MOST IMPORTANT QUESTIONS (BOOK - 1)

XII CBSE BOARD 2024- 25

Ch – 1 RELATIONS & FUNCTIONS

Q1. Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Q2. If $y = f(x) = \frac{x^2}{1+x^2}$, is the function one-one and onto provided $f: R \to R$?

Q3. If $f: R \to R$ be the function defined by $f(x) = 4x^3 + 7$, show that $f(x)$ is bijection.

Q4. Let $A = \{x \in Z: 0 \le x \le 12\}$. Show that that $R = \{(a, b): a, b \in A, |a - b|$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class.

Q5. Show that $f: N \to N$, given by $f(x) = \begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ is one –one and onto .

Q6. State the reason why the relation $R = \{(a, b) : a \le b^2\}$ on the set R of real numbers is not reflexive.

Q7. Show that the relation R in the set N x N defined by (a, b)R(c, d) if $a^2 + d^2 = b^2 + c^2 \forall$ a, b, c, d ∈ N, is an equivalence relation.

Q8. Show that the relation S defined on set $N\ x\ N\ by\ (a, b)S(c, d)\ if\ a + d = b + c$ is an equivalence relation.

Q9. Let N denote the set of all natural numbers and R be the relation on $N\ x\ N$ defined by $(a, b)R(c, d)\ if\ ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

Q10. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$. Then show that f is bijective.

Q11. Let $f: R \to$ R be defined by
(i) $f(x) = x + |x|$
(ii) $f(x) = x + 1$. Determine whether or not f is onto.

Q12. Write the domain of the relation R defined on the set Z of integers as follows: $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$.

Q13.Given a non – empty set X ,define the relation R in P(X) as follows :
For A , B ∈ $P(X)$, (A,B) ∈ R iff A ⊂ B .Prove that R is reflexive ,transitive and not symmetric.

Q14. Let N be the set of all natural numbers and R be a relation in N defined by $R = \{(a, b): a\ is\ a\ factor\ of\ b\}$, then show that R is reflexive and transitive but not symmetric.

Q15. Show that the function $f: R \to R$ such that $f(x) = \begin{cases} 1, if\ x\ is\ rational \\ -1, if\ x\ is\ irrational \end{cases}$ is many one and not onto.

Find : (i) f $\left(\frac{1}{2}\right)$ (ii) f $(\sqrt{2})$ (iii) f (iv) f $(2 + \sqrt{3})$

Q16.Let S be the set of all real numbers and let R be a relation in S , defined by R = { (a,b) : a≤ b) . Show that R is reflexive and transitive but not symmetric.

Q17. Let R be the relation defined in the Set A = { 1,2,3,4,5,6,7 } by R = { (a , b) : both a and b are either odd or even).Show that R is an equivalence relation .Hence ,find the elements of equivalence class [1].

Q18. Determine whether the relation R defined on the set R of all real numbers as R = { (a,b) : a,b $\in R$ and a – b + $\sqrt{3} \in$ S , where S in the set of all irrational numbers } , is reflexive ,symmetric and transitive .

Q19. Show that the function f in A = R - $\left\{\frac{2}{3}\right\}$ defined as f(x) = $\frac{4x+3}{6x-4}$ is one - one and onto .

Q20. Prove that a function f : [0 , ∞) → [-5 , ∞) defined as f(x) = $4x^2 + 4x - 5$ is both one – one and onto .

Ch – 2 INVERSE TRIGONOMETRIC FUNCTIONS

Q1. Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$.

Q2. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$; $-\frac{1}{\sqrt{2}} \le x \le 1$.

Q3. Find the domain and range of $sin^{-1}x$.

Q4. write in the simplest form : $tan^{-1}\sqrt{\frac{a-x}{a+x}}$.

Q5. Evaluate: $cos\left\{cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$

Q6. Simplify :$cos^{-1}\left(\frac{3}{5}\, cosx + \frac{4}{5}\sin x\right)$.

Q7. Find the value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$.

Q8. Write the simplest form of $tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$.

Q9. Express in the simplest form:
$\tan^{-1}\left[\frac{\cos x-\sin x}{cosx+\sin x}\right]$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$

Q10. Find the range of $f(x) = sin^{-1}x + tan^{-1}x + sec^{-1}x$.

Q11. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Q12. Show that $sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$.

Q13. What is the domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$.

Q14. Find the range of the function f(x) = $\tan^{-1} x + \frac{1}{2}\sin^{-1} x$.

Q15. Find the value of $cos^{-1}\left(\cos\frac{14\pi}{3}\right)$

Q16. Draw the graph of $\cos^{-1} x$,where x ϵ [-1 , 0] .Also ,write its range.

