter of $\Delta ABC = AB + BC + CA$

$\therefore$ Perimeter of ΔABC

$= (AF + FB) + (BD + CD) + (EC + AE)$

$= (AF + AE) + (BD + BF) + (EC + CD)$

$= 2(AF + BD + CE)$

$\Rightarrow$ $AF + BD + CE$

$= \frac{1}{2}$(perimeter of ΔABC)

So, $AF + BD + CE = AE + BF + CD$

$= \frac{1}{2}$(perimeter of ΔABC)

Hence proved.

A-31. Total number of cards = 96

Number of ways to draw one card = 96

(i) Let A be the event of number on the card is a perfect square.

Perfect squares are 4, 9, 16, 25, 36, 49, 64, 81

Outcomes favourable to A = 8

$\therefore$ $P(A) = \frac{8}{96} = \frac{1}{12}$

(ii) Let B be the event of number on the card is a multiple of 7.

Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 79, 77, 84, 91, 98

Outcomes favourable to B = 14

$\therefore$ $P(B) = \frac{14}{96} = \frac{7}{48}$

(iii) Let C be the event of number on the card is a prime number less than 30.

Prime numbers are 5, 7, 11, 13, 17, 19, 23, 29.

Outcomes favourable to C = 8

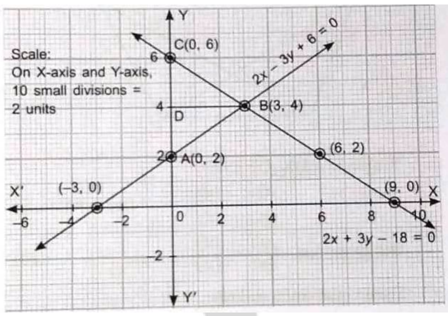
$\therefore$ $P(C) = \frac{8}{96} = \frac{1}{12}$

A-32. The solution table for $2x - 3y + 6 = 0$ is

x	0	–3	3
y	2	0	4

The solution table for $2x + 3y - 18 = 0$ is

x	0	9	6
y	6	0	2



Coordinates of the vertices of a triangle are A(0, 2), B(3, 4) and C(0, 6).

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ base × height

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4 \times 3$

= 6 units

OR

Let the digit at unit's place be x and the digt at ten's place be y.

$\therefore$ Required number = $10y + x$

When the digts are reversed, the number becomes $10x + y$

According to question,

$8(10y + x) = 3(10x + y)$

$\Rightarrow$ $80y + 8x = 30x + 3y$

$\Rightarrow$ $77y - 22x = 0 \Rightarrow 7y - 2x = 0$ ...(i)

Also $x - y = 5$ (keeing $x > y$) ...(ii)

Multiplying (ii) by 2 and adding to (i), we get

$y = 2$

Putting $y = 2$ in (ii), we get

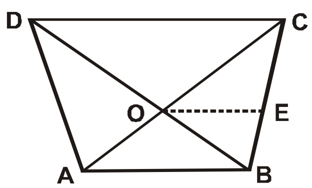
$x - 2 = 5$

$\Rightarrow$ $x = 7$

$\therefore$ Required number

$10y + x = 10 \times 2 + 7 = 27$

A-33. **Given :** A quadrilateral ABCD, whose diagonals intersect at O.



and $\frac{AO}{BO} = \frac{CO}{DO}$ or $\frac{AO}{OC} = \frac{BO}{DO}$

To prove : ABCD is a trapezium

Construction : Draw EO || AB

Proof : In ΔABC, OE || AB

$\therefore \quad \frac{AO}{OC} = \frac{BE}{EC}$ [By B.P.T.] ...(i)

But given that

$\frac{AO}{OC} = \frac{BO}{DO}$ ...(ii)

From equation (i) and (ii)

$\frac{BO}{DO} = \frac{BE}{EC}$

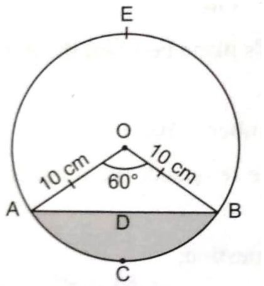
⇒ OE || DC

[By converse of B.P.T.]

OE || AB and OE || DC ⇒ AB || DC

∴ ABCD is a trapezium.

A-34.



Radius of the circle 10 cm

Central angle subtended by chord AB

= 60°

Area of minor sector OACB

$= \frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \times \frac{(10)^2 \times 60°}{360°}$

$= \frac{22}{7} \times \frac{10 \times 10}{6}$

$= \frac{1100}{21} cm^2 = 52.38\ cm^2$.

Area of equilateral triangle OAB formed by radii and chord

$= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (10)^2$

$= \frac{1.732}{4} \times 100$

$= 43.3\ cm^2$

∴ Area of minor segment ACBD

= Aea of sector OACB – Area of triangle OAB

$= (52.38 - 43.30)\ cm^2$

$= 9.08\ cm^2$

Areaw of circle $= \pi r^2$

$= \frac{22}{7} \times (10)^2$

$= \frac{22 \times 100}{7} cm^2$

$= 314.28\ cm^2$

∴ Area of major segment ADBE

= Area circle – Areaof minor segment

$= (314.28 - 9.08)\ cm^2$

$= 305.20\ cm^2$

OR

Radius of the circle 45 cm

Number of ribs = 8

Angle between two consecutive ribs

$= \frac{\text{central angle of the circle}}{\text{number of the sectors (ribs)}}$

$= \frac{360°}{8} = 45°$

Area between two consecutive ribs

= Area of one sector of circle

$= \frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \times \frac{45° \times 45° \times 45°}{360°}$

$= \frac{11 \times 45 \times 9 \times 5}{7 \times 4} cm^2$

$= \frac{22275}{28}\ cm^2$

A-35.

Classes	Frequency	Cumulative frequency
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50
	n = 50	

← Median Class

$\therefore \quad \frac{n}{2} = 25$

Median class = (60 – 80)

$l = 60$, f = 12, c.f. = 24, h = 20

$$\text{Median} = l + \frac{\frac{n}{2} - \text{c.f.}}{f} \times h$$

$$= 60 + \frac{25-24}{12} \times 20$$

$$= 60 + \frac{1 \times 5}{3} = \frac{180+5}{3}$$

$$= \frac{185}{3} = 61.6$$

Modal class = (60 – 80) as its frequency is 12

$h = 20, l = 60, f_1 = 12, f_0 = 10, f_2 = 6$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \frac{12-10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 65$$

Now, Mode = 3 Median – 2 Mean

65 = 3(61.6) – 2Mean

2 Mean = 184.8 – 65

2 Mean = 119.8

$\Rightarrow \quad \text{Mean} = \frac{119.8}{2} = 59.9$

$\therefore$ Mean = 59.9; Median = 61.6, Mode = 65

A-36. (i) As P divides AB in the ratio 1 : 2.

$\therefore$ coordinates of P are

$$\text{x-coordinate} = \frac{1(-2)+2(4)}{1+2}$$

$$= \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$\text{y-coordinate} = \frac{1(-3)+2(-2)}{1+2}$$

$$= \frac{-3-2}{3} = \frac{-5}{3}$$

Coordinate of P are $\left(2, \frac{-5}{3}\right)$

(ii) Coordinate of Q are as Q divides AB in the ratio 2 : 1

x-coordinate y-coordinate

$$= \frac{2(-3)+1(-1)}{1+2}$$

$$= \frac{-6-1}{3} = \frac{-7}{3}$$

Coordinate of Q are $\left(0, \frac{-7}{3}\right)$

(iii) P(2, –5/3) Q(0, –7/3)

Distance PQ

$$= \sqrt{(0-2)^2 + \left(\frac{-7}{3} + \frac{5}{3}\right)^2}$$

$$= \sqrt{(-2)^2 + \left(\frac{-2}{3}\right)^2}$$

$$= \sqrt{4 + \frac{4}{9}}$$

$$= \sqrt{\frac{40}{9}}$$

$$= \frac{1}{3}\sqrt{40} \text{ units}$$

OR

Distance AB

$$= \sqrt{(-2-4)^2 + (-3+1)^2}$$

$$= \sqrt{(-6)^2 + (-2)^2}$$

$$= \sqrt{36+4}$$

$$= \sqrt{40} \text{ units}$$

A-37. (i) $\frac{60}{x} + \frac{240}{y} = 4$

$\frac{100}{x} + \frac{200}{y} = 4\frac{1}{6}$

(ii) 80 km/h

(iii) $\frac{14}{3}$ hours OR 4 hours

A-38. (i) $\frac{1}{2}$

(ii) $\frac{3}{10}$

(iii) $\frac{1}{5}$ OR $\frac{4}{5}$

X – MATHEMATICS

SAMPLE PAPER – 9

Time Allowed : 3 Hours] **(SOLVED)** **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. What is the largest number that divides each one of 1152 and 1664 exactly ?

(a) 32 (b) 64 (c) 128 (d) 256

Q2. The roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is constant are

(a) $m, m + 3$ (b) $3 + 3, -m$ (c) $m, -(m + 3)$ (d) $-(m + 3), -m$

Q3. The numberof zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is / are

(a) 2 (b) 1 (c) 0 (d) 3

Q4. The pair of equations $2x - 3y = 1$ and $3x - 2y - 4$ has ________ solution

(a) one (b) two (c) no (d) many

Q5. A triangle with vertices (4, 0), (–1, –1) and (3, 5) is a/an

(a) equilateral triangle (b) right-angled triangle

(c) isosceles right-angled triangle (d) none of these

Q6. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then $\Delta ABC \sim \Delta EDF$, if

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

Q7. If θ is an acuate angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Q8. The line segment joining the points P(–3, 2) and Q(5, 7) is divided by the y-axis in the ratio

(a) 3 : 1 (b) 3 : 4 (c) 3 : 2 (d) 3 : 5

Q9. In the given figure $\frac{AD}{BD} = \frac{AE}{EC}$ and $\angle ADE = 70°$, $\angle BAC = 50°$, then angle $\angle BCA =$

(a) 70° (b) 50° (c) 80° (d) 60°

Q10. In the given figure, AD = 1.28 cm, BD = 2.56 cm, AE = 0.65 cm, DE will be parallel to BC, if EC =

(a) 1.28 cm (b) 2.56 cm (c) 0.64 cm (d) 0.32 cm

Q11. How many tangents can a circle have

(a) 1 (b) 2 (c) Infinity many (d) None of these

Q12. If the circumference and thearea of a circle are numerically equal, then the radius of the circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

Q13. The surface area of a sphere is 616 cm^2, its radius is

(a) 19 cm (b) 7 cm (c) – 7 cm (d) 14 cm

Q14. d_i is the deviation of x_i from assumed mean a.

If mean = $x + \frac{\Sigma f_i d_i}{\Sigma f_i}$, then x is

(a) class size (b) number of observation

(c) assumed mean (d) none of these

Q15. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(a) 23 (b) 24 (c) 50 (d) 60

Q16. Mean of 100 items is 49. It was observed that three items which should have been 60, 70, 80 were wrongly need as 40, 20, 50 respectively. The correct mean is

(a) 48 (b) 49 (c) 50 (d) 60

Q17. 2000 tickets of a lottery were sold and there are 16 prizes on these tickets. Abhinav has purchased one lottery ticket. The probability that Abhinav wins a prize is

(a) 10.08 (b) 00.07 (c) 0.0008 (d) 0.080

Q18. At sometimes, the length of a shadow of a tower is $\sqrt{3}$ times its height, then the angle of elevation of the Sun, at that time is

(a) 15° (b) 30° (c) 45° (d) 60°

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** The HCF of two number is 9 and their LCM is 2016. If one of the number is 306, then the other is 54.

Statement R (Reason) : For any positive integers a and b, we have : Product two numbers = HCF × LCM.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** The value of $\sin\theta = \frac{4}{3}$ in not posible.

Statement R (Reason) : Hypotentuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

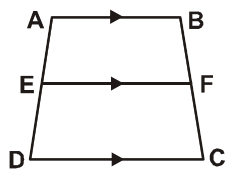
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

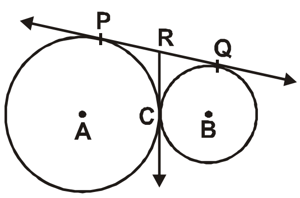
Section B consists of 5 questions of 2 marks each.

Q21. Find the sum of all multiples of 7 lying between 100 and 1000.

