PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 01 - CHAPTER 01 REAL NUMBERS (2024-25)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The exponent of 5 in the prime factorization of 3750 is
(a) 3 (b) 4 (c) 5 (d) 6

2. If two positive integers a and b are written as $a = x^2y^2$ and $b = xy^3$, where x and y are prime numbers, then the LCM (a, b) is:
(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^3

3. The HCF and the LCM of 12, 21, 15 respectively are
(a) 3, 140 (b) 12, 420 (c) 3, 420 (d) 420, 3

4. If the HCF of 65 and 117 is expressible in the form 65m –117, then the value of m is
(a) 4 (b) 2 (c) 11 (d) 3

5. Arnav has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?
(a) 4 cm as it is the HCF of 40 and 84 (b) 4 cm as it is the LCM of 40 and 84
(c) 12 cm as it is the LCM of 40 and 84 (d) 12 cm as it is the HCF of 40 and 84

6. The largest number which divides 70 and 125 leaving remainders 5 and 8 respectively is
(a) 13 (b) 65 (c) 875 (d) 1750

7. If $6370 = 2^m \times 5^n \times 7^k \times 13^p$, then the value of m + n + k + p is
(a) 2 (b) 3 (c) 4 (d) 5

8. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) $= 2^3 \times 3^2 \times 5$, then n is equal to
(a) 1 (b) 2 (c) 3 (d) 4

9. In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
Assertion (A): If product of two numbers is 5780 and their HCF is17, then their LCM is 340.
Reason (R): HCF is always a factor of LCM.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

10. In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
Assertion (A): 6^n ends with the digit zero, where n is natural number.
Reason (R): Any number ends with digit zero, if its prime factor is of the form $2^m \times 5^n$, where m, n are natural numbers.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Explain why $2 \times 3 \times 5 + 5$ and $5 \times 7 \times 11 + 7 \times 5$ are composite numbers.

12. Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers?

13. Show that any number of the form 6^n, where $n \in N$ can never end with digit 0. (2017)

14. The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find the HCF of the two numbers.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that $\sqrt{3}$ is an irrational number. (2023)

16. 4 Bells toll together at 9.00 am. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

17. Given that $\sqrt{3}$ is irrational, prove that $5 + 2\sqrt{3}$ is irrational. (CBSE Sample Paper 2022)

SECTION – D

Questions 18 carry 5 marks.

18. (a) Find the largest possible positive integer that divides 125, 162 and 259 leaving remainder 5, 6 and 7 respectively. (3)
(b) An army contingent of 678 soldiers is to march behind an army band of 36 members in a Republic Day parade. The two groups are to march in the same number of columns. What is the maximum number of columns they can march? (2)

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A morning walk may help improve your mental clarity and ability to focus throughout the day. A recent study found that amongst older adults, those who started their days with a morning walk improved their cognitive function, compared to those who remained sedentary. Walking may also help you think more creatively. In a morning walk three students step off together, their steps measure 80 cm, 85 cm and 90 cm respectively.

(i) What is the HCF of 80 and 90? (1)
(ii) Find the sum of exponents of the prime factors of total distance. (1)
(iii) What is the minimum distance each should walk so that he can cover the distance incomplete steps? (2)

20. A family room is an informal, all purpose room is a house. The family room is designed to be a place where family and guests gather for group recreation like talking, reading, watching TV and other family activities. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm.

(i) Determine the longest rod which can measure the three dimensions of the room exactly. (2)
(ii) What is LCM of the given three measurements? (1)
(iii) If the HCF (825 and 675) = 75, then find LCM (825 and 675). (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 01 - CHAPTER 01 REAL NUMBERS (2024-25)

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The exponent of 5 in the prime factorization of 3750 is

(a) 3 (b) 4 (c) 5 (d) 6

Ans: (b) 4

2. If two positive integers a and b are written as $a = x^2y^2$ and $b = xy^3$, where x and y are prime numbers, then the LCM (a, b) is:

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^3

Ans: (d) x^2y^3

Here, $a = x^2y^2$ and $b = xy^3$

$\therefore$ LCM(a, b) = x^2y^3

3. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420 (c) 3, 420 (d) 420, 3

Ans: (c) 3, 420

4. If the HCF of 65 and 117 is expressible in the form 65m –117, then the value of m is

(a) 4 (b) 2 (c) 11 (d) 3

Ans: (b) 2

65 = 5 x 13

117 = 3 × 3 × 13

Therefore, HCF of 65 and 117 is 13.

So, 65m – 117 = 13

$\Rightarrow$ 65m = 130 $\Rightarrow$ m = 2

5. Arnav has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

(a) 4 cm as it is the HCF of 40 and 84 (b) 4 cm as it is the LCM of 40 and 84

(c) 12 cm as it is the LCM of 40 and 84 (d) 12 cm as it is the HCF of 40 and 84

Ans: (a) 4 cm as it is the HCF of 40 and 84

6. The largest number which divides 70 and 125 leaving remainders 5 and 8 respectively is

(a) 13 (b) 65 (c) 875 (d) 1750

Ans: (a) 13

Number when divides 70 and 125 leaves remainders 5 and 8, then 70 - 5 = 65

125 - 8 = 117 then HCF of 65 and 117 is

65 = 5 x 13

$117 = 3 \times 3 \times 13$
Hence, HCF of 65 and 117 is 13.
13 is the largest number which divides 70 and 125 and leaves remainders 5 and 8.

7. If $6370 = 2^m \times 5^n \times 7^k \times 13^p$, then the value of $m + n + k + p$ is
(a) 2 (b) 3 (c) 4 (d) 5
Ans: (d) 5
$6370 = 2 \times 5 \times 7^2 \times 13$
On Comparing, we get $6370 = 2^m \times 5^n \times 7^k \times 13^p = 2^1 \times 5^1 \times 7^2 \times 13^1$
$m = 1, n = 1, k = 2, p = 1$
So, $m + n + k + p = 5$

8. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM $(a, b, c) = 2^3 \times 3^2 \times 5$, then n is equal to
(a) 1 (b) 2 (c) 3 (d) 4
Ans: (b) 2
LCM $(a, b, c) = 2^3 \times 3^2 \times 5$ ……. (1)
Now, $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, and $c = 3^n \times 5$
$\therefore$ LCM $(a, b, c) = 2^3 \times 3^n \times 5$ ……..(2)
Comparing (1) and (2), we will get
$2^3 \times 3^2 \times 5 = 2^3 \times 3^n \times 5$
$\Rightarrow n = 2$

9. In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
Assertion (A): If product of two numbers is 5780 and their HCF is17, then their LCM is 340.
Reason (R): HCF is always a factor of LCM.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.
Ans: (b) Both A and R are true but R is not the correct explanation of A.

10. In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
Assertion (A): 6^n ends with the digit zero, where n is natural number.
Reason (R): Any number ends with digit zero, if its prime factor is of the form $2^m \times 5^n$, where m, n are natural numbers.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.
Ans: (d) A is false but R is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Explain why $2 \times 3 \times 5 + 5$ and $5 \times 7 \times 11 + 7 \times 5$ are composite numbers.
Ans: We have, $2 \times 3 \times 5 + 5$ and $5 \times 7 \times 11 + 7 \times 5$.
We can write these numbers as :
$2 \times 3 \times 5 + 5 = 5 (2 \times 3 + 1)$
$= 5 \times 7$
and $5 \times 7 \times 11 + 7 \times 5 = 5 \times 7 (11 + 1)$
$= 5 \times 7 \times 12$

Since, on simplifying, we find that both the numbers have more than two factors. So, these are composite numbers.

12. Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers?
Ans: Let the two numbers be 2x and 3x.
LCM of 2x and 3x = 6x, HCF(2x, 3x) = x
Now, 6x = 180 [Given]
$\Rightarrow$ x = 180/6 = 30
$\therefore$ HCF (2x, 3x) = x = 30

13. Show that any number of the form 6^n, where n $\in$N can never end with digit 0. (2017)
Ans: For unit's digit to be 0, then 6^n should have 2 and 5 as its prime factors,
but $6^n = (2^n \times 3^n)$.
It does not contain 5 as one of its prime factors.
$\therefore$ 6^n will not end with digit 0 for n $\in$ N.

14. The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find the HCF of the two numbers.
Ans: Let a and b be two number such that
LCM (a, b) = 9.HCF (a, b) ...(i)
and LCM (a, b) + HCF (a, b) = 500 ...(ii)
Using (i) in (ii), we get
9HCF (a, b) + HCF (a, b) = 500
$\Rightarrow$ 10 HCF (a, b) = 500 $\Rightarrow$ HCF (a, b) = 50

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that $\sqrt{3}$ is an irrational number. (2023)

Ans: Let $\sqrt{3}$ is a rational number then we have $\sqrt{3} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow$ p = $\sqrt{3}$ q
Squaring both sides, we get $p^2 = 3q^2$
$\Rightarrow$ p^2 is divisible by 3 $\Rightarrow$ p is also divisible by 3
So, assume p = 3m where m is any integer.
Squaring both sides, we get $p^2 = 9m^2$
But $p^2 = 3q^2$
Therefore, $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$
$\Rightarrow$ q^2 is divisible by 3 $\Rightarrow$ q is also divisible by 3
From above we conclude that p and q have one common factor i.e. 3 which contradicts that p and q are co-primes.
Therefore, our assumption is wrong.
Hence, $\sqrt{3}$ is an irrational number.

16. 4 Bells toll together at 9.00 am. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?
Ans: 7 = 7 × 1
8 = 2 × 2 × 2
11 = 11 × 1
12 = 2 × 2 × 3
$\therefore$ LCM of 7, 8, 11, 12 = 2 × 2 × 2 × 3 × 7 × 11 = 1848
$\therefore$ Bells will toll together after every 1848 sec.

∴ In next 3 hrs, number of times the bells will toll together = $\frac{3\times3600}{1848}$ = 5.84

= 5 times.

17. Given that √3 is irrational, prove that 5 + 2√3 is irrational. (CBSE Sample Paper 2022)

Ans: Let 5 + 2√3 be a rational number such that

5 + 2√3 = *a*, where *a* is a non-zero rational number.

$\Rightarrow 2\sqrt{3} = a - 5 \Rightarrow \sqrt{3} = \frac{a-5}{2}$

Since 5 and 2 are integers and a is a rational number, therefore $\frac{a-5}{2}$ is a rational number

⇒ √3 is a rational number which contradicts the fact that √3 is an irrational number.

Therefore, our assumption is wrong.

Hence 5 + 2√3 is an irrational number

SECTION – D

Questions 18 carry 5 marks.

18. (a) Find the largest possible positive integer that divides 125, 162 and 259 leaving remainder 5, 6 and 7 respectively. (3)

Ans: It is given that the required number when divides 125, 162, 259 leaves the remainder 5, 6, 7 respectively.

This means that 125 – 5 = 120, 162 – 6 = 156, 259 – 7 = 252 are divisible by the required number.

The required number is HCF of all these numbers.

The prime factorisation of 120, 156, 252 are

120 = 2 × 2 × 2 × 3 × 5 = $2^3 \times 3 \times 5$

156 = 2 × 2 × 3 × 13 = $2^2 \times 3 \times 13$

252 = 2 × 2 × 3 × 3 × 7 = $2^2 \times 3^2 \times 7$

HCF (120, 156, 252) = $2^2 \times 3 = 12$

Hence, the required number is 12.

(b) An army contingent of 678 soldiers is to march behind an army band of 36 members in a Republic Day parade. The two groups are to march in the same number of columns. What is the maximum number of columns they can march? (2)

Ans: Number of soldiers in an army contingent = 678 = 2 × 3 × 113

Number of members in an army band = 36 = 2 × 2 × 3 × 3 = $2^2 \times 3^2$

The maximum number of columns such that two groups can march in same number of columns is HCF of 678 and 36.

HCF (678, 36) = 2 × 3 = 6

So, the maximum number of columns they can march is 6.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A morning walk may help improve your mental clarity and ability to focus throughout the day. A recent study found that amongst older adults, those who started their days with a morning walk improved their cognitive function, compared to those who remained sedentary. Walking may also help you think more creatively. In a morning walk three students step off together, their steps measure 80 cm, 85 cm and 90 cm respectively.

(i) What is the HCF of 80 and 90? (1)
(ii) Find the sum of exponents of the prime factors of total distance. (1)
(iii) What is the minimum distance each should walk so that he can cover the distance incomplete steps? (2)
Ans: (i) $80 = 2^4 \times 5$
and $90 = 2 \times 3^2 \times 5$
So, HCF (80, 90) $= 2 \times 5 = 10$.
(ii) Total distance = (80 + 85 + 90) cm = 255 cm
$\therefore$ The prime factors of $255 = 3 \times 5 \times 17$
Hence, the sum of exponents = 1 + 1 + 1 = 3.
(iii) We have to find the LCM of 80, 85 and 90 by using prime factorisation method.
$\therefore 80 = 2^4 \times 5$
$85 = 5 \times 17$
and $90 = 2 \times 3^2 \times 5$
$\therefore$ LCM (80, 85 and 90) $= 2^4 \times 3^2 \times 5 \times 17 = 12240$.
Hence, the minimum distance each should walk so that he can cover the distance 12240 cm or 122 m 40 cm.

20. A family room is an informal, all purpose room is a house. The family room is designed to be a place where family and guests gather for group recreation like talking, reading, watching TV and other family activities. The length, breadth and height of a room are8 m 25 cm, 6 m 75 cm and 4 m 50 cm.

(i) Determine the longest rod which can measure the three dimensions of the room exactly. (2)
(ii) What is LCM of the given three measurements? (1)
(iii) If the HCF (825 and 675) = 75, then find LCM (825 and 675). (1)
Ans: (i) Given, Length = 8 m 25 cm = 825 cm
Breadth = 6 cm 75 cm = 675 cm

and height = 4 m 50 cm = 450 cm
we have to find the HCF of 825, 675 and 450 by factorization method.
$825 = 3 \times 5^2 \times 11$
$675 = 3^3 \times 5^2$
and $450 = 2 \times 3^2 \times 5^2$
HCF (825, 675 and 450) $= 3 \times 5^2 = 75$
Hence, the longest rod which can measure the given dimensions of the room exactly 75 cm.
(ii) For LCM, taking the greatest exponent and raise each prime factor to the greatest exponent and multiply them.
LCM (825, 675 and 450) $= 2 \times 3^3 \times 5^2 \times 11 = 14850$.
(iii) By using fundamental theorem of arithmetic, we get
LCM (825 and 675) $= (825 \times 675)/75 = 7425$.

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 02 - CHAPTER 02 POLYNOMIALS (2024-25)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If the sum of the zeroes of the quadratic polynomial $kx^2 + 4x + 3k$ is equal to their product, then the value of k is
(a) -3/4 (b) 3/4 (c) 4/3 (d) -4/3

2. If α and β are the zeroes of $f(x) = 2x^2 + 8x - 8$, then
(a) $\alpha + \beta = \alpha\beta$ (b) $\alpha + \beta > \alpha\beta$ (c) $\alpha + \beta < \alpha\beta$ (d) $\alpha + \beta + \alpha\beta = 0$

3. The zeroes of the quadratic polynomial $x^2 + 25x + 156$ are
(a) both positive (b) both negative
(c) one positive and one negative (d) can't be determined

4. A quadratic polynomial whose one zero is 5 and product of zeroes is 0, is
(*a*) $x^2 - 5$ (*b*) $x^2 - 5x$ (*c*) $5x^2 + 1$ (*d*) $x^2 + 5x$

5. If the sum of the zeroes of the polynomial $p(x) = (p^2 - 23)x^2 - 2x - 12$ is 1, then *p* takes the value (s)
(*a*) $\sqrt{23}$ (*b*) -23 (*c*) 2 (*d*) ± 5

6. If α and β are the zeroes of the polynomial $x^2 + 5x + c$, and $\alpha - \beta = 3$, then $c =$
(*a*) 0 (*b*) 1 (*c*) 4 (*d*) 5

7. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal then
(*a*) *c* and *a* have opposite signs (*b*) *c* and *b* have opposite signs
(*c*) *c* and *a* have the same sign (*d*) *c* and *b* have the same sign

8. The value of *k* such that the quadratic polynomial $x^2 - (k + 6)x + 2(2k + 1)$ has sum of the zeroes as half of their product, is
(*a*) 2 (*b*) 3 (*c*) -5 (*d*) 5

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** 5x + 2 is a linear polynomial.
Reason (R): A polynomial of degree 1 is a linear polynomial.

10. **Assertion (*A*):** A quadratic polynomial having 5 and –3 as zeroes is $x^2 - 2x - 15$.
Reason (*R*): The quadratic polynomial having α and β as zeroes is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If the sum of the zeroes of the quadratic polynomial $ky^2 + 2y - 3k$ is equal to twice their product, find the value of k.

12. If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p.

13. α, β are zeroes of the polynomial $x^2 - 6x + a$. Find the value of a, if $3\alpha + 2\beta = 20$.

14. Find a quadratic polynomial whose one zero is 5 and product of zeroes is 30.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product.

16. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

17. If α, β re zeros of quadratic polynomial $x^2 - 6x + k$, find the value of k such that $(\alpha + \beta)^2 - 2\alpha\beta = 40$

SECTION – D

Questions 18 carry 5 marks.

18. If one zero of the polynomial $(k + 1)x^2 - 5x + 5$ is multiplicative inverse of the other, then find the zeroes of $kx^2 - 3kx + 9$, where k is constant.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case Study-1 : **Lusitania Bridge**

The below picture are few examples of natural parabolic which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.


Based on the above information, answer the following questions.

(*i*) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then find the value of k. (1)

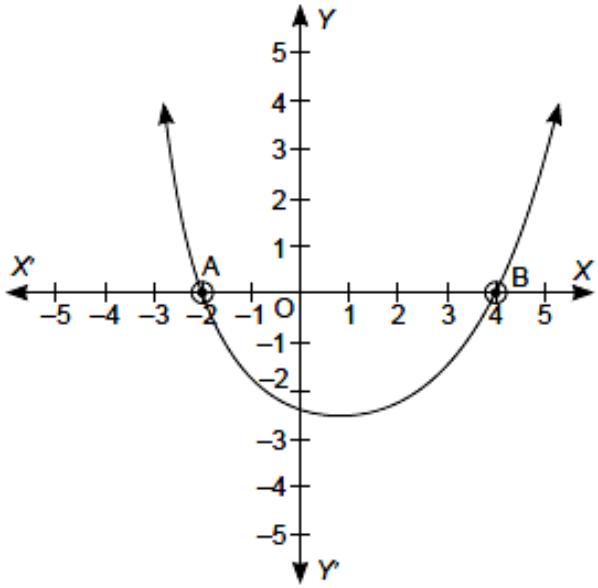
(*ii*) Find the sum of zeroes of $p(x) = kx^2 - kx + 5$. (1)

(*iii*) Write a quadratic polynomial whose one zero is 4 and product of zeroes is 0. (2)

OR

Find the zeroes of $p(x) = x^2 - 7x + 12$ (2)

20. In Maths activity period, Roma's Maths teacher told her to draw the graph of a polynomial having at most two zeroes. She draws the graph as shown below:



(*i*) Name the shape drawn by Roma. (1)

(*ii*) Find the zeroes of $p(x) = x^2 - 49$ (1)

(*iii*) If 1 is a zero of polynomial $p(x) = mx^2 - 3(m - 1)x + 4$, then find the value of m. (2)

OR

If α and β are zeroes of the polynomial $x^2 - px + q^2$, then find the value of $\alpha + \beta - \alpha\beta$. (2)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 02 - CHAPTER 02 POLYNOMIALS (2024-25)

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If the sum of the zeroes of the quadratic polynomial $kx^2 + 4x + 3k$ is equal to their product, then the value of k is

(a) -3/4 (b) 3/4 (c) 4/3 (d) -4/3

Ans: (d) -4/3

Let α and β be the zeroes of polynomial $kx^2 + 4x + 3k$ According to the question,

$$\alpha + \beta = \alpha\beta \Rightarrow \frac{-4}{k} = \frac{3k}{k} \Rightarrow -4 = 3k \Rightarrow k = \frac{-4}{3}$$

2. If α and β are the zeroes of $f(x) = 2x^2 + 8x - 8$, then

(a) α + β = αβ (b) α + β > αβ (c) α + β < αβ (d) α + β + αβ = 0

Ans: (*a*) α + β = αβ

Since a, b are the zeroes of $2x^2 + 8x - 8$

α + β = -8/2 = – 4 and αβ = -8/2 = – 4

Hence, α + β = αβ.

3. The zeroes of the quadratic polynomial $x^2 + 25x + 156$ are

(a) both positive (b) both negative
(c) one positive and one negative (d) can't be determined

Ans: (b) both negative

Let α and β be the zeroes of $x^2 + 25x + 156$.

Then, α + β = –25 and αβ = 156

This happens when α and β are both negative.

4. A quadratic polynomial whose one zero is 5 and product of zeroes is 0, is

(*a*) $x^2 - 5$ (*b*) $x^2 - 5x$ (*c*) $5x^2 + 1$ (*d*) $x^2 + 5x$

Ans: (*b*) $x^2 - 5x$

5. If the sum of the zeroes of the polynomial $p(x) = (p^2 - 23)x^2 - 2x - 12$ is 1, then *p* takes the value (s)

(*a*) √23 (*b*) –23 (*c*) 2 (*d*) ±5

Ans: (*d*) ±5

Let α and β be the zeroes of the polynomial $p(x) = (p^2 - 23)x^2 - 2x - 12$

Then $\alpha + \beta = -\frac{-2}{p^2 - 23} = \frac{2}{p^2 - 23}$

Also, sum of zeroes = α + β = 1

$\Rightarrow p^2 - 23 = 2 \qquad \Rightarrow p^2 = 25 \qquad \Rightarrow p = \pm 5.$

6. If α and β are the zeroes of the polynomial $x^2 + 5x + c$, and $\alpha - \beta = 3$, then c =
(*a*) 0 (*b*) 1 (*c*) 4 (*d*) 5
Ans:
Since α and β are zeroes of the polynomial $x^2 + 5x + c$
$\alpha + \beta = -5$ ...(*i*)
and $\alpha - \beta = 3$ (given) ...(*i*)
Solving (*i*) and (*ii*) we have $\alpha = -1$ and $\beta = -4$
Now, product of zeroes = $\alpha\beta = (-1)(-4) = 4$
$\Rightarrow c = 4$

7. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal then
(*a*) *c* and *a* have opposite signs (*b*) *c* and *b* have opposite signs
(*c*) *c* and *a* have the same sign (*d*) *c* and *b* have the same sign
Ans: (*c*) *c* and *a* have the same sign
The zeroes of the given quadratic polynomial
$ax^2 + bx + c$ where $c \neq 0$ are equal, if coefficient of x^2 and constant term have the same sign i.e. *c* and *a* have the same sign. While *b* i.e. coefficient of *x* can be positive or negative but not zero.

8. The value of *k* such that the quadratic polynomial $x^2 - (k + 6)x + 2(2k + 1)$ has sum of the zeroes as half of their product, is
(*a*) 2 (*b*) 3 (*c*) –5 (*d*) 5
Ans: (*d*) 5

$$\alpha + \beta = \frac{-[-(k+6)]}{1} = k + 6$$

$$\alpha\beta = \frac{2(2k+1)}{1} = 2(2k+1)$$

Now, $\frac{\alpha\beta}{2} = \alpha + \beta \Rightarrow \frac{2(2k+1)}{2} = k + 6 \Rightarrow k = 5$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** 5x + 2 is a linear polynomial.
Reason (R): A polynomial of degree 1 is a linear polynomial.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (*A*):** A quadratic polynomial having 5 and –3 as zeroes is $x^2 - 2x - 15$.
Reason (*R*): The quadratic polynomial having α and β as zeroes is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If the sum of the zeroes of the quadratic polynomial $ky^2 + 2y - 3k$ is equal to twice their product, find the value of k.

Ans: $p(y) = ky^2 + 2y - 3k$

$a = k, b = 2, c = -3k$

According to the question, Sum of zeroes = 2 × product of zeroes

$$\Rightarrow \frac{-b}{a} = 2 \times \frac{c}{a} \Rightarrow \frac{-2}{k} = 2 \times \frac{-3k}{k}$$

$$\Rightarrow \frac{2}{k} = 6 \Rightarrow k = \frac{1}{3}$$

12. If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p.

Ans: Since, 3 is a root (zero) of $p(x)$

$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$

$\Rightarrow 18 - 9 + p = 0 \Rightarrow p = -9$

Now $p(x) = 2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9$

$= 2x(x - 3) + 3(x - 3) = (x - 3)(2x + 3)$

For roots of polynomial, $p(x) = 0 \Rightarrow (x - 3)(2x + 3) = 0$

$\Rightarrow x = 3$ or $x = \frac{-3}{2}$, Other root = $\frac{-3}{2}$

13. α, β are zeroes of the polynomial $x^2 - 6x + a$. Find the value of a, if $3\alpha + 2\beta = 20$.

Ans: $\alpha + \beta = 6, \alpha\beta = a$

Now $3\alpha + 2\beta = 20$

$\Rightarrow \alpha + 2\alpha + 2\beta = 20 \Rightarrow \alpha + 2(\alpha + \beta) = 20$

$\Rightarrow \alpha + 2 \times 6 = 20 \Rightarrow \alpha = 20 - 12 = 8$

$\therefore \beta = -2$

Now $\alpha\beta = a$

$\therefore 8 \times (-2) = a \Rightarrow a = -16$

14. Find a quadratic polynomial whose one zero is 5 and product of zeroes is 30.

Ans: One zero = 5, Product of zeroes = 30

$\therefore$ Other zero = 30/5 = 6

$\therefore$ Sum of zeroes = 5 + 6 = 11

Quadratic polynomial is

$p(x) = x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})$

$p(x) = x^2 - 11x + 30$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product.

Ans: The given polynomial is $x^2 - (k + 6)x + 2(2k - 1)$

Let α and β be the zeroes of polynomial.

So, $\alpha + \beta = -\left[\frac{-(k+6)}{1}\right] = k + 6$

$\alpha\beta = \frac{2(2k-1)}{1} = 4k - 2$

$\therefore \quad \alpha + \beta = \frac{1}{2}\alpha\beta$

$\Rightarrow \quad k + 6 = \frac{1}{2}(4k - 2)$

$\Rightarrow \quad 2k + 12 = 4k - 2$

$\Rightarrow \quad 2k = 14 \Rightarrow k = 7$

16. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Ans:

Here $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of $p(y)$, $p(y) = 0$

$\Rightarrow \quad 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow \quad 21y^2 - 11y - 2 = 0$

$\Rightarrow \quad 21y^2 - 14y + 3y - 2 = 0$

$\Rightarrow \quad 7y(3y - 2) + 1(3y - 2) = 0$

$\Rightarrow \quad (7y + 1)(3y - 2) = 0$

$\Rightarrow \quad y = \frac{-1}{7}, \frac{2}{3}$

$\therefore$ zeroes are $\frac{-1}{7}$ and $\frac{2}{3}$

Also $a = 7$, $b = \frac{-11}{3}$, $c = \frac{-2}{3}$

Sum of zeroes $= \frac{-1}{7} + \frac{2}{3} = \frac{-3 + 14}{21} = \frac{11}{21}$

Also $\quad \frac{-b}{a} = \frac{-(-11/3)}{7} = \frac{11}{21}$

$\Rightarrow$ Sum of zeroes $= \frac{-b}{a}$

and product of zeroes $= \frac{-1}{7} \times \frac{2}{3} = \frac{-2}{21}$

Also $\quad \frac{c}{a} = \frac{\frac{-2}{3}}{7} = \frac{-2}{21}$

$\Rightarrow$ Product of zeroes $= \frac{c}{a}$

17. If α, β re zeros of quadratic polynomial $x^2 - 6x + k$, find the value of k such that $(\alpha + \beta)^2 - 2\alpha\beta = 40$

Ans: We know that $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$

Given, $x^2 - 6x + k = 0$

$\Rightarrow a = 1, b = -6, c = k$

Given that $(\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow (-b/a)^2 - 2c/a = 40$

$\Rightarrow b^2 - 2ca = 40a^2$ (Multiplying both sides by a^2)

$\Rightarrow (-6)^2 - 2k = 40(1)^2$

$\Rightarrow 36 - 2k = 40 \Rightarrow 2k = 36 - 40 = -4 \Rightarrow k = -2$

SECTION – D

Questions 18 carry 5 marks.

18. If one zero of the polynomial $(k + 1)x^2 - 5x + 5$ is multiplicative inverse of the other, then find the zeroes of $kx^2 - 3kx + 9$, where k is constant.

Ans: Here $f(x) = (k + 1)x^2 - 5x + 5$

$a = k + 1$, $b = -5$, $c = 5$

Let one zero = α

$\therefore$ According to the question, other zero = $\frac{1}{\alpha}$

Now product of zeroes = $\frac{c}{a}$

$$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{5}{k+1} \Rightarrow 1 = \frac{5}{k+1} \Rightarrow k+1 = 5 \Rightarrow k = 4$$

Now putting $k = 4$ in polynomial

$p(x) = kx^2 - 3kx + 9$

we get $p(x) = 4x^2 - 12x + 9$

For zeroes of $p(x)$, $4x^2 - 12x + 9 = 0 \Rightarrow (2x - 3)(2x - 3) = 0$

$$\Rightarrow x = \frac{3}{2}, x = \frac{3}{2}$$

Hence, Zeroes are $\frac{3}{2}, \frac{3}{2}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case Study-1 : **Lusitania Bridge**

The below picture are few examples of natural parabolic which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.


Based on the above information, answer the following questions.

(*i*) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then find the value of k. (1)

(*ii*) Find the sum of zeroes of $p(x) = kx^2 - kx + 5$. (1)

(*iii*) Write a quadratic polynomial whose one zero is 4 and product of zeroes is 0. (2)

OR

Find the zeroes of $p(x) = x^2 - 7x + 12$ (2)

Ans: (*i*) Let $p(x) = 2x^2 - x + 8k$

So, $a = 2$; $b = -1$; $c = 8k$

Product of zeroes = $\frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{8k}{2} \Rightarrow k = \frac{1}{4}$

(*ii*) $p(x) = kx^2 - kx + 5$

Here $a = k$, $b = -k$

How sum of zeroes = $\frac{-b}{a} = \frac{-(-k)}{k} = 1$

(*iii*) Let other zero is k.

Product of zeroes = 0

$4 \times k = 0$

$\Rightarrow k = 0$

Sum of zeroes = 4 + 0 = 4

$p(x) = x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$p(x) = x^2 - 4x + 0 = x^2 - 4x$

OR

For zeroes of $p(x)$,

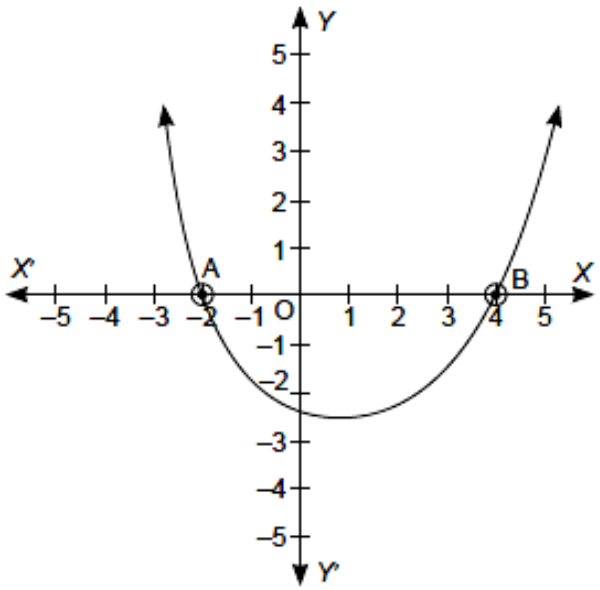
$x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 3x - 4x + 12 = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow x = 3, 4.$

20. In Maths activity period, Roma's Maths teacher told her to draw the graph of a polynomial having at most two zeroes. She draws the graph as shown below:



(*i*) Name the shape drawn by Roma. (1)

(*ii*) Find the zeroes of $p(x) = x^2 - 49$ (1)

(*iii*) If 1 is a zero of polynomial $p(x) = mx^2 - 3(m - 1)x + 4$, then find the value of m. (2)

OR

If α and β are zeroes of the polynomial $x^2 - px + q^2$, then find the value of $\alpha + \beta - \alpha\beta$. (2)

Ans: (*a*) parabola

(*b*) For zeroes of $p(x)$, $x^2 - 49 = 0$

$\Rightarrow x^2 = 49 \Rightarrow x = \pm 7$.

(*c*) As 1 is a zero of $p(x)$, then $p(1) = 0$

$m \times 1^2 - 3(m - 1)1 + 4 = 0$

$\Rightarrow \; m - 3m + 3 + 4 = 0 \quad \Rightarrow \quad m = \frac{7}{2}$.

OR

$f(x) = x^2 - px + q^2$

Here $a = 1$, $b = -p$, $c = q^2$

$\alpha\beta = \frac{c}{a} = q^2$

Now, $\alpha + \beta = \frac{-b}{a} = p$

$\alpha + \beta - \alpha\beta = p - q^2$.

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 03 (2024-25)

CHAPTER 03 LINEAR EQUATIONS IN TWO VARIABLES

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be

(a) $9x + 8y = 7$ (b) $-12x - 8y = 7$ (c) $-12x + 8y = 7$ (d) $12x + 8y = 7$

2. What is the value of k such that the following pair of equations have infinitely many solutions? $x - 2y = 3$ and $-3x + ky = -9$.

(a) (-6) (b) -3 (c) 3 (d) 6

3. The pair of linear equations $(3/2)x + (5/3)y = 7$ and $9x + 10y = 14$ is

(a) consistent (b) inconsistent

(c) consistent with one solution (d) consistent with many solutions

4. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k =

(a) -1 (b) 0 (c) 1 (d) 2

5. The values of x and y satisfying the two equations $32x + 33y = 34$, $33x + 32y = 31$ respectively are

(a) $-1, 2$ (b) $-1, 4$ (c) $1, -2$ (d) $-1, -4$

6. What is the value of q if $p/2 + 3q = 6$ and $2p - 2q = 10$?

(a) 1 (b) 4 (c) 6 (d) 16

7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-5/4$ (b) 2/5 (c) 15/4 (d) 3/2

8. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if :

(a) a = b (b) 3a = 2b (c) 2a = 3b (d) ab = 6

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If the pair of linear equations $3x + y = 3$ and $6x + ky = 8$ does not have a solution, then the value of $k = 2$.
Reason (R): If the pair of linear equations $x + y – 4 = 0$ and $2x + ky = 3$ does not have a solution, then the value of $k = 2$.

10. **Assertion (A):** If the equation $3x – y + 8 = 0$ and $6x – ky = –16$ represent coincident lines, then the value of $k = 2$.
Reason (R): If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is 15.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. For what value of k, will the following pair of equations have infinitely many solutions:
$2x + 3y = 7$ and $(k + 2)x – 3(1 – k)y = 5k + 1$

12. If $49x + 51y = 499$, $51x + 49y = 501$, then find the value of x and y.

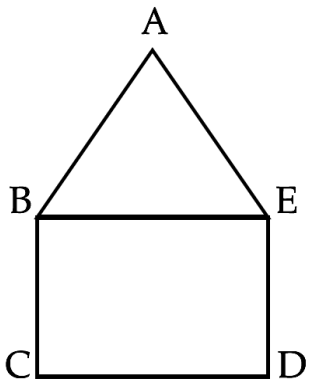
13. If $x = a$ and $y = b$ is the solution of the pair of equations $x – y = 2$ and $x + y = 4$, find the values of a and b.

14. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of the children. Find the age of the father.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In the figure, ABCDE is a pentagon with BE||CD and BC||DE. BC is perpendicular to CD. AE = AB = 5 cm, BE = 7 cm, BC = x – y and CD = x + y. If the perimeter of ABCDE is 27 cm. find the value of x and y, given $x, y \neq 0$.



16. A fraction becomes 1/3 when 2 is subtracted from the numerator and it becomes 1/2 when 1 is subtracted from the denominator Find the fraction.