Q17. Find the value of $\tan^{-1}\left[2\cos\left(2\, sin^{-1}\left\{\frac{1}{2}\right\}\right)\right]$.

Q18. Find the domain of y = $\sin^{-1}(x^2 - 4)$.

Q19. Write in simplest form : $\tan^{-1}\left(\frac{cos\,x-sin\,x}{cos\,x+sin\,x}\right)$, 0 < x < π .

Q20. Show that : $\cot^{-1}\left(\frac{\sqrt{1+sin\,x}+\sqrt{1-sin\,x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right) = \frac{x}{2}$, x $\epsilon \left(0, \frac{\pi}{4}\right)$.

Ch – 3 MATRICES

Q1.If A = $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}.

Q2. Find matrices X and Y if $2X + Y = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 0 & 3 & 5 \\ -2 & -4 & 1 \end{bmatrix}$.

Q3. Find matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$.

Q4. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Q5. If A= $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

Q6. Express the matrix A = $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$, as the sum of a symmetric and skew symmetric matrix.

Q7. Show that the matrix BAB is symmetric or skew symmetric accordingly when A is symmetric or skew-symmetric.

Q8. If A = diag [2 -1 3] and 6 = diag [3 0 -1], then find 4A + 2B.

Q9. For what value of x, is the matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$, a skew-symmetric matrix?

Q10. Find x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$, satisfies $A' = A^{-1}$.

Q11. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^2 = \begin{bmatrix} 7 & 12 \\ 18 & 31 \end{bmatrix}$,then find the value of (ad – bc) . (Given ad – bc < 0) .

Q12. If A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2 ,then show that $A^2 = 4A - 3I$ .Hence find A^{-1}.

Q13. Find a matrix A such that 2A – 3B + 5C = 0 , where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

Q14. Find the value of X and Y if X + Y = $\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$, X – Y = $\begin{bmatrix} 6 & 5 \\ 7 & 3 \end{bmatrix}$.

Q15. If $\begin{bmatrix} x^2 - 4x & x^2 \\ x^2 & x^3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -x + 2 & 1 \end{bmatrix}$, then find x .

Q16. If $\begin{bmatrix} a + 4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a + 2 & b + 2 \\ 8 & a - 8b \end{bmatrix}$, write the value of a – 2b .

Q17. Matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric ,find the values of a and b .

Q18. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$.Hence ,find the matrix P satisfying the matrix equation P $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Q19. Find the matrix X , so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ = $\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Q20. If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and (A + B)^2 = A^2 + B^2 , then find the values of a and b .

Ch – 4 DETERMINANTS

Q1.Given A = $\begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{7} \end{bmatrix}$, compute I^{-1} and show that 2A^{-1} = $9I - A$.

Q2. If A is a skew – symmetric matrix of order 3, then prove that $detA$ = 0.

Q3. If A is a square matrix satisfying $A'A = I$, write the value of |A|.

Q4. If A is a 3x3 matrix, $|A| \neq 0$ $|3A| = k|A|$, then write the value of k.

Q5. For what value of x, the given matrix A = $\begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular matrix?

Q6. If A,B are square matrices of the same order, then prove that $adj(AB)$ = $(adjB)(adjA)$.

Q7. f A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equation:

$3x + 4y + 7z = 14$,

$2x - y + 3z = 4$,

$x + 2y - 3z = 0$.

Q8. Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find the value of k if D(k, 0) is a point such that area of ΔABD is 3 square units.

Q9. If A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$

Q10. f A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

Q11. Find the adjoint of the matrix A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A. (adj A)=$|A|I_3$.

Q12. If A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$,then find the value of λ for which A^{-1} exists

Q13. If A is a square matrix of order 3 and I A I = 5 ,then find the value of I 2A' I .

Q14. If A and B are square matrices of the same order 3 ,such that IAI = 2 and AB = $2I$, write the value of I B I.

Q15. If A = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and B^{-1} = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find (AB)$^{-1}$.

Q16. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it solve the following system of equations :

x + 2y – 3z = 6 ,

3x + 2y – 2z = 3
2x – y + z = 2

Q17. Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equations :

x + 3z = 9 ,
-x + 2y – 2z = 4 ,
2x – 3y + 4z = -3

Q18. If $\begin{vmatrix}x+1 & x-1\\x-3 & x+2\end{vmatrix} = \begin{vmatrix}4 & -1\\1 & 3\end{vmatrix}$, then write the value of x .