Q22. In the given figure, ABCD is a trapezium in which AB || DC || EF. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.



Q23. In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

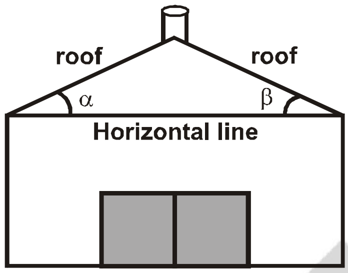


Q24. An arc of a circle of length 7π cm and the sector it bounds has an area 28π cm^3. Find the radius of the circle.

OR

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour ?

Q25. In some buildings especially in industries, the roof is inclined. This inclination of roof is the application of trigonometric functions. Here the roof of industry is inclined at angle α and β with horizontal line as shown. Determine the value of $\sin(\alpha+\beta)$, if $\operatorname{cosec}\alpha = \sqrt{2}$ and $\cot\beta = 1$, where both α and β are acute angles.



SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $3-2\sqrt{5}$ is irrational.

Q27. Solve for x : $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0;\ x \neq 3, \frac{-3}{2}$

Q28. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

OR

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Q29. If $\sec\theta = x+\frac{1}{4x}$, then prove that $\sec\theta-\tan\theta=\frac{1}{2x}$ or $2x$.

Q30. Prove that the line segment joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Q31. A game has 8 triangles of which 6 are blue and rest are green, 12 rectangles of which 3 are green and rest are blue, and 10 rhombuses of which 3 are blue and rest are green. One piece is lost at random. Find the probability that it is

(i) a rectangle (ii) a triangle of green colour

(iii) a rhombus of blue colour.

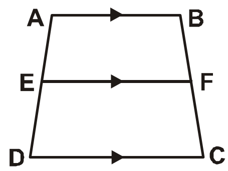
SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it.

Use the result to prove the following :

In the given figure, ABCD is a trapezium in which AB || DC || EF. Show that $\frac{AE}{ED}=\frac{BF}{FC}$.



Q33. At t minutes past 2 p.m. the time needed by the minutes hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

OR

At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age Nisha. Find the present age of btoh Asha and Nisha.

Q34. A circus tent is in the shape of a cylinder surmounted by a conical top of the same diameter. If their common diameter is 56m, the height of cylindrical part is 6m and the total height of the tent above the ground is 27m, find the area of canvas used in making the tent.

OR

A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid.

Q35. The marks of 80 students of class X in Mathematics test are given below. Find the mode of these marks obtained by the students in Mathematics test.

Marks	Frequency
0 – 10	2
10 – 20	6
20 – 30	12
30 – 40	16
40 – 50	13
50 – 60	20
60 – 70	5
70 – 80	1
80 – 90	4
90 – 100	1
Total	80

SECTION - E

Case study based questions are compoulsory.

Q36. Two friends Raj and Anuj have to travel to Shimla via Chandigarh from Gurgaon. When they reached the bus stand of Gurgaon, Raj got a call from his friend Ankit who was also on his way to bus stand. Ankit requested Raj to buy two tickets to Chandigarh and 3 tickets to Shimla also Anuj's friend Kamla asked Anuj to buy 3 tickets to Chandigarh and 4 tickets to Shimla. Raj purchased 2 tickets to Chandigarh and 3 tickets to Shimla for Rs. 3700, Anuj spent Rs. 5100 to buy 3 tickets to Chandigarh and 4 tickets to Shimla.

(i) If cost of one ticket to Chandigarh is Rs. x and cost of one ticket to Shimla is Rs. y then represent the situation algebraically.

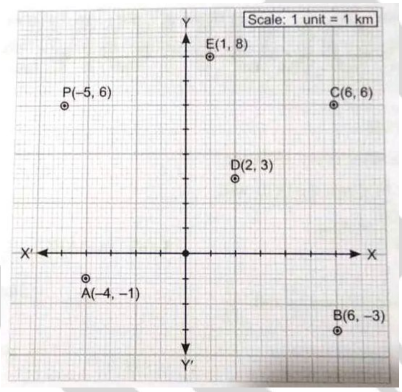
(ii) Find the cost of one ticket from Gurgaon to Chandigarh.

(iii) If Raj purchases 3 tickets to Chandigarh and 5 tickets to Shimla, how much amount he will pay ?

OR

If Anuj spends Rs. 5600 to buy tickets find how many total number of tickets he purchased ?

Q37. Five ships are positioned in the Indian Ocean. Their positions were plotted on a graph paper in reference to a rectangular coordinate axes.



An enemy ship is spotted at P(–5 6).

(i) What is the distance between P and E ?

(ii) Find the coordinate of mid-point of BD.

(iii) Ship D is moved to a position which is mid-point of AE. Find the distance moved by D.

OR

We find a rock at new position G such that B, G and C are in a straight line and BG : GC = 3 : 1 then fin dthe coordinates of G.

Q38. Group of friends playing with cards bearing numbers 5 to 50. All cards placed in a box and are mixed thoroughly one friend withdrawns the card from box at random and then replace it. Answer the questions based on above.

(i) What is the probability that the card withdrawn from the box bears a prime number less than 10 ?

(ii) What is the probability that the card withdrawn from the box bears a number which is a perfect square ?

(iii) What is the probability that the card withdrawn from the box bears a number which is multiple of 7 between 40 and 50 ?

OR

Find the probabilit of drawing a card bearing number from 5 and 50.

X – MATHEMATICS
SOLUTIONS : SAMPLE PAPER – 9

A-1. (c) 128

A-2. (b) $x^2 - 3x - m(m + 3) = 0$

$\Rightarrow x^2 + mx - (m + 3)x - m(m + 3) = 0$

$\Rightarrow x(x + m) - (m + 3)(x + m) = 0$

$\Rightarrow (x + m)[x - (m + 3)] = 0$

$\Rightarrow x + m = 0$ or $x - (m + 3) = 0$

$\Rightarrow x = -m$ or $x = m + 3$

A-3. (c) The given polynomial is

$f(x) = (x - 2)^2 + 4$

for zeroes,

$f(x) = 0$

$\Rightarrow (x - 2)^2 + 4 = 0$

$\Rightarrow (x - 2)^2 = -4$

Which is not possible.

Hence the polynomial has no zeroes.

A-4. (a) The given equations are $2x - 3y = 1$ and $3x - 2y = 4$

Here $\frac{a_1}{a_2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-2} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations has unique solution.

A-5. (c) Let coordinates of vertices be A(4, 0), B(–1, –1) and C(3, 5).

$AB = \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{36}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$AC = \sqrt{(3-4)^2 + (5-0)^2} = \sqrt{26}$

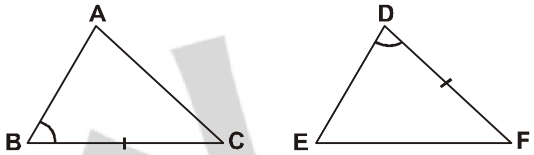
$\Rightarrow AB^2 + AC^2 = BC^2$

and AB = AC

Hence, triangle is an isosceles right-angled triangle.

A-6. (c) In ΔABC and ΔDEF

$$\frac{AB}{DE} = \frac{BC}{DF}$$



Also ΔABC ~ ΔEDF

This is possible when ∠D = ∠D.

A-7. (c)

A-8. (d)

A-9. (d) ∵ DE || BC

∴ ∠ABC = 70° (Corresponding ∠s)

Using angle sum property of triangle

∠ABC + ∠BCA + ∠BAC = 180°

⇒ ∠BCA = 180° – 70° – 50° = 60°

A-10. (a) DE || BC, if $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{1.28}{2.56} = \frac{0.64}{EC}$

$\Rightarrow$ EC = 1.28 cm

A-11. (c)

A-12. (a) Let radius of the circle be r units

Circumference of the circle = $2\pi r$

Area of the circle = πr^2

A.T.Q

Circumference of the circle = Area of the circle

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r = 2$ units

A-13. (b) Let radius of the sphere be a cm

∴ Surface area of sphere = $4\pi a^2$

$\therefore\ 4 \times \pi a^2 = 616$

$\therefore\ 4 \times \frac{2}{7} \times a^2 = 616$

$\Rightarrow\quad a^2 = \frac{616 \times 7}{22 \times 4} = 49$

$\Rightarrow\quad a = 7$ cm

A-14. (c) $\because$ Mean = assumed mean + $\frac{\Sigma f_i d_i}{\Sigma f_i}$

$\therefore\quad x =$ assumed mean.

A-15. (c) Circumference of smaller wheel

$= 30\pi$ cm

Circumference of bigger wheel

$= 50\pi$ cm

Now, $15 \times 50\pi$ = number of revolution $\times\ 30\pi$

$\Rightarrow$ number of revolutions = 25

A-16. (c) Sum of 100 observations

$= 100 \times 49 = 4900$

Correct sum $= 4900 - [40 + 20 + 50] + [60 + 70 + 80] = 5000$

$\therefore$ Correct mean $= \frac{5000}{100} = 50$

A-17. (c) Number of lottery ticket = 2000

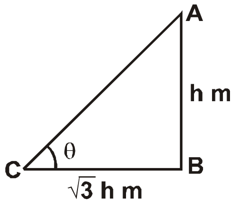
Total number of prizes = 16

$\therefore$ Probability that Abhinav wins a prize $= \frac{16}{2000} = \frac{1}{125} = 0.008$

A-18. (b) Here AB is tower of height h m.

Its shadow BC $= \sqrt{3}$ h m

Let θ be the angle of elevation



$\therefore$ In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

$\tan\theta = \tan 30°$

$\Rightarrow\quad \theta = 30°$

A-19. (d) Assertion (A) is false but Reason (R) is true.

A-20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-21. All multiples of 7 lying between 100 and 1000 are 105, 112, 119,..., 994

These numbers from an A.P.

Here a = 105, d = 112 – 105 = 7

Let $a_n = 994$

$\Rightarrow\ a + (n - 1)d = 994$

$\Rightarrow\quad n = 128$

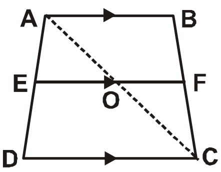
Now, $S_{128} = \frac{128}{2}(105 + 994)$

$= 70336$

A-22. **Given :** In trapezium ABCD,

AB || DC || EF

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$



Construction : Join AC, where point O is intersection of AC and EF.

Proof : In ΔADC and ΔAEO, EO || DC

$\Rightarrow\quad \frac{AE}{AD} = \frac{AO}{AC}$

$\Rightarrow\quad \frac{AD}{AE} = \frac{AC}{AO}$

$\Rightarrow\quad \frac{ED}{AE} = \frac{CO}{AO}$ ...(i)

In ΔCFO and ΔCBA, FO || BA

$$\Rightarrow \quad \frac{CF}{BC} = \frac{CO}{AC}$$

$$\Rightarrow \quad \frac{BC}{CF} = \frac{AC}{CO}$$

$$\Rightarrow \quad \frac{BF}{CF} = \frac{AO}{CO}$$

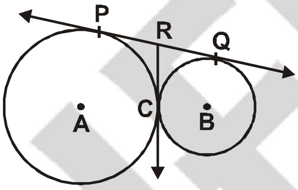
$$\Rightarrow \quad \frac{CF}{BF} = \frac{CO}{AO} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{ED}{AE} = \frac{CF}{BF}$$

$$\Rightarrow \quad \frac{AE}{ED} = \frac{BF}{CF} \qquad \text{Hence proved.}$$

A-23. **Given :** PQ and RC are common tangents to the two circles.



To prove : RC bisects PQ or R bisects PQ.

Proof : PR and RC are tangents to a circle with centre A.

$\therefore$ PR = RC [$\because$ Length of tangents drawn from an external point R to a circle are equal] ...(i)

Similarly, RQ and RC are tangents to a circle with centre B.

$\therefore$ RQ = RC ...(ii)

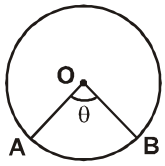
From (i) and (ii), we get

PR = RQ

$\therefore$ R bisects PQ. Hence proved.

A-24. Length of are AB = 7π cm,

Let $\angle AOB = \theta$



Now, length of an arc of a sector of angle

$$\theta = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\Rightarrow \quad 7\pi = \frac{\theta}{180^\circ} \times \pi r$$

$$\Rightarrow \quad \frac{1260^\circ}{r} = \theta$$

Now, area of the sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$$\Rightarrow \quad 28\pi = \frac{\frac{1260^\circ}{r}}{360^\circ} \times \pi r^2$$

$$\Rightarrow \quad r = 8 \text{ cm}$$

Radius of the circle 8 cm.