17. If $2x + y = 23$ and $4x – y = 19$, find the value of $(5y – 2x)$ and $\left(\frac{y}{x} – 2\right)$

SECTION – D

Questions 18 carry 5 marks.

18. Determine graphically the co-ordinates of the vertices of triangle, the equations of whose sides are given by $2y – x = 8$, $5y – x = 14$ and $y – 2x = 1$.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc. He takes a fixed charge for the first two days and an additional charge for subsequent days. Amruta paid ` 22 for a book and kept for 6 days; while Radhika paid `16 for keeping the book for 4 days.

Assume that the fixed charge be ` x and additional charge (per day) be ` y.

(a) Represent the situation of amount paid by Radhika and Amruta algebraically.

(b) What are the fixed charges for a book and the additional charges for each subsequent day for a book?

(c) What is the total amount paid by both, if both of them have kept the book for 2 more days?

20. Two friends purchased the same company car and same green colour. They are travelling in separate car which starts at place A and other car at place B. Place A and B are 100 km apart on a highway. One car starts from A and another from B at the same time.

Situation 1: If the cars travel in the same direction at different speeds, they meet in 5 hours.

Situation 1: If they travel towards each other, they meet in 1 hour.

(a) Assuming that the speed of first car and second car be x km/h and y km/h respectively, find the pair of linear equations representing the situations.

(b) What is the speed of car starts at A and speed of car starts at B?

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 03 (2024-25)

CHAPTER 03 LINEAR EQUATIONS IN TWO VARIABLES

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Two lines are given to be parallel. The equation of one of the lines is 3x – 2y = 5. The equation of the second line can be
(a) 9x + 8y = 7 (b) –12x – 8y = 7 (c) –12x + 8y = 7 (d) 12x + 8y = 7
Ans: (c) –12x + 8y = 7

2. What is the value of k such that the following pair of equations have infinitely many solutions? x – 2y = 3 and – 3x + ky = – 9.
(a) (–6) (b) –3 (c) 3 (d) 6
Ans: (d) 6

Here, $\frac{a_1}{a_2}=\frac{1}{-3}, \frac{b_1}{b_2}=\frac{-2}{k}$ and $\frac{c_1}{c_2}=\frac{-3}{9}$

For infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ For $\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{1}{-3}=\frac{-2}{k} \Rightarrow k=6$

and for $\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{-2}{k}=\frac{-3}{9} \Rightarrow 3k = 18 \Rightarrow k = 6$

3. The pair of linear equations (3/2)x + (5/3)y = 7 and 9x + 10y = 14 is
(a) consistent (b) inconsistent
(c) consistent with one solution (d) consistent with many solutions
Ans: (b) inconsistent

4. If the system of equations 3x + y = 1 and (2k – 1)x +(k – 1)y = 2k + 1 is inconsistent, then k =
(a) –1 (b) 0 (c) 1 (d) 2
Ans: (d) 2

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$

Either $\frac{3}{2k-1} = \frac{1}{k-1}$ or $\frac{1}{k-1} \neq \frac{1}{2k+1}$

$\Rightarrow 3k - 3 = 2k - 1$ or $2k + 1 \neq k - 1 \Rightarrow k = 2$ or $k \neq -2$

5. The values of x and y satisfying the two equations $32x + 33y = 34$, $33x + 32y = 31$ respectively are

(a) –1, 2 (b) –1, 4 (c) 1, –2 (d) –1, –4

Ans: (a) –1, 2

The given equations are, $32x + 33y = 34$ ...(i)

& $33x + 32y = 31$ ...(ii)

Subtract eq.(ii) from eq.(i)

$–x + y = 3 \Rightarrow y = 3 + x$

Put this value of y in (i), we get $32x + 33(3+x) = 34$

$\Rightarrow 32x + 99 + 33x = 34 \Rightarrow 65x = 34 – 99 \Rightarrow 65x = – 65 \Rightarrow x = –1$

Also, $y = 3 + x \Rightarrow y = 3 + (–1) = 3 – 1 = 2$

Hence, the correct solution is $x = –1$ and $y = 2$.

6. What is the value of q if $p/2 + 3q = 6$ and $2p – 2q = 10$?

(a) 1 (b) 4 (c) 6 (d) 16

Ans: (a) 1

7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-5/4$ (b) $2/5$ (c) $15/4$ (d) $3/2$

Ans: (c) $15/4$

8. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if :

(a) $a = b$ (b) $3a = 2b$ (c) $2a = 3b$ (d) $ab = 6$

Ans: (d) $ab = 6$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If the pair of linear equations $3x + y = 3$ and $6x + ky = 8$ does not have a solution, then the value of $k = 2$.

Reason (R): If the pair of linear equations $x + y – 4 = 0$ and $2x + ky = 3$ does not have a solution, then the value of $k = 2$.

Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. **Assertion (A):** If the equation $3x – y + 8 = 0$ and $6x – ky = –16$ represent coincident lines, then the value of $k = 2$.

Reason (R): If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is 15.

Ans: (c) Assertion (A) is true but reason (R) is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. For what value of k, will the following pair of equations have infinitely many solutions:

$2x + 3y = 7$ and $(k + 2)x – 3(1 – k)y = 5k + 1$

Ans: Here, $a_1 = 2$, $b_1 = 3$, $c_1 = 7$

and $a_2 = (k +2)$, $b_2 = –3(1 – k)$, $c_2 = 5k+1$

$\frac{a_1}{a_2} = \frac{2}{k+2}, \frac{b_1}{b_2} = \frac{3}{-3(1-k)}, \frac{c_1}{c_2} = \frac{7}{5k+1}$

For a pair of linear equations to have infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{k+2} = \frac{3}{-3(1-k)} = \frac{7}{5k+1}$

$\Rightarrow \frac{2}{k+2} = \frac{3}{-3(1-k)} \Rightarrow 2(1-k) = -(k+2)$

$\Rightarrow 2 - 2k = -k - 2 \Rightarrow k = 4$

Hence, for k = 4, the pair of linear equations has infinitely many solutions.

12. If 49x + 51y = 499, 51x + 49y = 501, then find the value of x and y.
Ans: Adding the two equations and dividing by 10, we get: x + y = 10
Subtracting the two equations and dividing by –2,
we get: x – y = 1
Solving these two new equations, we get,
x = 11/2 and y = 9/2

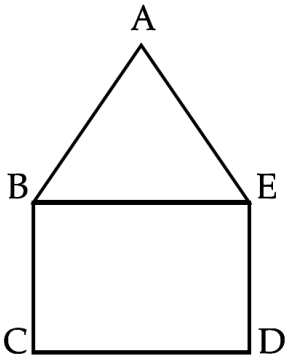
13. If x = a and y = b is the solution of the pair of equations x – y= 2 and x + y = 4, find the values of a and b.
Ans: Given equations are: x – y = 2 ...(i)
and x + y = 4 ...(ii)
Adding eq. (i) and (ii), we get
$2x = 6 \Rightarrow x = 3$
Substituting x = 3 in eq. (ii), we get
$3 + y = 4 \Rightarrow y = 4 - 3 = 1$
If x = a and y = b is the solution of given equations, then.
a = x = 3 and b = y = 1.
Hence, a = 3 and b = 1.

14. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of the children. Find the age of the father.
Ans: Let the sum of the ages of the 2 children be x and the age of the father be y years.
$\therefore$ y = 2x
$\Rightarrow$ 2x – y = 0 ...(i)
and 20 + y = x + 40
$\Rightarrow$ x – y = – 20 ...(ii)
Subtracting (ii) from (i), we get x = 20
From (i), $y = 2x = 2 \times 20 = 40$
$\Rightarrow$ y = 40
Hence, the age of the father = 40 years.

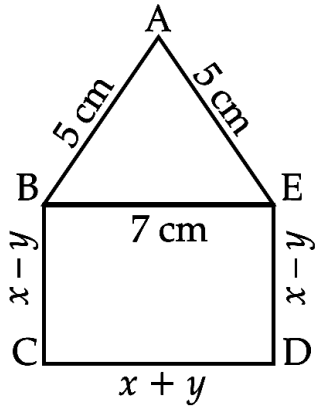
SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In the figure, ABCDE is a pentagon with BE||CD and BC||DE. BC is perpendicular to CD. AE = AB = 5 cm, BE = 7 cm, BC = x – y and CD = x + y. If the perimeter of ABCDE is 27 cm. find the value of x and y, given x, y $\neq$ 0.



Ans: Here, BCDE is a rectangle.
∴ x + y = 7 ..(i)
Perimeter of ABCDE = 27 cm (given)
∴ AB + BC + CD + DE + EA = 27
⇒ 3x – y = 27 – 10
⇒ 3x – y = 17 ...(ii)
Adding eq. (i) and (ii), we get
4x = 24 ⇒ x = 6
Substituting the value of x in eq. (i), we get 6 + y = 7
⇒ y = 7 – 6 = 1
Hence, x = 6 and y = 1.



16. A fraction becomes 1/3 when 2 is subtracted from the numerator and it becomes 1/2 when 1 is subtracted from the denominator Find the fraction.
Ans: Let the fraction be x/y.
According to the first condition, $\frac{x-2}{y}=\frac{1}{3}$
⇒ 3x – 6 = y
⇒ y = 3x – 6 ...(i)
According to the second condition, $\frac{x}{y-1}=\frac{1}{2}$
⇒ 2x = y – 1
⇒ y = 2x + 1 ...(ii)
From Eqs. (i) and (ii), we get 3x – 6 = 2x + 1
⇒ x = 7
Substitute value of x in Eq. (i), we get y = 3(7) – 6
⇒ y = 21 – 6 = 15
Hence, fraction is 7/15.

17. If 2x + y = 23 and 4x – y = 19, find the value of (5y – 2x) and $\left(\frac{y}{x}-2\right)$
Ans: Given, 2x + y = 23 ...(i)
and 4x – y = 19 ...(ii)
On adding Eq. (i) and (ii), we get
6x = 42 ⇒ x = 7
Putting the value of x in Eq. (i), we get 14 + y = 23
⇒ y = 23 – 14 = 9
Hence, 5y – 2x = 5 × 9 – 2 × 7 = 45 – 14 = 31
and $\frac{y}{x}-2=\frac{9}{7}-2=\frac{9-14}{7}=\frac{-5}{7}$

SECTION – D

Questions 18 carry 5 marks.

18. Determine graphically the co-ordinates of the vertices of triangle, the equations of whose sides are given by 2y – x = 8, 5y – x = 14 and y – 2x = 1.

Ans: Given, 2y – x = 8

⇒ x = 2y – 8

x	0	4	5
y	-8	0	2

5y – x = 14

⇒ x = 5y – 14

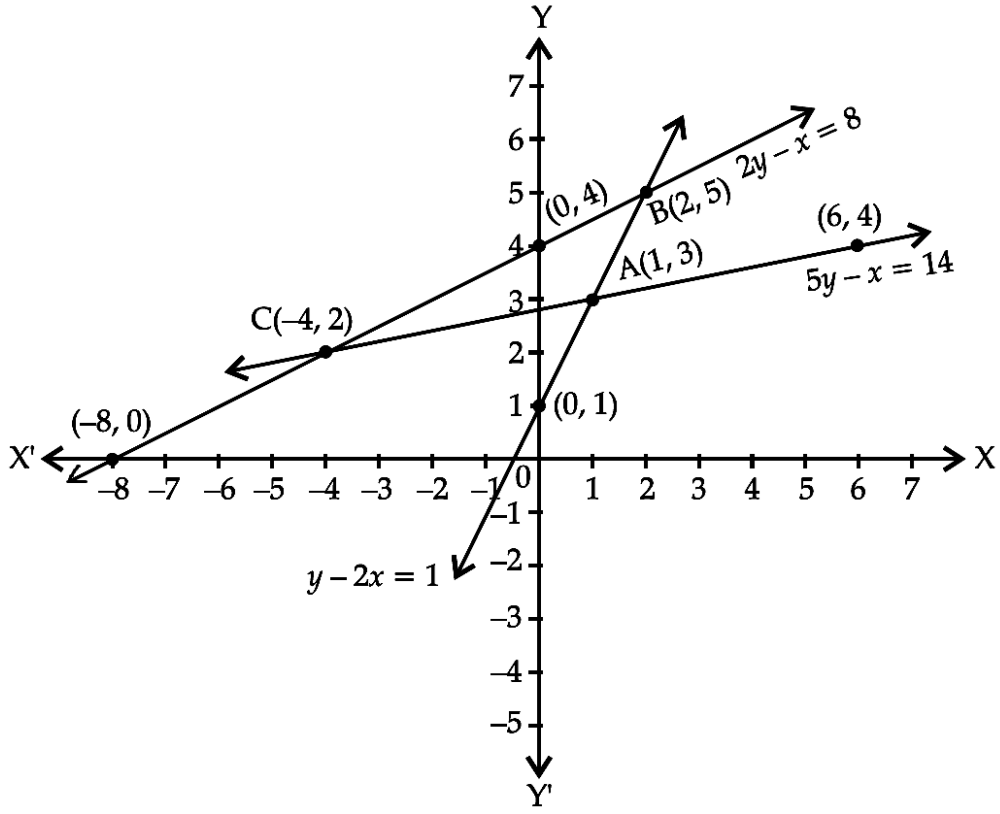
x	3	4	2
y	1	6	-4

and y – 2x = 1

⇒ y = 1 + 2x

x	0	1	2
y	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the co-ordinates of the vertices of the triangle ABC are A(1, 3), B(2, 5) and C(–4, 2).

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc. He takes a fixed charge for the first two days and an additional charge for subsequent days. Amruta paid ` 22 for a book and kept for 6 days; while Radhika paid `16 for keeping the book for 4 days.

Assume that the fixed charge be ` x and additional charge (per day) be ` y.
(a) Represent the situation of amount paid by Radhika and Amruta algebraically.
(b) What are the fixed charges for a book and the additional charges for each subsequent day for a book?
(c) What is the total amount paid by both, if both of them have kept the book for 2 more days?
Ans: (a) Let the fixed charge for two days be ` x and additional charge be ` y per day.
As Radhika has taken book for 4 days. It means that Radhika will pay fixed charge for first two days and pays additional charges for next two days.
$x + 2y = 16$.
As Amruta has taken book for 6 days. It means that Amruta will pay fixed charge for first two days and pays additional charges for next four days.
$x + 4y = 22$.
(b) $x + 2y = 16$ ...(i)
$x + 4y = 22$ ...(ii)
Subtracting (ii) from (i), we get
$y = 3$ and put this value of x in (i), we get $x = 10$.
Therefore, fixed charge x = ` 10 and additional charges, y = ` 3.
(c) For two more days price charged will be
$2y = 2 \times 3 = 6$
Total money paid by Amruta and Radhika = 22 + 16 + 6 + 6 = ` 50.

20. Two friends purchased the same company car and same green colour. They are travelling in separate car which starts at place A and other car at place B. Place A and B are 100 km apart on a highway. One car starts from A and another from B at the same time.
Situation 1: If the cars travel in the same direction at different speeds, they meet in 5 hours.
Situation 1: If they travel towards each other, they meet in 1 hour.

(a) Assuming that the speed of first car and second car be x km/h and y km/h respectively, find the pair of linear equations representing the situations.
(b) What is the speed of car starts at A and speed of car starts at B?
Ans: (a) Let the speed of car at A be x kmph
and the speed of car at B be y kmph
When the car travel in same direction Relative Speed is $x - y$
Distance = 100km and t = 5 hours

$\therefore$ Dist = Speed × Time
$\Rightarrow 100 = (x - y)5$
$\Rightarrow x - y = 20$
When the car travel in opposite direction Relative Speed is x + y
Distance = 100km and t = 1 hours
$\therefore$ Dist = Speed × Time
$\Rightarrow 100 = (x + y)1$
$\Rightarrow x + y = 100$
(b) $x - y = 20$ .....(1)
$x + y = 100$ .....(2)
Adding equations (1) and (2), we get
$2x = 120$
$\Rightarrow x = 60$ km/h
Substituting x = 60 in equation (2), we get
$60 + y = 100$
$\Rightarrow y = 100 - 60$
$\Rightarrow y = 40$ km/h

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32
PRACTICE PAPER 04 - CHAPTER 04 QUADRATIC EQUATIONS (2024-25)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : X **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Let p be a prime number. The quadratic equation having its roots as factors of p is
(a) $x^2 - px + p = 0$ (b) $x^2 - (p + 1)x + p = 0$
(c) $x^2 + (p + 1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$

2. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots, is:
(a) 0 only (b) 4 (c) 8 only (d) 0, 8

3. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
(a) 4 (b) ± 4 (c) – 4 (d) 0

4. Which of the following is not a quadratic equation?
(a) $2(x - 1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$
(c) $(\sqrt{2}\,x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

5. If α, β are roots of the equation $x^2 + 5x + 5 = 0$, then equation whose roots are α + 1 and β + 1 is
(a) $x^2 + 5x - 5 = 0$ (b) $x^2 + 3x + 5 = 0$ (c) $x^2 + 3x + 1 = 0$ (d) none of these

6. $(x^2 + 1)^2 - x^2 = 0$ has
(a) four real roots (b) two real roots (c) no real roots (d) one real root

7. If the difference of the roots of the equation $x^2 - bx + c = 0$ be 1, then
(a) $b^2 - 4c + 1 = 0$ (b) $b^2 + 4c = 0$ (c) $b^2 - 4c - 1 = 0$ (d) $b^2 - 4c = 0$

8. If the equation $x^2 - (2 + m)x + (-m^2 - 4m - 4) = 0$ has coincident roots, then
(a) m = 0, m = 1 (b) m = 2, m = 2 (c) m = –2, m = –2 (d) m = 6, m = 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A) :** The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.
Reason (R) : Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is called a quadratic equation.

10. Assertion (A) : The value of $k = 2$, if one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3.
Reason (R) : The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has two roots.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Solve the quadratic equation: $x^2 - 2ax + (a^2 - b^2) = 0$ for x.

12. Solve the quadratic equation: $x^2 + 2\sqrt{2}x - 6 = 0$ for x.

13. Find the value of 'k' for which the quadratic equation $2kx^2 - 40x + 25 = 0$ has real and equal roots.

14. If the sum of the roots of the quadratic equation $ky^2 - 11y + (k - 23) = 0$ is 13/21 more than the product of the roots, then find the value of k.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the value of 'p' for which the quadratic equation $p(x - 4)(x - 2) + (x - 1)^2 = 0$ has real and equal roots.

16. The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers.

17. If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2.

SECTION – D

Questions 18 carry 5 marks.

18. In a class test, the sum of Arun's marks in Hindi and English is 30. When he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case Study-1 : **Lusitania Bridge**

Japan's LO series Maglev is the fastest train in the world, with a speed record of 602 km/h. It could go the distance from New York City to Montreal in less than an hour. China has half of the eight fastest trains and the world's largest high speed railway network. Suppose a fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, then answer the following questions:

(a) Find the speed of slow train. (2)
(b) Find the speed of fast train. (1)
(c) How much time taken by the slow train to cover the distance 600 km? (1)

20. Generally, new methods such as aquaponics Raised-bed gardening raised beds and cultivation under glass are used. Marketing can be done locally in farmers markets, traditional markets or farmers can contract their whole crops to wholesalers, canners or retailers.
A farmer wishes to grow a 100 m^2 rectangular vegetable garden. Since he has with the only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side-fence.

(a) Represent given problem in quadratic form. (2)
(b) Find the length of the vegetable garden. (1)
(c) If length of the vegetable garden is 5 m, then find the breadth. (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 04 - CHAPTER 04 QUADRATIC EQUATIONS (2024-25)

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Let p be a prime number. The quadratic equation having its roots as factors of p is
(a) $x^2 - px + p = 0$ (b) $x^2 - (p + 1)x + p = 0$
(c) $x^2 + (p + 1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$
Ans: (b) $x^2 - (p + 1)x + p = 0$
Factors of $p = p \times 1$
$\therefore$ Roots are p and 1.
The quadratic equation is:
x^2 – (sum of roots)x + product of roots = 0
$\Rightarrow x^2 - (p + 1)x + p = 0$

2. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots, is:
(a) 0 only (b) 4 (c) 8 only (d) 0, 8
Ans: (d) 0, 8
Given equation is $2x^2 - kx + k = 0$
On comparing with $ax^2 + bx + c = 0$, $a = 2$, $b = -k$, $c = k$
For equal roots $b^2 - 4ac = 0$
$\Rightarrow (-k)^2 - 4(2)(k) = 0$
$\Rightarrow k^2 - 8k = 0$
$\Rightarrow k(k - 8) = 0$
$\Rightarrow k = 0, 8$
Hence, the required values of k are 0 and 8.

3. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
(a) 4 (b) ± 4 (c) – 4 (d) 0
Ans: (b) ± 4
Given: $2x^2 + kx + 2 = 0$
Comparing above equation with $ax^2 + bx + c = 0$,
$a = 2$, $b = k$ and $c = 2$
Condition for equal roots is: $D = 0$
i.e., $b^2 - 4ac = 0$
Substituting the values of a, b and c, we get
$k^2 - 4 \times 2 \times 2 = 0$
$\Rightarrow k^2 - 16 = 0$
$\Rightarrow [(k)^2 - (4)^2] = 0$
$\Rightarrow (k + 4)(k - 4) = 0$
$\Rightarrow k = 4$ or -4.

4. Which of the following is not a quadratic equation?
(a) $2(x-1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$
(c) $(\sqrt{2}\,x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$
Ans: (c) $(\sqrt{2}\,x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$
$2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$
$\Rightarrow\ 2\sqrt{6}x + 5x + 3 = 0$

5. If α, β are roots of the equation $x^2 + 5x + 5 = 0$, then equation whose roots are α + 1 and β + 1 is
(a) $x^2 + 5x - 5 = 0$ (b) $x^2 + 3x + 5 = 0$ (c) $x^2 + 3x + 1 = 0$ (d) none of these
Ans: (c) $x^2 + 3x + 1 = 0$
$\alpha + \beta = -5$, $\alpha\beta = 5$.
Required equation is
$x^2 - (\alpha + 1 + \beta + 1)x + (\alpha + 1)(\beta + 1) = 0$
$\Rightarrow x^2 - (\alpha + \beta + 2)x + (\alpha\beta + \alpha + \beta + 1) = 0$
$\Rightarrow x^2 - (-5 + 2)x + (5 - 5 + 1) = 0$
$\Rightarrow x^2 + 3x + 1 = 0$

6. $(x^2 + 1)^2 - x^2 = 0$ has
(a) four real roots (b) two real roots (c) no real roots (d) one real root
Ans: (c) no real roots

7. If the difference of the roots of the equation $x^2 - bx + c = 0$ be 1, then
(a) $b^2 - 4c + 1 = 0$ (b) $b^2 + 4c = 0$ (c) $b^2 - 4c - 1 = 0$ (d) $b^2 - 4c = 0$
Ans: (c) $b^2 - 4c - 1 = 0$
Let roots are α and β
$\Rightarrow \alpha - \beta = 1$
$\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$\Rightarrow 1 = \beta^2 - 4c \Rightarrow \beta^2 - 4c - 1 = 0$

8. If the equation $x^2 - (2 + m)x + (-m^2 - 4m - 4) = 0$ has coincident roots, then
(a) m = 0, m = 1 (b) m = 2, m = 2 (c) m = –2, m = –2 (d) m = 6, m = 1
Ans: (c) m = –2, m = –2
For coincident roots, D = 0
$\Rightarrow [-(2 + m)]^2 - 4 \times 1 \times (-m^2 - 4m - 4) = 0$
$\Rightarrow (2 + m)^2 + 4(m^2 + 4m + 4) = 0$
$\Rightarrow (2 + m)^2 + 4(m + 2)^2 = 0$
$\Rightarrow 5(2 + m)^2 = 0$
$\Rightarrow (2 + m)^2 = 0 \Rightarrow m = -2.$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A) :** The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.
Reason (R) : Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is called a quadratic equation.
Ans: We have, $x^2 + 3x + 1 = (x - 2)^2 = x^2 - 4x + 4$
$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow 7x - 3 = 0$, it is not of the form $ax^2 + bx + c = 0$.
So, A is false but R is true.
Hence, option (*d*) is correct.

10. Assertion (A) : The value of $k = 2$, if one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3.
Reason (R) : The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has two roots.
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Solve the quadratic equation: $x^2 - 2ax + (a^2 - b^2) = 0$ for x.
Ans: $x^2 - 2ax + (a^2 - b^2) = 0$
$\Rightarrow (x^2 - 2ax + a^2) - b^2 = 0$
$\Rightarrow (x - a)^2 - b^2 = 0$
$\Rightarrow (x - a + b)(x - a - b) = 0$
$\Rightarrow x - a + b = 0$ or $x - a - b = 0$
$\Rightarrow x = -(-a + b)$ or $x = -(-a - b)$
$\Rightarrow x = a - b$ or $x = a + b$.

12. Solve the quadratic equation: $x^2 + 2\sqrt{2}x - 6 = 0$ for x.
Ans: Given quadratic equation is $x^2 + 2\sqrt{2}x - 6 = 0$
$\Rightarrow x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$
$\Rightarrow x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$
$\Rightarrow (x + 3\sqrt{2})(x - \sqrt{2}) = 0$
$\Rightarrow x + 3\sqrt{2} = 0$ or $x - \sqrt{2} = 0$
$\Rightarrow x = -3\sqrt{2}$ or $x = \sqrt{2}$.

13. Find the value of 'k' for which the quadratic equation $2kx^2 - 40x + 25 = 0$ has real and equal roots.
Ans: Given quadratic equation is $2kx^2 - 40x + 25 = 0$
On comparing the above equation with $ax^2 + bx + c = 0$, we get
$a = 2k$, $b = -40$, $c = 25$
For real and equal roots, $D = 0$
i.e., $b^2 - 4ac = 0$
or, $(-40)^2 - 4(2k)(25) = 0$
$\Rightarrow 1600 - 200k = 0 \Rightarrow 200k = 1600 \Rightarrow k = 8$.

14. If the sum of the roots of the quadratic equation $ky^2 - 11y + (k - 23) = 0$ is 13/21 more than the product of the roots, then find the value of k.
Ans:
Let the roots of the given quadratic equation be α and β.
Now, Sum of roots, $\alpha + \beta = -(-11)/k$
$= 11/k$ ...(i)
and Product of roots, $\alpha\beta = (k - 23)/k$ ...(ii)
According to question, $\alpha + \beta = \alpha\beta + \frac{13}{21}$

$$\Rightarrow \frac{11}{k} = \frac{k-23}{k} + \frac{13}{21} \Rightarrow \frac{11}{k} - \frac{k-23}{k} = \frac{13}{21} \Rightarrow \frac{11-k+23}{k} = \frac{13}{21}$$

$\Rightarrow 21(34 - k) = 13k$
$\Rightarrow 34k = 714$
$\Rightarrow k = 21$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the value of 'p' for which the quadratic equation $p(x-4)(x-2)+(x-1)^2=0$ has real and equal roots.

Ans: Given quadratic equation is

$p(x-4)(x-2)+(x-1)^2=0$

$\Rightarrow p(x^2-4x-2x+8)+(x^2+1-2x)=0$

$\Rightarrow px^2-6px+8p+x^2+1-2x=0$

$\Rightarrow x^2(p+1)-2x(3p+1)+(8p+1)=0$

Comparing the above equation with $ax^2+bx+c=0$,

we get $a=p+1$, $b=-2(3p+1)$ and $c=8p+1$

For real and equal roots, $D=0 \Rightarrow b^2-4ac=0$

$\therefore [-2(3p+1)]^2-4(p+1)(8p+1)=0$

$\Rightarrow 4(3p+1)^2-4(8p^2+9p+1)=0$

$\Rightarrow 4(9p^2+1+6p)-32p^2-36p-4=0$

$\Rightarrow 36p^2+4+24p-32p^2-36p-4=0$

$\Rightarrow 4p^2-12p=0$

$\Rightarrow 4p(p-3)=0$

$\Rightarrow p=0$ or $p=3$

Hence, for $p=0$ or $p=3$, the given quadratic equation has real and equal roots.

16. The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers.

Ans: Let the first number be x and second number be y.

According to question, $x+y=34$

$\Rightarrow y=34-x$ ...(i)

and $(x-3)(y+2)=260$ ...(ii)

Substituting value of y from eq (i), in eq (ii), we get

$(x-3)(34-x+2)=260$

$\Rightarrow (x-3)(36-x)=260$

$\Rightarrow 36x-x^2-108+3x=260$

$\Rightarrow x^2-39x+368=0$

$\Rightarrow x^2-23x-16x+368=0$

$\Rightarrow x(x-23)-16(x-23)=0$

$\Rightarrow (x-23)(x-16)=0$

$\Rightarrow x=23$ or $x=16$

When $x=23$, $y=34-23=11$

When $x=16$, $y=34-16=18$

Hence, the numbers will be either 23 and 11 or 16 and 18.

17. If α and β are roots of the quadratic equation $x^2-7x+10=0$, find the quadratic equation whose roots are α^2 and β^2.

Ans: For the given Equation $x^2-7x+10=0$

$\Rightarrow x^2-5x-2x+10=0$

$\Rightarrow x(x-5)-2(x-5)=0$

$\Rightarrow (x-5)(x-2)=0$

$\Rightarrow x=5$ and $x=2$

Therefore, $\alpha=5$ and $\beta=2$

Thus, $\alpha^2=25$ and $\beta^2=4$

Quadratic equation whose roots are α^2 and β^2

$=x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2=0$

$=x^2-(25+4)x+25\times 4=0$

$=x^2-29x+100=0$

SECTION – D

Questions 18 carry 5 marks.

18. In a class test, the sum of Arun's marks in Hindi and English is 30. When he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

Ans: Let the marks in Hindi be x and the marks in English be y.

According to question, $x + y = 30 \Rightarrow y = 30 - x$ ...(i)

If he had got 2 marks more in Hindi, then his marks would be = $x + 2$

and if he had 3 marks less in English, then his marks would be = $y - 3$

According to question, $(x + 2)(y - 3) = 210$

$\Rightarrow (x + 2)(30 - x - 3) = 210$ (from Eq. (i)) **1½**

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow -x^2 + 25x - 156 = 0$

$\Rightarrow x^2 - 25x + 156 = 0$

$\Rightarrow x^2 - 13x - 12x + 156 = 0$

$\Rightarrow x(x - 13) - 12(x - 13) = 0$

$\Rightarrow (x - 12)(x - 13) = 0$

$\Rightarrow$ Either $x = 12$ or $x = 13$ **1½**

when $x = 12$, then $y = 30 - 12 = 18$

when $x = 13$, then $y = 30 - 13 = 17$

Hence, the marks in Hindi = 12 and marks in English = 18

or the marks in Hindi = 13 and marks in English = 17.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case Study-1 : **Lusitania Bridge**

Japan's LO series Maglev is the fastest train in the world, with a speed record of 602 km/h. It could go the distance from New York City to Montreal in less than an hour. China has half of the eight fastest trains and the world's largest high speed railway network. Suppose a fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, then answer the following questions:

(a) Find the speed of slow train. (2)

(b) Find the speed of fast train. (1)

(c) How much time taken by the slow train to cover the distance 600 km? (1)

Ans: (a) Let the speed of the slow train be x km/h, then

speed of the fast train is $(x + 10)$ km/h.

$\therefore$ Time taken by the slow train to cover 600 km = $600/x$ hr

and time taken by the fast train to cover 600 km = $600/(x + 10)$ hr

According to question, $\frac{600}{x}-\frac{600}{x+10}=3$

$\Rightarrow \frac{600x+6000-600x}{x(x+10)}=3 \Rightarrow \frac{6000}{x^2+10x}=3$

$\Rightarrow 3(x^2 + 10x) = 6000$

$\Rightarrow x^2 + 10x - 2000 = 0$

$\Rightarrow (x + 50)(x - 40) = 0$

$\Rightarrow x = -50$ or 40 [$\because$ speed cannot be negative]

Hence, Speed of slow train is 40 km/h.

(b) Speed of fast train = $(x + 10)$ km/h

= (40 + 10) km/h

= 50 km/h

(c) Time taken by the slow train to cover 600 km = 600/40 hr = 15 hr

20. Generally, new methods such as aquaponics Raised-bed gardening raised beds and cultivation under glass are used. Marketing can be done locally in farmers markets, traditional markets or farmers can contract their whole crops to wholesalers, canners or retailers.

A farmer wishes to grow a 100 m^2 rectangular vegetable garden. Since he has with the only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side-fence.

(a) Represent given problem in quadratic form. (2)

(b) Find the length of the vegetable garden. (1)

(c) If length of the vegetable garden is 5 m, then find the breadth. (1)

Ans: (a) Let the length of one side be x m and other side be y m, then $x + y + x = 30$

$\Rightarrow y = 30 - 2x$

Area of vegetable garden = 100 m^2

$\Rightarrow xy = 100$

$\Rightarrow x(30 - 2x) = 100$

$\Rightarrow 30x - 2x^2 - 100 = 0$

$\Rightarrow x^2 - 15x - 50 = 0$

(b) $x^2 - 15x - 50 = 0$

$\Rightarrow x^2 - 10x - 5x + 50 = 0$

$\Rightarrow x(x - 10) - 5(x - 10) = 0$

$\Rightarrow (x - 10)(x - 5) = 0$

$\Rightarrow x = 5$ or 10

Hence, length of the vegetable garden is 5 m or 10 m.

(c) Since, area = 100 m^2

Then. $l \times b = 100$

$\Rightarrow 5 \times b = 100$

$\Rightarrow b = 20$ m.

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 05 (2024-25)

CHAPTER 05 ARITHMETIC PROGRESSION

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The 21st term of the AP whose first two terms are –3 and 4 is
(a) 17 (b) 137 (c) 143 (d) –143

2. If k, $2k – 1$ and $2k + 1$ are three consecutive terms of an A.P., then the value of k is:
(a) 2 (b) 3 (c) –3 (d) 5

3. The sum of the first 16 terms of the A.P.: 10, 6, 2, ... is:
(a) –320 (b) 320 (c) –352 (d) –400

4. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?
(a) 30 (b) 33 (c) 37 (d) 38

5. Which term of the AP: 21, 42, 63, 84… is 210?
(a) 9th (b) 10th (c) 11th (d) 12th

6. The common difference of an A.P., whose nth term is $a_n = (3n + 7)$, is:
(a) 3 (b) 7 (c) 10 (d) 6

7. The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an A.P., is:
(a) – 1 (b) – 2 (c) 1 (d) 2

8. The number of terms of an A.P. 5, 9, 13, ... 185 is:
(a) 31 (b) 51 (c) 41 (d) 46

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Common difference of an A.P. in which $a_{21} – a_7 = 84$ is 14.
Reason (R): nth term of AP is given by $a_n = a + (n – 1)d$.

10. **Assertion (A):** If the second term of an A.P., is 13 and the fifth term is 25, then its 7th term is 33.
Reason (R): If the common difference of an A.P. is 5, then $a_{18} – a_{13}$ is 25.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Determine the A.P. whose third term is 16 and 7th term exceeds the 5th term by 12.

12. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?

13. If seven times the 7th term of an A.P. is equal to eleven times the 11th term, then what will be its 18th term?

14. Find how many integers between 200 and 500 are divisible by 8.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Solve for x : 1 + 5 + 9 + 13 + ... + x = 1326

16. Find the middle terms of the A.P. 7, 13, 19 .... 241.

17. The first term of an A.P. is –5 and the last term is 45. If the sum of the terms of the A.P. is 120, then find the number of terms and the common difference.

SECTION – D

Questions 18 carry 5 marks.

18. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle term is 7 : 15. Find the numbers.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A school auditorium has to be constructed with a capacity of 2000 people. The chairs in the auditorium are arranged in a concave shape facing towards the stage in such a way that each succeeding row has 5 seats more than the previous one.

(a) If the first row has 15 seats, then how many seats will be there in 12th row?
(b) If there are 15 rows in the auditorium, then how many seats will be there in the middle row?
(c) If total 1875 guests were there in the auditorium for a particular event, then how many rows will be needed to make all of them sit?

OR

(c) If total 1250 guests were there in the auditorium for a particular event, then how many rows will be left blank out of total 30 rows?

20. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete. Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day. Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week.
During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

(a) How many throws Sanjitha practiced on 11th day of the camp?
(b) What would be Sanjitha's throw distance at the end of 6 weeks?