Q19. If A = $\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 1\\1 & -3 & 1\end{bmatrix}$, find A^{-1}. Hence solve the system of equations:

$x + 2y + z = 4,$
$-x + y + z = 0,$
$x - 3y + z = 4.$

Q20. If A = $\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}$ and B = $\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\0 & -1 & 5\end{bmatrix}$ are square matrices, find $A \cdot B$ and hence solve the system of equations:

$x - y = 3,$
$2x + 3y + 4z = 17$
$y + 2z = 7$

Ch – 5 CONTINUITY AND DIFFERENTIABILITY

Q1. Find the value(s) of ' λ' , if the function continuous at x = 0.

F(x) = $\begin{cases}\frac{\sin^2\lambda}{x^2}, if\ x \neq 0\\ 1, if\ x = 0\end{cases}$

Q2. Find the values of p and q, for which f(x) = $\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x}, & x < \frac{\pi}{2}\\ p, if\ x = \frac{\pi}{2}\\ \frac{q(1-\sin x)}{(\pi-2x)^2}, if\ x > \frac{\pi}{2}\end{cases}$ is continuous at x = $\frac{\pi}{2}$.

Q3. If y = $(x)^{\cos x} + (\cos x)^{\sin x}$, then find $\frac{dy}{dx}$.

Q4. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\sin^{-1}\frac{2x}{1+x^2}$, if x ϵ (-1 , 1) .

Q5. If $\sqrt{1-x^2} + \sqrt{1-y^2}$ = a (x – y) , prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Q6. Show that the function $f(x) = |x-3|, x \in R$, is continuous but not differentiable at $x = 3$.

Q7. Find the value of k, for which f(x) = $\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, \text{if } -1 \leq x < 0\\ \frac{2x+1}{2x-1}, \text{if } 0 \leq x < 1\end{cases}$ is continuous at x = 0 .

Q8. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

Q9. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then show that $y^2 \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

Q10. If $y = e^{x+e^{x+e^{x}+\cdots to\infty}}$, prove that $\frac{dy}{dx} = \frac{y}{1-y}$.

Q11. Differentiate $tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$ w.r.t x .

Q12. If $y = \sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$.

Q13. If $x = \sqrt{a^{tan^{-1}(t)}}$, $y = \sqrt{a^{cot^{-1}(t)}}$, then show that $x\frac{dy}{dx} + y = 0$.

Q14. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t $\sin^{-1}(2x\sqrt{1-x^2})$.

Q15. If $(a + bx)e^{\frac{y}{x}} = x$ then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$.

Q16. If x = a sin t – b cos t , y = a cost + b sin t ,then prove that $\frac{d^2y}{dx^2} = -\left(\frac{x^2+y^2}{y^3}\right)$.

Q17. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, when\ x < 0 \\ a, when\ x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16+\sqrt{x}})-4}, when\ x > 0 \end{cases}$, and f is continuous at x = 0 , find the value of a .

Q18. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for $x \neq y$. Prove the following : $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

Q19. Show that the function f(x) = | x – 1 | + | x + 1 | , for all x ∈ R, is not all differentiable at the points x = -1 and x = 1 .

Q20. Differentate $x^{\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t x. x .

Ch – 6 APPLICATION OF DERIVATIVES

Q1. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

Q2. Find the maximum and minimum values of $f(x) = -|x-1| + 5\ for\ all\ x \in R$.

Q3. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width.

Q4. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

Q5. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given ,then show that the area of triangle is maximum ,when the angle between them is $\frac{\pi}{3}$.

Q6. Find the intervals in which the function F(x) = $\frac{x^4}{4} - x^3$ - $5x^2$ + $24x$ + 12 is

(i) strictly increasing,

(ii) strictly decreasing.

Q7. A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 cm^3 .If building of tank costs Rs. 100 per square metre for the base and Rs. 50 per square metre for the sides, find the cost of least expensive tank.

Q8. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

Q9. A man 16 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground . At what rate is the to of his shadow moving ?At what rate is his shadow lengthening ?

Q10. How that semi-vertical angle of a cone of maximum volume and given slant height is $cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Q11. If lengths of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum. Rate is the tip of his s

Q12. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

Q13. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

Q14. A particle moves along the curve 3y = ax^3 + 1 such that a point with x coordinate 1 , y – coordinate is changing twice as fast at x – coordinate .Find the value of a.

Q15. Find the maximum and minimum values of the function given by f(x) = 5 + sin 2x .

Q16. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s.Find the rate at which its side is increasing .

Q17. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec .How fast is the area decreasing when the two equal sides are equal to the base ?

Q18. Of all the closed right cylindrical cans of volume $128\pi cm^3$,find the dimensions of the can which has minimum surface area .

Q19.A window is the form of a semi – circle with a rectangle on its diameter .The total perimeter of the window is 10 m .Find the dimensions of the window to admit maximum light through the whole opening.

Q20. Find the point on the curve y^2 = 2x which is at minimum distance from the point (1,4).