OR

Given, diameter of the wheels of car = 80cm.

$\Rightarrow$ Radius = 40 cm

Circumference of the wheel

$$= 2\pi r = 2 \times \frac{22}{7} \times 40 \text{ cm}$$

Speed of the car = 66 km/h

Distance covere in 10 minutes

$$= \frac{66 \times 10}{60} = 11 \text{ km}$$

$$= 1100000 \text{ cm}$$

$\therefore$ Number of revolutions

$$= \frac{\text{Total distance in 10 minutes}}{\text{Circumference of the wheel}}$$

$$= \frac{1100000 \times 7}{2 \times 22 \times 40} = 4375$$

A-25. Given, $\operatorname{cosec}\alpha = \sqrt{2}$

$$\Rightarrow \quad \sin\alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \alpha = 45° \text{ and } \cot\beta = 1$$

$$\Rightarrow \quad \tan\beta = 1 \Rightarrow \beta = 45°$$

$$\therefore \quad \sin(\alpha + \beta) = \sin(45° + 45°)$$

$$= \sin 90° = 1$$

OR

$$\sin^6\theta - \cos^6\theta$$
$$= (\sin^3\theta)^2 - (\cos^3\theta)^2$$
$$= (\sin^3\theta - \cos^3\theta)(\sin^3\theta + \cos^3\theta)$$
$$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta) \times (\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$$
$$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$$
$$= (\sin^2\theta - \cos^2\theta)(1 + \sin\theta\cos\theta)(1 - \sin\theta\cos\theta)$$
$$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$$

A-26. Let us suppose that $3 - 2\sqrt{5}$ is irrational.

$\therefore$ $3 - 2\sqrt{5}$ can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

$$\Rightarrow \quad 3 - 2\sqrt{5} = \frac{p}{q} \quad \Rightarrow \quad 3 - \frac{p}{q} = 2\sqrt{5}$$

$$\Rightarrow \quad \frac{3q - p}{q} = 2\sqrt{5} \quad \Rightarrow \quad \frac{3q - p}{2q} = \sqrt{5}$$

Since p and q are integers, we get $\frac{3q - p}{2q}$ irrational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

$$\therefore \quad \frac{3q - p}{2q} \neq \sqrt{5}$$

So, our supposition is wrong.

Hence, $3 - 2\sqrt{5}$ is irrational.

A-27. $\frac{2x}{x - 3} + \frac{1}{2x + 3} + \frac{3x + 9}{(x - 3)(2x + 3)} = 0$

$$\Rightarrow \quad \frac{2x(2x + 3) + x - 3 + 3x + 9}{(x - 3)(2x + 3)} = 0$$

$$\Rightarrow \quad 4x^2 + 10 + 6 = 0$$

$$\Rightarrow \quad 2x^2 + 5x + 3 = 0$$

$$\Rightarrow \quad (x + 1)(2x + 3) = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = \frac{-3}{2}$$

When $x = \frac{-3}{2}$, given equation is not defined.

$\therefore$ $x = -1$

A-28. Let total number of pottery articles produced in a particular day be x.

Cost of production per article = Rs. $\frac{90}{x}$

ATQ $\quad 2x + 3 = \frac{90}{x}$

$$\Rightarrow \quad x(2x + 3) = 90$$

$$\Rightarrow \quad 2x^2 + 3x = 90$$

$$\Rightarrow \quad 2x^2 + 3x - 90 = 0$$

$$\Rightarrow \quad (2x + 15)(x - 6) = 0$$

$$\Rightarrow \quad 2x = -15 \text{ or } x - 6 = 0$$

$$\Rightarrow \quad x = -\frac{15}{2} \text{ (rejected) or } x = 6$$

$\therefore$ Number of articles produced in a particular day = 6

Cost of production per article

$$= \frac{90}{6} = \text{Rs. } 15$$

OR

Given $a_{11} = 38$ and $a_{16} = 73$

$\Rightarrow$ $a + 10d = 38$

and $a + 15d = 73$

$\Rightarrow$ $a + 15d - a - 10d = 73 - 38$

$\Rightarrow$ $5d = 35$

$\Rightarrow$ $d = 7$

$\therefore$ $a_{11} = a + 10 \times 7 = 38$

$\Rightarrow$ $a = 38 - 70 = -32$

$\therefore$ $a_{31} = a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210 = 178$

A-29. Given $\sec\theta = x + \frac{1}{x}$

squaring both sides, we get

$$\sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \sec^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \tan\theta = \left(x - \frac{1}{4x}\right) \text{ or } -\left(x - \frac{1}{4x}\right)$$

Consider LHS = $\sec\theta - \tan\theta$

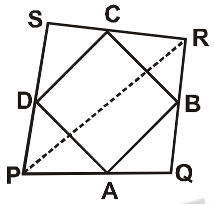
$$= x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$\text{or} \quad x + \frac{1}{4x} + \left(x - \frac{1}{4x}\right) = \frac{1}{2x} \text{ or } 2x$$

= RHS

$\therefore$ LHS = RHS

A-30. **Given :** In a quadrilateral PQRS, A, B, C and D are the mid-points of sides PQ, QR, RS and SP respectively.



To prove : ABCD is a parallelogram.

Construction : Join PR.

Proof : In ΔPQR, A and B are mid-points of sides PQ and QR respectively.

$\therefore$ AB || PR (Using mid-point theorem) ...(i)

In ΔPSR, D and C are mid-pionts of sides PS and SR respectively.

$\therefore$ DC || PR (Using mid-point theorem) ...(ii)

From (i) and (ii), we get

AB || DC

Similarly, we have AD || BC

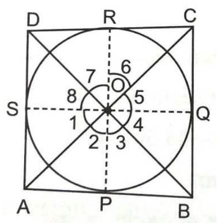
$\therefore$ In quadrilateral ABCD, AB || CD and AD || BC.

$\therefore$ ABCD is a parallelogram, because both pairs of opposite sides ofa quadrilateral ABCD are parallel.

OR

AB touches at P and BC, CD and DA touch the circle at Q, R and S.

Construction : Join OA, OB, OC, OD and OP, OQ, OR, OS.



$\therefore$ $\angle 1 = \angle 2$ [OA bisects $\angle$POS]

Similarly, $\angle 4 = \angle 3$;

$\angle 5 = \angle 6$;

$\angle 8 = \angle 7$

$2[\angle 1 + \angle 4 + \angle 5 + \angle 8] = 360°$

$(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 180°$

$\angle AOD + \angle BOC = 180°$

Similarly $\angle AOB + \angle COD = 180°$

Hence, opposite sides of quadrilateral circumscribing a circle subtend supplementary angles at the centre of a circle.

A-31. Total number of objects = 12+8+10 = 30

Number of blue triangles = 6

Number of green triangles = 8 – 6 = 2

Number of green rectangles = 3

Number of blue rectangles = 12 – 3 = 9

Number of blue rhombus = 3

Number of green rhombuses = 10 – 3 = 7

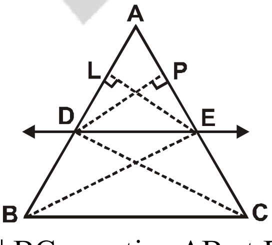
(i) Probability that one piece lost is a rectangle = $\frac{12}{30} = \frac{2}{5}$

(ii) Probability that one piece lost is a tiangle of green colour = $\frac{2}{30} = \frac{1}{15}$

(iii) Probability that one piece lost is a rhombus of blue colour = $\frac{3}{30} = \frac{1}{10}$

A-32. First part :

Given : A triangle ABC



DE || BC, meeting AB at D and AC at E

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EL ⊥ AD.

Proof : ΔBDE and ΔCDE are on the same base and between the same parallel BC and DE, hence equal in area, i.e,

$ar(\Delta BDE) = ar(\Delta CDE)$ ...(i)

Now, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}.AD.EL}{\frac{1}{2}.BD.EL} = \frac{AD}{BD}$ ...(ii)

Similarly, $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}.AE.DP}{\frac{1}{2}.EC.DP} = \frac{AE}{EC}$ ...(iii)

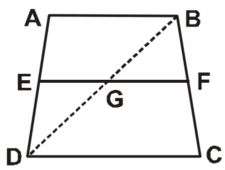
Also, $\frac{ar(\Delta ADE)}{\Delta(BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$ [Using (i)]

$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$ [From (ii) and (iii)]

Second Part :

Join intersecting EF at G.

In ΔDAB, EG || AB



$\therefore \frac{AE}{DE} = \frac{BG}{GD}$ [Using B.P.T.) ...(i)

In ΔDBC, GF || DC

$\therefore \frac{BG}{GD} = \frac{BF}{FC}$ ...(ii)

From (i) and (ii)

$\frac{AE}{DE} = \frac{BF}{FC}$

A-33. ATQ $(60 - t) = \frac{t^2}{4} - 3$

$\Rightarrow 240 - 4t = t^2 - 12$

$\Rightarrow t^2 + 4t - 252 = 0$

$\Rightarrow t^2 + 18t - 14t - 252 = 0$

$\Rightarrow (t + 18)(t - 14) = 0$

$\Rightarrow t = 14, - 18$ [rejected]

$\Rightarrow t = 14$ minutes.

OR

Let present age of Asha be x years

and present age of Nisha be ye years

ATQ $x = y^2 + 2$

Difference in ages = (x – y) years

Mother's age after (x – y) yers is

$x + (x - y) = 10y - 1$

$\Rightarrow 2x - y - 10y + 1 = 0$

$\Rightarrow 2(y^2 + 2) - 11y + 1 = 0$

$\Rightarrow 2y^2 + 4 - 11y + 1 = 0$

$\Rightarrow 2y^2 - 11y + 5 = 0$

$\Rightarrow 2y^2 - 10y - y + 5 = 0$

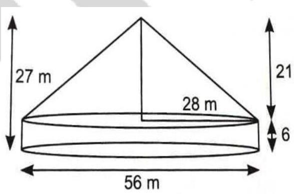
$\Rightarrow (y - 5)(2y - 1) = 0$

$\Rightarrow y = 5$ or $y = \frac{1}{2}$ (rejecting)

Neha's present age = 5 years

Asha's present age = $5^2 + 2 = 27$ years.

A-34.



Let l be the slant height of conical part of tent.

Radius of conical part (r) = 28 m

Height of conical part (h) = 21 m

Now, $l = \sqrt{(28)^2 + (21)^2}$

$= \sqrt{784 + 441}$

$= \sqrt{1225} = 35$ m

Curved surface area of conical part

$= \pi r l = \pi(28)35$

$m^2 = 980\pi\ m^2$

Radius of cylindrical part = 28 m

Height of cylindrical part = 6m

Curved surface area of cylindrical part

$= 2\pi rh = 2\pi(28)6$

$= 336\pi\ m^2$

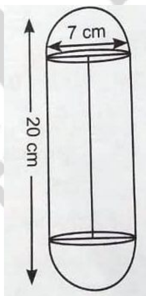
Total curved surface area = $980\pi + 336\pi$

$= 1316\pi\ m^2 = \frac{1316 \times 22}{7}$

$= 4136\ m^2$

$\therefore$ Area of canvas used = $4136\ m^2$

OR



Diameter of cylinder = diameter of the hemisphere

= 7 cm

$\therefore$ Radius of cylinder = $\frac{7}{2}$ cm

Total height of the solid = 20 cm

Height of the cylinder = $20 - \left(\frac{7}{2} + \frac{7}{2}\right)$

= 13 cm

Volume of the solid = Volume of the cylinder + 2 × vol. of one hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(13 + \frac{4}{3} \times \frac{7}{2}\right) cm^3$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\left(13+\frac{14}{3}\right)$$

$= 680.167\ cm^3$.

A-35.

Marks	Frequency
0 – 10	2
10 – 20	6
20 – 30	12
30 – 40	16
40 – 50	13
50 – 60	20
60 – 70	5
70 – 80	1
80 – 90	4
90 – 100	1
Total	80

Here, frequency of the class 50 – 60 is maximum.

$\therefore$ Modal class is 50 – 60

Also, $l = 50$, $f_0 = 13$, $f_1 = 20$, $f_2 = 5$, $h = 10$

$$\therefore \quad \text{Mode} = l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$$

$$= 50+\left(\frac{20-13}{2\times 20-13-5}\right)\times 10$$

$$= 50+\frac{7}{22}\times 10$$

$= 50 + 3.18 = 53.18$

So, the mode marks are 53.18.