OR

(b) When will she be able to achieve a throw of 11.16 m?
(c) How many throws did she do during the entire camp of 15 days?

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 05 (2024-25)

CHAPTER 05 ARITHMETIC PROGRESSION

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The 21st term of the AP whose first two terms are –3 and 4 is
(a) 17 (b) 137 (c) 143 (d) –143
Ans: (b) 137
First two terms of an AP are a = – 3 and $a_2 = 4$.
We know, nth term of an AP is $a_n = a + (n – 1)d$
Where, a = first term, an is nth term and d is the common difference
$a_2 = a + d \Rightarrow 4 = – 3 + d \Rightarrow d = 7$
Common difference, d = 7
$\therefore a_{21} = a + 20d = – 3 + (20)(7) = 137$

2. If k, $2k – 1$ and $2k + 1$ are three consecutive terms of an A.P., then the value of k is:
(a) 2 (b) 3 (c) –3 (d) 5
Ans. (b) 3
If a, b, c are in AP then $2b = a + c$
So, $a = k$, $b = 2k – 1$, $c = 2k + 1$
$a + c = 2b$
Here, $k + (2k + 1) = 2 (2k – 1)$
i.e., $3k + 1 = 4k – 2 \Rightarrow k = 3$

3. The sum of the first 16 terms of the A.P.: 10, 6, 2, ... is:
(a) –320 (b) 320 (c) –352 (d) –400
Ans. (a) –320
The given series of A.P. is 10, 6, 2 ...
Here, the first term, $a = 10$ and common difference, $d = a_2 – a_1 = 6 – 10 = –4$
We know that, $S_n = \frac{n}{2}[2a + (n – 1)d]$
$\Rightarrow S_{16} = \frac{16}{2}[2(10) + (16 – 1)(–4)]$
$= 8[20 + 15(–4)] = 8[20 – 60] = 8(–40) = –320 \Rightarrow S16 = –320$
Hence, the sum of the first 16 terms of the AP is –320.

4. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?
(a) 30 (b) 33 (c) 37 (d) 38
Ans: (b) 33

We know that the nth term of an AP is an = a + (n – 1)d
$\Rightarrow a_2 = a + d = 13$ …..(1)
and $a_5 = a + 4d = 25$ …… (2)
Solving the above equations, we get d = 4 and a = 9
$\therefore a_7 = a + 6d = 9 + 6(4) = 9 + 24 = 33$

5. Which term of the AP: 21, 42, 63, 84… is 210?
(a) 9th (b) 10th (c) 11th (d) 12th
Ans: (b) 10th
Let nth term of the given AP be 210.
According to question, first term, a = 21
common difference, d = 42 – 21 = 21 and an = 210
We know that the nth term of an AP is an = a + (n – 1)d
⇒ 210 = 21 + (n – 1)21
⇒ 189 = (n – 1)21 ⇒ n – 1 = 9 ⇒ n = 10
So, 10th term of an AP is 210.

6. The common difference of an A.P., whose *n*th term is $a_n = (3n + 7)$, is:
(a) 3 (b) 7 (c) 10 (d) 6
Ans. (a) 3
Here, $a_n = 3n + 7$
$\Rightarrow a_1 = 3 \times 1 + 7 = 10$
and $a_2 = 3 \times 2 + 7 = 13$
So, $d = a_2 - a_1 = 13 - 10 = 3$
Thus common difference of an AP = 3.

7. The value of *p* for which (2*p* + 1), 10 and (5*p* + 5) are three consecutive terms of an A.P., is:
(a) – 1 (b) – 2 (c) 1 (d) 2
Ans: (d) 2
The terms (2p + 1), 10 and (5p + 5) are consecutive terms of A.P., when
2 × 10 = (2p + 1) + (5p + 5)
⇒ 20 = 7p + 6 ⇒ 7p = 14 ⇒ p = 2

8. The number of terms of an A.P. 5, 9, 13, ... 185 is:
(a) 31 (b) 51 (c) 41 (d) 46
Ans. (d) 46
The given A.P. is 5, 9, 13, ... 185
Here, $a = 5$, $d = 9 - 5 = 4$ and $l = 185$
We know, last term of an A.P. is given by,
$l = a + (n - 1)d$
⇒ 185 = 5 + (*n* – 1)4
⇒ 180 = (*n* – 1)4 ⇒ *n* – 1 = 45 ⇒ *n* = 46
Thus, there are 46 terms in the given A.P.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Common difference of an A.P. in which $a_{21} - a_7 = 84$ is 14.
Reason (R): nth term of AP is given by $a_n = a + (n - 1)d$.
Ans. (d) Assertion (A) is false but reason (R) is true.

We have, $a_n = a + (n - 1)\, d$
$a_{21} - a_7 = 84$ [given]
$\Rightarrow [a + (21 - 1)d] - [a + (7 - 1)d] = 84$
$\Rightarrow a + 20\, d - a - 6d = 84$
$\Rightarrow 14d = 84$
$\Rightarrow d = 84/14 = 6$

10. Assertion (A): If the second term of an A.P., is 13 and the fifth term is 25, then its 7th term is 33.
Reason (R): If the common difference of an A.P. is 5, then $a_{18} - a_{13}$ is 25.
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
In the given A.P., $a_2 = 13$ and $a_5 = 25$
$\Rightarrow a + d = 13$ and $a + 4d = 25$
Solving these equations, we get $a = 9$ and $d = 4$
Thus, $a_7 = a + 6d = 9 + 6(4) = 33$
$\therefore$ Assertion is true.
In case of reason:
In the given A.P., $d = 5$ Thus, $a_{18} - a_{13} = a + 17d - a - 12d = 5d = 25$
$\therefore$ Reason is true.
Hence, both assertion and reason are true but reason is not the correct explanation for assertion.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Determine the A.P. whose third term is 16 and 7th term exceeds the 5th term by 12.
Ans. Let, the first term of an A.P. be 'a' and common difference be 'd'.
Given, $a_3 = 16$ [General term $a_n = a + (n - 1)d$]
$\Rightarrow a + 2d = 16$ ...(i)
and $a_7 = a_5 + 12$ (Given)
$\Rightarrow a + 6d = a + 4d + 12 \Rightarrow 2d = 12 \Rightarrow d = 6$
Put the value of d in equation (i), we get
$a + 2 \times 6 = 16 \Rightarrow a = 4$
$\therefore$ The first term of the A.P is 4 and its common difference is 6.
Hence, the required A.P. is 4, 10, 16, 22.....

12. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?
Ans. Given: A.P. is 3, 15, 27, 39, ...
Here, the first term, $a = 3$
Common difference, $d = 15 - 3 = 27 - 15 = 12$
Now, 21st term : $an = a + (n - 1)d$
$a_{21} = 3 + (21 - 1) \times 12 = 3 + 20 \times 12 = 3 + 240 = 243$
Let nth term of the given A.P. be 120 more than its 21st term.
Then, according to the given condition: $a_n = a_{21} + 120$
$a_n = 243 + 120$
$\Rightarrow a + (n - 1)d = 363$
$\Rightarrow 3 + (n - 1) \times 12 = 363$
$\Rightarrow (n - 1) = 360/12 \Rightarrow n = 31$
Hence, the 31st term is 120 more than its 21st term.

13. If seven times the 7th term of an A.P. is equal to eleven times the 11th term, then what will be its 18th term?
Ans: Let the first term of the A.P. be 'a' and its common difference be 'd'.
Given, $7a_7 = 11a_{11}$
Then, $7(a + 6d) = 11(a + 10d)$

[∵ $a_n = a + (n-1)d$]
$\Rightarrow 7a + 42d = 11a + 110d$
$\Rightarrow 7a - 11a = 110d - 42d$
$\Rightarrow -4a = 68d \Rightarrow a = -17d$ ...(i)
Now, 18th term of A.P., $a_{18} = a + 17d = -17d + 17d$ [Using (i)]
$\Rightarrow a_{18} = 0$
Hence, the 18th term of the A.P. is 0.

14. Find how many integers between 200 and 500 are divisible by 8.
Ans. Integers between 200 and 500 divisible by 8 are 208, 216, 224 ,..., 496.
This series forms an A.P., where first term, a = 208, common difference, d = 8 and last term, l = 496.
Let, the number of such integers be 'n'.
Then, nth term = a_n (last term l) = a + (n – 1)d
$\Rightarrow 496 = 208 + (n-1) \times 8$
$\Rightarrow 496 - 208 = (n-1) \times 8$
$\Rightarrow (n-1) \times 8 = 496 - 208$
$\Rightarrow (n-1) = 288/8 = 36$
$\Rightarrow n = 37$
Hence, the number of terms is 37.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Solve for x : 1 + 5 + 9 + 13 + ... + x = 1326
Ans: In the given series, we have 5 – 1 = 4, 9 – 5 = 4, 13 – 9 = 4 and so on i.e. the difference between any two consecutive terms is same.
So, this series is an A.P. with a = 1, d = 4, an = x and Sn = 1326.
We know, an = a + (n – 1)d
$\Rightarrow x = 1 + (n-1)(4)$
$\Rightarrow x = 4n - 3 \Rightarrow n = \frac{x+3}{4}$
Further, $S_n = \frac{n}{2}[a + a_n]$
$\Rightarrow 1326 = \frac{x+3}{8}[1+x]$
$\Rightarrow (1 + x)(x + 3) - 10608 = 0$
$\Rightarrow x^2 + 4x - 10605 = 0 \Rightarrow x^2 + 105x - 101x - 10605 = 0$
$\Rightarrow x(x + 105) - 101(x + 105) = 0$
$\Rightarrow (x + 105)(x - 101) = 0 \Rightarrow x - 101 = 0 \Rightarrow x = 101$
(Since, $x + 105 \neq 0$, as this A.P. is an increasing series, so x cannot be negative)
Thus, the value of x is 101.

16. Find the middle terms of the A.P. 7, 13, 19 .... 241.
Ans: Given A.P. 7, 13, 19, ... 241
$\therefore$ a = 7, a_n = 241, d = 13 – 7 = 6
We know that, $a_n = a + (n-1)d$
$\Rightarrow 241 = 7 + (n-1)6$
$\Rightarrow 241 - 7 = (n-1)6$
$\Rightarrow n - 1 = 234/6$
$\Rightarrow n - 1 = 39$
$\Rightarrow n = 40$
$\Rightarrow n/2 = 20$ ..... (middle term)

$\Rightarrow$ n/2 + 1 = 21 ..... (middle term)
$\therefore a_{20} = 7 + (20 - 1)6$
$\therefore a_{20} = 7 + 19 \times 6 = 121$
$\therefore a_{21} = 121 + 6 = 127$
$\therefore$ Middle terms are 121 and 127.

17. The first term of an A.P. is –5 and the last term is 45. If the sum of the terms of the A.P. is 120, then find the number of terms and the common difference.
Ans: Let a be the first term, d be the common difference, *l* be the last term and n be the number of terms of the given A.P.
So, a = –5, *l* = 45 and Sn = 120 [Given]
We know that, if the last term of an A.P. is known, then the sum of n terms of an A.P. is

$S_n = \frac{n}{2}(a + l) \Rightarrow 120 = \frac{n}{2}(-5+45) \Rightarrow 120 \times 2 = 40 \times n \Rightarrow n = 6$

Also, $l = a + (n - 1)d$
$\Rightarrow 45 = -5 + (6 - 1)d$
$\Rightarrow 50 = 5d$
$\Rightarrow d = 10$
Hence, number of terms = 6 and common difference = 10

SECTION – D

Questions 18 carry 5 marks.

18. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle term is 7 : 15. Find the numbers.
Ans: Let '*a*' be the first term and '*d*' be the common difference of the A.P.
Also, let $a - 3d, a - d, a + d, a + 3d$ be the four consecutive terms of the A.P.
As per the question, $(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$
$\Rightarrow 4a = 32$, or $a = 8$ ...(i)
and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$

$\Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$
$\Rightarrow 8a^2 = 128\, d^2$
Using (i), we have: $8 \times 8^2 = 128\, d^2$
$\Rightarrow d^2 = 4$, or $d = \pm 2$
Thus, the four numbers are 2, 6, 10, 14 or 14, 10, 6, 2.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A school auditorium has to be constructed with a capacity of 2000 people. The chairs in the auditorium are arranged in a concave shape facing towards the stage in such a way that each succeeding row has 5 seats more than the previous one.
(a) If the first row has 15 seats, then how many seats will be there in 12th row?
(b) If there are 15 rows in the auditorium, then how many seats will be there in the middle row?
(c) If total 1875 guests were there in the auditorium for a particular event, then how many rows will be needed to make all of them sit?

OR

(c) If total 1250 guests were there in the auditorium for a particular event, then how many rows will be left blank out of total 30 rows?

Ans: (a) a = 15, d = 5
a_{12} = 15 + 11 × 5 = 70
(b) n = 15
Middle row = 8th row
a_8 = 15 + 7 × 5 = 50
(c) 1875 = n/2 [2 × 15 + (n − 1) × 5]
⇒ n^2 + 5n − 750 = 0
(n + 30)(n − 25) = 0 ⇒ n = 25
∴ Total number of rows required = 25

OR

(c) 1250 = n/2 [2 × 15 + (n − 1) × 5]
⇒ n^2 + 5n − 500 = 0
(n + 25)(n − 20) = 0 ⇒ n = 20
∴ Number of rows left = 30 − 20 = 10

20. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete. Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day. Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week.
During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

(a) How many throws Sanjitha practiced on 11th day of the camp?
(b) What would be Sanjitha's throw distance at the end of 6 weeks?

OR

(b) When will she be able to achieve a throw of 11.16 m?

(c) How many throws did she do during the entire camp of 15 days?

Ans: (a) Number of throws during camp. a = 40; d = 12
a_{11} = a + 10d = 40 + 10 × 12 = 160 throws
(b) a = 7.56 m; d = 9 cm = 0.09 m; n = 6 weeks
a_n = a + (n – 1)d = 7.56 + 6(0.09) = 7.56 + 0.54 = 8.10 m
Sanjitha's throw distance at the end of 6 weeks = 8.1 m

OR

(b) a = 7.56 m; d = 9 cm = 0.09 m
a_n =11.16 m
⇒ a_n = a + (n – 1)d
⇒ 11.16 = 7.56 + (n – 1) (0.09)
⇒ 3.6 = (n – 1) (0.09)
⇒ n – 1 = 36/0.09 = 40
⇒ n = 41
Sanjitha will be able to throw 11.16 m in 41 weeks.
(c) a = 40; d = 12; n = 15
S_n = n/2 [2a + (n – 1)d]
S_n = 15/2 [2(40) + (15 – 1) (12)]
= 15/2 [80 + 168]
= 15/2 [248] = 1860 throws

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 06 (2024-25)

CHAPTER 06 TRIANGLES

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

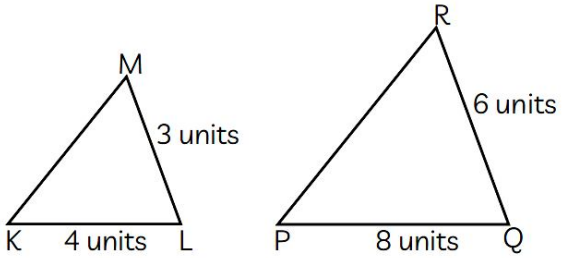
General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

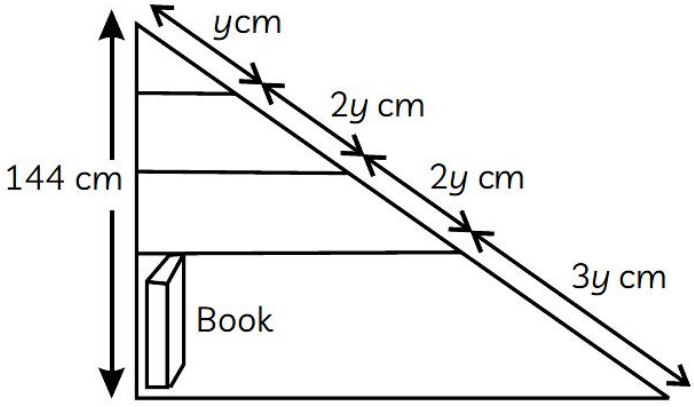
Questions 1 to 10 carry 1 mark each.

1. Shown below are two triangles such that length of two sides of each is known. Along with the given information, which of these is sufficient to conclude whether ΔKLM is similar to ΔPQR?



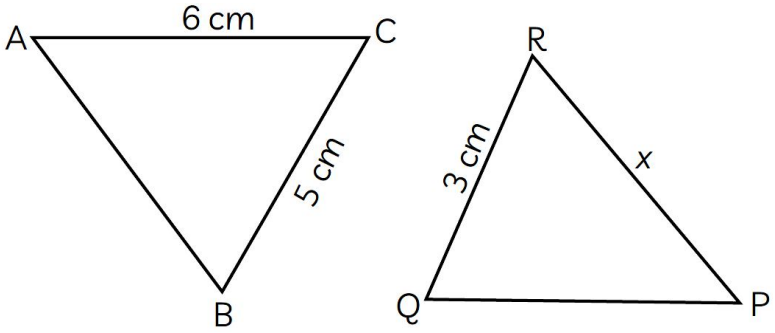
(I) ∠KLM = ∠PQR
(II) Ratio of KM : PR = 1 : 2
Options:
(a) only (I)
(b) only (II)
(c) either (I) or (II)
(d) the given information is enough to conclude that ΔKLM ~ ΔPQR as ratio of sides is known

2. Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below.



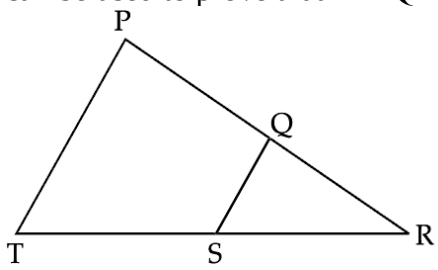
The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom-most shelf?
(a) 18 cm (b) 36 cm (c) 54 cm (d) 86.4 cm

3. In the given figure, ΔABC ~ ΔQPR. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x; then the value of x is:



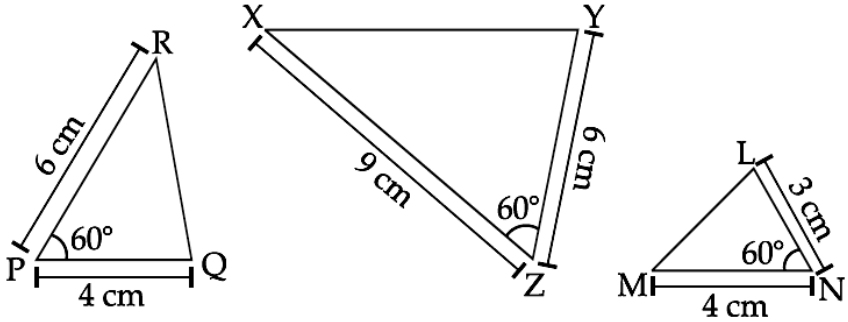
(a) 3.6 cm (b) 2.5 cm (c) 10 cm (d) 3.2 cm

4. In the following figure, Q is a point on PR and S is a point on TR. QS is drawn and ∠RPT = ∠RQS. Which of these criteria can be used to prove that ΔRSQ is similar to ΔRTP?



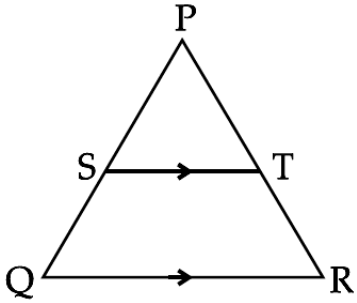
(a) AAA similarity criterion (b) SAS similarity criterion
(c) SSS similarity criterion (d) RHS similarity criterion

5. Shown below are three triangles. The measures of two adjacent sides and included angle are given for each triangle.



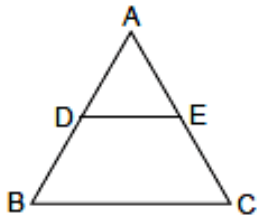
(a) ΔRPQ and Δ XZY
(b) ΔRPQ and Δ MNL
(c) ΔXZY and Δ MNL
(d) ΔRPQ, Δ XZY and Δ MNL are similar to one another.

6. In the following figure, ST || QR, point S divides PQ in the ratio 4 : 5. If ST = 1.6 cm, what is the length of QR?



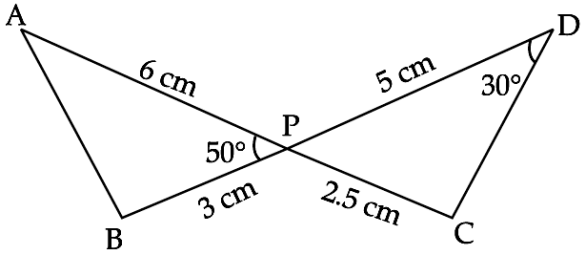
(a) 0.71 cm (b) 2 cm (c) 3.6 cm (d) cannot be calculated from the given data.

7. In the given figure, DE || BC, AE = a units, EC = b units, DE = x units and BC = y units. Which of the following is true?



(a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

8. In the figure given below, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then, ∠PBA is equal to:



(a) 50° (b) 30° (c) 60° (d) 100°

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

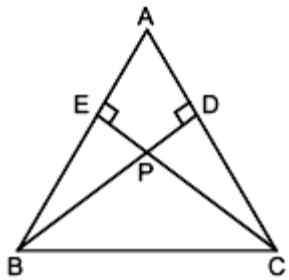
9. **Assertion (A):** D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC then the value of x is 4, when AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm.
Reason (R): If a line is parallel to one side of a triangle then it divides the other two sides in the same ratio.

10. **Assertion (A):** D and E are points on the sides AB and AC respectively of a ΔABC such that AB = 10.8cm, AD = 6.3cm, AC = 9.6cm and EC = 4cm then DE is not parallel to BC.
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.

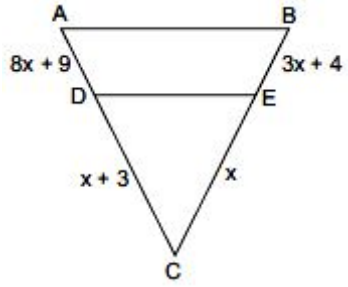
SECTION – B

Questions 11 to 14 carry 2 marks each.

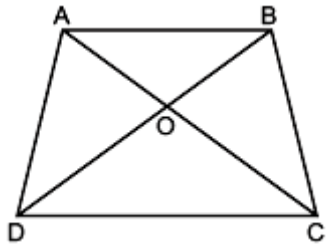
11. In the given figure, considering triangles BEP and CPD, prove that BP × PD = EP × PC.



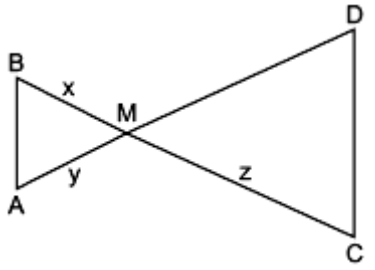
12. What value(s) of *x* will make DE || AB in the given figure?



13. In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 4 cm. Find the value of DC.



14. In the given figure, ΔAMB ~ ΔCMD.

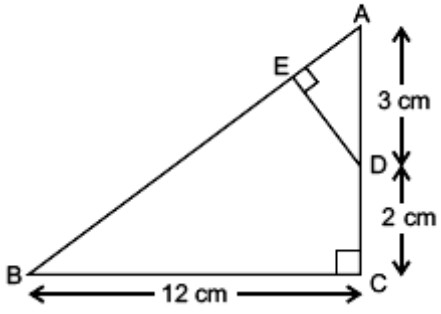


Determine MD in terms of *x, y* and *z.*

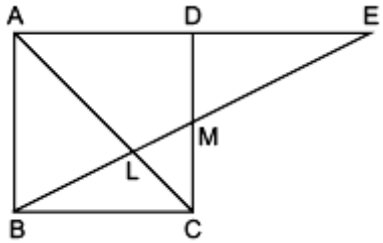
SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In figure, ΔABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE.



16. In figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2BL.



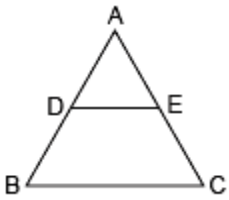
17. In an isosceles ΔABC, the base AB is produced both ways to P and Q such that AP × BQ = $(AC)^2$. Prove that ΔACP ~ ΔBCQ.

SECTION – D

Questions 18 carry 5 marks.

18. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following :

In figure, D and E are points on AB and AC respectively, such that DE || BC. If AD = $\frac{1}{3}$ BD, AE = 4.5 cm, find EC.



SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

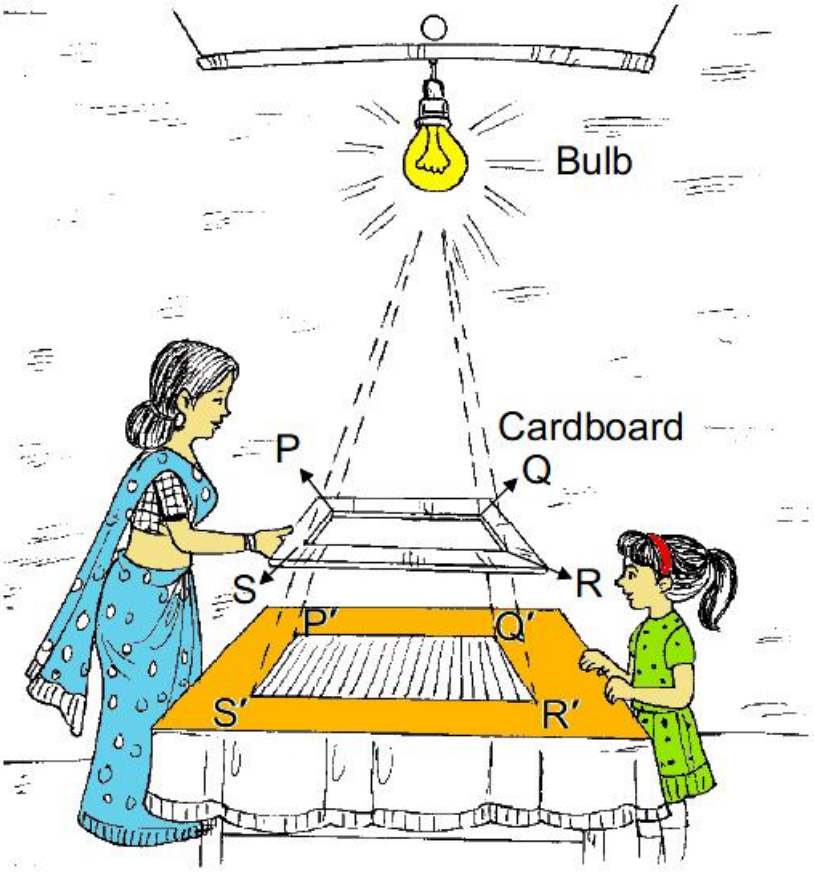
19. While browsing through the catalogue of wooden shelves, Karthik came across this beautiful triangular shaped shelf. In the shelf, DE is parallel to the base BC could be used for displaying small plants and showpieces.

(a) Find the relation between the sides AD, DB, AE and EC. Also, mention the theorem used. (1)
(b) With measurement AE = 1.8 cm, BD = 7.2 cm and CE = 5.4 cm. Karthik thought of finding the length of side AD from the given figure of shelf. How he will find the length. (1)
(c) Find the value of x if AD = (x + 3) cm, BD = (3x + 19) cm, AE = x cm and EC = (3x + 4) cm. (2)

OR

(c) If AB = 9 cm, AC = 18 cm, AD = 2 cm and AE = 4 cm, then prove that DE || BC. (2)

20. Anjali placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P′Q′R′S′. Quadrilateral P′Q′R′S′ is an enlargement of the quadrilateral PQRS with scale factor 1 : 3. Given that PQ = 2.5 cm, QR = 3.5 cm. RS = 3.4 cm and *PS* = 3.1 cm; ∠P = 115°, ∠Q = 95°, ∠R = 65° and ∠S = 85°.



(a) Find the length of R′S′. (1)

(b) Find the measurement of ∠Q′ (1)

(c) Find the ratio of sides P′Q′ and Q′R′. (2)

OR

(c) Find the sum of the lengths Q′R′ and P′S′. (2)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 07 (2024-25)

CHAPTER 06 TRIANGLES (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

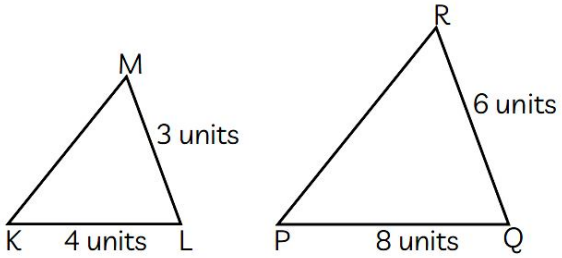
General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

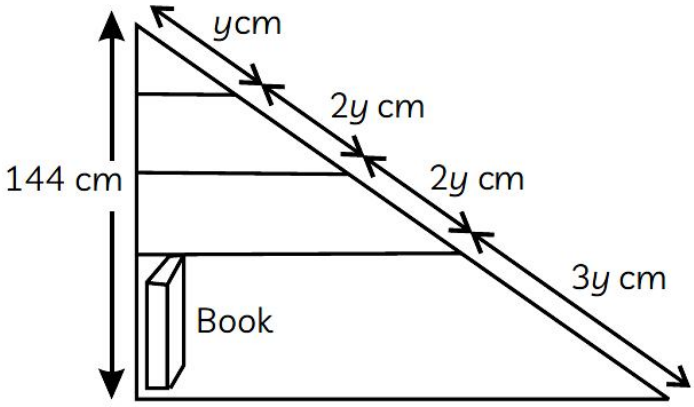
Questions 1 to 10 carry 1 mark each.

1. Shown below are two triangles such that length of two sides of each is known. Along with the given information, which of these is sufficient to conclude whether ΔKLM is similar to ΔPQR?



(I) ∠KLM = ∠PQR
(II) Ratio of KM : PR = 1 : 2
Options:
(a) only (I)
(b) only (II)
(c) either (I) or (II)
(d) the given information is enough to conclude that ΔKLM ~ ΔPQR as ratio of sides is known
Ans. (c) either (I) or (II)

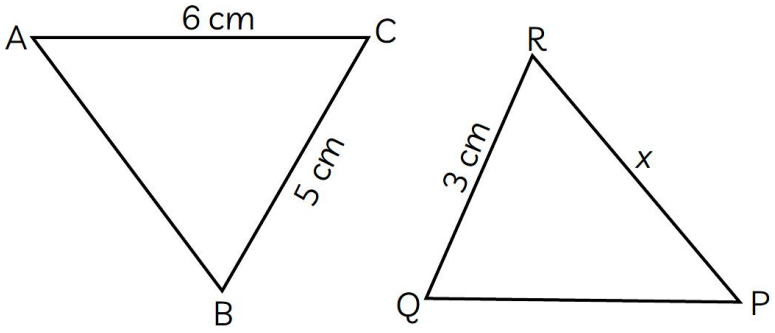
2. Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below.



The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom-most shelf?
(a) 18 cm (b) 36 cm (c) 54 cm (d) 86.4 cm

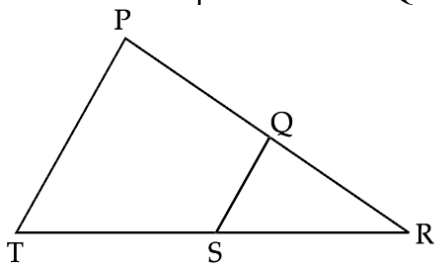
Ans. (c) 54 cm

3. In the given figure, ΔABC ~ ΔQPR. If AC = 6 cm, BC = 5 cm, QR = 3 cm and PR = x; then the value of x is:



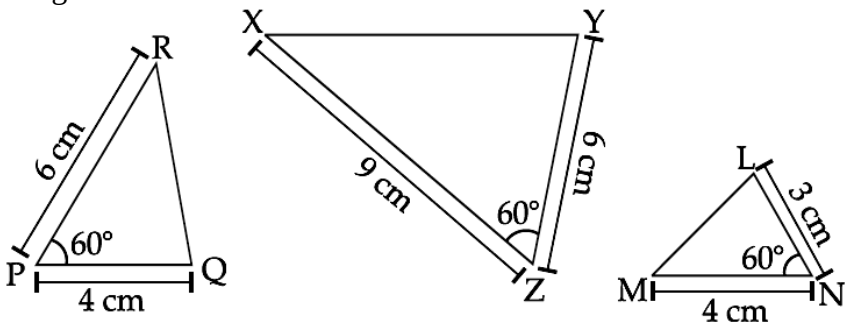
(a) 3.6 cm (b) 2.5 cm (c) 10 cm (d) 3.2 cm
Ans. (b) 2.5 cm

4. In the following figure, Q is a point on PR and S is a point on TR. QS is drawn and ∠RPT = ∠RQS. Which of these criteria can be used to prove that ΔRSQ is similar to ΔRTP?



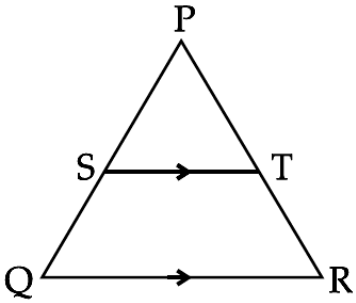
(a) AAA similarity criterion (b) SAS similarity criterion
(c) SSS similarity criterion (d) RHS similarity criterion
Ans: (a) AAA similarity criterion

5. Shown below are three triangles. The measures of two adjacent sides and included angle are given for each triangle.



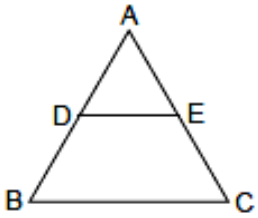
(a) ΔRPQ and Δ XZY
(b) ΔRPQ and Δ MNL
(c) ΔXZY and Δ MNL
(d) ΔRPQ, Δ XZY and Δ MNL are similar to one another.
Ans: (a) ΔRPQ and Δ XZY

6. In the following figure, ST || QR, point S divides PQ in the ratio 4 : 5. If ST = 1.6 cm, what is the length of QR?



(a) 0.71 cm (b) 2 cm (c) 3.6 cm (d) cannot be calculated from the given data.
Ans. (c) 3.6 cm

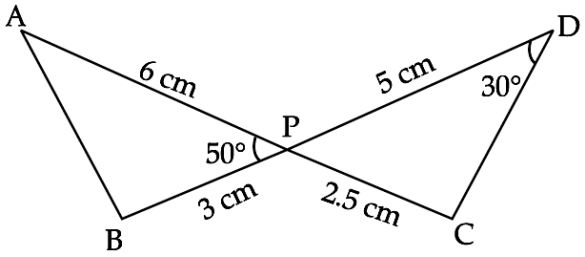
7. In the given figure, DE || BC, AE = a units, EC = b units, DE = x units and BC = y units. Which of the following is true?



(a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

Ans: (c) $x = \frac{ay}{a+b}$

8. In the figure given below, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then, ∠PBA is equal to:



(a) 50° (b) 30° (c) 60° (d) 100°
Ans: (d) 100°

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

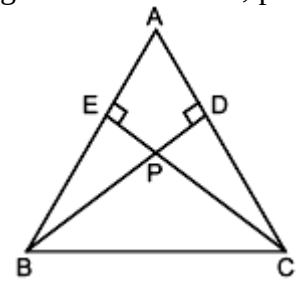
9. **Assertion (A):** D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC then the value of x is 4, when AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm.
Reason (R): If a line is parallel to one side of a triangle then it divides the other two sides in the same ratio.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. Assertion (A): D and E are points on the sides AB and AC respectively of a ΔABC such that AB = 10.8cm, AD = 6.3cm, AC = 9.6cm and EC = 4cm then DE is not parallel to BC.
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.
Ans: (d) Assertion (A) is false but reason (R) is true.

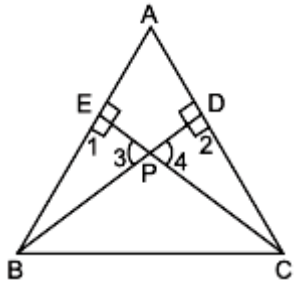
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, considering triangles BEP and CPD, prove that BP × PD = EP × PC.

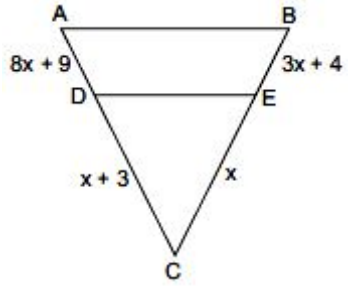


Ans: In ΔBEP and ΔCPD



$\angle 1 = \angle 2 = 90°$
$\angle 3 = \angle 4$ (Vertically Opposite Angles)
$\Rightarrow$ ΔBEP ~ ΔCDP (By AA similarity)
$\Rightarrow \frac{BP}{CP} = \frac{EP}{DP} \Rightarrow \frac{BP}{EP} = \frac{CP}{DP}$
$\Rightarrow$ BP × PD = EP × CP

12. What value(s) of x will make DE || AB in the given figure?



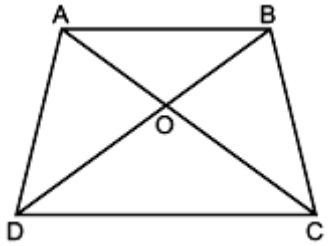
Ans: DE will be parallel to AB
Only, if $\frac{CD}{AD} = \frac{CE}{BE}$ [Converse of Basic Proportionality Theorem]
$\Rightarrow \frac{x+3}{8x+9} = \frac{x}{3x+4}$
$\Rightarrow (x + 3)(3x + 4) = x(8x + 9)$
$\Rightarrow 3x^2 + 9x + 4x + 12 = 8x^2 + 9x$
$\Rightarrow 5x^2 - 4x - 12 = 0$
$\Rightarrow 5x^2 - 10x + 6x - 12 = 0$
$\Rightarrow 5x(x - 2) + 6(x - 2) = 0$
$\Rightarrow (x - 2)(5x + 6) = 0$
$\Rightarrow$ either $x = 2$ or $5x = -6$

$\Rightarrow x = \frac{-6}{5}$ (Rejecting as side is never negative)

$\Rightarrow x = 2$

if $x = 2$ then DE || AB.

13. In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 4 cm. Find the value of DC.



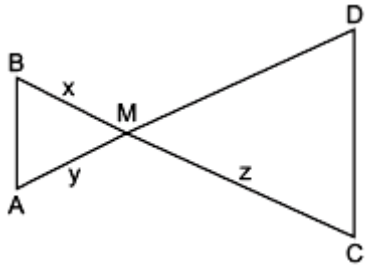
Ans: In ΔAOB and ΔCOD, $\frac{AO}{OC} = \frac{BO}{OD}$ and ∠AOB = ∠COD (Vertically Opposite Angles)

∴ ΔAOB ~ ΔCOD (SAS similarity)

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD} \Rightarrow \frac{1}{2} = \frac{4}{CD}$

$\Rightarrow$ CD = 8 cm.

14. In the given figure, ΔAMB ~ ΔCMD.



Determine MD in terms of *x, y* and *z*.

Ans: Given that ΔAMB ~ ΔCMD

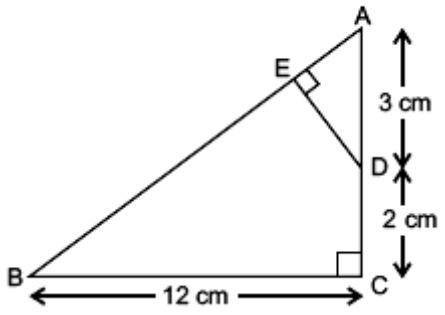
$\Rightarrow \frac{BM}{AM} = \frac{MD}{CM} \Rightarrow \frac{BM}{MD} = \frac{AM}{CM}$ (Corresponding sides of similar triangles are proportional)

$\Rightarrow \frac{x}{y} = \frac{MD}{z} \Rightarrow MD = \frac{xz}{y}$

SECTION – C

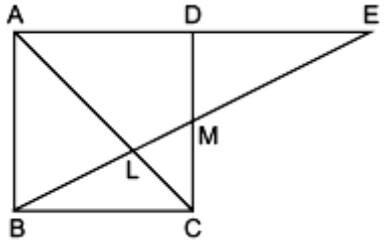
Questions 15 to 17 carry 3 marks each.

15. In figure, ΔABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE.

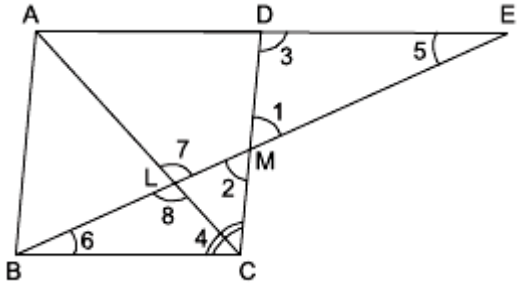


Ans: In ΔABC and ΔADE
$\angle C = \angle E = 90°$ [each]
$\angle A = \angle A$ (Common angle)
ΔABC ~ ΔADE (By AA similarity)
In ΔABC, $AB^2 = AC^2 + BC^2$ (By pythagoras theorem)
$AB^2 = 25 + 144 = 169 \Rightarrow AB = 13$
then, $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$
$\Rightarrow \frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$
then, AE = $\frac{15}{13}$ cm and DE = $\frac{36}{13}$ cm

16. In figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2BL.



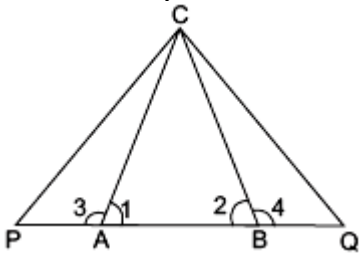
Ans: In ΔMDE and ΔMCB



DM = CM (Given)
$\angle 1 = \angle 2$ (Vertically opposite)
$\angle 3 = \angle 4$ (∵ BC || AE and DC is a transversal) (Alt. int $\angle$s)
∴ ΔMDE ≅ ΔMCB (ASA Congruency)
DE = BC (CPCT) ... (i)
Also BC = AD ... (ii)
(Opposite sides of the parallelogram)
∴ AD = DE [On equating (i) and (ii)]
Now, AE = AD + DE ⇒ AE = 2 AD (Put DE = AD)
In ΔBLC and ΔELA,
$\angle 5 = \angle 6$ (Alt. int. angles)
and $\angle 7 = \angle 8$ (Vertically opposite angles)
∴ ΔBLC ~ ΔELA (AA similarity)
$\Rightarrow \frac{BL}{EL} = \frac{LC}{LA} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{2AD}$
$\Rightarrow \frac{BL}{EL} = \frac{AD}{2AD}$ (∵ BC = AD)
$\Rightarrow \frac{BL}{EL} = \frac{1}{2} \Rightarrow$ EL = 2 BL

17. In an isosceles ΔABC, the base AB is produced both ways to P and Q such that AP × BQ = $(AC)^2$. Prove that ΔACP ~ ΔBCQ.

Ans: Given that in ΔABC, AC = BC and AB is produced to P and Q such that AP × BQ = $(AC)^2$



Now, ∠1 + ∠3 = ∠2 + ∠4 (Each linear pair)
AC = BC (Given)
As ∠1 = ∠2 (Opposite angles of isosceles Δ)
∴ ∠3 = ∠4
Also, AP × BQ = $(AC)^2$
⇒ AP × BQ = AC × BC [∵ AC = BC]

$$\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$$

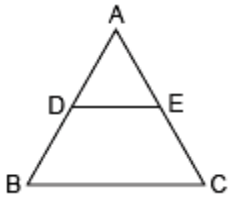
∠3 = ∠4 (Proved above)
∴ ΔACP ~ ΔBCQ (SAS similarity)

SECTION – D

Questions 18 carry 5 marks.

18. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following :

In figure, D and E are points on AB and AC respectively, such that DE || BC. If AD = $\frac{1}{3}$ BD, AE = 4.5 cm, find EC.



Ans: Given - ½ mark
To prove - ½ mark
Figure - ½ mark
Construction - ½ mark
Proof – 2 marks
Second part - 1 mark

Here AD = $\frac{1}{3}$ BD,
AE = 4.5 cm, DE || BC

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$$ (using Basic Proportionality Theorem)

$$\Rightarrow \frac{1}{3} = \frac{4.5}{EC} \Rightarrow EC = 13.5cm$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. While browsing through the catalogue of wooden shelves, Karthik came across this beautiful triangular shaped shelf. In the shelf, DE is parallel to the base BC could be used for displaying small plants and showpieces.

(a) Find the relation between the sides AD, DB, AE and EC. Also, mention the theorem used. (1)
(b) With measurement AE = 1.8 cm, BD = 7.2 cm and CE = 5.4 cm. Karthik thought of finding the length of side AD from the given figure of shelf. How he will find the length. (1)
(c) Find the value of x if AD = (x + 3) cm, BD = (3x + 19) cm, AE = x cm and EC = (3x + 4) cm. (2)

OR

(c) If AB = 9 cm, AC = 18 cm, AD = 2 cm and AE = 4 cm, then prove that DE || BC. (2)

Ans: (a) Since DE is parallel to BC, so by Basic Proportionality theorem AD/BD = AE/CE

(b) As DE || BC, then by Thales theorem, we have AD/DB = AE/EC

$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} = \frac{1}{3} \Rightarrow AD = \frac{7.2}{3} = 2.4cm$$

(c) Using basic proportionality theorem, we have AD/DB = AE/EC

$$\Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$\Rightarrow (x + 3)(3x + 4) = x(3x + 19)$

$\Rightarrow 3x^2 + 13x + 12 = 3x^2 + 19x$

$\Rightarrow 13x + 12 = 19x \Rightarrow -6x = -12 \Rightarrow x = 2$

OR

(c) Given, AB = 9 cm, AC =18 cm, AD = 2 cm and AE = 4 cm

Now, DB = AB – AD = 9 – 2 = 7 cm

EC = AC – AE = 18 – 4 = 14 cm

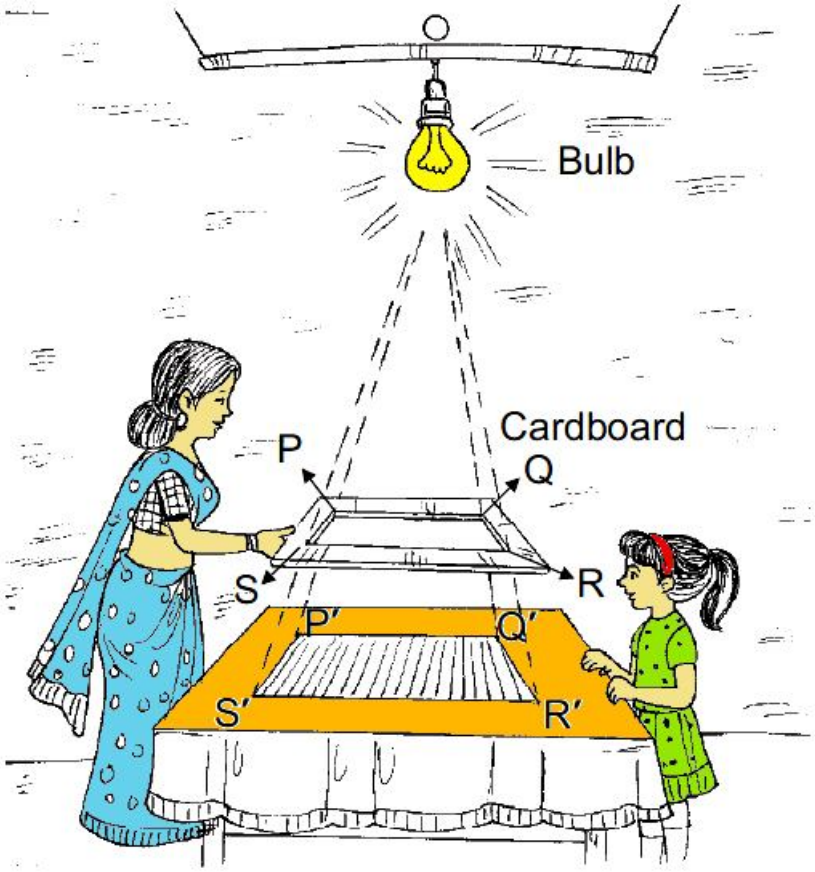
Now, AD/DB = 2/7

And, AE/EC = 4/14 = 2/7

∴ AD/DB = AE/EC

Therefore, DE || BC [by converse of basic proportionality theorem]

20. Anjali placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P′Q′R′S′. Quadrilateral P′Q′R′S′ is an enlargement of the quadrilateral PQRS with scale factor 1 : 3. Given that PQ = 2.5 cm, QR = 3.5 cm. RS = 3.4 cm and *PS* = 3.1 cm; ∠P = 115°, ∠Q = 95°, ∠R = 65° and ∠S = 85°.



(a) Find the length of R′S′. (1)
(b) Find the measurement of ∠Q′ (1)
(c) Find the ratio of sides P′Q′ and Q′R′. (2)

OR

(c) Find the sum of the lengths Q′R′ and P′S′. (2)

Ans: (a) Given, scale factor is 1 : 3.
R ′S ′ = 3RS
∴ R ′S ′ = 3 × 3.4 = 10.2 cm
(b) Quadrilateral P ′Q ′R ′S ′ is similar to PQRS
∴ ∠Q′ = ∠Q = 95°
(c) Since, P′Q′ = 3 PQ = 3 × 2.5 = 7.5 cm
and Q′R′ = 3 QR = 3 × 3.5 = 10.5 cm
∴ P′Q′/Q′R′ = 7.5/10.5 = 5 : 7

OR

(c) Q′R ′ = 3 QR = 3 × 3.5 = 10.5 cm
and P′S′ = 3 PS = 3 × 3.1 = 9.3 cm
∴ Q′R ′ + P′S ′ = 10.5 + 9.3 = 19.8 cm

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 06 (2024-25)

CHAPTER 07 COORDINATE GEOMETRY

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Three vertices of a parallelogram ABCD are A(1, 4), B(–2, 3) and C(5, 8). The ordinate of the fourth vertex D is

(a) 8 (b) 9 (c) 7 (d) 6

2. Points A(–1, y) and B(5, 7) lie on a circle with centre O(2, –3y). The values of y are

(a) 1, –7 (b) –1, 7 (c) 2, 7 (d) –2, –7

3. If A(4, –2), B(7, –2) and C(7, 9) are the vertices of a ΔABC, then ΔABC is

(a) equilateral triangle (b) isosceles triangle

(c) right angled triangle (d) isosceles right angled triangle

4. If (a, b) is the mid point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, the values of k is

(a) 30 (b) 22 (c) 4 (d) 40

5. The coordinate of point P on X-axis equidistant from the points A (–1, 0) and B (5, 0) is

(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

6. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

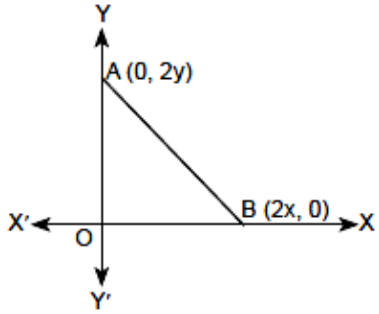
(a) $\left(-\frac{3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, -\frac{1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

7. If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertices of a parallelogram PQRS, then

(a) a = 2, b = 4 (b) a = 3, b = 4(c) a = 2, b = 3 (d) a = 3, b = 5

8. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is

(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$



In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Mid-point of a line segment divides line in the ratio 1 : 1.
Reason (R): The ratio in which the point (–3, k) divides the line segment joining the points (–5, 4) and (–2, 3) is 1 : 2.

10. **Assertion (A):** The origin is the only point equidistant from (2, 3) and (-2, -3).
Reason (R): The origin is the mid-point of the line joining (2, 3) and (-2, -3).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The line segment AB joining the points A(3, –4) and B(1, 2) is trisected at the points P(p, –2) and Q(5/3, q). Find the values of p and q.

12. Find the point on x-axis which is equidistant from the points (2, – 5) and (–2, 9).

13. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, –1), (1, 3) and (x, 8) respectively.

14. Find the coordinates of the point of trisection of the line segment joining (1, –2) and (–3, 4).
Ans: Let the points P and Q trisect AB.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Show that the points A(3, 5), B(6, 0), C(1, – 3) and D(–2, 2) are the vertices of a square ABCD.

16. In what ratio does the line $x - y - 2 = 0$ divide the line segment joining (3, –1) and (8, 9)?

17. Show that points A(7, 5), B(2, 3) and C(6, – 7) are the vertices of a right triangle. Also find its area.

OR

Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y.

SECTION – D

Questions 18 carry 5 marks.

18. Find the centre of a circle passing through (5, – 8), (2, – 9) and (2, 1).

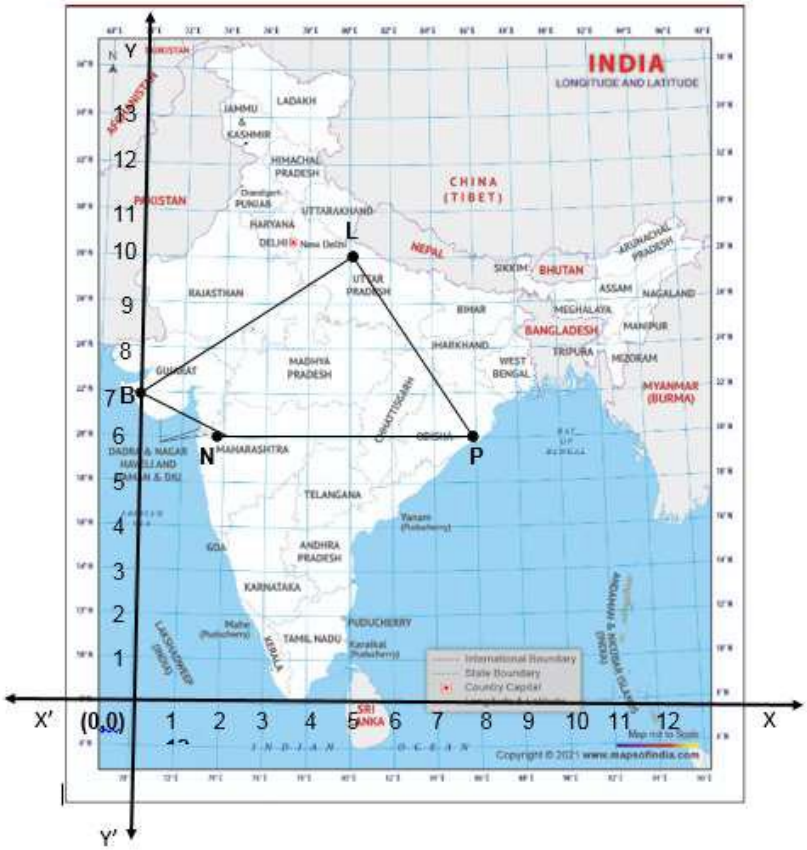
OR

If the points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.



Based on the above information answer the following questions using the coordinate geometry.

(i) Find the distance between Lucknow (L) to Bhuj(B).

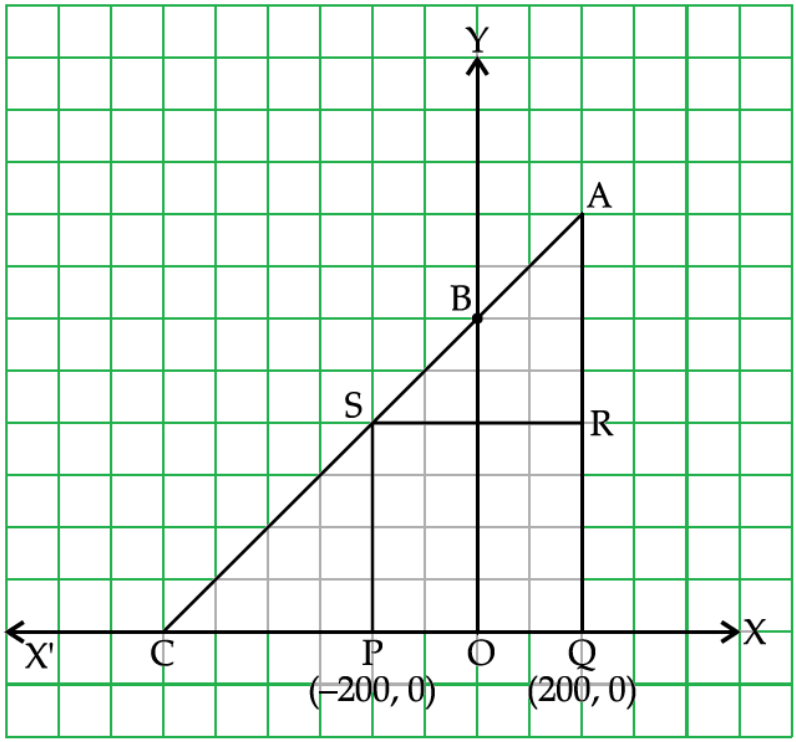
(ii) If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

(iii) Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

OR

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).

20. Jagdhish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field from growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

(ii) (a) What is the area of square PQRS ?

OR

(b) What is the length of diagonal PR in square PQRS?

(iii) If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800) ?

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 06 (2024-25)

CHAPTER 07 COORDINATE GEOMETRY

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Three vertices of a parallelogram ABCD are A(1, 4), B(–2, 3) and C(5, 8). The ordinate of the fourth vertex D is

(a) 8 (b) 9 (c) 7 (d) 6

Ans: (b) 9

2. Points A(–1, y) and B(5, 7) lie on a circle with centre O(2, –3y). The values of y are

(a) 1, –7 (b) –1, 7 (c) 2, 7 (d) –2, –7

Ans: (b) –1, 7

3. If A(4, –2), B(7, –2) and C(7, 9) are the vertices of a ΔABC, then ΔABC is

(a) equilateral triangle (b) isosceles triangle

(c) right angled triangle (d) isosceles right angled triangle

Ans: (c) right angled triangle

4. If (a, b) is the mid point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, the values of k is

(a) 30 (b) 22 (c) 4 (d) 40

Ans: (b) 22

5. The coordinate of point P on X-axis equidistant from the points A (–1, 0) and B (5, 0) is

(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

Ans: (a) (2, 0)

6. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

(a) $\left(-\frac{3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, -\frac{1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

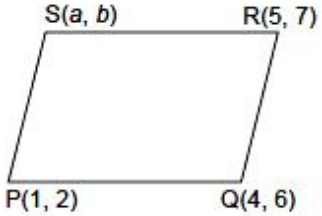
Ans: (d) $\left(-6, \frac{5}{2}\right)$

Distance of $\left(-6, \frac{5}{2}\right)$ from centre of the circle *i.e.*, (0, 0)

$$= \sqrt{(0+6)^2 + \left(0-\frac{5}{2}\right)^2} = \sqrt{36+\frac{25}{4}}$$

$$= \sqrt{\frac{144+25}{4}} = \frac{13}{2} = \text{radius of circle.}$$

7. If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertices of a parallelogram PQRS, then
(a) a = 2, b = 4 (b) a = 3, b = 4(c) a = 2, b = 3 (d) a = 3, b = 5
Ans: (c) a = 2, b = 3



Mid-point of PR $= \left(\frac{1+5}{2}, \frac{2+7}{2}\right) = \left(3, \frac{9}{2}\right)$

Mid-points of SQ $= \left(\frac{4+a}{2}, \frac{6+b}{2}\right)$

Diagonals of parallelogram bisect each other.

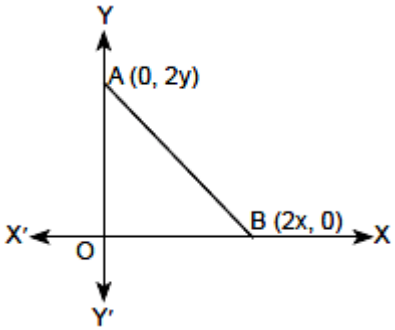
$$\therefore \quad \left(3, \frac{9}{2}\right) = \left(\frac{4+a}{2}, \frac{6+b}{2}\right)$$

$$\Rightarrow \quad 3 = \frac{4+a}{2}, \ \frac{9}{2} = \frac{6+b}{2}$$

$$\Rightarrow \quad a = 2, \quad b = 3.$$

8. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is

(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$



Ans: (a) (x, y)
∵ AOB is a right triangle.
∴ Mid-point of AB is equidistant from A, O and B.

Mid-point of AB $= \left(\frac{0+2x}{2}, \frac{2y+0}{2}\right) = (x, y)$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

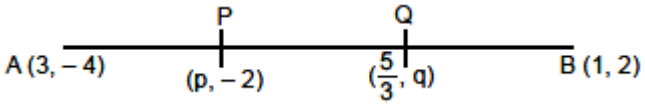
9. **Assertion (A):** Mid-point of a line segment divides line in the ratio 1 : 1.
Reason (R): The ratio in which the point (–3, k) divides the line segment joining the points (–5, 4) and (–2, 3) is 1 : 2.
Ans. (c) Assertion (A) is true but reason (R) is false.

10. **Assertion (A):** The origin is the only point equidistant from (2, 3) and (-2, -3).
Reason (R): The origin is the mid-point of the line joining (2, 3) and (-2, -3).
Ans. (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The line segment AB joining the points A(3, –4) and B(1, 2) is trisected at the points P(p, –2) and Q(5/3, q). Find the values of p and q.
Ans: Now AP : PB = 1 : 2



$\therefore\ p = \dfrac{1\times1+2\times3}{1+2} \Rightarrow p = \dfrac{7}{3}$

Also AQ : QB = 2 : 1 $\Rightarrow q = \dfrac{2\times2+1\times(-4)}{1+2} = 0$

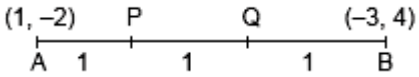
12. Find the point on x-axis which is equidistant from the points (2, – 5) and (–2, 9).
Ans: Let point on x-axis be P(a, 0) and given that A(2, –5) and B(–2, 9) are equidistant.
$\therefore$ PA = PB
$\Rightarrow \sqrt{(a-2)^2+25} = \sqrt{(a+2)^2+81}$
Squaring both sides, we get
$a^2 + 4 - 4a + 25 = a^2 + 4 + 4a + 81$
$\Rightarrow -8a = 56 \Rightarrow a = -7$
Hence the required point is (–7, 0)

13. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, –1), (1, 3) and (x, 8) respectively.
Ans: Here, P (6, – 1), Q (1, 3) and R (x, 8)
Given PQ = QR
$\Rightarrow \sqrt{(6-1)^2+(-1-3)^2} = \sqrt{(1-x)^2+(3-8)^2}$
$\Rightarrow \sqrt{5^2+(-4)^2} = \sqrt{1^2+x^2-2x+(-5)^2}$
$\Rightarrow \sqrt{25+16} = \sqrt{1+x^2-2x+25}$
$\Rightarrow \sqrt{41} = \sqrt{x^2-2x+26}$
Squaring both sides, we get
$41 = x^2 - 2x + 26 \Rightarrow x^2 - 2x + 26 - 41 = 0 \Rightarrow x^2 - 2x - 15 = 0$
$\Rightarrow x^2 - 5x + 3x - 15 = 0 \Rightarrow x(x-5) + 3(x-5) = 0$
$\Rightarrow (x-5)(x+3) = 0$
either $x - 5 = 0$ or $x + 3 = 0$
$x = 5$ or $x = -3$
So $x = 5$ or -3

14. Find the coordinates of the point of trisection of the line segment joining (1, –2) and (–3, 4).

Ans: Let the points P and Q trisect AB.

$\Rightarrow$ AP : PB = 1 : 2 and AQ : QB = 2 : 1



Using section formula coordinates of P are

$x = \frac{1\times(-3)+2\times1}{1+2} = \frac{-3+2}{3} = \frac{-1}{3}$ and $y = \frac{1\times4+2\times(-2)}{1+2} = \frac{4+(-4)}{3} = \frac{0}{3} = 0$

Thus, P is $\left(\frac{-1}{3}, 0\right)$,

Coordinates of Q are $x = \frac{2\times(-3)+1\times1}{1+2} = \frac{-6+2}{3} = \frac{-5}{3}$

$y = \frac{2\times4+1\times(-2)}{1+2} = \frac{8+(-2)}{3} = \frac{6}{3} = 2$

Thus, Q is $\left(\frac{-5}{3}, 2\right)$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Show that the points A(3, 5), B(6, 0), C(1, – 3) and D(–2, 2) are the vertices of a square ABCD.

Ans:

$AB = \sqrt{(6-3)^2+(0-5)^2}$

$= \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(6-1)^2+(0+3)^2}$

$= \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(1+2)^2+(-3-2)^2}$

$= \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(-2-3)^2+(2-5)^2}$

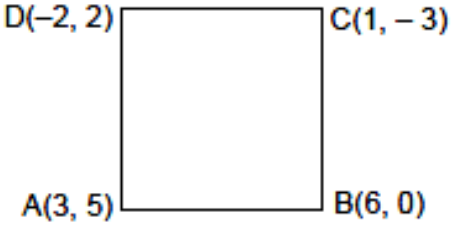
$= \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1-3)^2+(-3-5)^2}$

$= \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(6+2)^2+(0-2)^2}$

$= \sqrt{64+4} = \sqrt{68}$



AB = BC = CD = DA = $\sqrt{34}$

Diagonal AC = diagonal BD = $\sqrt{68}$

Hence A, B, C and D are vertices of a square.

16. In what ratio does the line $x – y – 2 = 0$ divide the line segment joining (3, –1) and (8, 9)?

Ans: Let the line $x – y – 2 = 0$, divides the line segment joining (3, – 1) and (8, 9) in the ratio k : 1 and let the coordinates of the required point be (x_1, y_1).

Then $x_1 = \frac{8k+3}{k+1}$

and $y_1 = \frac{9\times k+1\times(-1)}{k+1} = \frac{9k-1}{k+1}$

This point (x_1, y_1) lies on the line whose equation is $x – y – 2 = 0$.

$\therefore$ It must satisfy the equation of the given line

$\Rightarrow \frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$

$\Rightarrow 8k + 3 – (9k – 1) – 2(k + 1) = 0$

$\Rightarrow 8k + 3 – 9k + 1 – 2k – 2 = 0$

$\Rightarrow -3k + 2 = 0 \Rightarrow k = \frac{2}{3}$

Therefore, the required ratio is $k : 1 = \frac{2}{3} : 1$ or 2 : 3.

17. Show that points A(7, 5), B(2, 3) and C(6, – 7) are the vertices of a right triangle. Also find its area.

Ans:

$$AB = \sqrt{(2-7)^2+(3-5)^2} = \sqrt{25+4} = \sqrt{29}$$

$$BC = \sqrt{(6-2)^2+(-7-3)^2} = \sqrt{16+100} = \sqrt{116}$$

$$CA = \sqrt{(7-6)^2+(5+7)^2} = \sqrt{1+144} = \sqrt{145}$$

Since $AB^2 + BC^2 = 29 + 116 = 145 = CA^2$.

$\therefore$ ΔABC is right angled at B.

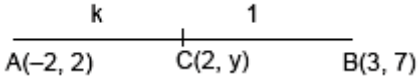
Area =

$$\frac{1}{2}AB \times BC = \frac{1}{2}\sqrt{29}.\sqrt{116} = \frac{1}{2}\sqrt{29}.2.\sqrt{29} = 29.$$

OR

Find the ratio in which the point (2, *y*) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of *y*.

Ans: Let C divides AB in the ratio *k* : 1



$\therefore$ *x* coordinate of C = $\frac{k \times 3 + 1 \times (-2)}{k+1}$

$\Rightarrow 2 = \frac{3k-2}{k+1} \Rightarrow 2k + 2 = 3k - 2 \Rightarrow k = 4$

$\therefore$ C divides AB in the ratio 4 : 1

Now *y* coordinate of C = $\frac{4 \times 7 + 1 \times 2}{4+1}$ [$\because k = 4$]

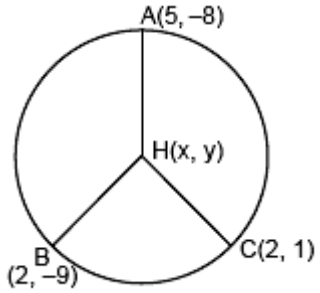
$\Rightarrow y = \frac{28+2}{5} = \frac{30}{5} = 6$

SECTION – D

Questions 18 carry 5 marks.

18. Find the centre of a circle passing through (5, – 8), (2, – 9) and (2, 1).

Ans: Let H(*x*, *y*) is centre of circle passing through A, B and C. Since AH, BH and CH are radius of circle.



$\therefore$ AH = BH and BH = CH

Also $AH^2 = BH^2$ and $BH^2 = CH^2$

$AH^2 = (x-5)^2 + (y+8)^2 = x^2 + 25 - 10x + y^2 + 64 + 16y$

$BH^2 = (x-2)^2 + (y+9)^2 = x^2 + 4 - 4x + y^2 + 81 + 18y$

$CH^2 = (x-2)^2 + (y-1)^2 = x^2 + 4 - 4x + y^2 + 1 - 2y$

$\because$ $AH^2 = BH^2$ [Radii of a circle]

$\therefore$ $x^2 + 25 - 10x + y^2 + 64 + 16y = x^2 + 4 - 4x + y^2 + 81 + 18y$

$\Rightarrow -10x + 4x + 16y - 18y = -4$

$\Rightarrow -6x - 2y = -4 \Rightarrow 3x + y = 2$ ...(i)

Also $BH^2 = CH^2$

$\therefore$ $x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 4 - 4x + y^2 + 1 - 2y$

$\Rightarrow 18y + 2y = 1 - 81$

$\Rightarrow 20y = -80 \Rightarrow y = -4$

Putting value of y in (i), we get

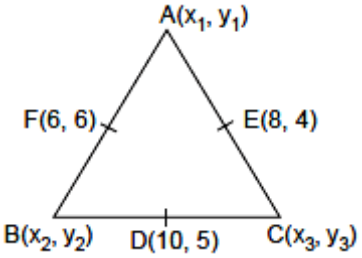
$3x + (-4) = 2 \Rightarrow 3x = 2 + 4 \Rightarrow 3x = 6 \Rightarrow x = 2$

$\therefore$ Coordinates of centre are (2, –4).

OR

If the points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

Ans: Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle D(10, 5), E(8, 4) and F(6,6) are mid-points of sides BC, CA and AB respectively.



Therefore, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (6, 6)$

$\Rightarrow x_1 + x_2 = 12$ ...(i)

and $y_1 + y_2 = 12$ ...(ii)

$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (10, 5)$

$x_2 + x_3 = 20$ ...(iii)

and $y_2 + y_3 = 10$ ...(iv)

and $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) = (8, 4)$

$\Rightarrow x_1 + x_3 = 16$ ...(v)

and $y_1 + y_3 = 8$ ...(vi)

Adding (i), (iii) and (v), we get $2(x_1 + x_2 + x_3) = 48$ ...(vii)

$\Rightarrow x_1 + x_2 + x_3 = 24$

From (i), (iii), (v) and (vii), we get $x_1 = 4$, $x_2 = 8$, $x_3 = 12$ ...(viii)

Adding (ii), (iv) and (vi), we get $2(y_1 + y_2 + y_3) = 30$

$y_1 + y_2 + y_3 = 15$ ...(ix)

From (ii), (iv), (vi) and (ix), we get $y_1 = 5$, $y_2 = 7$, $y_3 = 3$ ...(x)

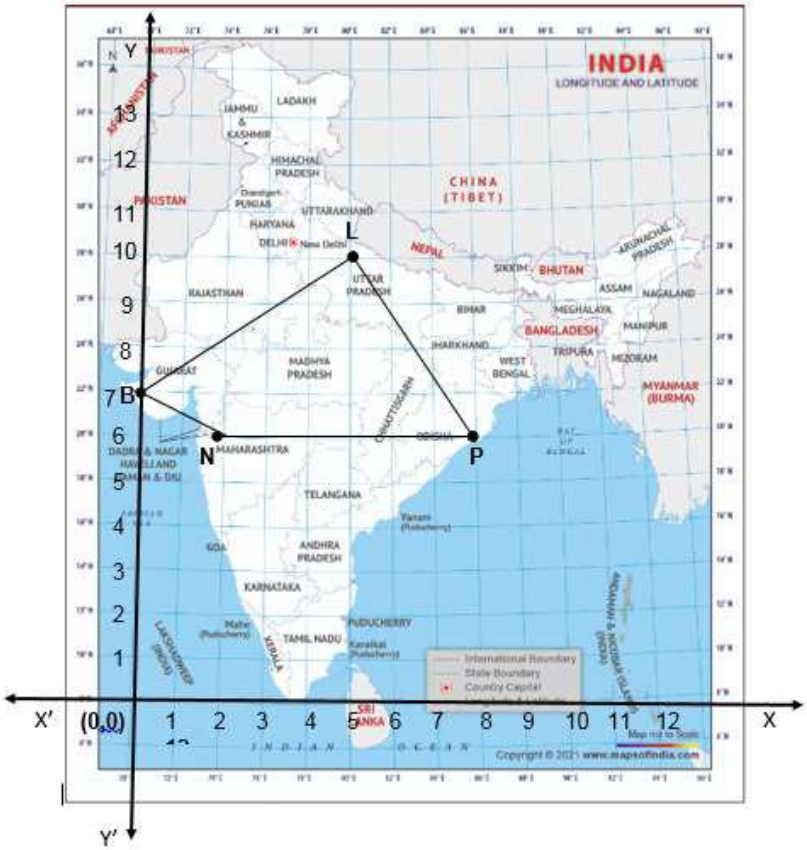
From (viii) and (x), we get

Coordinates of vertices are A (4, 5), B (8, 7) and C (12, 3).