A-36. (i) $\sqrt{40}$ km

(ii) (5, 1)

(iii) $\dfrac{\sqrt{50}}{2}$ km or $\left(6, \dfrac{15}{4}\right)$

A-37. (i) $2x + 3y = 3700$, $3x + 4y = 5100$

(ii) Rs. 500

(iii) Rs. 6000 **OR** 8

A-38. (i) Prime number from 5 to 10 are 5 and 7 only.

$\therefore$ number of favourable cases = 2

Total possible outcomes = 46

P(prime number less than 10)

$$= \frac{2}{46} = \frac{1}{23}$$

(ii) Perfect squares from 5 to 50 are 9, 16, 25, 36, 49

Number of favourable cases = 5

Total possible outcomes = 46

P(a perfect square number from 5 to 50) = $\dfrac{5}{46}$

(iii) Multiple of 7 between 40 and 50 are 42 and 49

Number of favourable outcomes = 2

Total possible outcomes = 46

P(multiple of 7 between 40 and 50)

$$= \frac{2}{46} = \frac{1}{23}$$

OR

P(from 5 and 50) = $\dfrac{2}{46} = \dfrac{1}{23}$

MATHEMATICS – X

SAMPLE PAPER – 10

Time Allowed : 3 Hours] **(SOLVED)** **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. What is the largest number that divides each one of 1152 and 1664 exactly ?

(a) 32 (b) 64 (c) 128 (d) 256

Q2. The roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is constant, are

(a) $m, m + 3$ (b) $m + 3, -m$ (c) $m, -(m + 3)$ (d) $-(m + 3), -m$

Q3. The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is / are

(a) 2 (b) 1 (c) 0 (d) 3

Q4. The pair of equation $2x - 3y = 1$ and $3x - 2y = 4$ has ________ solution.

(a) unique (b) two (c) no (d) many

Q5. A triangle with vertices (4, 0), (–1, –1) and (3, 5) is a/an

(a) equilateral triangle (b) right-angled triangle

(c) isosceles right-angled triangle (d) none of these

Q6. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then $\Delta ABC \sim \Delta EDF$, if

(a) $\angle B = \angle C$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

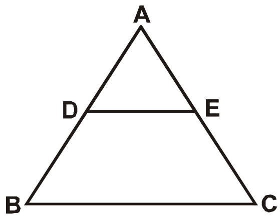
Q7. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^2\theta + \cos^2\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Q8. The line segment the points P(–3, 2) and Q(5, 7) is divided by the y-axis in the ratio

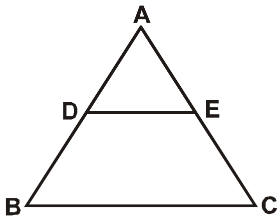
(a) 3: 1 (b) 3 : 4 (c) 3 : 2 (d) 3 : 5

Q9. In the given figure, $\frac{AD}{BD} = \frac{AE}{EC}$ and $\angle ADE = 70°$, $\angle BAC = 50°$, then angle $\angle BCA$ is equal to



(a) 70° (b) 50° (c) 80° (d) 60°

Q10. In the given figure, AD = 1.28 cm, DB = 2.56 cm, AE = 0.64 cm. DE will be parallel to BC, if EC =



(a) 1.28 cm (b) 2.56 cm (c) 0.64 cm (d) 0.32 cm

Q11. How many tangents can a circle have

(a) 1 (b) 2 (c) Infinitely many (d) None of these

Q12. If the circumference and the area of a circle are numerically equal, then the radius of the circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

Q13. The surface area of a sphere is 616 cm^2, its radius is

(a) 19 cm (b) 7 cm (c) – 7 cm (d) 14 cm

Q14. d, is the deviation of x, from assumed mean a.

If mean = $x + \frac{\Sigma f_i d_i}{\Sigma f_i}$, then x is

(a) class size (b) number of observations

(c) assumed mean (d) none of these

Q15. A toothed wheel of diameter 50 cml is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(a) 23 (b) 24 (c) 25 (d) 26

Q16. Mean of 100 items is 49. It was observed that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(a) 48 (b) 49 (c) 50 (d) 60

Q17. 2000 tickets of a lottery were sold and there are 16 prizes on these tickets. Abhinav has purchased one lottery ticket. The probability that Abhinav wins a prize is

(a) 10.08 (b) 00.07 (c) 0.008 (d) 0.080

Q18. At sometime, the length of a shadow of a tower is $\sqrt{3}$ times its height, then the angle of elevation of the Sun at that time is

(a) 15° (b) 30° (c) 45° (d) 60°

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** The HCF of two number is 9 and their LCM is 2016. If one of the number is 306, then the other is 54.

Statement R (Reason) : For any positive integers a and b, we have : Product two number = HCF × LCM.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is the false but reason (R) is true.

Q20. **Statement A (Assertion) :** The value of $\sin\theta = \frac{4}{3}$ in not possible.

Statement R (Reason) : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

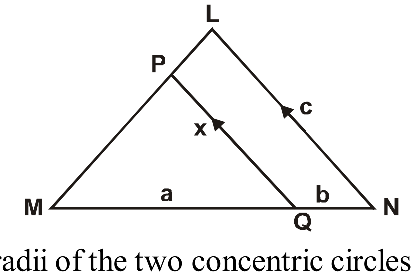
(d) Assertion (A) is the false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. If $2x + 3y = 2$ and $4x - 9y = -1$, then find $x + y$.

Q22. In the given figure, if PQ || LN, then express x in terms of a, b and c.



Q23. If radii of the two concentric circles are 15cm and 17cm, then find the length of the chord of one circle which is tangent to the other.

Q24. If $\sin\theta + \cos\theta = \frac{\sqrt{3}+1}{2}$, then show that $\sin\theta = \sqrt{3}\cos\theta$.

OR

Prove that $\frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta} = \cot\theta$, where $0 \le \theta \le 90°$.

Q25. If area of a sector of a circle having radius 7 cm is $\frac{77}{3}\text{cm}^3$, find length of corresponding arc.

OR

Find the area of corresponding minor segment formed by chord of length equal to radius of the circle that is 6 cm.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Find the product of the least number divisible by 18, 24 and 36 and the greatest number which divides 18, 24 and 36.

Q27. If α and β are the zeroes of polynomial $x^2 + 7x + 8$, find the polynomial whose zeroes are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

Q28. The area of a rectangle gets reduced by 9 sq units, if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle.

OR

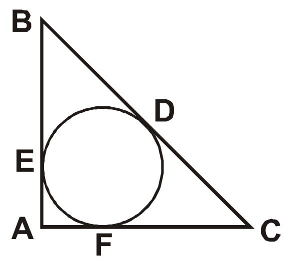
A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby getting sum of Rs. 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs. 1028. Find the cost price of saree and marked price of the sweater.

Q29. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$

Q30. A circle touches the sides of a quadrilateral ABCD at P, O, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

OR

In the given figure, ABC is a right-angle triangle right-angled at A. A circle is inscribed in the triangle and touches the sides of triangle at points D, E and F. If BD = 30 cm, CD = 7 cm. Find the other two sides of the triangle.



Q31. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another. What is the probability that both will visit the shop on (i) the same day ? (ii) different day ? (iii) consecutive days ?

SECTION - D

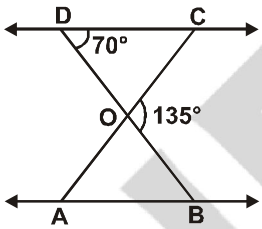
Section D consists of 4 questions 5 marks each.

Q32. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

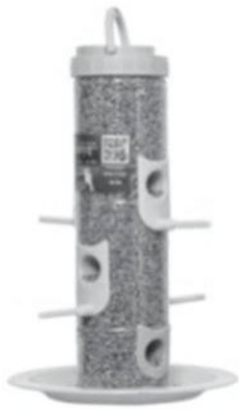
OR

To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool ?

Q33. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 135°$ and $\angle CDO = 70°$ find the angles $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Q34. A bird feeder tube has a diameter of a 8 cm and height of 28 cm. The tube has 7 circular openings of 2 m diameter each for the birds to eat from. The tube can hold a maximum of 3 kg of bird food.



(**Note :** The image is for visual representation only).

If the birds eat an average 75g of food per hour, what will be the height of the food in the tube after 5 hours ? Show you work.

(Note : Take π as 22/7)

OR

Water is a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes, if 8 cm standing water is needed ?

Q35. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

SECTION - E

Case study based questions are compulsory.

Q36. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(i) If the first circular row has 30 seats, how many seats will be there in the 10th row ? **1**

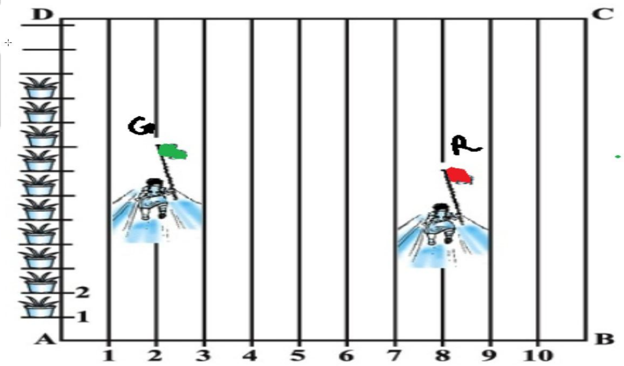
(ii) For 1500 seats in the auditorium, how many rows need to be there ? **2**

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row ?

(iii) If there were 17 rows in the auditorium, how many seats are still left to the put after 10th row ? **1**

Q37. In order to conduct Sports Day activities in your School lines have been drawn with chalk powder at a distance of 1m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1m from each other along AD, as shown in given figure below. Niharika runs 1/4th the distance AD on the 2nd line and posts a green (G) flag. Preet runs 1/5th distance AD on the eighth line and posts a red (R) flag.



(i) Find the position of green flag ? **1**

(ii) Find the position of red flag ? **1**

(iii) What the distance between both the flags ? **2**

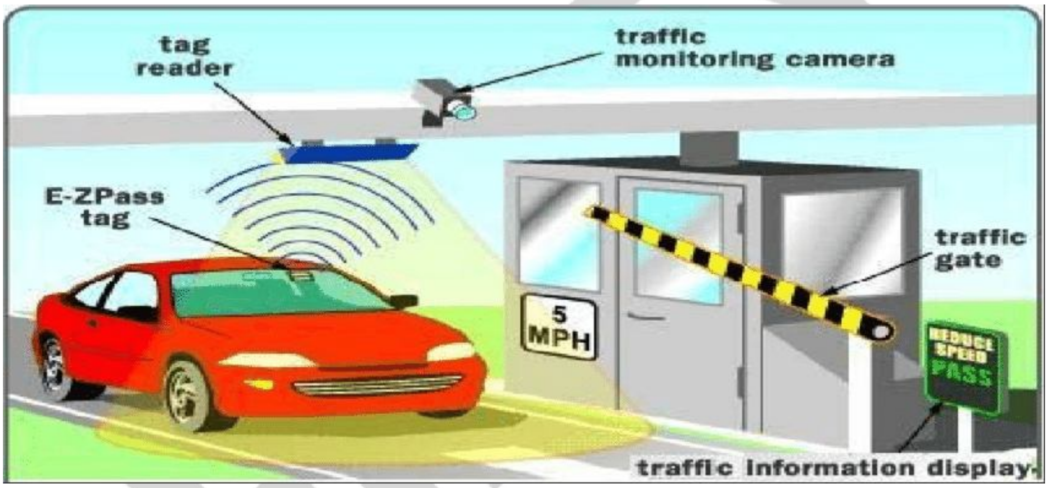
OR

If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags. Where should she post her flag ?

Q38. At a toll plaza, an electric toll collection system has been installed. FAST Tag can be used to pay the fare. The tag can be pasted on the windscreen of a car.

At the toll plaza a tag scanner is placed at a height of 6 m from the ground. The scanner reads the information on the tag of the vehicle and debits the desired toll amount from a linked bank account.

For the tag scanner to function properly the speed of a car needs to be less than 30 km per hour. A car with a tag installed at a height of 1.5 m from the ground enters the scanner zone.



(i) The scanner gets activated when the car's tag is at a distance of 5 m from it.

Give one trigonometric ratio for the angle between the horizontal and the line between the car tag and the scanner ?