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.



Based on the above information answer the following questions using the coordinate geometry.

(i) Find the distance between Lucknow (L) to Bhuj(B).

(ii) If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

(iii) Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

OR

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).

(i)

$LB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow LB = \sqrt{(0 - 5)^2 + (7 - 10)^2}$

$LB = \sqrt{(5)^2 + (3)^2} \Rightarrow LB = \sqrt{25 + 9}$ $LB = \sqrt{34}$

(ii) Coordinates of Kota (K)= $\left(\frac{3\times0+2\times5}{3+2}, \frac{3\times7+2\times10}{3+2}\right) = \left(\frac{10}{5}, \frac{41}{5}\right) = \left(2, \frac{41}{5}\right)$

(iii)

L(5, 10), N(2,6), P(8,6)

$LN = \sqrt{(2 - 5)^2 + (6 - 10)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$NP = \sqrt{(8 - 2)^2 + (6 - 6)^2} = \sqrt{(4)^2 + (0)^2} = 4$

$PL = \sqrt{(8 - 5)^2 + (6 - 10)^2} = \sqrt{(3)^2 + (4)^2} \Rightarrow LB = \sqrt{9 + 16} = \sqrt{25} = 5$

as LN = PL ≠ NP, so Δ LNP is an isosceles triangle.

OR

Let A (0, b) be a point on the y – axis then AL = AP

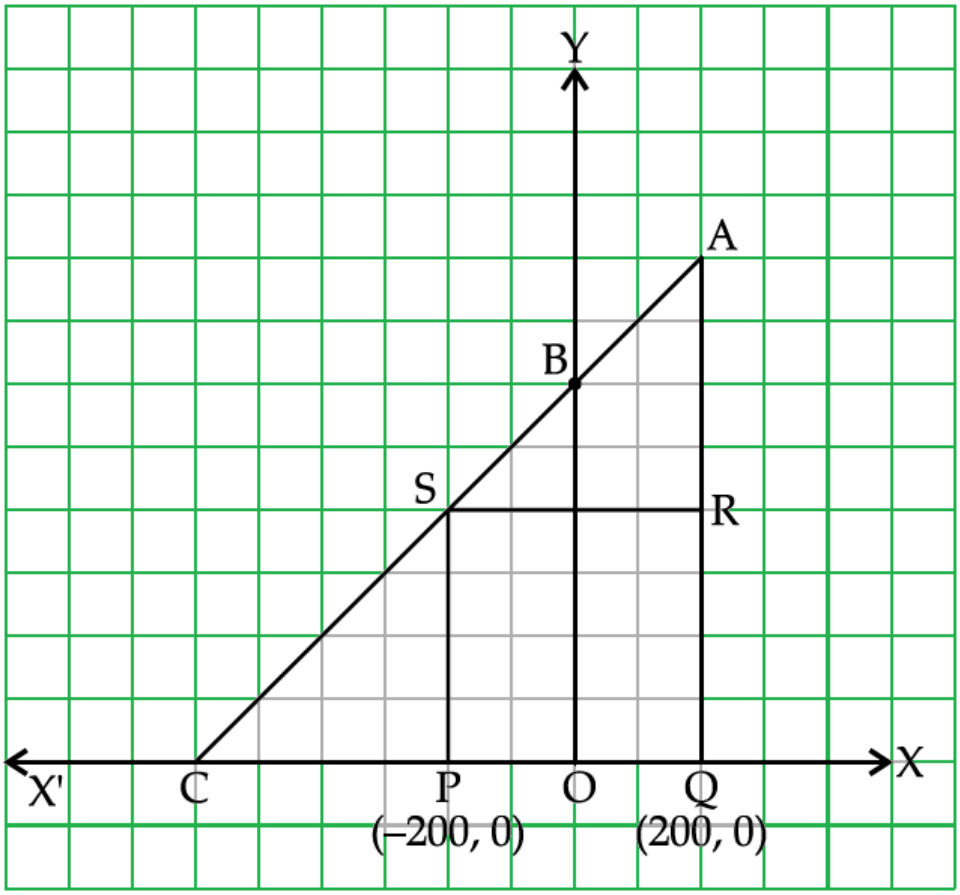
$\Rightarrow \sqrt{(5 - 0)^2 + (10 - b)^2} = \sqrt{(8 - 0)^2 + (6 - b)^2}$

$\Rightarrow (5)^2 + (10 - b)^2 = (8)^2 + (6 - b)^2$

$\Rightarrow 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$

So, the coordinate on y axis is $\left(0, \frac{25}{8}\right)$

20. Jagdhish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field from growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

(ii) (a) What is the area of square PQRS ?

OR

(b) What is the length of diagonal PR in square PQRS?

(iii) If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800) ?

Ans: (i) Coordinates of R = (200, 400)

Coordinates of S = (– 200, 400)

(ii) Since, side of square PQRS = 400

Thus, area of square PQRS = $(\text{side})^2$

= $(400)^2$ = 160000 unit^2

OR

We know that, diagonal of square = 2 × side

∴ Diagonal PR of square PQRS = 2 × 400

= 400 √2 units

(iii) Let the ratio be k : 1.

Using section formula, $-200 = \dfrac{200k + 1 \times (-600)}{k + 1}$

⇒ – 200 k – 200 = 200 k – 600

⇒ – 400 k = – 400

⇒ k = 1

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 09 (2024-25)

CHAPTER 08 INTRODUCTION TO TRIGONOMETRY

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ is equal to:

(a) –1 (b) 1 (c) 0 (d) 2

2. In Δ ABC right angled at B, $\sin A = \frac{7}{25}$, then the value of cos C is ……

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{7}{24}$ (d) $\frac{24}{7}$

3. If $5\tan\theta = 4$, then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is

(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

4. If $\operatorname{cosec} A = 13/12$, then the value of $\frac{2\sin A - 3\cos A}{4\sin A - 9\cos A}$

(a) 4 (b) 5 (c) 6 (d) 3

5. Given that $\sin\alpha = 1/2$ and $\cos\beta = 1/2$, then the value of $(\beta - \alpha)$ is

(a) 0° (b) 30° (c) 60° (d) 90°

6. If $\tan\theta = 1$, then the value of $\sec\theta + \operatorname{cosec}\theta$ is:

(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

7. If $\sin 2A = \frac{1}{2}\tan^2 45°$ where A is an acute angle, then the value of A is

(a) 60° (b) 45° (c) 30° (d) 15°

8. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

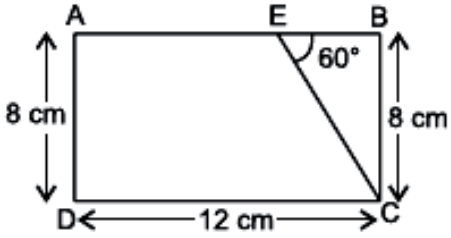
9. **Assertion (A):** In a right ΔABC, right angled at B, if tan A = 1, then 2 sin A. cos A = 1.
Reason (R): tan 45° = 1 and sin 45° = cos 45° = $1/\sqrt{2}$

10. **Assertion (A):** sin (A + B) = sin A + sin B
Reason (R): For any value of θ, $1 + \tan^2\theta = \sec^2\theta$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of 60° with AB, intersecting AB at E. Find the length of CE and BE.



12. If sin (A + B) = $\sqrt{3}/2$ and sin (A – B) = $\frac{1}{2}$, $0 \le A + B \le 90°$ and A > B, then find A and B.

13. Evaluate: $3\cos^2 60° \sec^2 30° - 2\sin^2 30° \tan^2 60°$.

14. Simplify: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

OR

If $7\sin^2 A + 3\cos^2 A = 4$, then find tan A

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If cosecθ + cotθ = p, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$

16. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$

OR

If sin θ + cos θ = $\sqrt{3}$, then prove that tan θ + cot θ = 1.

17. Prove that: $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^2\theta}{1-\cot\theta} = 1 + \sin\theta\cos\theta$

OR

If cos θ + sin θ = $\sqrt{2}$ cos θ, show that cos θ – sin θ = $\sqrt{2}$ sin θ.

SECTION – D

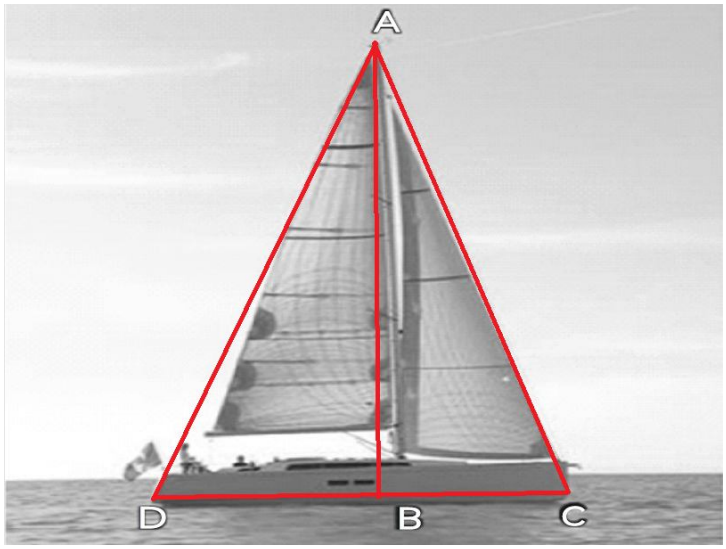
Questions 18 carry 5 marks.

18. (a) Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ [3]
(b) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\sin\theta$ then find $x^2 + y^2$. [2]

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance BC = 1 m and BD = 2 m and height AB = 4 m.



Based on the given in formation, answer the following questions:

(a) Find the value of sec D. [1]

(b) Find the value of cosec C. [1]

(c) Find the value of tan D + cot C. [1]

(d) Find the value of $\sin^2 C + \cos^2 D$ [1]

20. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river "Ganges" or "Ganga". A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.

Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.732$)

(a) Find the value of sin A. [1]

(b) Find the value of sin C. [1]

(c) Find the value of tan A – cot C. [1]

(d) Find the value of $\text{cosec}^2 C$. [1]

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 09 (2024-25)

CHAPTER 08 INTRODUCTION TO TRIGONOMETRY (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$ is equal to:

(a) –1 (b) 1 (c) 0 (d) 2

Ans. (b) 1

$(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$

$= \tan^2\theta \times \cot^2\theta$

$= \tan^2\theta \times 1/\tan^2\theta = 1$

2. In Δ ABC right angled at B, $\sin A = \frac{7}{25}$, then the value of cos C is …….

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{7}{24}$ (d) $\frac{24}{7}$

Ans: (a) $\frac{7}{25}$

3. If $5\tan\theta = 4$, then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is

(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

Ans: (a) 1/6

4. If cosec A = 13/12, then the value of $\frac{2\sin A - 3\cos A}{4\sin A - 9\cos A}$

(a) 4 (b) 5 (c) 6 (d) 3

Ans: (d) 3

Given cosec A = 13/12,

$\sin A = \frac{12}{13}, \cos A = \frac{5}{13}$

$$Now, \frac{2\sin A - 3\cos A}{4\sin A - 9\cos A} = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)} = \frac{24-15}{48-45} = \frac{9}{3} = 3$$

5. Given that $\sin\alpha = 1/2$ and $\cos\beta = 1/2$, then the value of $(\beta - \alpha)$ is

(a) 0° (b) 30° (c) 60° (d) 90°

Ans: (b) 30°

6. If $\tan\theta = 1$, then the value of $\sec\theta + \operatorname{cosec}\theta$ is:
(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$
Ans: (c) $2\sqrt{2}$
Given, $\tan\theta = 1$, we have $\theta = 45°$
So, $\sec\theta + \operatorname{cosec}\theta = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

7. If $\sin 2A = \frac{1}{2}\tan^2 45°$ where A is an acute angle, then the value of A is
(a) 60° (b) 45° (c) 30° (d) 15°
Ans: (d) 15°

$$\sin 2A = \frac{1}{2}\tan^2 45^0 = \frac{1}{2}\times 1^2 = \frac{1}{2} = \sin 30^0 \Rightarrow 2A = 30^0 \Rightarrow A = 15^0$$

8. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$
Ans: (c) $\frac{\sqrt{2}}{2}$

$$\tan\theta + \cot\theta = 2 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2 \Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$$

$$\Rightarrow (\tan\theta - 1)^2 = 0 \Rightarrow \tan\theta = 1 = \tan 45^0 \Rightarrow \theta = 45^0$$

$$Now, \sin^3\theta + \cos^3\theta = \sin^3 45^0 + \cos^3 45^0 = \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

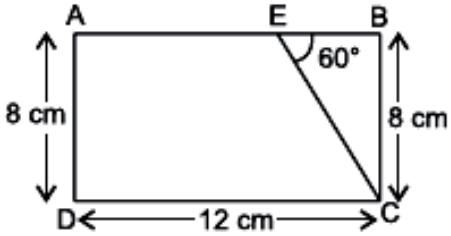
9. Assertion (A): In a right ΔABC, right angled at B, if $\tan A = 1$, then $2\sin A.\cos A = 1$.
Reason (R): $\tan 45° = 1$ and $\sin 45° = \cos 45° = 1/\sqrt{2}$
Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. Assertion (A): $\sin(A + B) = \sin A + \sin B$
Reason (R): For any value of θ, $1 + \tan^2\theta = \sec^2\theta$
Ans. (d) Assertion (A) is false but reason (R) is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of 60° with AB, intersecting AB at E. Find the length of CE and BE.



Ans: In ΔCBE, we have $\tan 60^0 = \frac{CB}{BE}$

$\Rightarrow \sqrt{3} = \frac{8}{BE} \Rightarrow BE = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} cm$

and $\sin 60^0 = \frac{CB}{CE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{CE} \Rightarrow CE = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3} cm$

12. If sin (A + B) = √3/2 and sin (A – B) = $\frac{1}{2}$, $0 \le A + B \le 90°$ and A > B, then find A and B.

Ans: sin(A + B) = √3/2 = sin 60º
⇒ A + B = 60º .....(i)
sin (A - B) = 1/2 = sin 30º
⇒ A - B = 30º .....(ii)
Solving eq. (i) and (ii), A = 45º and B = 15º

13. Evaluate: $3 \cos^2 60° \sec^2 30° – 2 \sin^2 30° \tan^2 60°$.

Ans: $3 \cos^2 60° \sec^2 30° – 2 \sin^2 30° \tan^2 60°$

$= 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2 - 2\left(\frac{1}{2}\right)^2\left(\sqrt{3}\right)^2 = \frac{3}{4}\times\frac{4}{3} - 2\times\frac{1}{4}\times 3 = 1 - \frac{3}{2} = -\frac{1}{2}$

14. Simplify: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

Ans: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\cos ec^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}\times\frac{\cos^2\theta}{1} + \frac{\cos^2\theta}{\sin^2\theta}\times\frac{\sin^2\theta}{1} = \sin^2\theta + \cos^2\theta = 1$

OR

If $7 \sin^2A + 3 \cos^2A = 4$, then find tan A
Ans: Given, $7\sin^2A + 3\cos^2A = 4$
Dividing both sides by $\cos^2A$, we get
$7 \tan2A + 3 = 4 \sec^2A$ [∵ $\sec^2\theta = 1 + \tan^2\theta$]
⇒ $7 \tan^2A + 3 = 4(1 + \tan^2A)$
⇒ $7 \tan^2A + 3 = 4 + 4 \tan^2A$
⇒ $3\tan^2A = 1 \Rightarrow \tan^2A = 1/3 \Rightarrow \tan A = 1/\sqrt{3}$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If cosecθ + cotθ = p, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$

Ans: Given cosecθ + cotθ = p ….. (1)
$\Rightarrow (cosec\theta - \cot\theta)(cosec\theta + \cot\theta) = 1 \Rightarrow (cosec\theta - \cot\theta)\,p = 1$

$\Rightarrow \cos ec\theta - \cot\theta = \frac{1}{p}$ ……. (2)

Adding (1) and (2), we get

$cosec\theta = \frac{p + \frac{1}{p}}{2} = \frac{p^2+1}{2p}; \cot\theta = \frac{p - \frac{1}{p}}{2} = \frac{p^2-1}{2p}$

Now, $\cos\theta = \dfrac{\cot\theta}{\operatorname{cosec}\theta} = \dfrac{\frac{p^2-1}{2p}}{\frac{p^2+1}{2p}} = \dfrac{p^2-1}{p^2+1}$

16. Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$

Ans: LHS = $\dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ (Dividing numerator and denominator by cos□)

$= \dfrac{\tan\theta + \sec\theta - 1}{\tan\theta + 1 - \sec\theta}$

$= \dfrac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta + 1 - \sec\theta}$

$= \dfrac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta + 1 - \sec\theta} = \sec\theta + \tan\theta$ = RHS

OR

If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans: $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow 2\sin\theta\cos\theta = 2$

$\Rightarrow \sin\theta\cos\theta = 1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow 1 = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \tan\theta + \cot\theta \Rightarrow \tan\theta + \cot\theta = 1$

17. Prove that: $\dfrac{\cos^2\theta}{1 - \tan\theta} + \dfrac{\sin^2\theta}{1 - \cot\theta} = 1 + \sin\theta\cos\theta$

Ans: $LHS = \dfrac{\cos^2\theta}{1 - \tan\theta} + \dfrac{\sin^2\theta}{1 - \cot\theta}$

$= \dfrac{\cos^3\theta}{\cos\theta - \sin\theta} - \dfrac{\sin^3\theta}{\cos\theta - \sin\theta}$

$= \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = \dfrac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)}{\cos\theta - \sin\theta}$

$= \cos^2\theta + \sin^2\theta + \cos\theta\sin\theta = 1 + \sin\theta\cos\theta = RHS$

OR

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Ans: Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta)$

$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ [3]

(b) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\sin\theta$ then find $x^2 + y^2$. [2]

Ans: (a) L.H.S = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \times 1/\sin A + 2\cos A \times 1/\cos A$

Since, $(\sin^2 A + \cos^2 A = 1)$

$(\sec^2 A = 1 + \tan^2 A,\ \operatorname{cosec}^2 A = 1 + \cot^2 A)$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A = RHS$

(b)

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

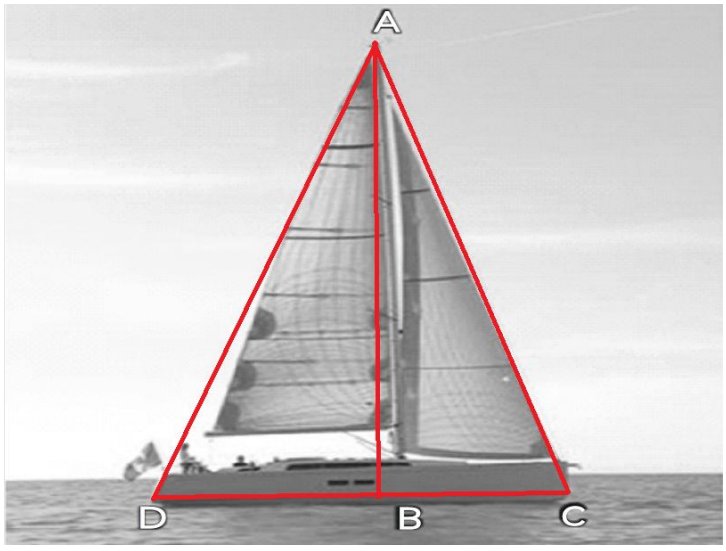
Now, $x\sin\theta = y\cos\theta \Rightarrow \cos\theta\sin\theta = y\cos\theta \Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance BC = 1 m and BD = 2 m and height AB = 4 m.



Based on the given in formation, answer the following questions:

(a) Find the value of sec D. [1]

(b) Find the value of cosec C. [1]

(c) Find the value of tan D + cot C. [1]

(d) Find the value of $\sin^2 C + \cos^2 D$ [1]

Ans. (a) In ΔABD, sec D = AD/BD

by using Pythagoras theorem in right triangle ABD.

$AD^2 = BD^2 + AB^2 = 2^2 + 4^2 = 20$

$\Rightarrow AD = \sqrt{20} = 2\sqrt{5}$m

$\therefore$ sec D = AD/BD = $2\sqrt{5}/2 = \sqrt{5}$

(b) In ΔABC, cosec C = AC/AB

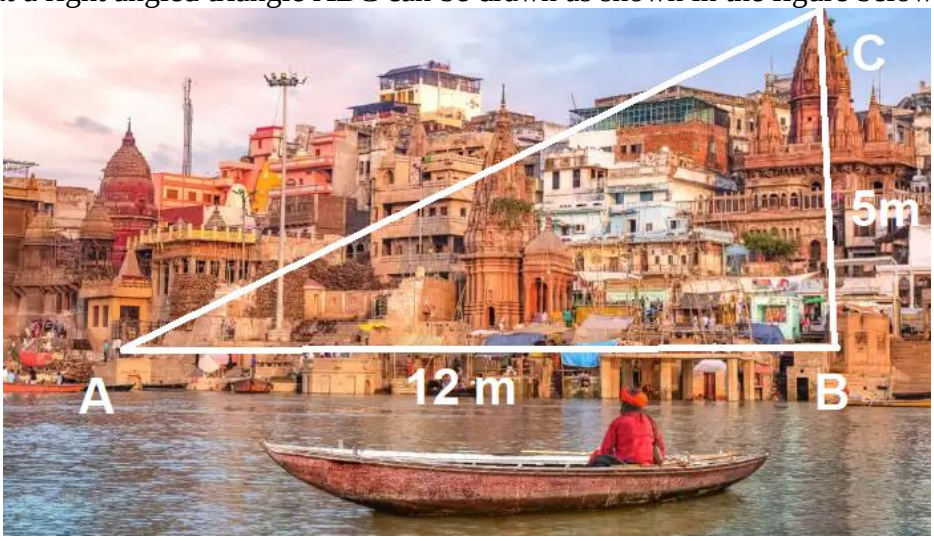
by using Pythagoras theorem in right triangle ABC.

$AC^2 = AB^2 + BC^2 = 4^2 + 1^2 = 17$

$\Rightarrow AC = \sqrt{17}$ m

∴ cosec C = AC/AB = √17/ 4
(c) In ΔABD, tanD = AB/BD = 4/2 = 2
In ΔABC, cotC = BC/AB = 1/4
∴ tanD + cotC = 2 + 1/4 = 9/4
(d) In ΔABC, sinC = AB/AC = 4/√17
In ΔABD, cosD = BD/AD = 1/√5
∴ $\sin^2C + \cos^2D = 16/17 + 1/5 = 97/85$

20. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river "Ganges" or "Ganga". A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.



Based on the above information, answer the following questions. (Take √3 =1.732)
(a) Find the value of sin A. [1]
(b) Find the value of sin C. [1]
(c) Find the value of tan A – cot C. [1]
(d) Find the value of cosec^2C. [1]
Ans. (a) In ΔABC, sinA = BC/AC
by using Pythagoras theorem in right triangle ABC.
$AC^2 = AB^2 + BC^2 = 12^2 + 5^2 = 144 + 25 = 169$
⇒ AD = 13 m
∴ sinA = BC/AC = 5/13
(b) In ΔABC, sinC = AB/AC
⇒ sinC = AB/AC = 12/13
(c) In ΔABC, tanA = BC/AB = 5/12
⇒ cotC = BC/AB = 5/12
Therefore, tan A – cot C = 0
(d) In ΔABC, sinC = AB/AC = 12/13
cosecC = 1/sinC = 13/12
Therefore, $\text{cosec}^2C = 169/144$

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 10 (2024-25)

CHAPTER 01 to 08

SUBJECT: MATHEMATICS **MAX. MARKS : 50**

CLASS : X **DURATION : 2 hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**3** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 5 questions of **2 marks** each. **Section C** comprises of 4 questions of **3 marks** each. **Section D** comprises of 5 questions of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

2. If $p - 1, p + 3, 3p - 1$ are in AP, then p is equal to _____.

(a) 3 (b) 4 (c) 2 (d) none of these

3. If the distance between the points (4, p) and (1, 0) is 5 units, then the value of p is

(a) 4 only (b) ± 4 (c) –4 only (d) 0

4. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is –3, then the value of k is

(a) 4/3 (b) -4/3 (c) 2/3 (d) -2/3

5. If the zeroes of the quadratic polynomial $x^2 + (a + 1)\,x + b$ are 2 and –3, then

(a) a = –7, b = –1 (b) a = 5, b = –1 (c) a = 2, b = –6 (d) a = 0, b = –6

6. If $3 \cot \theta = 2$, then the value of $\tan \theta$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{\sqrt{13}}$ (d) $\frac{2}{\sqrt{13}}$

7. If ΔABC is right angled at C, then the value of sin (A + B) is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

8. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then the HCF (a, b) is:

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC then the value of x is 11, when AD = 4cm, DB = (x – 4) cm, AE = 8cm and EC = (3x – 19) cm.
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.

10. **Assertion (A):** The value of k for which the system of linear equations $3x - 4y = 7$ and $6x - 8y = k$ have infinite number of solution is 14.
Reason (R): The system of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

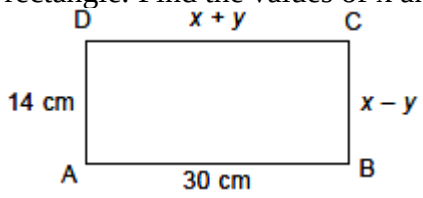
SECTION – B

Questions 11 to 15 carry 2 marks each.

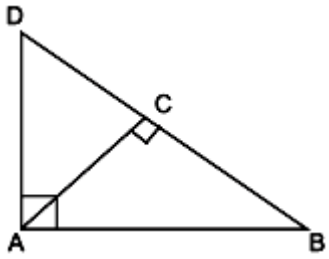
11. Can we have any $n \in N$, where 12^n ends with the digit zero? Explain

12. Find the value of α such that the quadratic equation $(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$, has equal roots.

13. In the below Figure, ABCD is a rectangle. Find the values of x and y.



14. In figure, ΔABD is a right triangle, right angled at A and AC ⊥ BD. Prove that $AB^2 = BC.BD$.



15. Find the value of x if $\tan 3x = \sin 45° . \cos 45° + \sin 30°$.

SECTION – C

Questions 16 to 19 carry 3 marks each.

16. Prove that √5 is an irrational number.

17. The sum of the digits of a two digit number is 9. The number obtained by reversing the order of digits of the given number exceeds the given number by 27. Find the given number.

18. Find the zeroes of $p(x) = 4x^2 + 24x + 36$ quadratic polynomials and verify the relationship between the zeroes and their coefficients.

19. Prove that: $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{2\sec^2\theta}{\tan^2\theta - 1}$

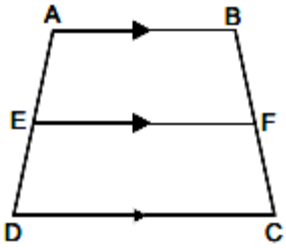
SECTION – D

Questions 20 to 21 carry 5 marks.

20. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

21. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following :

In the given figure, if ABCD is a trapezium in which AB || DC || EF, then $\frac{AE}{ED} = \frac{BF}{FC}$



SECTION – E (Case Study Based Questions)

Questions 22 to 23 carry 4 marks each.

22. In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

Based on the above information answer the following questions.

(i) Find the production in the 1st year. (1)

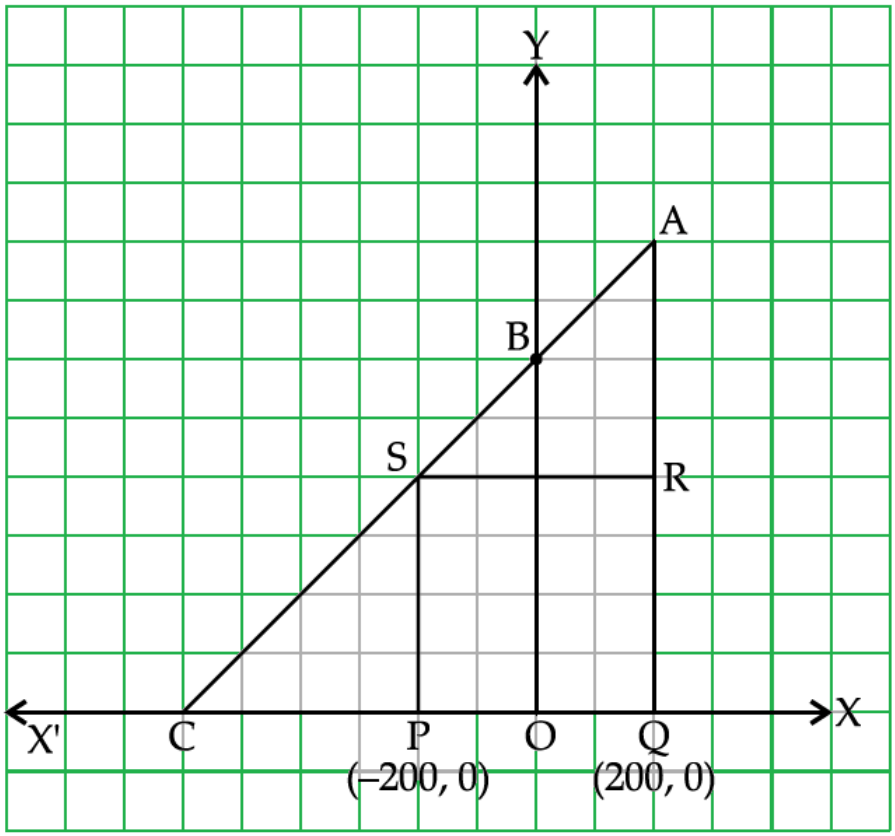
(ii) Find the production in the 12th year. (1)

(iii) Find the total production in first 10 years. (2)

OR

(iii) In how many years will the total production reach 31200 cars? (2)

23. Jagdhish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field from growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

(ii) (a) What is the area of square PQRS ?

OR

(b) What is the length of diagonal PR in square PQRS?

(iii) If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800) ?

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 10 (2024-25)

CHAPTER 01 to 08 (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 50**

CLASS : X **DURATION : 2 hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**3** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 5 questions of **2 marks** each. **Section C** comprises of 4 questions of **3 marks** each. **Section D** comprises of 5 questions of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

Ans: (b) $x^2 + x - 5 = 0$ as $D > 0$

2. If $p - 1$, $p + 3$, $3p - 1$ are in AP, then p is equal to _____.

(a) 3 (b) 4 (c) 2 (d) none of these

Ans: ∵ $p - 1$, $p + 3$ and $3p - 1$ are in AP.

∴ $2(p + 3) = p - 1 + 3p - 1$

⇒ $2p + 6 = 4p - 2$.

⇒ $-2p = -8 \Rightarrow p = 4$.

3. If the distance between the points (4, p) and (1, 0) is 5 units, then the value of p is

(a) 4 only (b) ± 4 (c) –4 only (d) 0

Ans: (b) ± 4

$\sqrt{(4-1)^2 + (p-0)^2} = 5$

⇒ $3^2 + p^2 = 5^2 \Rightarrow p^2 = 25 - 9 = 16 \Rightarrow p = \pm 4$

4. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is –3, then the value of k is

(a) 4/3 (b) -4/3 (c) 2/3 (d) -2/3

Ans: (a) $(k - 1)x^2 + kx + 1$

One zero is – 3, so it must satisfy the equation and make it zero.

∴ $(k - 1)(-3)^2 + k(-3) + 1 = 0$

⇒ $9k - 9 - 3k + 1 = 0$

⇒ $6k - 8 = 0 \Rightarrow k = \frac{8}{6} = \frac{4}{3}$

5. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then

(a) a = –7, b = –1 (b) a = 5, b = –1 (c) a = 2, b = –6 (d) a = 0, b = –6

Ans: (d) $x^2 + (a + 1)x + b$

∵ x = 2 is a zero and x = – 3 is another zero

∴ $(2)^2 + (a + 1)^2 + b = 0$

and $(-3)^2 + (a + 1)(-3) + b = 0$

⇒ $4 + 2a + 2 + b = 0$ and $9 - 3a - 3 + b = 0$

$\Rightarrow 2a + b = -6$ ...(i) and $-3a + b = -6$ ...(ii)
Solving (i) and (ii), we get $5a = 0$
$\Rightarrow a = 0$ and $b = -6$.

6. If $3 \cot \theta = 2$, then the value of $\tan \theta$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{\sqrt{13}}$ (d) $\frac{2}{\sqrt{13}}$

Ans: (b) $\frac{3}{2}$

$3 \cot \theta = 2 \Rightarrow \cot \theta = \frac{2}{3} \Rightarrow \tan \theta = \frac{3}{2}$

7. If ΔABC is right angled at C, then the value of $\sin (A + B)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Ans: (b) 1
ΔABC is right angled at C,
$\therefore A + B + C = 180°$
$A + B = 180° - 90° = 90°$ ($\because \angle C = 90°$)
$\sin (A + B) = \sin 90° = 1$

8. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers, then the HCF (a, b) is:

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

Ans: (b) Here, $a = x^3y^2$ and $b = xy^3$
$\Rightarrow a = x \times x \times x \times y \times y$ and $b = xy \times y \times y$
$\therefore$ HCF(a, b) $= x \times y \times y = x \times y^2 = xy^2$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** D and E are points on the sides AB and AC respectively of a ΔABC such that $DE \parallel BC$ then the value of x is 11, when AD = 4cm, DB = (x – 4) cm, AE = 8cm and EC = (3x – 19) cm.
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.
Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. **Assertion (A):** The value of k for which the system of linear equations $3x - 4y = 7$ and $6x - 8y = k$ have infinite number of solution is 14.
Reason (R): The system of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Ans: We know that the system of linear equations

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

So, Reason is not correct

For Assertion, we have, $a_1 = 3$, $b_1 = -4$, $c_1 = -7$, $a_2 = 6$, $b_2 = -8$ and $c_2 = -k$

Now, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-4}{-8} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-7}{-k}$

$\Rightarrow \frac{-7}{-k} = \frac{1}{2} \Rightarrow k = 14$

So, Assertion is correct.

Correct option is (c) Assertion (A) is true but reason (R) is false.

SECTION – B

Questions 11 to 15 carry 2 marks each.

11. Can we have any $n \in N$, where 12^n ends with the digit zero? Explain

Ans : $12^n = (2\times2\times3)^n = 2^n \times 2^n \times 3^n$

For units digit to be 0, 12^n should have 2 and 5 as its prime factors, but 12^n does not contain 5 as its prime factor.

Hence 12^n will not end with digit 0 for $n \in N$.

12. Find the value of α such that the quadratic equation $(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$, has equal roots.

Ans: Here, $a = \alpha - 12$, $b = 2(\alpha - 12)$, $c = 2$

For equal roots, $D = 0 \Rightarrow b^2 - 4ac = 0$

$\Rightarrow [2(\alpha - 12)]^2 - 4 \times [2(\alpha - 12)] = 0$

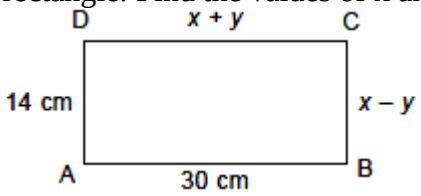
$2(\alpha - 12)\,[2(\alpha - 12) - 4] = 0$

$\Rightarrow (\alpha - 12)\,(2\alpha - 28) = 0$

$\Rightarrow \alpha = 12, 14$

$\alpha = 12$ not possible, take $\alpha = 14$

13. In the below Figure, ABCD is a rectangle. Find the values of x and y.



Ans: We know that the opposite sides of rectangle are equal.

$\therefore x + y = 30$ and $x - y = 14$

Adding both equations we get, $2x = 44$

$\Rightarrow x = 22$ cm

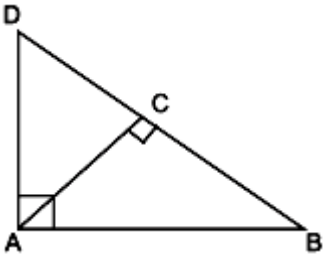
Putting $x = 22$ in eq. (i), we have

$22 + y = 30$

$\Rightarrow y = 30 - 22 = 8$

$\therefore x = 22$ cm and $y = 8$ cm

14. In figure, ΔABD is a right triangle, right angled at A and $AC \perp BD$. Prove that $AB^2 = BC.BD$.



Ans: In Δ DAB, and ΔACB

$\angle DAB = \angle ACB = 90°$

$\angle B = \angle B$ (common)

$\therefore$ $\Delta DAB \sim \Delta ACB$

$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC} = \frac{BD}{AB} \Rightarrow \frac{AB}{BC} = \frac{BD}{AB}$

$\Rightarrow$ $AB^2 = BC.BD$ Hence proved.

15. Find the value of x if $\tan 3x = \sin 45° . \cos 45° + \sin 30°$.

Ans: $\tan 3x = \sin 45° . \cos 45° + \sin 30°$

$\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \tan 3x = 1 = \tan 45°$

$\Rightarrow 3x = 45° \Rightarrow x = 15°$

SECTION – C

Questions 16 to 19 carry 3 marks each.

16. Prove that $\sqrt{5}$ is an irrational number.

Ans: Let $\sqrt{5}$ is a rational number then we have $\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get $p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5 $\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2 \Rightarrow q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5 $\Rightarrow$ q is also divisible by 5

From above we conclude that p and q have one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore, our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

17. The sum of the digits of a two digit number is 9. The number obtained by reversing the order of digits of the given number exceeds the given number by 27. Find the given number.

Ans: Let the tens digit be x and unit place digit be y.

Number = 10x + y

According to the Question, x + y = 9 ...(i)

and 10y + x = 10x + y + 27 –9x + 9y = 27

– x + y = 3 ...(ii)

Adding (i) and (ii), we get 2y = 12

$\Rightarrow$ y = 6

Putting value of y in equation (i), we get x + 6 = 9

$\Rightarrow$ x = 9 – 6

$\Rightarrow$ x = 3

So, the given number is 36.

18. Find the zeroes of $p(x) = 4x^2 + 24x + 36$ quadratic polynomials and verify the relationship between the zeroes and their coefficients.

Ans: $p(x) = 4x^2 + 24x + 36$

For zeroes, $p(x) = 0$

$\Rightarrow 4x^2 + 24x + 36 = 0 \Rightarrow 4(x^2 + 6x + 9) = 0$

$\Rightarrow 4(x^2 + 3x + 3x + 9) = 0 \Rightarrow (x + 3)(x + 3) = 0$

$\Rightarrow$ $x + 3 = 0$ or $x + 3 = 0$ $\Rightarrow$ $x = -3, x = -3$

$\therefore$ Zeroes are –3, –3.

Now $a = 4$, $b = 24$, $c = 36$

$\frac{-b}{a} = \frac{-24}{4} = -6$

Sum of zeroes = –3 + (–3) = –6

$\Rightarrow$ Sum of zeroes = $\frac{-b}{a}$

Also, $\frac{c}{a} = \frac{36}{4} = 9$

and Product of zeroes = (–3) × (–3) = 9

$\Rightarrow$ Product of zeroes = $\frac{c}{a}$

19. Prove that: $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sec^2\theta}{\tan^2\theta-1}$

Ans: LHS = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$=\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta}{\sin^2\theta-\cos^2\theta}$

$=\frac{\sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=\frac{1+1}{\sin^2\theta-\cos^2\theta}=\frac{2}{\sin^2\theta-\cos^2\theta}$

$=\frac{\frac{2}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}-\frac{\cos^2\theta}{\cos^2\theta}}=\frac{2\sec^2\theta}{\tan^2\theta-1}$ = RHS

SECTION – D

Questions 20 to 21 carry 5 marks.

20. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

Ans: Let original speed of the aircraft be x km/hr

Reduced speed = (x – 200) km/hr

According to given condition, $\frac{600}{x-200}-\frac{600}{x}=\frac{30}{60}=\frac{1}{2}$

$\Rightarrow\frac{600x-600x+120000}{x(x-200)}=\frac{1}{2}\Rightarrow\frac{120000}{x^2-200x}=\frac{1}{2}$

$\Rightarrow x^2 - 200x = 240000$

$\Rightarrow x^2 - 200x - 240000 = 0$

$\Rightarrow x^2 - 600x + 400x - 240000 = 0$

$\Rightarrow x(x - 600) + 400(x - 600) = 0$

$\Rightarrow (x + 400)(x - 600) = 0$

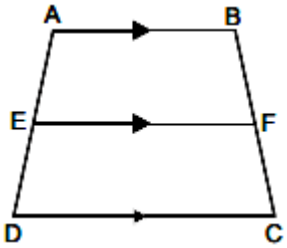
$\Rightarrow x + 400 = 0$ or $x - 600 = 0 \Rightarrow x = -400$ (rejected) or $x = 600$

$\therefore$ original speed = 600 km/hr

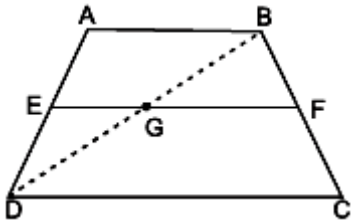
$\therefore$ original duration of flight = $\frac{600}{600}$ = 1 hour

21. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it. Use this result to prove the following :

In the given figure, if ABCD is a trapezium in which AB || DC || EF, then $\frac{AE}{ED} = \frac{BF}{FC}$



Ans: Given - ½ mark
To prove - ½ mark
Figure - ½ mark
Construction - ½ mark
Proof – 2 marks



Second part - 1 mark
Join BD intersecting EF at G.
In ΔDAB, EG || AB
$\therefore \frac{AE}{ED} = \frac{BG}{GD}$ (Using B.P.T.) ...(i)
In ΔDBC, GF || DC
$\therefore \frac{BG}{GD} = \frac{BF}{FC}$ ...(ii)
From (i) and (ii) $\frac{AE}{ED} = \frac{BF}{FC}$

SECTION – E (Case Study Based Questions)

Questions 22 to 23 carry 4 marks each.

22. In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

Based on the above information answer the following questions.

(i) Find the production in the 1st year. (1)
(ii) Find the production in the 12th year. (1)
(iii) Find the total production in first 10 years. (2)

OR

(iii) In how many years will the total production reach 31200 cars? (2)

Ans: (i) Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP.

So, a + 3d = 1800 & a + 7d = 2600

So d = 200 & a = 1200

(ii) $a_{12} = a + 11d \Rightarrow a_{30} = 1200 + 11 \times 200$

$\Rightarrow a_{12} = 3400$

(iii) $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2\times1200 + (10-1)\times200]$

$\Rightarrow S_{10} = 5[2400 + 1800] = 5\times4200 = 21000$

OR

$S_n = \frac{n}{2}[2a + (n-1)d] = 31200$

$\Rightarrow \frac{n}{2}[2\times1200 + (n-1)\times200] = 31200$

$\Rightarrow \frac{n}{2}\times200[12 + (n-1)] = 31200$

$\Rightarrow n[12 + (n-1)] = 312$

$\Rightarrow n^2 + 11n - 312 = 0$

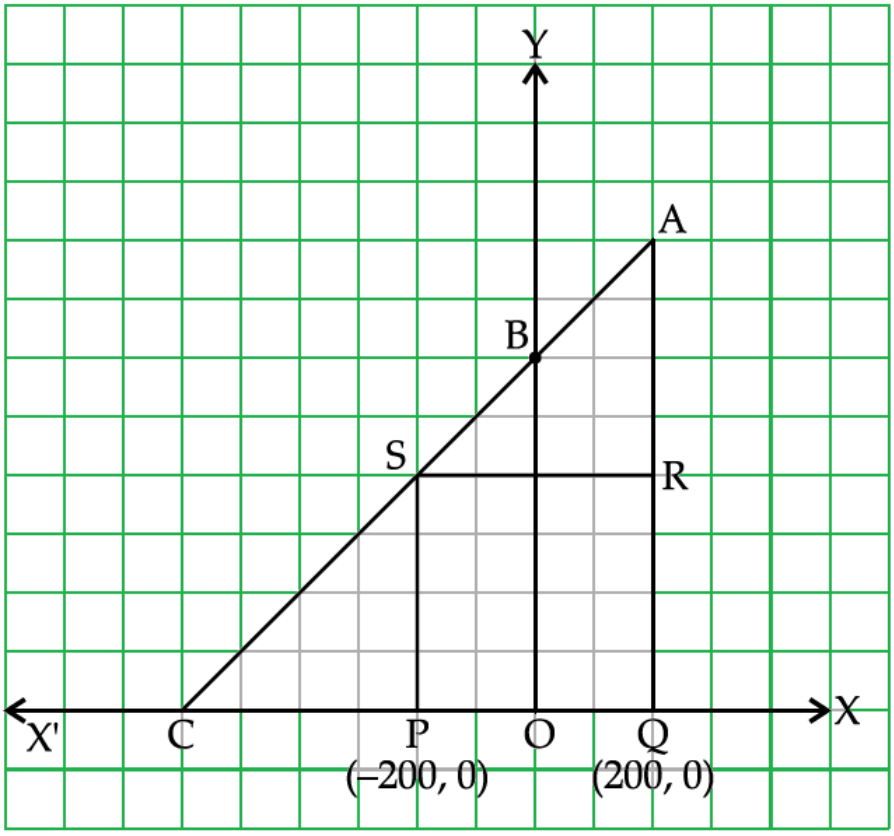
$\Rightarrow n^2 + 24n - 13n - 312 = 0$

$\Rightarrow (n+24)(n-13) = 0$

$\Rightarrow n = 13$ or -24.

As n can't be negative. So n = 13

23. Jagdhish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field from growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(i) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

(ii) (a) What is the area of square PQRS ?

OR

(b) What is the length of diagonal PR in square PQRS?

(iii) If S divides CA in the ratio K : 1, what is the value of K, where point A is (200, 800) ?

Ans: (i) Coordinates of R = (200, 400)

Coordinates of S = (– 200, 400)

(ii) Since, side of square PQRS = 400

Thus, area of square PQRS = $(\text{side})^2$

= $(400)^2$ = 160000 unit^2

OR

We know that, diagonal of square = 2 × side

∴ Diagonal PR of square PQRS = 2 × 400

= 400 √2 units

(iii) Let the ratio be k : 1.

Using section formula, $-200 = \dfrac{200k + 1 \times (-600)}{k + 1}$

⇒ – 200 k – 200 = 200 k – 600

⇒ – 400 k = – 400

⇒ k = 1

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD–32

PRACTICE PAPER 11 (2024-25)

CHAPTER 09 SOME APPLICATIONS OF TRIGONOMETRY

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If 300 m high pole makes an angle of elevation at a point on ground which is 300 m away from its foot, then the angle of elevation is:
(a) 60° (b) 90° (c) 30° (d) 45°

2. The angle of depression of a bike parked on the road from the top of a 90 m high pole is 60 degrees. The distance of the bike from the pole is:
(a) $20\sqrt{3}$ m (b) 90 m (c) $15\sqrt{3}$ m (d) $30\sqrt{3}$ m

3. A stone is $15\sqrt{3}$ m away from a tower 15 m high, then the angle of elevation of the top of the tower from the stone is:
(a) 45° (b) 60° (c) 30° (d) 90°

4. The ratio of the length of a tower and its shadow is $\sqrt{3} : 1$. The altitude of the sun is:
(a) 0° (b) 60° (c) 30° (d) 45°

5. The tops of the poles of height 16 m and 10 m are connected by a wire of length l meters. If the wire makes an angle of 30° with the horizontal, then l =
(a) 26 m (b) 16 m (c) 12 m (d) 10 m

6. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is
(a) 8 m (b) 10 m (c) 12 m (d) 14 m

7. If the angle of depression of an object from a temple is 30°, and the distance of the object from the temple is 45 m, then the height of the temple is:
(a) $45\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) 20 m (d) $20\sqrt{3}$ m

8. If two towers of heights h_1 and h_2 subtend angles of 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2$ =
(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45º.
Reason (R): According to Pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base.

10. **Assertion (A):** The ladder 20 m long makes an angle 60° with the wall, then the height of the point where the ladder touches the wall is 15 m.
Reason (R): For an angle θ, $\cos\theta = \frac{Adjacent\ \ Side}{Hypotenuse}$

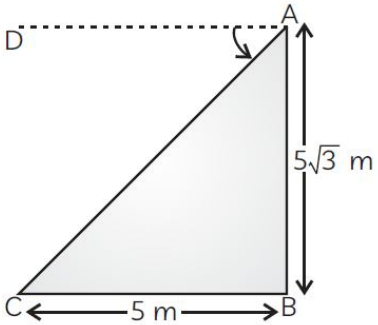
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The angle of depression of a car standing on the ground, from the top of a 85 m high tower is 45º. Find the distance of the car from the base of the tower.

12. A pole casts a shadow of length $2\sqrt{3}$ m on ground, when the sun's elevation is 60°. Find the height of the pole.

13. The figure shows the observation of point C from point A. Find the angle of depression from A.



14. The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sunrays meet the ground at an angle of 60°. Find the angle between the sunrays and the ground at the time of longer shadow.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3}$ = 1.732)

16. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

17. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30º and 45º. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use √3 = 1.732]

SECTION – D

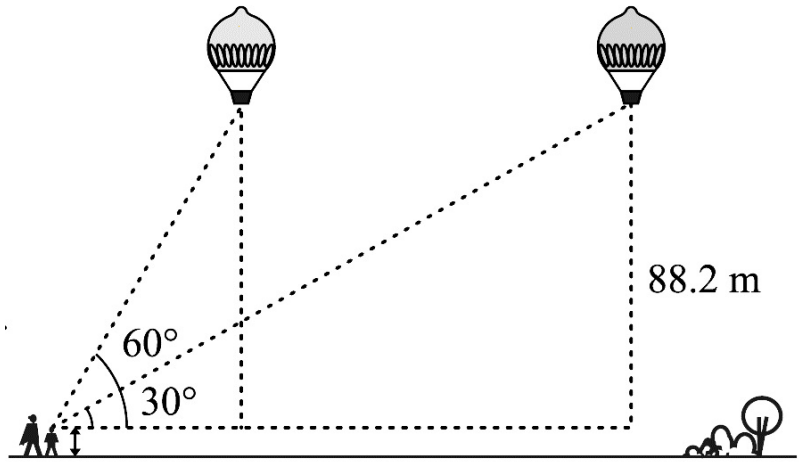
Questions 18 carry 5 marks.

18. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After 30 seconds, the angle of elevation reduces to 30° (see the below figure).



Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.732$)

(i) Find the distance travelled by the balloon during the interval. (2)

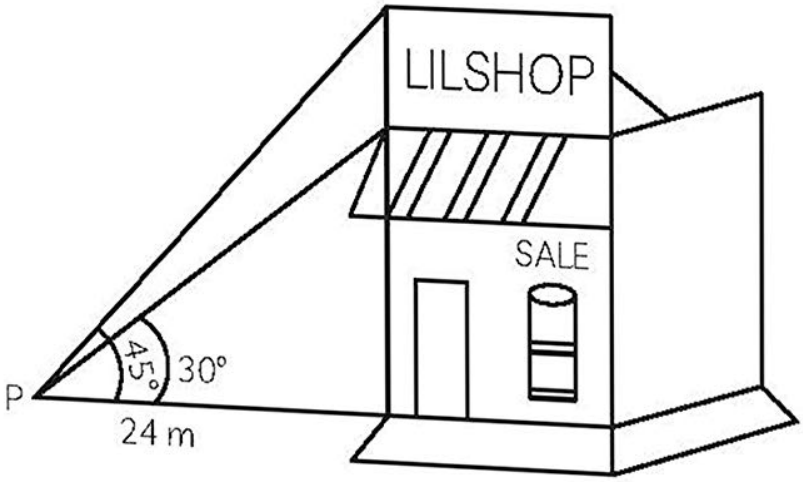
(ii) Find the speed of the balloon. (2)

OR

(ii) If the elevation of the sun at a given time is 30°, then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

20. Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.

From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.



On the basis of the above information, answer the following questions:

(i) Find the height of the building (without the sign board). (2)

OR

Find the height of the building (with the sign board) (2)

(ii) Find the height of the sign board. (1)

(iii) Find the distance of the point P from the top of the sign board. (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD–32

PRACTICE PAPER 11 (2024-25)

CHAPTER 09 SOME APPLICATIONS OF TRIGONOMETRY

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If 300 m high pole makes an angle of elevation at a point on ground which is 300 m away from its foot, then the angle of elevation is:

(a) 60° (b) 90° (c) 30° (d) 45°

Ans. (d) 45°

2. The angle of depression of a bike parked on the road from the top of a 90 m high pole is 60 degrees. The distance of the bike from the pole is:

(a) $20\sqrt{3}$ m (b) 90 m (c) $15\sqrt{3}$ m (d) $30\sqrt{3}$ m

Ans. (d) $30\sqrt{3}$ m

3. A stone is $15\sqrt{3}$ m away from a tower 15 m high, then the angle of elevation of the top of the tower from the stone is:

(a) 45° (b) 60° (c) 30° (d) 90°

Ans. (c) 30°

4. The ratio of the length of a tower and its shadow is $\sqrt{3} : 1$. The altitude of the sun is:

(a) 0° (b) 60° (c) 30° (d) 45°

Ans. (b) 60°

5. The tops of the poles of height 16 m and 10 m are connected by a wire of length l meters. If the wire makes an angle of 30° with the horizontal, then l =

(a) 26 m (b) 16 m (c) 12 m (d) 10 m

Ans. (c) 12 m

6. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 8 m (b) 10 m (c) 12 m (d) 14 m

Ans. (c) 12 m

7. If the angle of depression of an object from a temple is 30°, and the distance of the object from the temple is 45 m, then the height of the temple is:

(a) $45\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) 20 m (d) $20\sqrt{3}$ m

Ans. (b) $15\sqrt{3}$ m

8. If two towers of heights h_1 and h_2 subtend angles of 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2$ =
(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1
Ans. 3 : 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

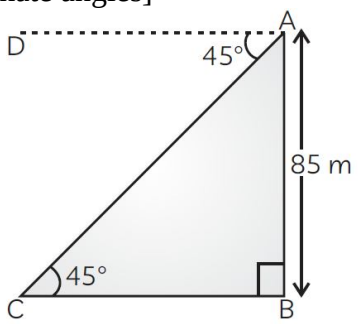
9. Assertion (A): If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45º.
Reason (R): According to Pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base.
Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. Assertion (A): The ladder 20 m long makes an angle 60° with the wall, then the height of the point where the ladder touches the wall is 15 m.
Reason (R): For an angle θ, $\cos\theta = \frac{Adjacent\ \ Side}{Hypotenuse}$
Ans. (d) Assertion (A) is false but reason (R) is true.

SECTION – B

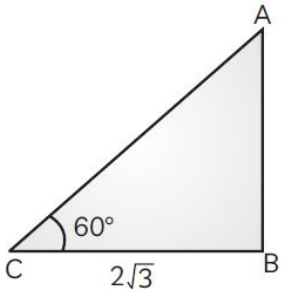
Questions 11 to 14 carry 2 marks each.

11. The angle of depression of a car standing on the ground, from the top of a 85 m high tower is 45º. Find the distance of the car from the base of the tower.
Ans. Let AB = 85 m be the height of the tower and angle of depression is ∠DAC = 45°.
Then, ∠ACB = ∠DAC = 45° [alternate angles]



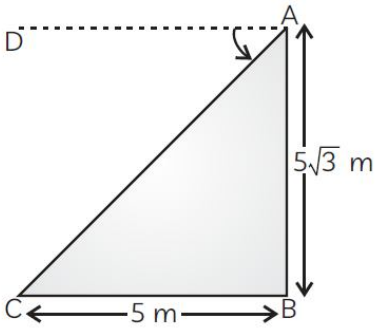
Now, in right–angled ΔABC, tan45° = AB/BC
⇒ 1 = 85/BC
⇒ BC = 85 m
Hence, the distance of the car from the base of the tower is 85 m.

12. A pole casts a shadow of length $2\sqrt{3}$ m on ground, when the sun's elevation is 60°. Find the height of the pole.
Ans. Let AB be the pole and BC be its shadow.



∴ In ABC, tan 60° = AB/BC
⇒ √3 = AB/ 2√3
⇒ AB = 6 m
Hence, the height of the pole is 6 m.

13. The figure shows the observation of point C from point A. Find the angle of depression from A.

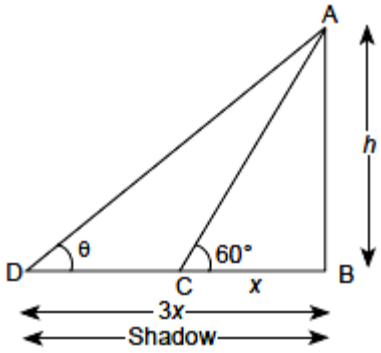


Ans. In right–angled ΔABC, ∠B = 90°
Let ∠DAC = θ
Then ∠DAC = ∠ACB = θ [alternate angles]
Now, tan θ = AB/BC = 5√3 / 5
⇒ tan θ = √3
⇒ tan θ = tan 60°
⇒ θ = 60°
Hence, the angle of depression from A is 60º.

14. The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sunrays meet the ground at an angle of 60°. Find the angle between the sunrays and the ground at the time of longer shadow.

Ans. In ΔABC, $\tan 60° = \frac{AB}{BC} = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$



In ΔABD, $\tan\theta = \frac{AB}{BD} \Rightarrow \tan\theta = \frac{h}{3x}$

$\Rightarrow \tan\theta = \frac{\sqrt{3}x}{3x} = \frac{1}{\sqrt{3}} = \tan 30^0 \Rightarrow \theta = 30^0$

SECTION – C

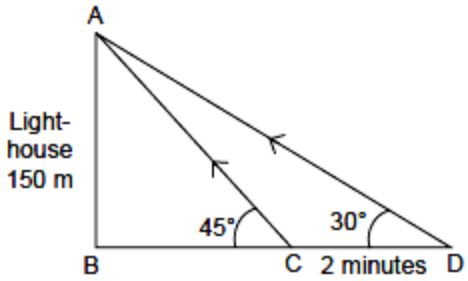
Questions 15 to 17 carry 3 marks each.

15. A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3}$ = 1.732)

Ans. Let AB is lighthouse.

∴ AB = 150 m

Initially boat is at C and after 2 minutes it reaches at D.



In right Δ ABC, $\frac{AB}{BC} = \tan 45°$

$\Rightarrow \frac{150}{BC} = 1 \Rightarrow$ BC = 150 m

In right Δ ABD, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{150}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 150\sqrt{3}m$

Distance covered in 2 minutes = BD – BC = 150 $\sqrt{3}$ – 150 = 150 ($\sqrt{3}$ – 1) m

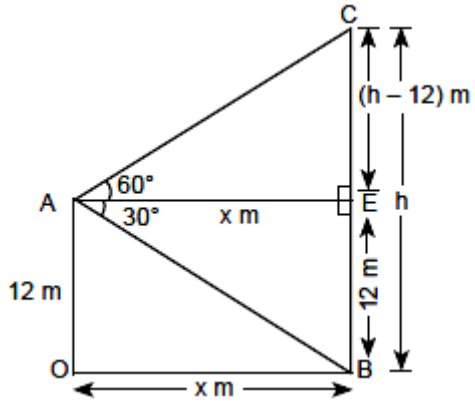
∴ Speed = $\frac{\text{Distance covered}}{\text{time taken}} = \frac{150(\sqrt{3}-1)}{2}$

= 75 × (1.732 – 1) = 54.9 m/minutes

16. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

Ans. A is the position of the man, OA = 12m, BC is cliff.

Let height of the cliff BC = *h* m and CE = (*h* – 12) m.



Let AE = OB = *x* m

In right angled triangle AEB, $\frac{AE}{BE} = \cot 30° \Rightarrow$ AE = 12 × $\sqrt{3}$

= 12 × 1.732 m = 20.78 m.

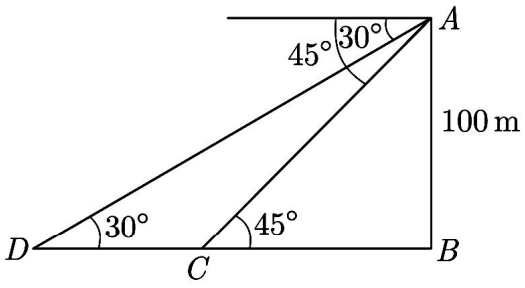
∴ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC, $\frac{CE}{AE} = \tan 60^0 \Rightarrow \frac{h-12}{12\sqrt{3}} = \sqrt{3} \Rightarrow h - 12 = 36 \Rightarrow h = 48$ m

$\therefore$ Height of the cliff = 48 m

17. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30º and 45º. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Ans: Let AB be the tower and ships are at points C and D. As per question statement we have shown diagram below.



Now in ΔABC we have $\tan 45^\circ = \frac{AB}{AC}$

$$\Rightarrow \frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$\Rightarrow AB + CD = \sqrt{3}\,AB$$

$$\Rightarrow CD = AB(\sqrt{3} - 1) = 100 \times (1.732 - 1) = 73.2 \text{ m}$$

Distance between two ships is 73.2 m.

SECTION – D

Questions 18 carry 5 marks.

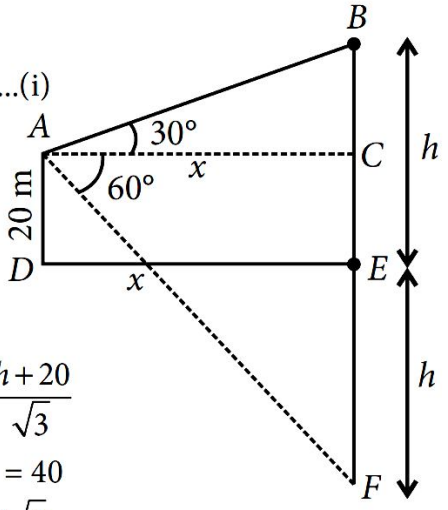
18. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

Ans: Let DE be the level of water and cloud be at position B which is h m above the level of water and reflection of cloud be at F and AC = DE = x m.

$\therefore$ BC = (h – 20)m, CF = (h + 20) m

In ΔABC, $\tan 30^\circ = \frac{BC}{AC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-20}{x} \Rightarrow x = \sqrt{3}(h-20) \quad ...(i)$$



In ΔACF,

$$\tan 60^\circ = \frac{CF}{AC} \Rightarrow \sqrt{3} = \frac{h+20}{x}$$

$$\Rightarrow x = \frac{h+20}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we get $\sqrt{3}(h-20) = \frac{h+20}{\sqrt{3}}$

$\Rightarrow 3h - 60 = h + 20 \Rightarrow 2h = 80 \Rightarrow h = 40$

From (i), we have $x = \sqrt{3}(40-20) = 20\sqrt{3}$

Applying Pythagoras theorem in ΔABC,

$AB^2 = BC^2 + AC^2 = (20)^2 + (20\sqrt{3})^2$

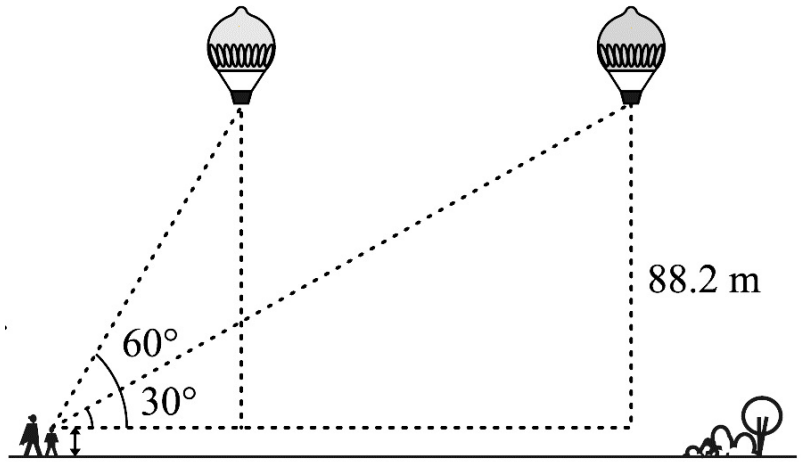
$= 400 + 1200 = 1600 \Rightarrow AB = \sqrt{1600} = 40$ m

$\therefore$ Distance of the cloud from point A = 40 m

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After 30 seconds, the angle of elevation reduces to 30° (see the below figure).



Based on the above information, answer the following questions. (Take $\sqrt{3}$ =1.732)

(i) Find the distance travelled by the balloon during the interval. (2)

(ii) Find the speed of the balloon. (2)

OR

(ii) If the elevation of the sun at a given time is 30°, then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

Ans: (i) In the figure, let C be the position of the observer (the girl).

A and P are two positions of the balloon.

CD is the horizontal line from the eyes of the (observer) girl.

Here PD = AB = 88.2 m − 1.2 m = 87 m

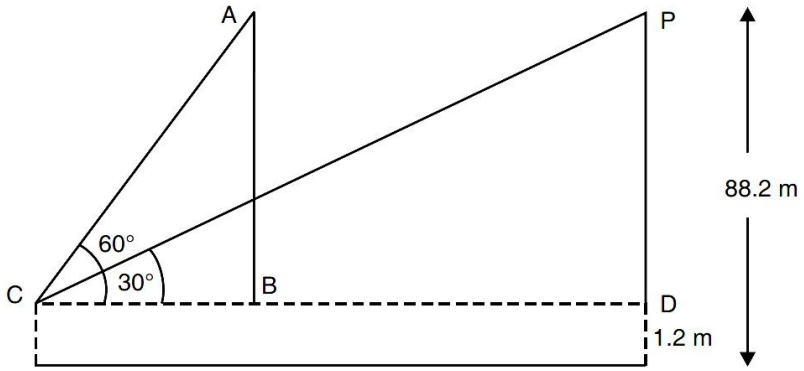
In right ΔABC, we have $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{87}{BC} = \sqrt{3} \Rightarrow BC = \frac{87}{\sqrt{3}}$ m

In right ΔPDC, we have $\frac{PD}{CD} = \tan 30°$

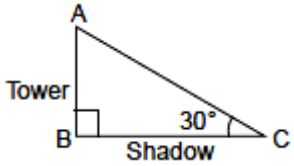
$\Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = 87\sqrt{3}$

Now, $BD = CD - BC = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 58\sqrt{3}$ m



Thus, the required distance between the two positions of the balloon = 58 √3 m
= 58 x 1.732 = 100.46 m (approx.)
(ii) Speed of the balloon = Distance/time = 100.46/30 = 3.35 m/s (approx.)

OR

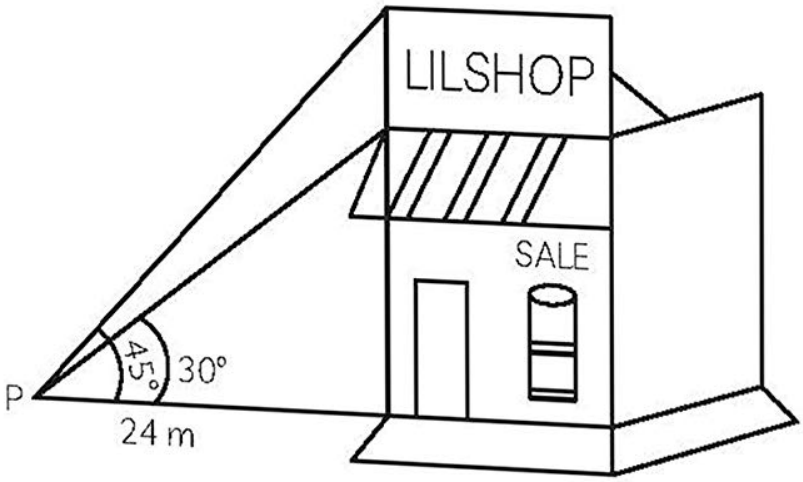


In right ∆ABC

$$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{150}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 150\sqrt{3} \text{ feet}$$

20. Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.

From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.



On the basis of the above information, answer the following questions:

(i) Find the height of the building (without the sign board). (2)

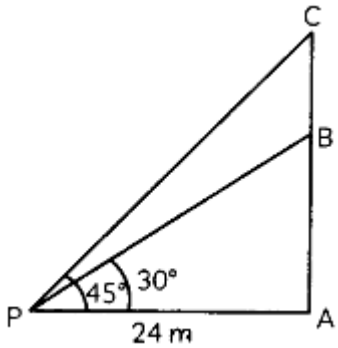
OR

Find the height of the building (with the sign board) (2)

(ii) Find the height of the sign board. (1)

(iii) Find the distance of the point P from the top of the sign board. (1)

Ans: (i) In ΔAPC,



tan 30° = AB/AP

⇒ $1/\sqrt{3}$ = AB/24

⇒ AB = $24/\sqrt{3}$ m = 13.85 m = 14 m (approx)

OR

Considering, the diagram in the above question, AC as the new height of the shop including the sign-baard.

In ΔAPC,

tan 45° = AC/AP

⇒ 1 = AC/24

⇒ AC = 24 m

(ii) From Q (i) and Q (ii).

Length of sign board, BC = AC – AB

= 24 – 14

= 10 m

(iii) In ΔAPC,

cos 45° = AP/AC

⇒ $1/\sqrt{2}$ = 24/AC

⇒ PC = $24\sqrt{2}$ m

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 08 (2024-25)

CHAPTER 10 CIRCLES

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

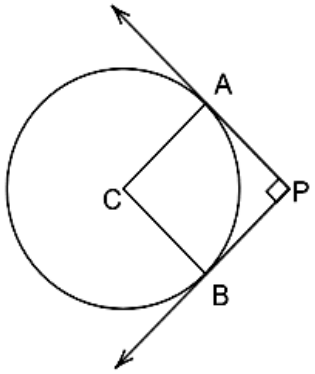
(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

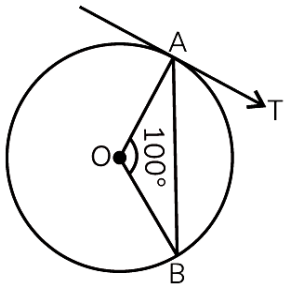
Questions 1 to 10 carry 1 mark each.

1. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4cm. If PA ⊥ PB, then the length of each tangent is:



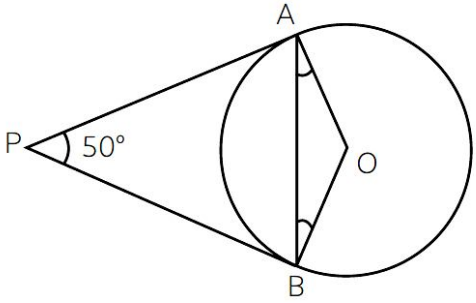
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

2. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then ∠BAT is equal to:



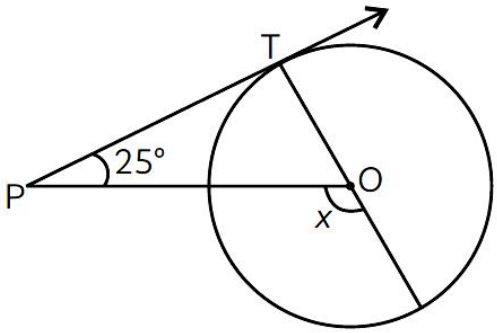
(a) 100° (b) 40° (c) 50° (d) 90°

3. In the figure, if PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is:



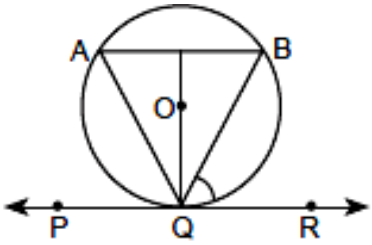
(a) 25º (b) 30º (c) 40º (d) 50º

4. In the given figure, PT is a tangent at T to the circle with centre O. If ∠TPO = 25°, then x is equal to:



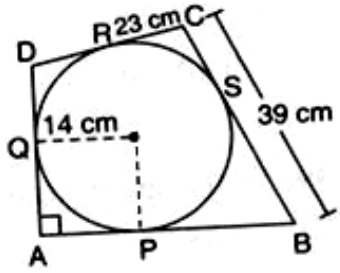
(a) 25° (b) 65° (c) 90° (d) 115°

5. In figure if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to



(a) 20° (b) 40° (c) 35° (d) 45°

6. In the given figure, quadrilateral ABCD is circumscribed, touching the circle at P, Q, R and S such that ∠DAB = 90°, If CR = 23 cm and CB = 39 cm and the radius of the circle is 14 cm, then the measure of AB is



(a) 37 cm (b) 16cm (c) 30 cm (d) 39 cm

7. A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is

(a) 8 units (b) 10 units (c) $10\sqrt{2}$ units (d) $8\sqrt{2}$ units

8. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 π cm^2, then AB is equal to

(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

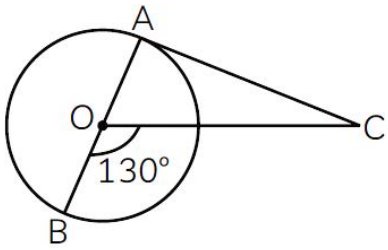
9. **Assertion (A):** The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.
Reason (R): If the angle between two radii of a circle is 130°, then the angle between the tangents at the end points of radii at their point of intersection is 50°.

10. **Assertion (A):** A circle can have infinitely many tangents.
Reason (R): The tangent at any point of a circle is perpendicular to the radius through the point of contact.

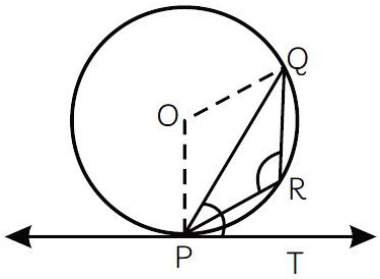
SECTION – B

Questions 11 to 14 carry 2 marks each.

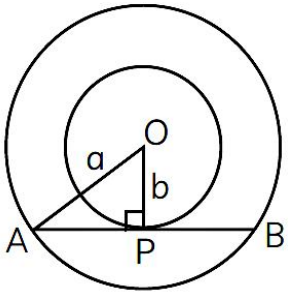
11. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130º, then find ∠ACO.



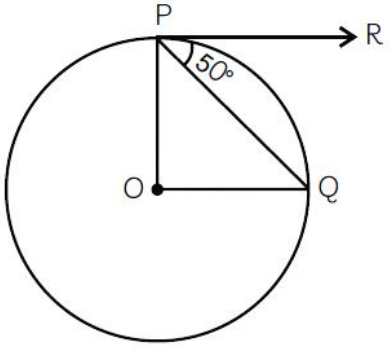
12. In figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ.



13. Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.



14. In the figure, if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find ∠POQ.

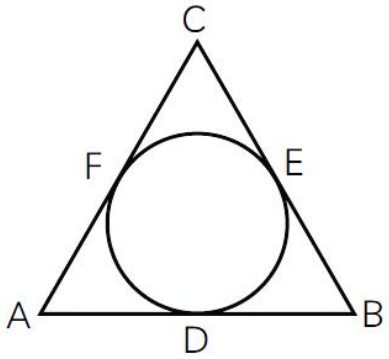


SECTION – C

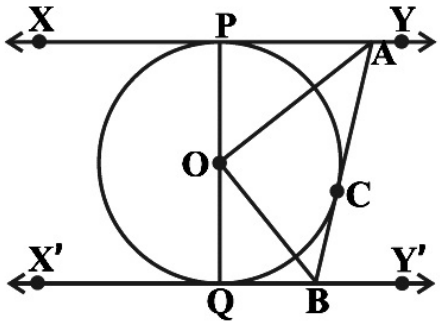
Questions 15 to 17 carry 3 marks each.

15. Prove that the rectangle circumscribing a circle is a square.

16. In the figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the length of AD, BE and CF.



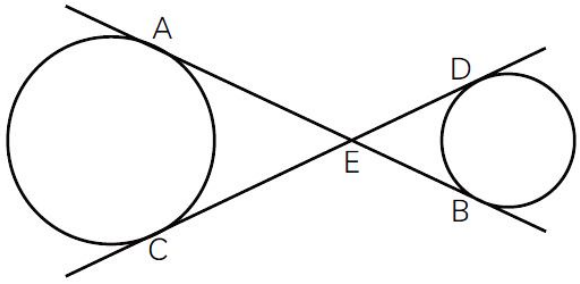
17. In the below figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠AOB = 90°.



SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (4)
(b) In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD. (1)

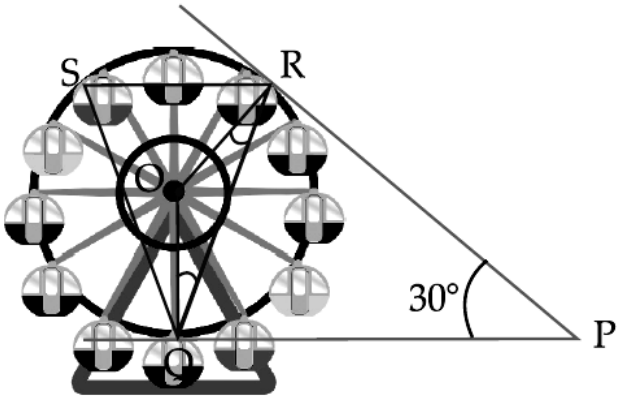


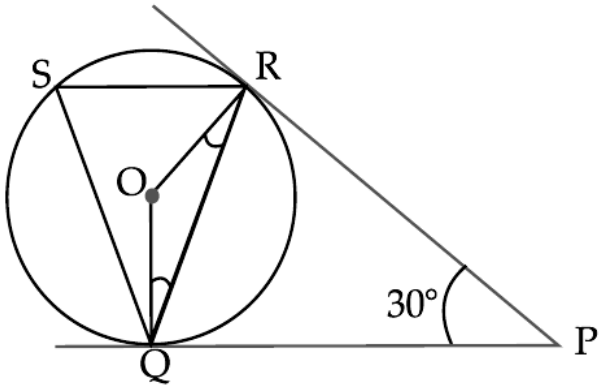
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Monika came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





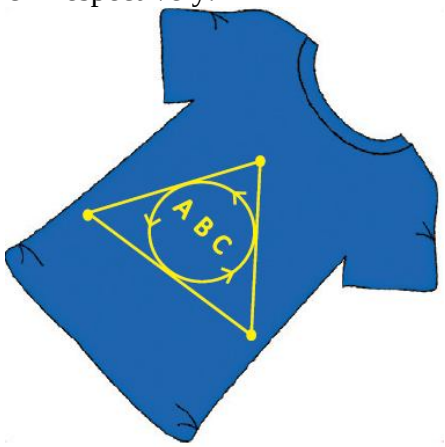
(a) In the given figure, find ∠ROQ. (2)

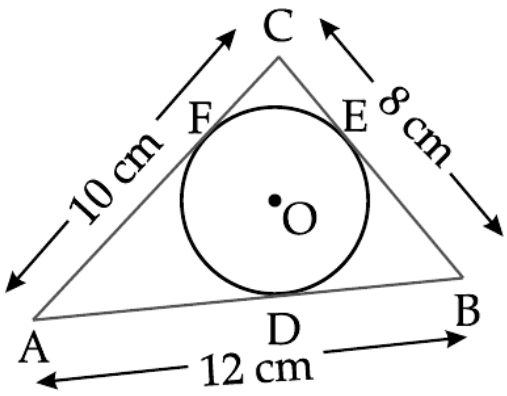
(b) Find the measure of ∠RQP. (2)

OR

(b) Find measure of ∠RSQ. Also, find the sum of ∠ORP and ∠OQP. (2)

20. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo is designed in different geometry and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.





(a) Find the length of AD and BE. (2)

OR

If the radius of the circle is 4 cm, find the area of ΔOAB.

(b) Find the perimeter of ΔABC. (1)

(c) Find the length of CF. (1)

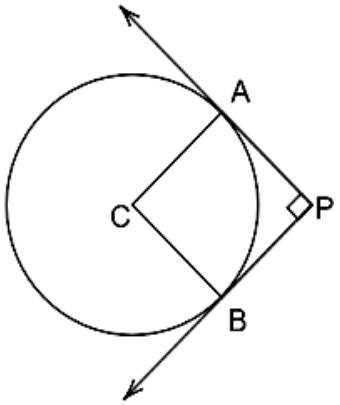
PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD–32
PRACTICE PAPER 08 (2024-25)
CHAPTER 10 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : X **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

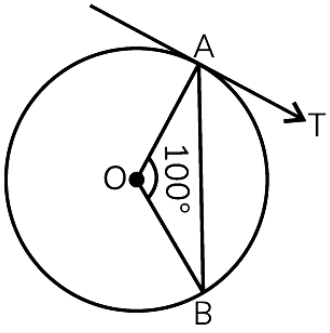
1. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4cm. If PA ⊥ PB, then the length of each tangent is:



(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Ans. (b) 4 cm
CA ⊥ AP and CB ⊥ AP [As tangent to a circle is ⊥ to radius]
Also, ∠APB = 90°
Now, in quad. APBC, ∠APB + ∠CAP + ∠CBP + ∠ACB = 360°
⇒ 90° + 90° + 90° + ∠ACB = 360° ⇒ ∠ACB = 90°
Now, each angle of quadrilateral APBC is 90° and AP = PB.
∴ APCB is a square.
∴ AP = BP = BC = CA = 4 cm
Thus, the length of each tangent is 4 cm.

2. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then ∠BAT is equal to:



(a) 100° (b) 40° (c) 50° (d) 90°

Ans. (c) 50°

Here, ∠AOB = 100°

∠OAT = 90° [As tangent at a point to a circle is perpendicular to the radius]

In ΔOAB, OA = OB [Radii of the circle]

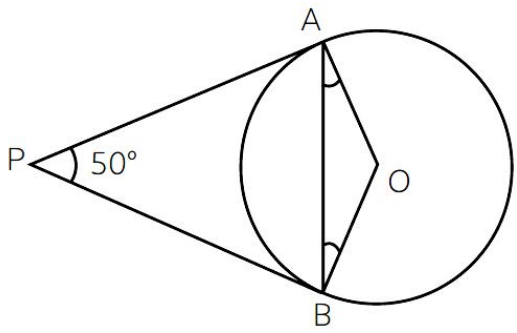
⇒ ∠OBA = ∠OAB [∵ angle opposite to equal sides are equal]

Since, sum of angles in a triangle is 180°.

∴ ∠OBA = ∠OAB = (180° − ∠ AOB)/2 = (180° − 100°)/2 = 40°

Now, ∠BAT = ∠OAT – ∠OAB = 90° – 40° = 50°

3. In the figure, if PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is:



(a) 25º (b) 30º (c) 40º (d) 50º

Ans. (a) 25º

We know that the radius and tangent are perpendicular at their point of contact

∵ ∠OBP = ∠OAP = 90º

Now, In quadrilateral AOBP

∠AOB + ∠OBP + ∠APB + ∠OAP = 360º [Angle sum property of a quadrilateral]

⇒ ∠AOB + 90º + 50º + 90º = 360º

⇒ 230º + ∠AOB = 360º ⇒ ∠AOB = 130º

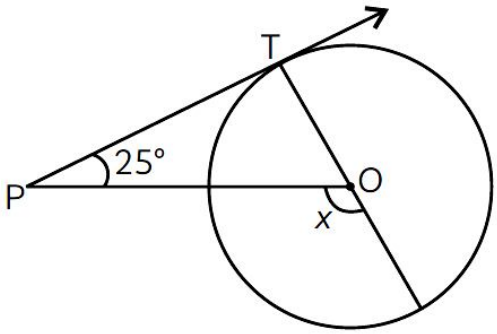
Now, In isosceles triangle AOB

∠AOB + ∠OAB + ∠OBA = 180º [Angle sum property of a triangle]

⇒ 130º + 2∠OAB = 180º [∵ ∠OAB = ∠OBA]

⇒ ∠OAB = 25º

4. In the given figure, PT is a tangent at T to the circle with centre O. If ∠TPO = 25°, then x is equal to:



(a) 25° (b) 65° (c) 90° (d) 115°

Ans. (d) 115°

Given, ∠TPO = 25°

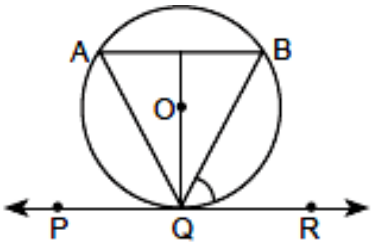
As we know, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ ∠PTO = 90°

Now, Exterior angle, x = ∠PTO + ∠TPO

⇒ x = 90° + 25° ⇒ x = 115°

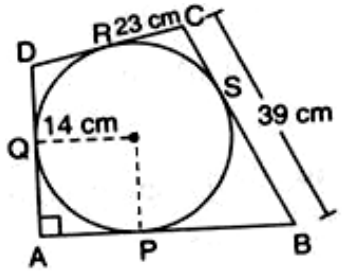
5. In figure if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to



(a) 20° (b) 40° (c) 35° (d) 45°

Ans. (b) 40°
AB || PR
∴ ∠ABQ = ∠BQR [Alternate interior angles]
⇒ ∠ABQ = 70°
Also, ∠BQR = ∠BAQ [Angles in alternate segment]
⇒ ∠BAQ = 70°
In ΔAQB,
∠BAQ + ∠ABQ + ∠AQB = 180°
⇒ 70° + 70° + ∠AQB = 180° ⇒ ∠AQB = 180° – 140° = 40°.

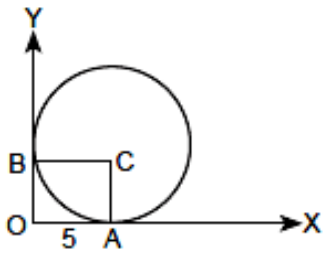
6. In the given figure, quadrilateral ABCD is circumscribed, touching the circle at P, Q, R and S such that ∠DAB = 90°, If CR = 23 cm and CB = 39 cm and the radius of the circle is 14 cm, then the measure of AB is



(a) 37 cm (b) 16cm (c) 30 cm (d) 39 cm
Ans: (c) 30 cm
∵ Tangent is perpendicular to the radius through the point of contact.
∠OQA = ∠OPA = 90° and OQ = OP [Radii]
∴ OQAP is a square.
⇒ AP = 14cm
Now, CR = CS = 23 cm [Tangents from an external point to a circle are equal]
∴ BS = 39 - 23 = 16cm
And BS = BP = 16 cm [Tangents from an external point to a circle are equal]
Now, AB = AP + BP = 14 + 16 = 30cm

7. A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is

(a) 8 units (b) 10 units (c) $10\sqrt{2}$ units (d) $8\sqrt{2}$ units
Ans. (b) 10 units
OA = OB ⇒ OB = 5



AC = BC [Radii]
⇒ OACB is a square.
⇒ AC = OA = 5 ⇒ Diameter = 10 units.

8. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 π cm^2, then AB is equal to
(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm
Ans. (c) 10 cm
Let radius of circle = R
∴ $\pi R^2 = 25\pi$
⇒ R = 5 cm
∴ Distance between two parallel tangents = diameter = 2 × 5 = 10 cm.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

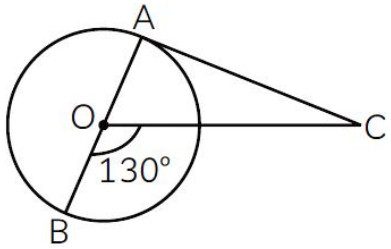
9. Assertion (A): The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.
Reason (R): If the angle between two radii of a circle is 130°, then the angle between the tangents at the end points of radii at their point of intersection is 50°.
Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. Assertion (A): A circle can have infinitely many tangents.
Reason (R): The tangent at any point of a circle is perpendicular to the radius through the point of contact.
Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

SECTION – B

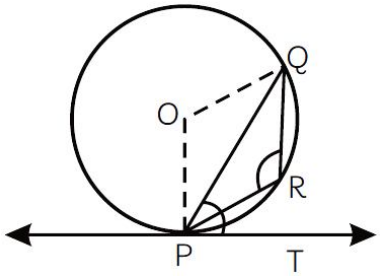
Questions 11 to 14 carry 2 marks each.

11. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130º, then find ∠ACO.



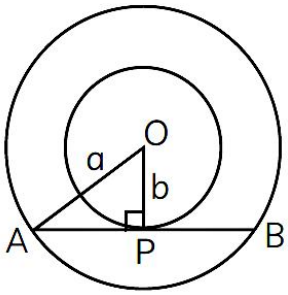
Ans. Given, ∠BOC = 130º
Since, AOB is the diameter of the circle.
Then, ∠AOB = 180º
⇒ ∠BOC + ∠AOC = 180°
⇒ 130º + ∠AOC = 180º
⇒ ∠AOC = 50º
Now, ∠OAC = 90º [Since a tangent at any point on a circle is perpendicular to the radius]
In ΔOAC, ∠AOC + ∠OAC + ∠ACO = 180º
∠ACO = 180º – (90º + 50º)
= 180º – 140º = 40º

12. In figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ.



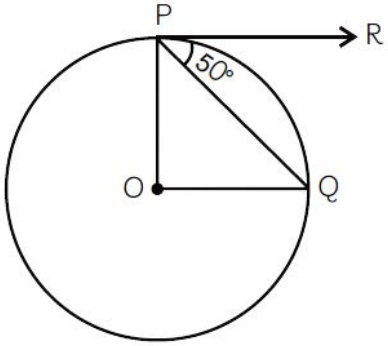
Ans. Given, ∠QPT = 60°
Since, OP is the radius of the circle.
Now, ∠OPT = 90°
∴ ∠OPQ = ∠OPT – ∠QPT = 90° – 60° = 30°
In ΔOPQ, OP = OQ [radii of circle]
∠OQP = ∠POQ = 30° [∵ Angles opposite to equal sides are equal]
∴ ∠POQ = 180° – (30° + 30°) = 120°
∴ Reflex ∠POQ = 360° – 120° = 240°
We know that, angle subtended by an arc at the centre double the angle subtended by it on the remaining part of the circle.
∴ ∠PRQ = $\frac{1}{2}$ Reflex ∠POQ = 240°/2 = 120°

13. Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.



Ans. Let O be the centre of the concentric circles and AB be the chord for bigger circle and tangent to the smaller circle.
Let P be the point where AB meets smaller circle.
∴ OA = a and OP = b
Now, ∠OPA = 90° [As, tangent at any point is perpendicular to the radius]
Now, in ΔOPA, by Pythagoras theorem
$OA^2 = OP^2 + AP^2 \Rightarrow AP = \sqrt{a^2+b^2}$
Now, AB = 2AP [as perpendicular from centre to the chord bisects the chord]
$\Rightarrow AB = 2\sqrt{a^2+b^2}$

14. In the figure, if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find ∠POQ.

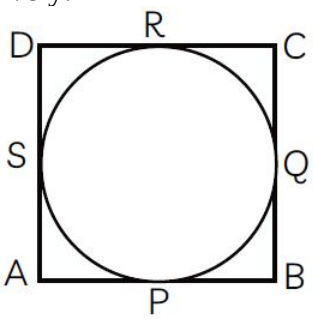


Ans. Given, ∠RPQ = 50°
Now, ∠OPR = 90° [As tangent makes an angle of 90° with radius]
⇒ ∠OPQ + ∠QPR = 90°.
⇒ ∠OPQ = 90° – 50° = 40°
In ΔOPQ, OP = OQ [Radii of a circle]
⇒ ∠OQP = ∠OPQ = 40°
So, ∠POQ = 180° – (40° + 40°)
⇒ ∠POQ = 100°

SECTION – C

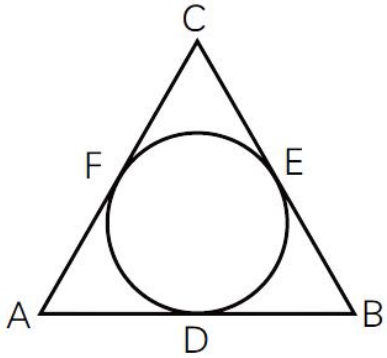
Questions 15 to 17 carry 3 marks each.

15. Prove that the rectangle circumscribing a circle is a square.
Ans. Consider a rectangle ABCD circumscribing a circle such that if touches the sides AB, BC, CD and DA at P, Q, R and S respectively.

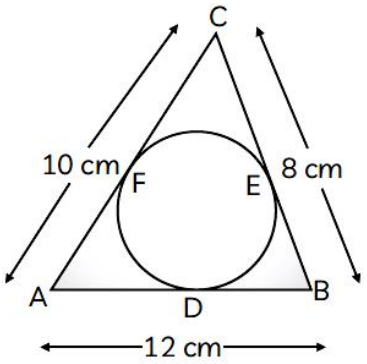


Now, we know lengths of tangents drawn from an external point to a circle are equal.
∴ AP = AS, BP = BQ, CR = CQ and DR = DS
Adding the above equations, we get
⇒ AP + BP + CR + DR = AS + BQ + CQ + DS
⇒ (AP + BP) + (CR + DR) = (AS +DS) + (BQ + CQ)
⇒ AB + CD = AD + CB
But AB = CD and AD = CB [Since, opposite sides of rectangle are equal]
⇒ AB = AD
i.e., Adjacent sides of rectangle ABCD are equal.
Hence, ABCD is a square.

16. In the figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the length of AD, BE and CF.

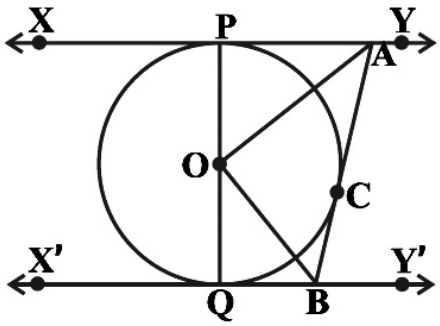


Ans. Given, A circle inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively.

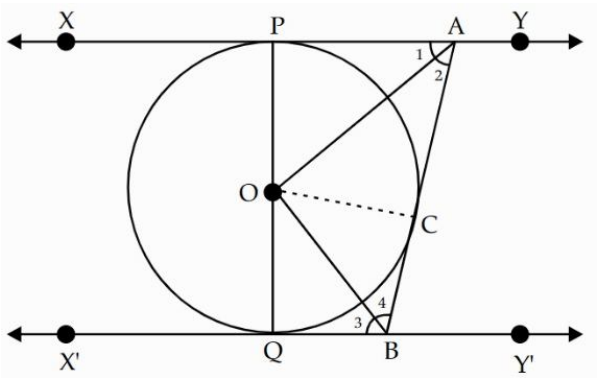


Also, AB = 12 cm, BC = 8 cm and CA = 10 cm.
Since, the lengths of tangents drawn from an external point to a circle are equal, therefore
AD = AF = x (say)
BD = BE = y (say)
CE = CF = z (say)
Then, AD + BD = AB
$\Rightarrow x + y = 12$ ...(i)
Also, BE + EC = BC
$\Rightarrow y + z = 8$ ...(ii)
and CF + AF = AC
$\Rightarrow z + x = 10$ ...(iii)
Adding equations (i), (ii) and (iii), we get
$2(x + y + z) = 30$
$\Rightarrow x + y + z = 15$ ...(iv)
Subtracting eq. (i) from eq. (iv), we get z = 3
Subtracting eq. (ii) from eq. (iv), we get x = 7
And, subtracting eq (iii) from eg (iv), we get y = 5
Hence, the lengths of AD, BE and CF are 7 cm, 5 cm and 3 cm, respectively.

17. In the below figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠AOB = 90°.



Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal.
∴ AP = AC



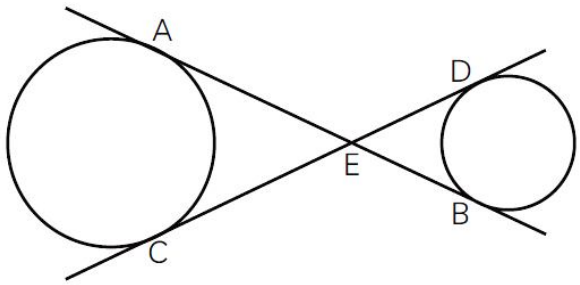
In Δ PAO and Δ AOC, we have:
AO = AO [Common]
OP = OC [Radii of the same circle]

AP = AC
⇒ Δ PAO ≅ Δ AOC [SSS Congruency]
∴ ∠PAO = ∠CAO = ∠1
∠PAC = 2 ∠1 ...(1)
Similarly ∠CBQ = 2 ∠2 ...(2)
Again, we know that sum of internal angles on the same side of a transversal is 180°.
∴ ∠PAC + ∠CBQ = 180°
⇒ 2 ∠1 + 2 ∠2 = 180° [From (1) and (2)]
⇒ ∠1 + ∠2 = 180°/2 = 90° ...(3)
Also ∠1 + ∠2 + ∠AOB = 180° [Sum of angles of a triangle]
⇒ 90° + ∠AOB = 180°
⇒ ∠AOB = 180° − 90° ⇒ ∠AOB = 90°.

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (4)
(b) In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD. (1)

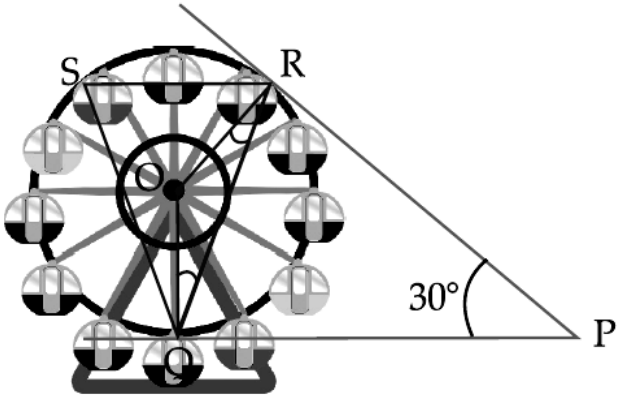


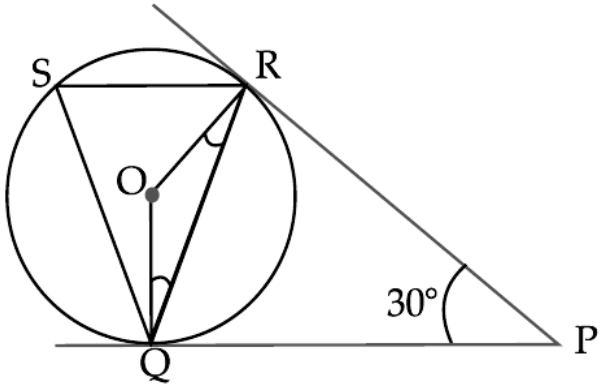
Ans. (a) Given, To prove, Construction and figure of 2 marks
Proof of 2 marks
(b) We know that lengths of tangents drawn from an external point to a circle is equal.
∴ EB = ED and EA = EC
On adding, we get EA + EB = EC + ED ⇒ AB = CD

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.
After taking a ride in Ferris wheel, Monika came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





(a) In the given figure, find ∠ROQ. (2)
(b) Find the measure of ∠RQP. (2)

OR

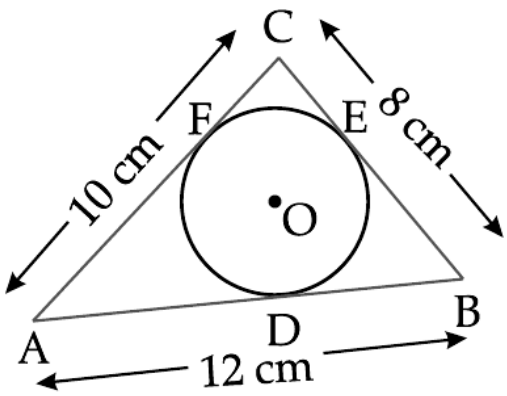
(b) Find measure of ∠RSQ. Also, find the sum of ∠ORP and ∠OQP. (2)

Ans. (a) ∠ORP = 90° = ∠OQP
[∵ radius of circle is perpendicular to tangent]
∴ ∠ROQ + ∠ORP + ∠OQP + ∠QPR = 360°
⇒ ∠ROQ + 90° + 90° + 30° = 360°
⇒ ∠ROQ + 210° = 360°
⇒ ∠ROQ = 360° – 210°
⇒ ∠ROQ = 150°.
(b) In ΔOQR, ∠OQR = ∠ORQ
∠ROQ = 150°
and ∠ROQ + ∠OQR + ∠ORQ = 180°
⇒ 150° + 2 ∠ORQ = 180°
⇒ 2 ∠ORQ = 30°
⇒ ∠ORQ = 15°
∴ ∠OQR = ∠ORQ = 15°
Now ∠RQP = ∠OQP – ∠OQR = 90° – 15° = 75°.

OR

(b) $\angle RSQ = \frac{1}{2}\angle ROQ = 75°$. (Angle subtended at the centre is double)
From the figure, ∠ORP = ∠OQP = 90° (Radius is perpendicular to tangent)
∴ ∠ORP + ∠OQP = 90° + 90° = 180°.

20. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo is designed in different geometry and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



(a) Find the length of AD and BE. (2)

OR

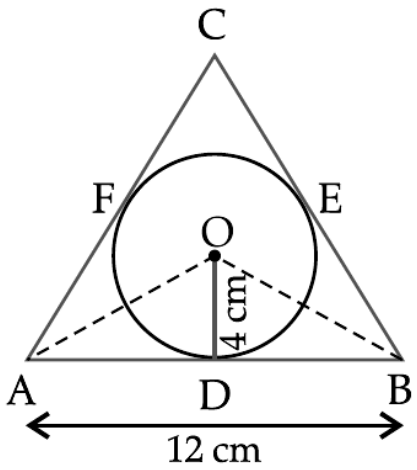
If the radius of the circle is 4 cm, find the area of ΔOAB.
(b) Find the perimeter of ΔABC. (1)
(c) Find the length of CF. (1)

Ans. (a) Let AD be x cm, then DB = (12 – x) cm
∵ AD = AF, CF = CE, DB = BE [Tangents to a circle from external points]
∴ AF = x cm,
then CF = (10 – x) cm
BE = (12 – x) cm,
then CE = 8 – (12 – x) = (x – 4) cm
Now CF = CE

$\Rightarrow 10 - x = x - 4$
$\Rightarrow 2x = 14$
$\Rightarrow x = 7$. ...(i)
Hence, AD = 7cm.
Since, BE = (12 – x) cm = (12 – 7) cm [From (i)]
= 5 cm

OR

Radius, OD = 4 cm and AB = 12 cm



Then, area of $\Delta OAB = 1/2 \times OD \times AB$
$= 1/2 \times 4 \times 12$
$= 24\ cm^2$.
(b) Perimeter of $\Delta ABC = AB + BC + CA$
= (12 + 8 + 10) cm = 30 cm.
(c) From question (a), CF = (10 – x cm)
= (10 – 7) cm = 3 cm.

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 12 (2024-25)

CHAPTER 11 AREAS RELATED TO CIRCLES

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

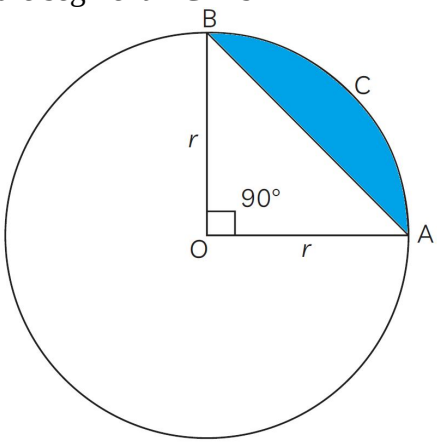
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. What is the area of a semi-circle of diameter 'd'?

(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$ (c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$

2. In the given figure, the area of the segment ACB is



(a) $\frac{r^2}{4}(\pi-2)$ (b) $\frac{r^2}{4}(\pi+2)$ (c) $\frac{r^2}{4}(\pi-1)$ (d) $\frac{r^2}{4}(\pi+1)$

3. The area of the circle that can be inscribed in a square of 6 cm is:

(a) 36π cm² (b) 18π cm² (c) 12π cm² (d) 9π cm²

4. The minute hand of a clock is 84 cm long. The distance covered by the tip of minute hand from 10:10 am to 10:25 am is :

(a) 44 cm (b) 88 cm (c) 132 cm (d) 176 cm

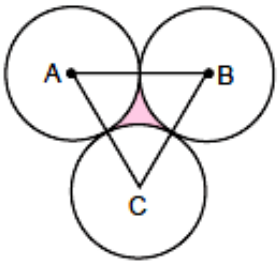
5. An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm². Then the radius of the circle is:

(a) 4 cm (b) 8 cm (c) 12 cm (d) 16 cm

6. If a square ABCD is inscribed in a circle of radius 'r' and AB = 4 cm, then the value of r is:

(a) 2 cm (b) 2 √2 cm (c) 4 cm (d) 4 √2 cm

7. ABC is an equilateral triangle. The area of the shaded region if the radius of each of the circle is 1 cm, is



(a) $2-\frac{\pi}{3}$ (b) $\sqrt{3}-\pi$ (c) $\sqrt{3}-\frac{\pi}{2}$ (d) $\sqrt{3}-\frac{\pi}{4}$

8. If the difference between the circumference and the radius of of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(a) 154 (b) 44 (c) 14 (d) 7

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If the radius of an arc is 8 cm and the central angle is 40°, then the length of an arc is 5.59 cm.

Reason (R): Length of arc = $\pi r^2 \times \frac{\theta}{360^0}$

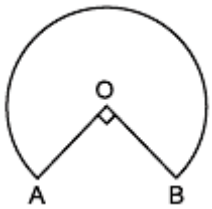
10. Assertion (A): In a circle of radius 4 cm, the angle of a sector is 45°, then the area of the sector is 44/7 cm².

Reason (R): Area of sector = $\pi r^2 \times \frac{\theta}{360^0}$

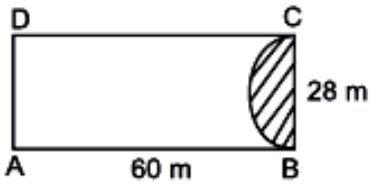
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, the shape of the top of a table is that a sector of a circle with centre O and ∠AOB = 90°. If AO = OB = 42 cm, then find the perimeter of the top of the table. [Use $\pi = \frac{22}{7}$]

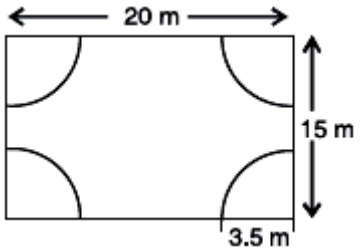


12. A plot is in the form of a rectangle ABCD having semicircle on BC as shown in the figure. The semicircle portion is grassy while the remaining plot is without grass. Find the area of the plot without grass where AB = 60 m and BC = 28 m. [Use $\pi = \frac{22}{7}$]



13. The measure of the minor arc of a circle is 1/5 of the measure of the corresponding major arc. If the radius of the circle is 10.5 cm, find the area of the sector corresponding to the major arc. [π = 22/7]

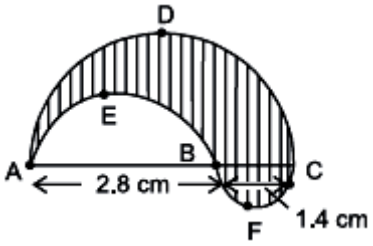
14. A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.