(ii) The scanner reads the complete information on the car's tag while the angle between tag and scanner changes from 30° to 60° due to car movement. What is the distance moved by car ? **2**

OR

A vehicle with a tag pasted at a height of 2 m from the ground stops in the scanner zone. The scanner reads the data at a angle of 45°. What is the distance between the tag and the scanner ?

(iii) Which trigonometric ratio in the right triangle vary from 0 to 1 ? **1**

— Notes —

MATHEMATICS – X

SOL: SAMPLE PAPER –10

A-1. (c) HCF of 1152 and 1664 = 128

A-2. (b) $x^2 - 3x - m(m + 3) = 0$

$x^2 + mx - (m + 3)x - m(m + 3) = 0$

$\Rightarrow x(x + m) - (m + 3)(x + m) = 0$

$\Rightarrow (x + m)[x - (m + 3)] = 0$

$\Rightarrow x + m = 0 \text{ or } x - (m + 3) = 0$

$\Rightarrow x = -m \text{ or } x = m + 3$

A-3. (c) The given polynomial

$f(x) = (x - 2)^2 + 4$

for zeroes $f(x) = 0$

$\Rightarrow (x - 2)^2 + 4 = 0$

$\Rightarrow (x - 2)^2 = -4$

Which is not possible,

Hence the polynomial has no zeroes.

A-4 (a) The given equation are $2x - 3y = 1$ and $3x - 2y = 4$

Here $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-3}{-2} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations has unique solution.

A-5. (c) Let coordinates of vertices be A(4, 0), B(–1, –1) and C (3, 5)

$AB = \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{26}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

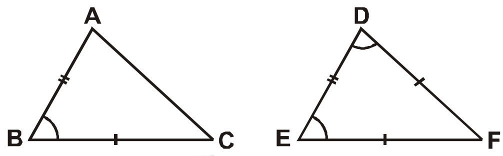
$AC = \sqrt{(3-4)^2 + (5-0)^2} = \sqrt{26}$

Since $AB^2 + AC^2 = BC^2$

and AB = AC

Hence, triangle is an isosceles right-angled triangle.

A-6. (c) In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{DF}$



Also ΔABC ~ ΔEDF

This is only possible when ∠B = ∠D

A-7. (c) Given, $\tan\theta + \cot\theta = 2$

Let $\tan\theta = x$

$\therefore x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0$

On solving the quadratic equation

$x = 1 \Rightarrow \tan\theta = 1$

$\therefore \theta = 45°$

$\therefore$ The value of $\sin^3\theta + \cos^3\theta$

$= (\sin 45°)^3 + (\cos 45°)^3$

$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

A-8. (d) Let the y-axis divides in k : 1.

Now, according to the question,

$x = \frac{k \times (5) + 1 \times (-3)}{(k+1)}$

$\Rightarrow 0 = \frac{5k-3}{(k+1)} \Rightarrow 5k - 3 = 0$

$\Rightarrow k = \frac{3}{5} = 3 : 5$

A-9. (d) $\because$ DE || BC

$\therefore$ ∠ABC = 70°

(Corresponding angles)

Using angle sum property of triangle in ΔABC.

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\Rightarrow \angle BCA = 180° - 70° - 50° = 60°$

A-10. (a) DE || BC, if $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{1.28}{2.56} = \frac{0.64}{EC}$

$\Rightarrow$ EC = 1.28 cm

A-11. (c) Infinitely many

A-12. (a) Let radius of the circle be r units

Circumference of the circle = $2\pi r$

Area of the circle πr^2

A.T.Q

Circumference of the circle = Area of circle

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r = 2$ units

A-13. (b) Let radius of sphere be a cm

$\therefore$ Surface area of sphere = $4\pi a^2$

$\therefore 4 \times \frac{22}{7} \times a^2 = 616$

$\Rightarrow a^2 = \frac{616 \times 7}{22 \times 4} = 49$

$\Rightarrow a = 7$ cm

A-14. (c) $\because$ Mean = assumed mean + $\frac{\Sigma f_i d_i}{\Sigma f_i}$

$\therefore$ x = assumed mean

A-15. (c) Circumference of smaller wheel

= 30π cm

Circumference of bigger wheel

= 50π cm

Now, $15 \times 50\pi$ = number of revolution $\times 30\pi$

$\Rightarrow$ number of revolutions = 25

A-16. (c) Sum of 100 observations

$= 100 \times 49 = 4900$

Correct sum = 4900 − [40 + 20 + 50] + [60 + 70 + 80] = 5000

$\therefore$ Correct mean = $\frac{5000}{100} = 50$

A-17. (c) Number of lottery tickets = 2000

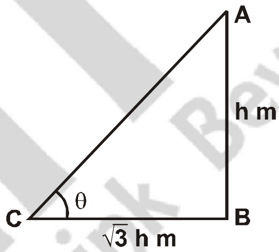
Total number of prizes = 16

$\therefore$ Probability that Abhinav wins a prize = $\frac{16}{2000} = \frac{1}{125} = 0.008$

A-18. (b) Here AB is tower of height h m.

Its shadow BC = $\sqrt{3}h$ m

Let θ be the angle of elevation



$\therefore$ In ΔABC,

$\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30°$

$\Rightarrow \theta = 30°$

A-19. (d) Assertion (A) is false but Reason (R) is true.

A-20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-21. $2x + 3y = 2$ ...(i)

$4x - 9y = -1$ ...(ii)

Now multiplying equation (i) by (2) and subtracting equation (ii) from it, we get

$4x + 6y = 4$ ...(iii)

$4x - 9y = -1$ ...(iv)

$-\quad +\quad +$

$15y = 5$

$\Rightarrow \quad y = \frac{1}{3}$

Putting the value of y in equation (i), we get

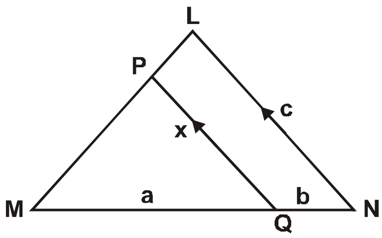
$2x + 1 = 2$

$\Rightarrow \quad 2x = 1$

$\Rightarrow \quad x = \frac{1}{2}$

$\Rightarrow \quad x + y = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

A-22.



PQ || LN

$\therefore \quad \Delta MPQ \sim \Delta MLN$ [AA similarity]

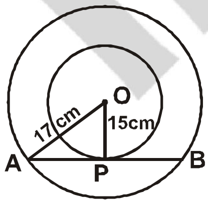
$\Rightarrow \quad \frac{MQ}{MN} = \frac{PQ}{LN}$

[Corresponding sides of similar Δs]

$\Rightarrow \quad \frac{a}{a+b} = \frac{x}{c}$

$\therefore \quad x = \frac{ac}{a+b}$

A-23.



$OP \perp AB$

[The tangent is perpendicular to radius drawn through point of contact]

In ΔOPA,

$\angle OPA = 90°$

$OA^2 = OP^2 + PA^2$

(Pythagoras theorem)

$\Rightarrow \quad 17^2 = 15^2 + PA^2$

$\Rightarrow \quad 289 - 225 = PA^2$

$PA^2 = 64$

$PA = 8$ cm

Similarly $\quad PB = 8$ cm

$AB = PA + PB = 16$ cm

A-24. $\sin\theta + \cos\theta = \frac{\sqrt{3}+1}{2}$

$\Rightarrow \quad \sin\theta + \cos\theta = \frac{\sqrt{3}}{2} + \frac{1}{2}$

On comparing, we get

$\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

$LHS = \sin\theta = \sin 60°$

$= \frac{\sqrt{3}}{2}$

$= \sqrt{3} \times \frac{1}{2} = \sqrt{3}.\cos 60°$

$= \sqrt{3}\cos\theta = RHS$

Alternate Method :

Consider $\sin\theta = \sqrt{3}\cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

Now taking

$LHS = \sin\theta + \cos\theta$

$= \sin 60° + \cos 60°$

$[\because \theta = 60°]$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

RHS

Therefore, if $\sin\theta + \cos\theta = \dfrac{\sqrt{3}+1}{2}$

then $\sin\theta = \sqrt{3}\cos\theta$

OR

$$\text{LHS} = \frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta}$$

$$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(1-2\sin^2\theta)}$$

$$= \frac{\cos[2(1-\sin^2\theta)-1]}{\sin\theta(1-2\sin^2\theta)}$$

$$= \frac{\cos\theta(2-2\sin^2\theta-1)}{\sin\theta(1-2\sin^2\theta)}$$

$$= \frac{\cos\theta(1-\sin^2\theta)}{\sin\theta(1-2\sin^2\theta)}$$

$= \cot\theta = \text{RHS}$

A-25. Area of sector $= \dfrac{77}{3}\text{cm}^2$

$$\Rightarrow \pi r^2 \frac{\theta}{360^\circ} = \frac{77}{3}$$

$$\Rightarrow \pi \times 7 \times 7 \times \frac{\theta}{360^\circ} = \frac{77}{3}$$

$$\Rightarrow \theta = \frac{77}{3} \times \frac{360^\circ}{\pi \times 7 \times 7}$$

$$\Rightarrow \theta = \frac{11 \times 120^\circ}{\frac{22}{7} \times 7}$$

$$\Rightarrow \theta = 60^\circ$$

Length of arc $= \pi r \dfrac{\theta}{180^\circ}$

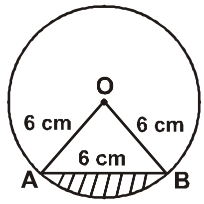
$$= \frac{22}{7} \times 7 \times \frac{60^\circ}{180^\circ}$$

$$= \frac{22}{3}\text{cm}$$

OR

We have AB = AO = BO = 6 cm

$\therefore$ ΔAOB is an equilateral triangle



Area of minor segment

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{22}{7} \times 6 \times 6 \times \frac{60^\circ}{360^\circ} - \frac{1}{2} \times 6 \times 6 \times \sin 60^\circ$$

$$= \frac{132}{7} - \frac{36\sqrt{3}}{4} = \left(\frac{132}{7} - 9\sqrt{3}\right)\text{cm}^2$$

A-26.

2	18
3	9
3	3
	1

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	6
3	3
	1

18 = 2 × 3 × 3

24 = 2 × 2 × 2 × 3

36 = 2 × 2 × 3 × 3

HCF = 2 × 3 × 6

LCM = 2 × 3 × 2 × 3 × 2 = 72

The least number divisible by 18, 24 and 36 is 72. The greatest number which divides 18, 24 and 36 is 6.

Product = 72 × 6 = 432

A-27. The given polynomial is $x^2 + 7x + 8$

$$\alpha + \beta = -7 \text{ and } \alpha\beta = 8$$

Now $\alpha^2 + \beta^2 = (\alpha - \beta)^2 - 2\alpha\beta$

$= (-7)^2 - 2 \times 8$

$= 49 - 16 = 33$

The polynomial whose zeroes are $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$ is

$$k\left[x^2-\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x+\frac{1}{(\alpha\beta)^2}\right]$$

$$= k\left[x^2-\left(\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}\right)x+\frac{1}{\alpha^2\beta^2}\right]$$

$$= k\left[x^2-\frac{33}{64}x+\frac{1}{64}\right]$$

$$= \frac{k}{64}(64x^2-33x+1)$$

$= k'[64x^2-33x+11]$, where k' s any real number.

A-28. Let length of rectangle be x units and breadth of rectangle be y units.

Area = *xy* sq. units

According to first condition

(x – 5)(y – 3) = xy – 9

⇒ xy + 3x – 5y – 15 = xy – 9

⇒ 3x – 5y = 6 ...(i)

According to second condition

(x + 3)(y + 2) = xy + 67

⇒ xy + 2x + 3y + 6 = xy + 67

⇒ 2x + 3y = 61 ...(ii)

On solving (i) and (ii), we get x = 17 and y = 9

length of rectangle = 17 units

and breath of rectangle = 9 units

OR

Let the cost of price of saree be Rs. x and marked price of sweater be Rs. y.

According to first condition

Selling price of saree = Rs. x + 8% of x

$= \text{Rs. } x + \text{Rs. } \frac{8x}{100} = \text{Rs. } \frac{27x}{25}$

Selling price of sweater

$= \text{Rs. } y - \text{Rs. } 10\% \text{ of } y = \text{Rs. } \frac{9y}{10}$

Now $\frac{27x}{25}+\frac{9y}{10}=1008$

or $\frac{3x}{25}+\frac{y}{10}=112$

or 6x + 5y = 5600 ...(i)

According to second equation

Selling price of same Rs. x + 10% of Rs.x

$= \frac{11x}{10}$

= Rs. y – 8% of Rs. y

$= \text{Rs. } y - \text{Rs. } \frac{8y}{100} = \text{Rs. } \frac{23y}{25}$

$\Rightarrow \frac{11}{10}x+\frac{23}{25}y=1028$

⇒ 55x + 46y = 51400 ...(ii)

On solving (i) and (ii), we get

y = 400 and x = 600

So cost price of saree = Rs. 600

Market price of sweater = Rs. 400

A-29. LHS

$$= \frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$= \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

[Since $\sec^2\theta-\tan^2\theta=1$]

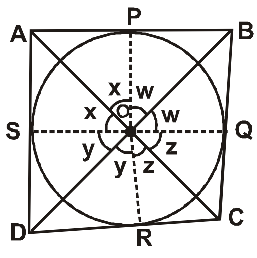
$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)(1+\tan\theta-\sec\theta)}{(\tan\theta-\sec q+1)}$$

= tan θ + sec θ

$$= \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{\sin\theta+1}{\cos\theta} = \text{RHS}$$

A-30. **Given :** circle with O as centre and circle touches the sides of quadrilateral at points P, Q, R and S. ∠AOB, ∠BOC, ∠DOC, ∠AOD are the angles made by sides AB, BC, CD and DA at centre respectively.



To prove : $\angle AOB + \angle DOC = 180°$

$\angle AOD + \angle BOC = 180°$

Construction : Join PO, QO, RO and SO.

Proof : In ΔAPO and ΔASO

AP = AS (Tangents from an external point)

$\angle APO = \angle ASO = 90°$

(Tangent is perpendicular to radius)

AO = AO (Common)

So, $\Delta APO \cong \Delta ASO$ (RHS)

$\angle AOP = \angle AOS$ (CPCT)

Let $\angle AOP = \angle AOS = x$

Similarly

$\angle DOS = \angle DOR = y$ (say)

$\angle COR = \angle COQ = z$ (say)

$\angle BOP = \angle BOQ = w$ (say

Now $\angle AOP + \angle AOS + \angle DOS + \angle DOR + \angle COR + \angle COQ + \angle BOP + \angle BOQ = 360°$

$\Rightarrow x + x + y + y + z + z + w + w = 360°$

$\Rightarrow 2x + 2y + 2z + 2w = 360°$

$\Rightarrow x + y + z + w = 180°$

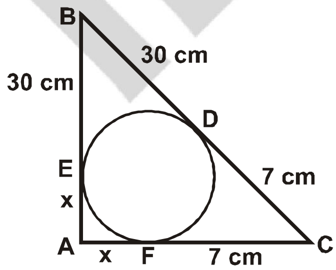
$\Rightarrow (x + w) + (y + z) = 180°$

$\angle AOD + \angle BOC = 180°$

Hence Proved.

OR

As, tangents from an external to a circle are equal in lengths.



BE = BD = 30 cm

DC = FC = 7 cm

Let AF = AE = x cm

Now using Pythagoras theorem

$(30 + x)^2 + (7 + x)^2 = 37^2$

$\Rightarrow 90 + x^2 + 60x + 49 + x^2 + 14x = 1369$

$\Rightarrow 2x^2 + 74x - 420 = 0$

$\Rightarrow x^2 + 37x - 210 = 0$

$\Rightarrow (x + 42)(x - 5) = 0$

$\Rightarrow x + 42 = 0$ or $x - 5 = 0$

$x = -42$ (rejected) or $x = 5$

So AB = 30 cm + 5 cm = 35 cm

AC = 5 cm + 7 cm = 12 cm

A-31.

	Mon	Tue	Wed	Thr	Fri	Sat
Mon	M,M	M,T	M,W	M,Th	M,F	M,S
Tue	T,M	T,T	T,W	T,Th	T,F	T,S
Wed	W,M	W,T	W,W	W,Th	W,F	W,S
Thr	Th,M	Th,T	Th,W	Th,Th	Th,F	Th,S
Fri	F,M	F,T	F,W	F,Th	F,F	F,S
Sat	S,M	S,T	S,W	S,Th	S,F	S,S

Total elements in sample space = 36

(i) Chances to visit shop on same day = 6

P (both will visit the shop on the same day) $= \frac{6}{36} = \frac{1}{6}$

(ii) P (to visit the shop on different days)

$= 1 - \frac{1}{6} = \frac{5}{6}$

(iii) Chances to visit on consecutive days = 10

P (both will visit the shop on the same days) $= \frac{10}{36} = \frac{5}{18}$

A-32. Let the time taken by the smaller diameter tap = x h

$\therefore$ Time for the larger diameter tap = (x – 2) h

Total time taken $= 1\frac{7}{8} = \frac{15}{8}$ h

Portion filled one hour by smaller diameter tap = 1/x

and by larger diameter tap $= \frac{1}{x-2}$

According to the problem

$\Rightarrow \quad \frac{1}{x}+\frac{1}{x-2}=\frac{8}{15}$

$\Rightarrow \quad \frac{x-2x+x}{x(x-2)}=\frac{8}{15}$

$\Rightarrow$ 15(2x – 2) = 8x(x – 2)

$\Rightarrow$ $30x - 30 = 8x^2 - 16x$

$\Rightarrow$ $8x^2 - 46x + 30 = 0$

$\Rightarrow$ $4x^2 - 23x + 15 = 0$

$\Rightarrow$ $4x^2 - 20x - 3x + 15 = 0$

$\Rightarrow$ 4x(x – 5) – 3(x – 50 = 0

$\Rightarrow$ (4x – 3)(x – 5) = 0

$\Rightarrow$ $x = \frac{3}{4}$ or x = 5

If $x = \frac{3}{4}$, then $x - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$

Since, time cannot be negative, we neglect $x = \frac{3}{4}$

Therefore, x = 5 and x – 2 = 5 – 2 = 3

Hence, time taken by larger diameter tap = 3 hours and time taken by smaller diameter tap = 5 hours.

OR

Let the time taken by larger pipe done alone to fill the tank = x hours.

Therefore, the time taken by the smaller pipe = x + 10 hours.

Water filled by larger pipe running for 4 hours = $\frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours = $\frac{9}{x+10}$ litres

Acording to questions,

$$\frac{4}{x}+\frac{9}{x+10}=\frac{1}{2}$$

Which on simplification gives

$x^2 - 16x - 80 = 0$

$x^2 - 20x + 4x - 80 = 0$

x(x – 20) + 4(x – 20) = 0

(x + 4)(x – 20) = 0

x = – 4, 20

z cannot be negative

Thus x = 20

x + 10 = 20

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

A-33. From the figure given in question, it is clear the DOB is a straight line.

$\therefore$ ∠DOC + ∠COB = 180°

[by linear pair axiom]

$\Rightarrow$ ∠DOC + 135 = 180°

$\Rightarrow$ ∠DOC = 180° – 135° = 45°

In ΔDOC,

∠DCO + ∠CDO + ∠DOC = 180°

[by angle sum property of a triangle]

$\Rightarrow$ ∠DCO + 70° + 45° = 180°

$\Rightarrow$ ∠DCO = 180° – 115° = 65°

[∵ ∠CDO = 70° and ∠DOC = 45°]

Given, ΔODC ~ ΔOBA

$\Rightarrow$ ΔOAB = ∠OCD = ∠DCO

$\Rightarrow$ ∠OAB = 65°

Hence, ∠DOC = 45°, ∠DCO = 65° and ∠OAB = 65°.

A-34. Volume of feeder tube = $\pi r^2 h$

$= \frac{22}{7}\times 4\times 4\times 28 = 1408\ cm^3$

Birds can eat 75 g in an hour.

So, Bird's can eat in 5 hours

= 75 × 5 = 375 g

Total capacity of tube = 3 kg = 300 g

So, volume of feeder tube = capacity of tube

$1408\ cm^3 = 3000$ g

$\therefore$ $375\ g = 176\ cm^3$

Volume of tube after 5 hours

$= 1408 - 176 = 1232\ cm^3$

i.e., $\pi r^2 h = 1232$

$$h = \frac{(1232\times 7)}{(22\times 4\times 4)} = 24.5$$

Thus, height of food in tube after 5 hours = 24.5 cm.

OR

Canal is the shape of cuboid, where

Breadth = 6 m

Depth = 1.5 m

and Speed of water = 10 km/h

Length of water moved in 60 minutes

= 10 km

Length of water moved in 1 minute

$= \frac{1}{60} \times 10$ km

Length of water moved in 30 minutes

$= \frac{30}{60} \times 10 = 5$ km = 5000 m

Now, volume of water in canal

= Length × Breadth × Depth

= 5000 × 6 × 1.5 m^3

Now, volume of water in canal

= volume of water in area irrigated

5000×6×1.5 m^3 = Area of irrigated × 8m

5000×6×1.5 m^3=Area of irrigated× $\frac{8}{100}$ m

∴ Area of irrigated

$= \frac{5000 \times 6 \times 1.5 \times 100}{8} m^2$

= 5.625 × 10^5 m^2.

35.

Class	Frequency(f)	Cumulative Frequency (c.f.)
0 – 10	f_1	f_1
10 – 20	5	$f_1 + 5$
20 – 30	9	$f_1 + 14$
30 – 40	12	$f_1 + 26$
40 – 50	f_2	$f_1 + f_2 + 26$
50 – 60	3	$f_1 + f_2 + 29$
60 – 70	2	$f_1 + f_2 + 31$
	$N = \Sigma f = 40$	

Now, $f_1 + f_2 + 31 = 40$

$\Rightarrow \quad f_1 + f_2 = 9$

$\Rightarrow \quad f_2 = 9 - f_1$ ...(i)

Given the median is 32.5, which lies in 30 – 40

Hence, median class 30 - 40

Here $l = 30$, $\frac{N}{2} = \frac{40}{2} = 20$, $f = 12$ and

c.f. = $14 + f_1$

Now, median = 32.5

$$\Rightarrow l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h = 32.5$$

$$\Rightarrow 30 + \left[\frac{20 - (14 + f_1)}{12}\right] \times 10 = 32.5$$

$$\Rightarrow \left(\frac{6 - f_1}{12}\right) \times 10 = 2.5$$

$$\Rightarrow \frac{60 - 10f_1}{12} = 2.5$$

$$\Rightarrow 60 - 10f_1 = 30$$

$$\Rightarrow 10f_1 = 30$$

$$\Rightarrow f_1 = 3$$

From eqn. (i), we get $f_2 = 9 - 3 = 6$

Hence, $f_1 = 3$ and $f_2 = 6$

A-36. (i) Since each row is increased by 10 seats, so it is an A.P. with first term a = 30 and common difference d = 10.

So, number of seats in 10th row

$= a_{10}$

= a + 9d

= 30 + 90 × 10 = 120

(ii) $S_n = \frac{n}{2}[2a + (n - 1)d]$

$1500 = \frac{n}{2}[2 \times 30 + (n - 1)10]$

$3000 = 50n + 10n^2$

$n^2 + 5n - 300 = 0$

$n^2 + 20n - 15n - 300 = 0$

$(n + 20)(n - 15) = 0$

Rejecting the negative value, n = 15.

OR

No. of seats already put up to the 10th row = S_{10}

$$S_{10} = \frac{10}{2}\{2 \times 30 + (10-1)10\}$$

$$= 5(60 + 90) = 750$$

So, the number of seats still required to be put are 1500 – 750 = 750.

(iii) Given, no. of rows = 17

Then the middle row si the 9th row.

$$a_9 = a + 8d$$

$$= 30 + 80$$

$$= 110 \text{ seats.}$$

A-37. (i) As Niharika runs $\frac{1}{4}$th distance of AD = y coordinate = $\frac{1}{4} \times 100 = 25$

And, x coordinate = 2

$\therefore$ Position of green flag = (2, 25)

(ii) As Preet runs 1/5th distance of AD = y coordinates = $\frac{1}{5} \times 100 = 20$

And, x coordinates = 8

$\therefore$ Position of red flag = (8, 20)

(iii) Distance between both the flags

$$= \sqrt{(x_2 - x_1) + (y_2 - y_1)}$$

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{6^2 - (-5)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61}$$

OR

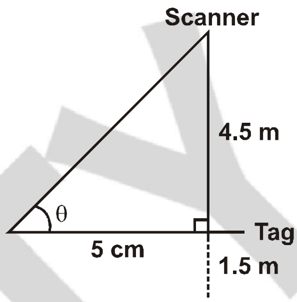
According to mid-point formula

$$= \frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$$

$$= \left[\frac{(2+8)}{2}, \frac{(20+28)}{2}\right]$$

$= (5, 22.5)$

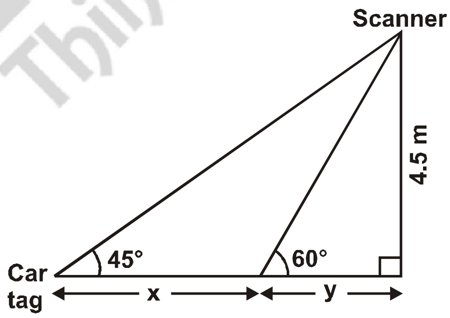
A-38. (i) One trigonometric ratio is



$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \quad \sin\theta = \frac{4.5}{5}$$

(ii) Let the distance moved by the car



be x m, then

$$\tan 30° = \frac{4.5}{x+y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{4.5}{x+y}$$

$$\Rightarrow \quad x + y = 4.5\sqrt{3} \quad ...(i)$$

$$\text{Now } \tan 60° = \frac{4.5}{y}$$

$$\Rightarrow \quad \sqrt{3} = \frac{4.5}{y}$$

$$\Rightarrow \quad y = \frac{4.5}{\sqrt{3}} \quad ...(ii)$$

From eqn. (i), we get

$$\Rightarrow \quad x + \frac{4.5}{\sqrt{3}} = 4.5\sqrt{3}$$

$$\Rightarrow \quad x = 4.5\sqrt{3} - \frac{4.5}{\sqrt{3}}$$

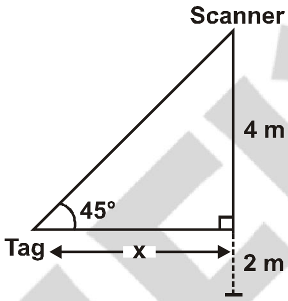
$$= \frac{3 \times 4.5 - 4.5}{\sqrt{3}}$$

$$= \frac{2 \times 4.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 3\sqrt{3} \text{ m}$$

OR

Let the distance between the tag and the scanner be x m, then according to the figure.



$$\tan 45° = \frac{4}{x}$$

$$\Rightarrow \quad 1 = \frac{4}{x}$$

$$\Rightarrow \quad x = 4 \text{ m}$$

(iii) The values of sin and cos vary from 0 to 1.

MATHEMATICS – X

PRACTICE PAPER –11

Time Allowed : 3 Hours] **(UNSOLVED)** **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF (a, b) = p^mq^n and LCM (a, b) = p^rs^s, then $(m + n)(r + s) =$

(a) 15 (b) 30 (c) 35 (d) 72

Q2. Let p be a prime number. The quadratic equation having its roots as factor of p is

(a) $x^2 - px + p = 0$ (b) $x^2 - (p + 1)x + p = 0$

(c) $x^2 + (p + 1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$

Q3. If α and β are the zeroes of polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

Q4. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k =

(a) – 1 (b) 0 (c) 1 (d) 2

Q5. If the vertices of a parallelogram PQRS taken in order are P(3, 4), Q(–2, 3) and R(–3, –2), then the coordinates of its fourth vertex S are

(a) (–2, –1) (b) (–2, – 3) (c) (2, –1) (d) (1, 2)

Q6. $\Delta ABC \sim \Delta PQR$. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM : PN =

(a) 3 : 2 (b) 16 : 81 (c) 4 : 9 (d) 2 : 3

Q7. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then x =

(a) cos 30° (b) tan 30° (c) sin 30° (d) cot 30°

Q8. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

(a) 1 (b) 2 (c) 3 (d) 4

Q9. In the given figure, DE || BC, AE = a units, EC = b units, DE = x units and BC = y units. Which of the following is true ?

(a) $x = \frac{a+b}{ay}$ (b) $x = \frac{ax}{a+b}$

(c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

Q10. ABCD is a trapezium with AD || BC and AD = 4cm. If the diagonals AC and BD intersect each other at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC =

(a) 6 cm (b) 7 m (c) 8 cm (d) 9 cm

Q11. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3cm, then the length of each tangent is equal to

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 3 cm (c) 6 cm (d) $3\sqrt{3}$ cm

Q12. The area of the circle that can be inscribed in a square of 6 cm is

(a) 36π cm² (b) 18π cm² (c) 12π cm² (d) 9π cm²

Q13. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is

(a) 48 cm² (b) 72 cm² (c) 96 cm² (d) 108 cm²

Q14. If the difference of Mode and Median of a data is 24, then the difference of Median and Mean is

(a) 8 (b) 12 (c) 24 (d) 36

Q15. The number of revolutions made by a circular wheel of radius 0.25 m in rolling a distance of 11 km is

(a) 2800 (b) 4000 (c) 5500 (d) 7000

Q16. For the following distribution

Class	Frequency
0 – 5	10
5 – 10	15
10 – 15	12
15 – 20	20
20 – 25	9

the sum of the lower limits of the median and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

Q17. Two dice are rolled simultaneously. What is the probability that 6 will come up at least once?

(a) $\frac{1}{6}$ (b) $\frac{7}{36}$ (c) $\frac{11}{36}$ (d) $\frac{13}{36}$

Q18. If $5\tan\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) 6

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** If produced of two numbers is 5780 and their HCF is 17, then their LCM is 340.

Statement (Reason) : HCF is always a factor of LCM.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Assertion (A) :** If the coordinates of the mid-points of the sides AB and AC of ΔABC are D(3, 5) and E(–3, –3) respectively, then BC = 20 units.

Reason (R) : The line joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. Find the number of solution of the following pair of linear equation :

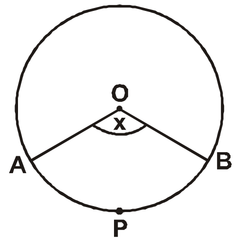
$2x + 3y + 17 = 0$

$6y + 4x + 15 = 0$

Q22. If the mid-point of the line segment joining the points P(6, a – 2) and Q(– 2, 4) is (2, – 4), then find the value of a.

Q23. The length of a line segment is 13 units and the coordinates of one end point are (–6, 7). If the ordinate of the other end point is –1, find the abscissa of the other and point.

Q24. In the given figure, O is the centre of the circle. The area of sector OAPB is $\frac{7}{18}$ times of the area of the circle. Find the value of x.



OR

The perimeter of a sector of a circle of a radius 5.2 cm is 16.4 cm. Find its area.

Q25. If cosec A = $\frac{15}{7}$ and A + B = 90°, find the value of sec B.

OR

If $7\sin^2\theta + 3\cos^2\theta = 4$, then show that $\tan\theta = \pm\frac{1}{\sqrt{3}}$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $7 - 2\sqrt{3}$ is an irrational number.

Q27. Solve the following pair of equations for x and y : $\frac{15}{x-y} + \frac{22}{x+y} = 5, \frac{40}{x-y} + \frac{55}{x+y} = 13,$ $x \neq y, x \neq -y.$

Q28. In an A.P. the sum of first ten terms is – 150 and the sum of its next ten terms is – 550. Find the AP.

OR

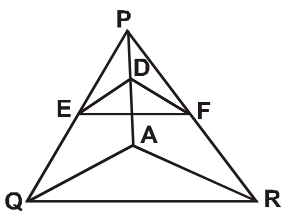
Using AP, find the sum of all 3-digit natural numbers which are the multiple of 7.

Q29. If cos A = $\frac{17}{18}$, evaluate $\frac{(1+\sin A)(1-\sin A)}{(1+\cos A)(1-\cos A)}$.

Q30. AB is a diameter of a circle AH and BK are perpendiculars from A and B respectively to the tangent at P. Prove that AH + BK = AB.

OR

In the given figure, DE || AQ and DF || AR. Prove that EF || QR.



Q31. A card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is

(i) either a heart or a queen (b) a black king

(iii) neither an ace nor a jack.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. The percentage age of a girl is 3 years more than three times the age of her sister. Three years hence, the girl's age will be 10 years more than twice the age of her sister. Find their present age.

OR

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Q33. Sides AB, AC and median AD of a ΔABC are respectively proportional to sides PQ, PR and median PS of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Q34. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume and surface area of remaining solid.

OR

A right-angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two ways. Find the ratio of volume of two comes so formed. Also, find the difference of their curved surface area. ($\pi = 3.14$).

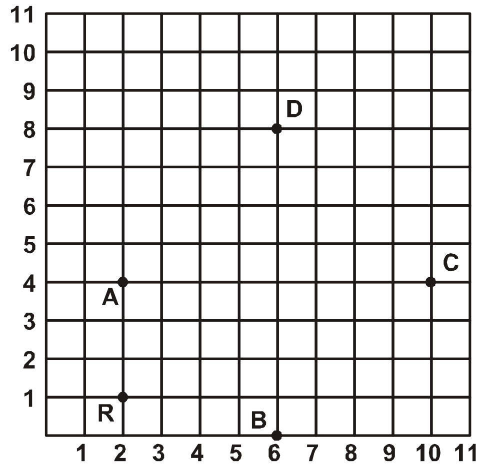
Q35. Compare the modal ages of two groups of students appearing for an entrance examination.

Age (in years)	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

SECTION - E

Case study based questions are compoulsory.

Q36. For annual day practice students of a class are standing in rows and columns. Four students Ashish, Bipin, Cintha and Damodar are holding flags, their position is shown as in the figure.



(i) What are the coordinates of D ?

(ii) What is the distance of A from D ?

(iii) Ram is positioned at R. He moves from R and take his position such that he is equidistant from A and C. What are the coordinates of Ram in its new position ?

OR

How much distance is covered by Ram to move to the new position ?

Q37. General form of pair of linear equations in two variables is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. If graph of pairs of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. Represent two intersecting lines then point of intersection is the solution of pair of linear equations. If graph represent two parallel lines then pair of linear equations has no common solutions. If graph represents two coincident lines then pair of linear equations has infinitely many solutions.

Answer the questions based on above.

(i) The pair of equations x = 5 and y = 5 graphically intersect at which point ?

(ii) For the linear equation 2x + 5y – 8 = 0, find the another linear equation in two variables such that the graphical representation of the pair so formed represents parallel lines.

(iii) The graph of linear equations x = 3 and x = 5 and 2x – y – 4 = 0 and x-axis represents which type of figure ?

OR

Find the area of the region formed by lines x = 2, y = 5, x = 0 and y = 0.

Q38. Agarima organised a stall to play 'Lucky Number' at the school fest. She kept a ticket of Rs.

20 to play the game. She will give coin to the player. On tossing the coin if head appears. Agarima will throw a die and player will get the money equivalent to 5 times of the number appearing on the die. If tail appears, player losses the game.

(i) Find the total number of possible outcomes.

(ii) If a plays get a head and on die there is number 2. How much money he will lose or win (count the money paid to play the game) ?

(iii) What is probability of getting Rs. 30 on throwing the die ?

OR

What is the probability of losing the game ?

ANSWERS

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)
5. (c)	**6.** (d)	**7.** (b)	**8.** (b)
9. (c)	**10.** (c)	**11.** (d)	**12.** (d)
13. (c)	**14.** (b)	**15.** (d)	**16.** (b)
17. (c)	**18.** (a)	**19.** (b)	**20.** (a)

21. unique solution

22. a = – 10

23. $\frac{-12+\sqrt{420}}{2}$ or $\frac{-12-\sqrt{420}}{2}$

24. x = 140° **OR** 15.6 cm^2.

25. $\sin B = \frac{15}{7}$ **OR** $\pm\frac{1}{\sqrt{3}}$

26. Prove

27. x = 8, y = 3

28. –3, –1, –5, – 9,... **OR** 70336

29. $\frac{289}{324}$

30. Prove

31. (i) $\frac{4}{13}$ (ii) $\frac{1}{26}$ (iii) $\frac{11}{13}$

32. 33 years, 10 years **OR** 64

33. Prove

34. $\frac{6}{7}(310-11\sqrt{58})\,\text{cm}^2$ **OR** 15.70 cm^2

35. Modal age of group B is 0.1 year less than the age of group A

36. (i) 6.8 (ii) $4\sqrt{2}$ units (iii) (6, 4) **OR** 5 units

37. (i) (5, 5) (ii) may vary (iii) represent quadrilateral **OR** 10 sq. units

38. (i) 7 (ii) player will get ₹10 when the outcome is H2. (iii) $\frac{1}{7}$ **OR** $\frac{1}{7}$

MATHEMATICS – X

PRACTICE PAPER–12(UNSOLVED)

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. If one of the zero of polynomial $p(x) = x^2 - kx - 3$ is 3, then the value of k is

(a) – 3 (b) 2 (c) – 2 (d) 0

Q2. The HCF of the smallest l2-digit composite number and 2-digit largest prime number is

(a) 2 (b) 0 (c) 10 (d) 1

Q3. The solution of pair of linear equation $x + y = 7$ and $x - y = 1$ is

(a) $x = 6, y = 1$ (b) $x = 5, y = 2$ (c) $x = 5, y = 4$ (d) $x = 4, y = 3$

Q4. Which of them is not a similarity criterion for two triangles ?

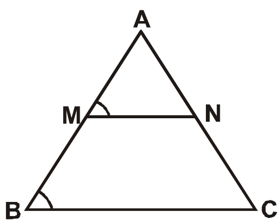
(a) SSS similarity criterion (b) AA similarity criterion

(c) SAS similarity criterion (d) SSA similarity criterion

Q5. For what value of k, the quadratic equation $9x^2 + 3kx + 4 = 0$ has real and equal roots ?

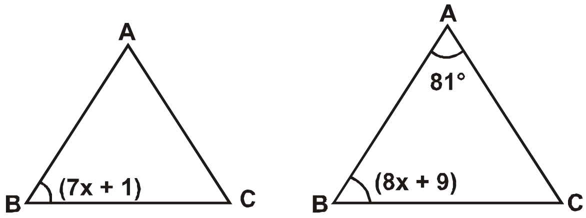
(a) ± 3 (b) ± 6 (c) ± 4 (d) 0

Q6. In the given figure, ∠AMN = ∠ABC, if AB = 8 cm, AM = 3 cm, AC = 12 cm, then NC is equal to



(a) 9 cm (b) 4 cm (c) 7.5 cm (d) 8.5 cm

Q7. In the given figure, if ΔABC ~ ΔPQR, then value of ∠C is



(a) 70° (b) 50° (c) 29° (d) 81°

Q8. The ratio of CSA (curved surface area) to the total surface area of a cylinder having height equal to radius of its base is

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 2 : 3

Q9. If mean given distribution is 6.1, then value of x is

x_i	1	3	5	7	9
f_i	$x+1$	$x+4$	$x+7$	$x+8$	$x+10$

(a) 4 (b) 2 (c) 10 (d) 0

Q10. For some data, mean : mode = 5 : 3, then median : mode is

(a) 13 : 1 (b) 8 : 3 (c) 5 : 3 (d) 13 : 9

Q11. A basket contains 7 apples and some oranges. If the probability of drawing an apple is 1/3rd of an orange, then number of oranges in the basket is

(a) 7 (b) 21 (c) 10 (d) 14

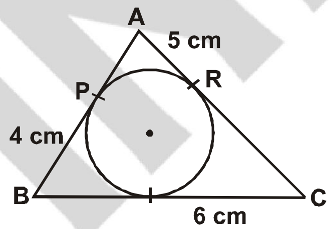
Q12. The volume of metal used for making a pipe of length 14 cm, having outer and inner radii as 2.2 cm and 1.8 cm is

(a) 140.8 cm^3 (b) 35.2 cm^3 (c) 17.6 cm^3 (d) 70.4 cm^3

Q13. If the length of tangent drawn from an external point is $\sqrt{3}$ times the radius of the circle, then angle between two tangents drawn from same point is

(a) 60° (b) 30° (c) 45° (d) 120°

Q14. In figure, the perimeter of ΔABC is



(a) 15 cm (b) 30 cm (c) 32 cm (d) 120 cm

Q15. If $\sin\theta + \cos\theta = \sqrt{2}$, then value of $\sin\theta - \cos\theta$ is

(a) $\sqrt{2}$ (b) $\pm\frac{1}{\sqrt{2}}$ (c) 0 (d) $\pm\sqrt{2}$

Q16. The length of shadow of a vertical pole of height 12 m, when altitude of Sun is 30° is

(a) $12\sqrt{3}$m (b) $4\sqrt{3}$m (c) 12 m (d) $6\sqrt{3}$m

Q17. The ratio of length of arcs formed by two chords which makes the angle of 30° and 60° respectively with centre of the circle is

(a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 1 : 1

Q18. If point C $\left(\frac{q}{2}, 4\right)$ is the mid point of A(p, 0) and B(0, q), then value of $\frac{p}{q}+\frac{q}{p}$ is

(a) 4 (b) 2 (c) 0 (d) 6

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** (3 × 3 × 2 × 2 + 7) is a composite number.

Statement R (Reason) : A number having more than 2 factors is called a composite number.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is the false but reason (R) is true.

Q20. **Statement A (Assertion) :** y-axis divides the line joining points A(–3, 5) and B(3, 1) at P(0, 3), where P is the mid point of line AB.

Statement R (Reason) : The mid-point of a line segment divides the line segment in the ratio of 1 : 1. So, the coordinates of mid-point p of the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is the false but reason (R) is true.

SECTION - B

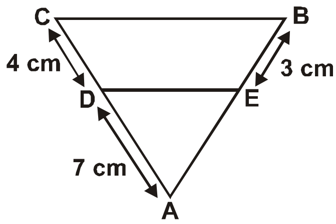
Section B consists of 5 questions of 2 marks each.

Q21. If 2 is a root of the equation $x^2 + bx + 12 = 0$, then find the value of b.

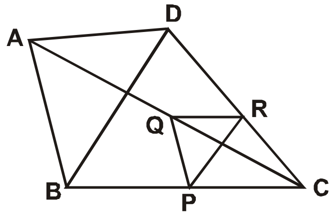
OR

If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, then find the value of $\alpha+\beta+\alpha\beta$.

Q22. In the given figure, DE || CB. Find the length of AE.



Q23. In the given figure, two triangles ABC and DBC lie on the same base BC. P is a point BC such that PQ || BA and PR || BD. Prove that QR || ADs.



Q24. A cone and a sphere have equal radii and equal volume. What is the ratio of the diameter of the sphere to the height of the cone ?

OR

Probability of getting ₹2 coin

From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining solid.

Q25. In ΔABC, right-angled at C, prove that $\sqrt{\dfrac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \dfrac{\sin^2 B}{\cos^2 A}} = \tan A$

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Show that $3\sqrt{3}$ is an irrational number.

Q27. Solve for x : $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}=\dfrac{1}{a+b+x}$; $a \neq 0,\ b \neq 0,\ x \neq 0$

Q28. Determine graphically the vertices of a triangle, the equations of whose sides are given below : $2y - x = 8,\ 5y - x = 14,\ -x + \dfrac{y}{2} = \dfrac{1}{2}$

OR

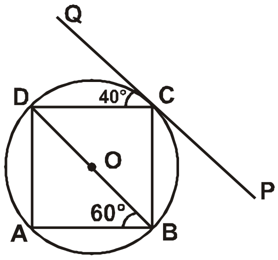
Seven times a two-digit number is equal to four times the number obtained by reversing the order of the digits. If the difference of the digits is 3, determine the number.

Q29. A vertical pillar stands on the plane ground and is surmounted by a flagstaff of height 5m. From a point on the ground, the angle of elevation of the bottom of the flagstaff is 45° and that of the top of the flagstaff is 60°. Find the height of the pillar. (Use $\sqrt{3}$ = 1.732).

Q30. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

OR

In the given figure, ABCD is a cyclic quadrilateral and PQ is tangent to the circle at C. If BD is a diameter, $\angle DCQ = 40°$ and $\angle ABD = 60°$, find $\angle BCP$.



Q31. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card

(iii) the queen of diamond.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. Divide 39 into two parts such that their product is 324.

OR

The difference of square of two natural numbers is 45. The square of smaller number is four times the larger number. Find the numbers.

Q33. For going to a city A, there is a route via city C such that $AC \perp CB$, AC = 2 km and CB = 2(x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and b. Find how much distance will be saved in reaching city B from city B after the construction of the highway.

Q34. An iron pipe 20 cm long has exterior diameter equal to 25 cm. If the thickness of the pipe is 1cm, find the whole surface area of the pipe.

OR

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

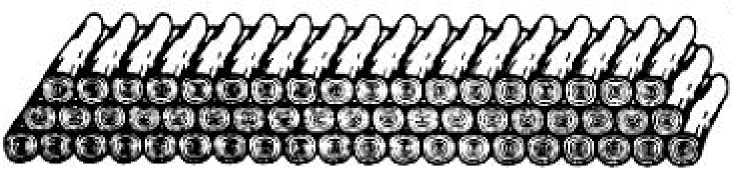
Q35. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the mean monthly expenditure of the families.

Expenditure (in ₹)	Number of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

SECTION - E

Case study based questions are compoulsory.

Q36. 200 logs are staked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the next row to it and so on in a timber factory by the crane.



(i) Find the number of logs in 6th row.

(ii) How many rows are there in which 200 logs can placed ?

(iii) Find the difference of logs in top most row and 1st row ?

OR

Find the difference of logs between 14th and 10th row.

Q37. In a classroom, 4 friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and observe for a few minutes now help them :

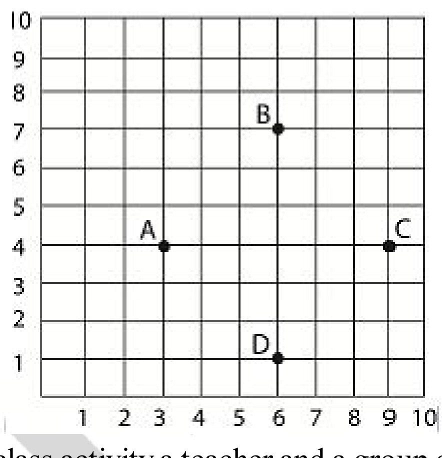
(i) to find the distance between A and B; A and D.

(ii) to find the distance between B and C; D and C.

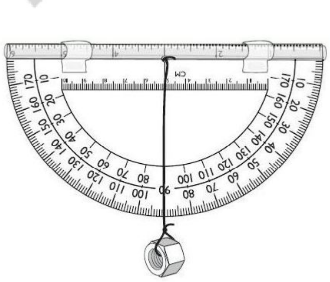
(iii) to show that ABCD is a square using distance formula.

OR

Find the coordinates of point where diagonals of a square ABCD intersect.



Q38. In a class activity a teacher and a group of 12 students went to see Qutub Minar and with the help of clinometer (A device used in measuring angle) they observed that the angle of elevation of top of Qutub Minar is 60°.



Answer the following :

(i) If they are 140 feet away from the foot of the Minar, find height of Qutub Minar. (Use $\sqrt{3} = 1.7$)

(ii) Find the distance they walked away from this point if now the angle of elevation is 45°.

(iii) If the Sun's altitude reduced from 60° to 30°, then find the increase in the length of the shadow.

OR

How trigonometry is useful for us ? Give any 2 points.

ANSWERS

1. (b) **2.** (d) **3.** (d) **4.** (d)

5. (c) **6.** (c) **7.** (a) **8.** (c)

9. (b) **10.** (d) **11.** (b) **12.** (d)

13. (a) **14.** (b) **15.** (c) **16.** (a)

17. (a) **18.** (b) **19.** (d) **20.** (a)

21. b = – 8 **OR** –1 **22.** 5.25 cm **23.** Prove **24.** 1 : 2 **OR** 1056 cm^2

25. Prove **26.** Prove **27.** x = – a, – b

28. (2, 5), (1, 3), (– 4, 2) **OR** 36 **29.** 6.83 m **30.** Prove **OR** 50°

31. (i) $\frac{1}{26}$ (ii) $\frac{3}{13}$ (iii) $\frac{1}{52}$ **32.** 12 and 27 **OR** 9 and 6

33. 8 km **34.** 3168 cm^2 **OR** π cm^3 **35.** ₹2662.50

36. (i) 6th row = 15 (ii) no. of rows = 16 (iii) 15 **OR** 4

37. (i) $3\sqrt{2}$ units (ii) $3\sqrt{2}$ units (iii) ABCD is square **OR** (6, 4)

38. (i) 238 feet (ii) 98 feet (iii) 280 feet