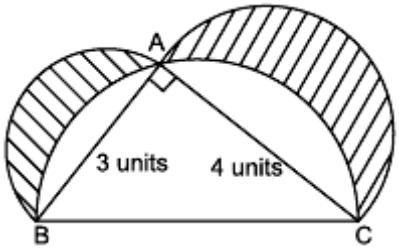
SECTION – C

Questions 15 to 17 carry 3 marks each.

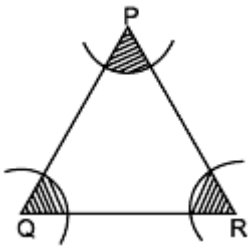
15. In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.



16. In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



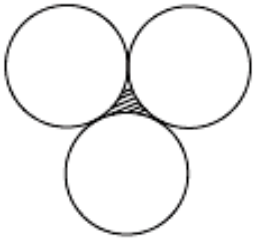
17. In figure arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



SECTION – D

Questions 18 carry 5 marks.

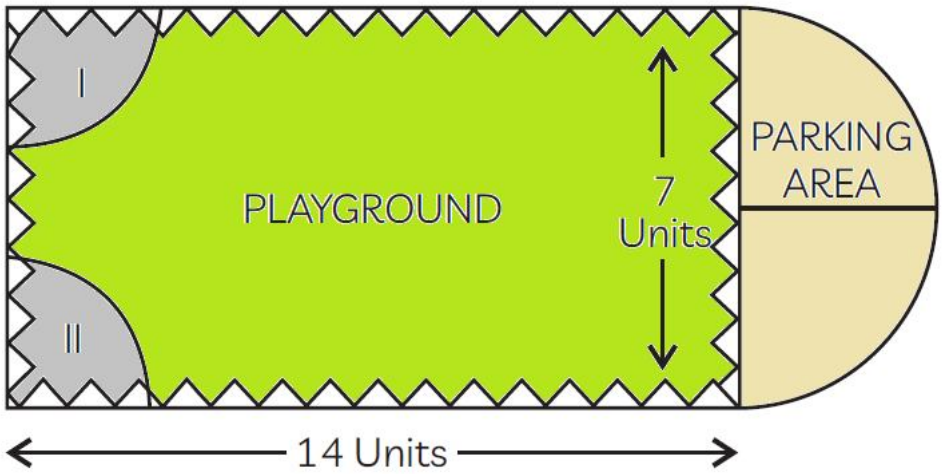
18. In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area of shaded region enclosed between these three circles. [Use $\pi = 22/7$ and $\sqrt{3} = 1.732$]



SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.
After survey, it was decided to build rectangular playground, with a semi–circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

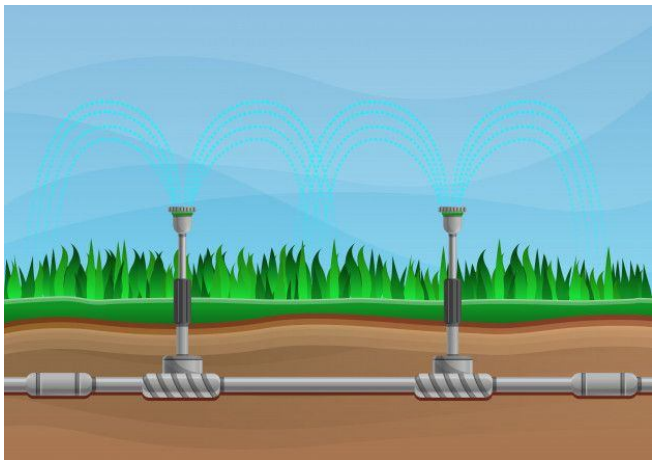


Based on the above information, answer the following questions:
(a) What is the total perimeter of the parking area? (1)
(b) What is the total area of parking and the two quadrants? (2)

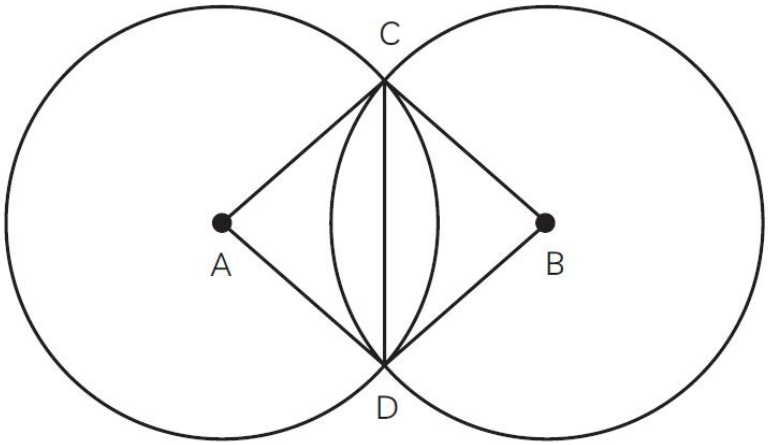
OR

What is the ratio of area of playground to the area of parking area?
(c) Find the cost of fencing the playground and parking area at the rate of ₹ 2 per unit. (1)

20. Sprinklers are crop irrigation equipment which rotate around a center and spay water on the crops in the circular region.
Two such high powers sprinklers, occupying negligible area are installed in a straight line in a field such that they spray water on an common area. Shown below are the side and top views where points A and B are the sprinklers.



Side view of the sprinklers



Top view of the region sprayed

(Note: The figures are not to scale.)

Both the sprinklers spray over an equal area. It is given that, CD = 400 m and ∠CAD = ∠CBD = 90°.

(a) Find the radius of the circular region sprayed by the sprinkler. (1)

(b) Find the area of the overlapping region. (2)

(c) Find the perimeter of the overlapping region. (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 12 (2024-25)

CHAPTER 11 AREAS RELATED TO CIRCLES

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

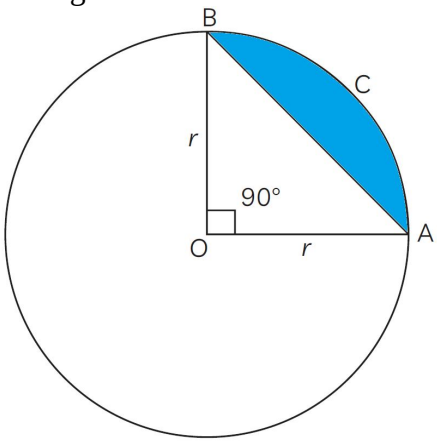
1. What is the area of a semi-circle of diameter 'd'?

(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$ (c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$

Ans. (c) $\frac{1}{8}\pi d^2$

Area of semi-circle = $\frac{1}{2}\pi r^2 \; \frac{1}{2}\pi\left(\frac{1}{2}d\right)^2 = \frac{1}{2}\pi\frac{1}{4}d^2 = \frac{1}{8}\pi d^2$

2. In the given figure, the area of the segment ACB is



(a) $\frac{r^2}{4}(\pi-2)$ (b) $\frac{r^2}{4}(\pi+2)$ (c) $\frac{r^2}{4}(\pi-1)$ (d) $\frac{r^2}{4}(\pi+1)$

Ans. (a) $\frac{r^2}{4}(\pi-2)$

Area of segment ACB = Area of sector AOBCA – Area of ΔAOB

$= \frac{\theta}{360°}\times\pi r^2 - \frac{1}{2}\times r\times r$

$= \frac{90°}{360°}\times\pi\times r^2 - \frac{1}{2}\times r^2 = \frac{1}{4}\pi r^2 - \frac{1}{2}r^2 = \frac{1}{4}r^2\left[\pi-2\right]$

3. The area of the circle that can be inscribed in a square of 6 cm is:

(a) 36π cm² (b) 18π cm² (c) 12π cm² (d) 9π cm²

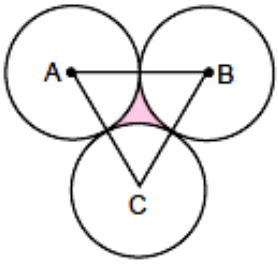
Ans. (d) 9π cm^2
Diameter of circle = side of square = 6 cm
$\Rightarrow$ Radius = 6/2 = 3 cm
Area of circle = $\pi r^2 = \pi(3)^2 = 9\pi$
Therefore, the area of the circle is 9p square cm.

4. The minute hand of a clock is 84 cm long. The distance covered by the tip of minute hand from 10:10 am to 10:25 am is :
(a) 44 cm (b) 88 cm (c) 132 cm (d) 176 cm
Ans. (c) 132 cm

5. An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. Then the radius of the circle is:
(a) 4 cm (b) 8 cm (c) 12 cm (d) 16 cm
Ans. (b) 8 cm

6. If a square ABCD is inscribed in a circle of radius 'r' and AB = 4 cm, then the value of r is:
(a) 2 cm (b) $2\sqrt{2}$ cm (c) 4 cm (d) $4\sqrt{2}$ cm
Ans. (b) $2\sqrt{2}$ cm

7. ABC is an equilateral triangle. The area of the shaded region if the radius of each of the circle is 1 cm, is



(a) $2-\frac{\pi}{3}$ (b) $\sqrt{3}-\pi$ (c) $\sqrt{3}-\frac{\pi}{2}$ (d) $\sqrt{3}-\frac{\pi}{4}$
Ans.
Side of triangle = 2 cm

$$\therefore \text{Area of triangle} = \frac{\sqrt{3}}{4}\times 2^2 = \sqrt{3}\ \text{cm}^2$$

$$\text{Area of one sector} = \frac{60^\circ}{360^\circ}\times \pi \times 1^2 = \frac{1}{6}\pi\ \text{cm}^2$$

$$\text{Area of 3 sectors} = 3\times\frac{1}{6}\pi = \frac{1}{2}\pi\ \text{cm}^2$$

$$\text{Area of shaded region} = \left(\sqrt{3}-\frac{1}{2}\pi\right)\text{cm}^2$$

8. If the difference between the circumference and the radius of of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:
(a) 154 (b) 44 (c) 14 (d) 7
Ans. (b) 44
A.T.Q.

$$2\pi r - r = 37 \Rightarrow 2\times\frac{22}{7}r - r = 37$$

$$\frac{37}{7}r = 37 \Rightarrow r = 7\ \text{cm}$$

$$\therefore \text{Circumference} = 2\times\frac{22}{7}\times 7 = 44\ \text{cm}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If the radius of an arc is 8 cm and the central angle is 40°, then the length of an arc is 5.59 cm.

Reason (R): Length of arc = $\pi r^2 \times \frac{\theta}{360^0}$

Ans. (c) Assertion (A) is true but reason (R) is false.

10. Assertion (A): In a circle of radius 4 cm, the angle of a sector is 45°, then the area of the sector is 44/7 cm².

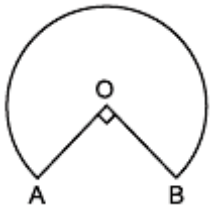
Reason (R): Area of sector = $\pi r^2 \times \frac{\theta}{360^0}$

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

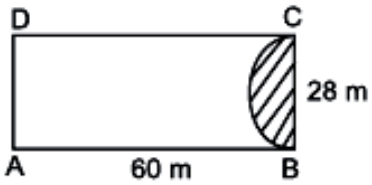
11. In the given figure, the shape of the top of a table is that a sector of a circle with centre O and ∠ AOB = 90°. If AO = OB = 42 cm, then find the perimeter of the top of the table. [Use $\pi = \frac{22}{7}$]



Ans. Perimeter = length of major arc + 2r

$= \frac{270^0}{360^0} \times 2 \times \pi r + 2r = \frac{3}{2} \times \frac{22}{7} \times 42 + 2 \times 42 = 198 + 84 = 284$ cm

12. A plot is in the form of a rectangle ABCD having semicircle on BC as shown in the figure. The semicircle portion is grassy while the remaining plot is without grass. Find the area of the plot without grass where AB = 60 m and BC = 28 m. [Use $\pi = \frac{22}{7}$]



Ans. Length of the rectangle = AB = 60 m
Breadth of the rectangle = BC = 28 m
Diameter of the shaded portion = 28 m
$\Rightarrow$ Radius of the shaded portion = $\frac{28}{2}$ =14 m

Grass portion = shaded portion

$\Rightarrow$ Area of the plot without grass = area of the rectangle ABCD – area of the shaded portion

$= \left[60\times 28-\frac{1}{2}\times\frac{22}{7}\times 14^2\right]m^2 = (1680 - 308)\text{m}^2 = 1372\ \text{m}^2$

13. The measure of the minor arc of a circle is 1/5 of the measure of the corresponding major arc. If the radius of the circle is 10.5 cm, find the area of the sector corresponding to the major arc. [π = 22/7]

Ans. Let measure of minor arc = $x°$

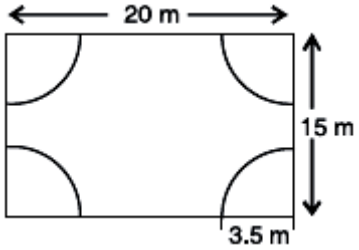
$\therefore$ Measure of major arc = $5x°$

$\Rightarrow x° + 5x° = 360° \Rightarrow 6x° = 360° \Rightarrow x = 60°$

$\therefore$ Measure of major arc = $5 \times 60° = 300°$

Area of sector = $\frac{300^0}{360^0}\times\frac{22}{7}\times 10.5^2 = 288.75\ \text{cm}^2$

14. A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.



Ans. Angle of each quadrant = 90°

Radius of each quadrant = 3.5 m

Area of each quadrant = $\frac{90^0}{360^0}\times\pi\times(3.5)^2\,\text{m}^2 = \frac{1}{4}\times\pi\times(3.5)^2\,\text{m}^2$

$\therefore$ Area of the 4 quadrants = $4\times\frac{1}{4}\times\pi\times(3.5)^2 = \frac{22}{7}\times(3.5)^2 = 38.5\ \text{m}^2$

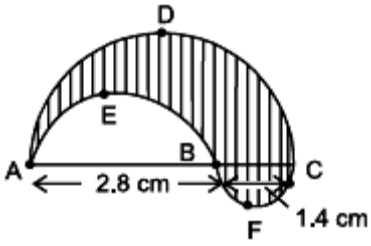
Area of the rectangle = 20 m × 15 m = 300 m^2

$\therefore$ Area of the remaining portion = $(300 - 38.5)\text{m}^2 = 261.5\ \text{m}^2$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In the fig., find the perimeter of shaded region where ADC, AEB and BFC are semicircles on diameters AC, AB and BC respectively.



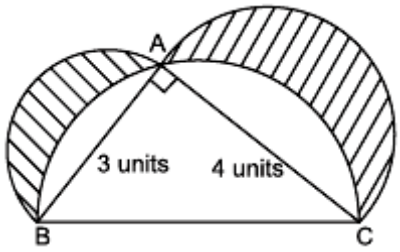
Ans Length of semicircle ADC = $\pi\times\frac{4.2}{2}cm = 2.1\pi cm$

Length of semicircle AEB = $\pi\times\frac{2.8}{2}cm = 1.4\pi cm$

Length of semicircle BFC = $\pi\times\frac{1.4}{2}cm = 0.7\pi cm$

$\therefore$ Perimeter of shaded region = $2.1\pi + 1.4\pi + 0.7\pi = 4.2\pi$ cm = 13.2 cm

16. In fig., ABC is a right-angled triangle, right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Ans. In right-angled ΔABC, $AB^2 + AC^2 = BC^2$ [By Pythagoras theorem]

$(3)^2 + (4)^2 = BC^2 \Rightarrow 9 + 16 = BC^2$

$\therefore$ BC = 5 units

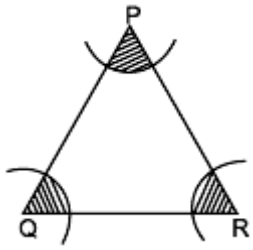
Now, Area of shaded region = area of semicircle on side AB + area of semicircle on side AC – area of semicircle on side BC + area of ΔABC.

$$= \frac{1}{2}\pi \times \frac{3}{2} \times \frac{3}{2} + \frac{1}{2}\pi \times \frac{4}{2} \times \frac{4}{2} - \frac{1}{2}\pi \times \frac{5}{2} \times \frac{5}{2} + \frac{1}{2} \times 3 \times 4$$

$$= \frac{1}{2}\pi\left(\frac{9}{4} + \frac{16}{4} - \frac{25}{4}\right) + 6 = \frac{1}{2}\pi\left(\frac{9+16-25}{4}\right) + 6 = 0 + 6 = 6$$

$\therefore$ Area of shaded region = 6 square units.

17. In figure arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Ans. Area of sector on P = $\frac{\angle P}{360^0} \times \pi(14)^2$

Area of sector on Q = $\frac{\angle Q}{360^0} \times \pi(14)^2$

Area of sector on R = $\frac{\angle R}{360^0} \times \pi(14)^2$

Area of shaded region = adding area of all three sectors

$$= \frac{\angle P}{360^0} \times \pi(14)^2 + \frac{\angle Q}{360^0} \times \pi(14)^2 + \frac{\angle R}{360^0} \times \pi(14)^2$$

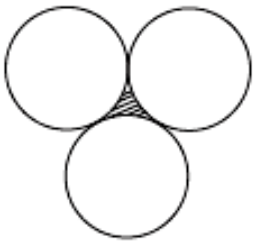
$$= \frac{\pi(14)^2}{360^0}(\angle P + \angle Q + \angle R) = \frac{\pi(14)^2}{360^0} \times 180^0 \text{ [Angle sum property of triangle]}$$

$$= \frac{22}{7} \times \frac{196}{360^0} \times 180^0 = \frac{22}{7} \times 98 = 308 cm^2$$

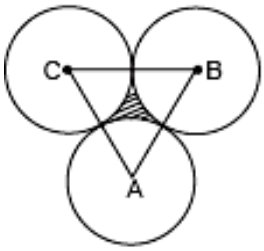
SECTION – D

Questions 18 carry 5 marks.

18. In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area of shaded region enclosed between these three circles. [Use $\pi = 22/7$ and $\sqrt{3} = 1.732$]



Ans. The ΔABC is an equilateral triangle each of whose side is of length = 3.5 + 3.5 = 7 cm
∴ ∠A = ∠B = ∠C = 60°



$\text{ar}(\Delta ABC) = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(7^2) = \frac{\sqrt{3}}{4} \times 49 cm^2$

$\text{Area of 3 sectors} = 3 \times \frac{60^0}{360^0} \times \pi r^2 = 3 \times \frac{60^0}{360^0} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{77}{4} cm^2$

Area of shaded region = area of ΔABC – area of 3 sectors

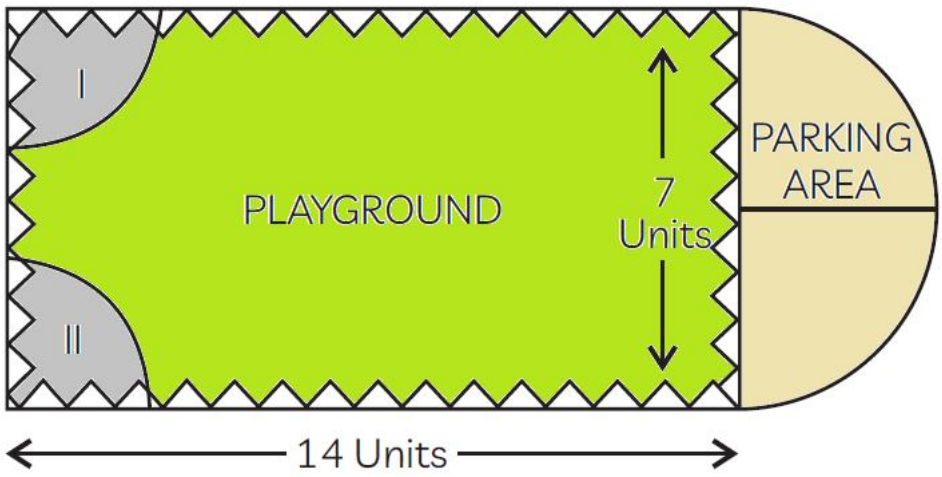
$= \frac{\sqrt{3}}{4} \times 49 - \frac{77}{4} = \frac{1}{4}(49\sqrt{3} - 77) cm^2$

$= \frac{1}{4}(49 \times 1.732 - 77) cm^2 = \frac{1}{4}(84.868 - 77) = \frac{1}{4} \times 7.868 = 1.967 cm^2$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.
After survey, it was decided to build rectangular playground, with a semi–circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.



Based on the above information, answer the following questions:
(a) What is the total perimeter of the parking area? (1)
(b) What is the total area of parking and the two quadrants? (2)

OR

What is the ratio of area of playground to the area of parking area?

(c) Find the cost of fencing the playground and parking area at the rate of ₹ 2 per unit. (1)

Ans. (a) Perimeter of parking area = $\pi r + 2r = r(\pi + 2) = \frac{7}{2}\left(\frac{22}{7} + 2\right) = \frac{7}{2} \times \frac{36}{7}$ = 18 units

(b) Total area of parking and the two quadrants = area of semi-circular region + area of 2 quadrants

$$= \frac{\pi R^2}{2} + 2 \times \frac{1}{4} \times \pi r^2 = \frac{\pi}{2}[R^2 + r^2] = \frac{22}{7 \times 2}\left[\left(\frac{7}{2}\right)^2 + (2)^2\right]$$

$$= \frac{11}{7}\left[\frac{49}{4} + 4\right] = \frac{11}{7}\left[\frac{49+16}{4}\right] = \frac{11}{7}\left[\frac{65}{4}\right]$$

$$= \frac{715}{28} \text{ unit}^2 = 25.54 \text{ unit}^2 \text{ (approx)}$$

OR

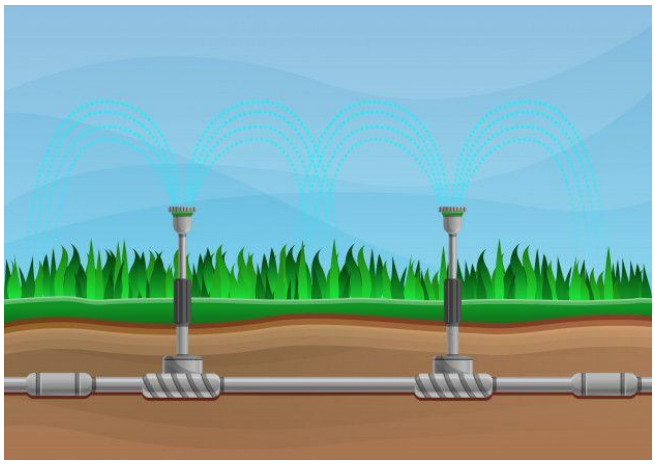
Area of playground = 14 × 7 units²

Area of parking = $\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4}$ units²

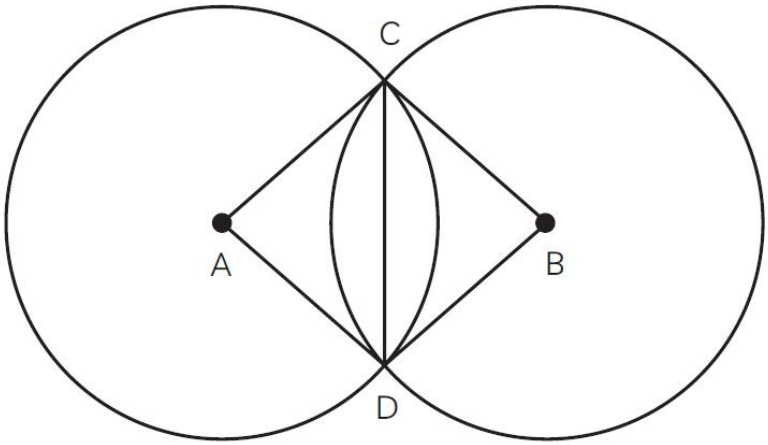
∴ Ratio of area of playground to the area of parking area = $\frac{14 \times 7}{\frac{77}{4}} = \frac{56}{11} = 56 : 11$

(c) Length of fencing = Perimeter of rectangular playground + Perimeter of parking area
= 2(l + b) + 2πr/2
= 2(14 + 7) + 22/7 × 7/2
= 42 + 11 = 53 units
Cost of fencing = 53 × 2 = ₹ 106

20. Sprinklers are crop irrigation equipment which rotate around a center and spay water on the crops in the circular region.
Two such high powers sprinklers, occupying negligible area are installed in a straight line in a field such that they spray water on an common area. Shown below are the side and top views where points A and B are the sprinklers.



Side view of the sprinklers



Top view of the region sprayed

(Note: The figures are not to scale.)

Both the sprinklers spray over an equal area. It is given that, CD = 400 m and ∠CAD = ∠CBD = 90°.

(a) Find the radius of the circular region sprayed by the sprinkler. (1)

(b) Find the area of the overlapping region. (2)

(c) Find the perimeter of the overlapping region. (1)

Ans. (a) Using Pythagoras Theorem in ΔACD to find the length of the AC as:

$CD^2 = AC^2 + AD^2$

$\Rightarrow 160000 = 2AC^2$

$\Rightarrow 200\sqrt{2} = AC$

(b) Area of sector CAD = area of sector CBD = $\frac{90^0}{360^0} \times 3.14 \times (200\sqrt{2})^2$

$= 62800 \text{ m}^2$

Area of ΔCAD = area of ΔCBD = $\frac{1}{2} \times 200\sqrt{2} \times 200\sqrt{2}$

$= 40000 \text{ m}^2$.

Area of the overlapping region = 62800 + 62800 – 40000 – 40000 = 45600 m^2.

(c) Perimeter of overlapping region = sum of the length of the both side arc CD

$= 2 \times \frac{90^0}{360^0} \times 2\pi \times (200\sqrt{2}) = 200\sqrt{2} \text{ m}$

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 13 – SAMPLE PAPER PT2 (2024-25)

CHAPTER 07 to 11

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

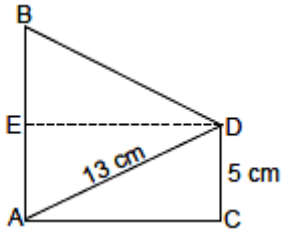
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 (b) 12 (c) 11 (d) $7 + \sqrt{5}$

2. In the given figure, if AB = 14 cm, then the value of tan B is:



(a) 4/3 (b) 14/3 (c) 5/3 (d) 13/3

3. If $\sin\theta = \sqrt{3}\cos\theta$, $0° < \theta < 90°$, then θ is equal to

(a) 30° (b) 45° (c) 60° (d) 90°

4. If secx + tanx = x, then tanA =

(a) $\frac{2}{x}$ (b) $\frac{1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{2x}{x^2-1}$

5. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 8 m (b) 10 m (c) 12 m (d) 14 m

6. The angle of elevation of the top of a tower from a point 20 metres away from its base is 45°. The height of the tower is

(a) 10 m (b) 20 m (c) 30 m (d) $20\sqrt{3}$ m

7. A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is

(a) 8 units (b) 10 units (c) $10\sqrt{2}$ units (d) $8\sqrt{2}$ units

8. The ratio of the areas of the incircle and circumcircle of a square is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) $1:\sqrt{2}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.
Reason (R): If the angle between two radii of a circle is 130°, then the angle between the tangents at the end points of radii at their point of intersection is 50°.

10. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is 77/6 cm^2.
Reason (R): The length of an arc of a sector of angle q and radius r is given by $l = \frac{\theta}{360^0} \times 2\pi r$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If (1 + cos A) (1 – cos A) = 3/4 , find the value of tan A.

12. A rope by which a cow is tethered is increased from 16mto 23m. How much additional ground does it have now to graze?

13. Find A and B, if sin (A + 2B) = $\sqrt{3}/2$ and cos (A + B) = 1/2.

14. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are vertices of a parallelogram ABCD. Find the values of a and b.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. [Use $\pi = \frac{22}{7}$]

16. Prove that: $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

OR

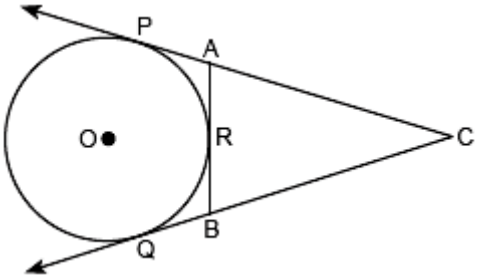
Prove that: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$

17. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the center

SECTION – D

Questions 18 carry 5 marks.

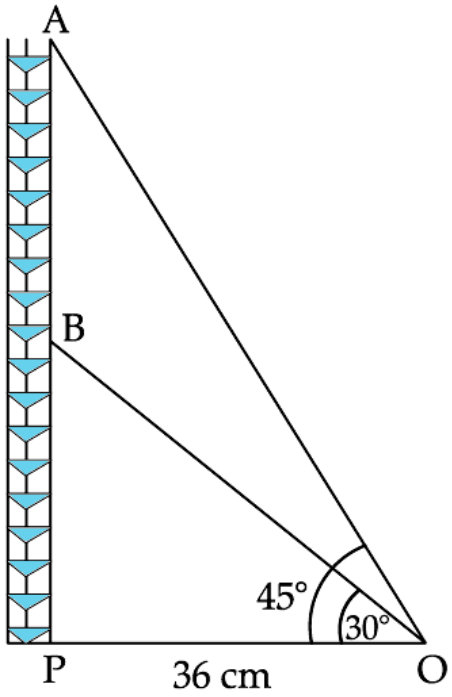
18. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (3)
(b) In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR. (2)



SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.



Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.

Based on the above information, answer the following questions:

(i) Find the length of the wire from the point O to the top of section B.

(ii) Find the distance AB.

OR

Find the area of ΔOPB.

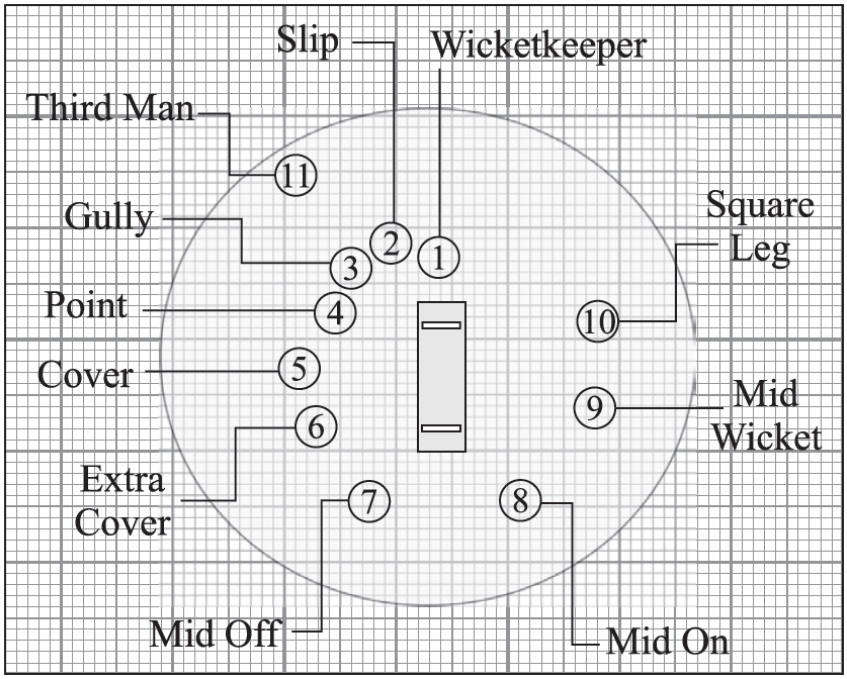
(iii) Find the height of the Section A from the base of the tower.

20. In the sport of cricket the Captain sets the field according to a plan. He instructs the players to take a position at a particular place. There are two reasons to set a cricket field—to take wickets and to stop runs being scored.

The following graph shows the position of players during a cricket match.

(i) Find the coordinate of the point on y-axis which are equidistant from the points representing the players at Cover P(2, –5) and Mid-wicket Q(–2, 9)

(ii) Find the ratio in which x-axis divides the line segment joining the points Extra Cover S(3, –3) and Fine Leg (–2, 7).


Slip
Wicketkeeper
Third Man
11
Gully
2
1
Square
Leg
3
Point
4
10
Cover
5
9
Mid
Wicket
6
Extra
Cover
7
8
Mid Off
Mid On

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS, HYD-32

PRACTICE PAPER 13 – SAMPLE PAPER PT2 (2024-25)

CHAPTER 07 to 11 (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

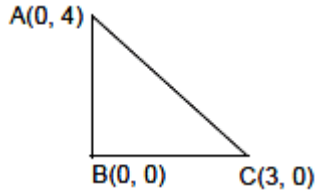
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 (b) 12 (c) 11 (d) 7 + $\sqrt{5}$

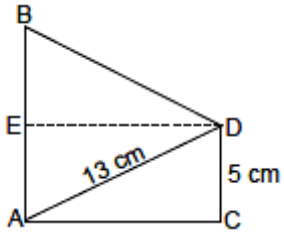
Ans. (b) 12



Perimeter of ΔABC = AB + BC + AC

= 4 + 3 + $\sqrt{4^2+3^2}$ = 7 + $\sqrt{25}$ = 7 + 5 = 12

2. In the given figure, if AB = 14 cm, then the value of tan B is:



(a) 4/3 (b) 14/3 (c) 5/3 (d) 13/3

Ans.

(*a*) Hint: AC = $\sqrt{13^2-5^2}$ = 12

DE = AC = 12 cm, BE = 14 cm – 5 cm = 9 cm

In ΔBED, tan B = $\frac{DE}{BE} = \frac{12}{9} = \frac{4}{3}$.

3. If sin θ = $\sqrt{3}$ cos θ, 0° < θ < 90°, then θ is equal to

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. (c) 60°

$\sin\theta = \sqrt{3}\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$

4. If secx + tanx = x, then tanA =

(a) $\frac{2}{x}$ (b) $\frac{1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{2x}{x^2-1}$

Ans. (c) $\frac{x^2-1}{2x}$

$\sec A + \tan A = x$

Also $\sec^2 A - \tan^2 A = 1$

$\Rightarrow (\sec A - \tan A)(\sec A + \tan A) = 1$

$\Rightarrow x(\sec A - \tan A) = 1$

$\therefore \sec A - \tan A = \frac{1}{x}$

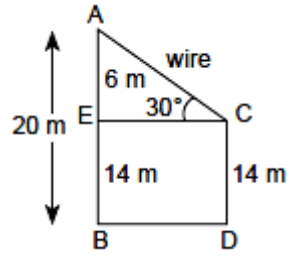
Now, subtracting (ii) from (i), we have

$$\tan A = \frac{x^2-1}{2x}$$

5. The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 8 m (b) 10 m (c) 12 m (d) 14 m

Ans. (c) 12 m



$$\frac{AE}{AC} = \sin 30°$$

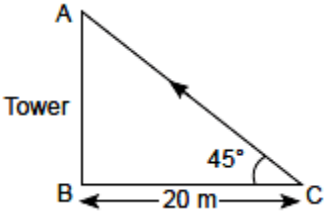
$$\Rightarrow \frac{6}{AC} = \frac{1}{2}$$

$$\Rightarrow AC = 12 \text{ m}$$

6. The angle of elevation of the top of a tower from a point 20 metres away from its base is 45°. The height of the tower is

(a) 10 m (b) 20 m (c) 30 m (d) $20\sqrt{3}$ m

Ans.



$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \frac{AB}{20} = 1$$

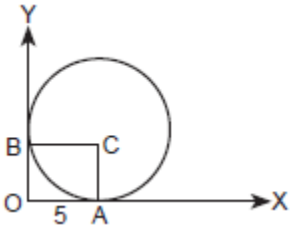
$$\Rightarrow AB = 20 \text{ m}.$$

7. A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is

(a) 8 units (b) 10 units (c) $10\sqrt{2}$ units (d) $8\sqrt{2}$ units

Ans. (b) 10 units

OA = OB ⇒ OB = 5



AC = BC [Radii]

⇒ OACB is a square.

⇒ AC = OA = 5

⇒ Diameter = 10 units

8. The ratio of the areas of the incircle and circumcircle of a square is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) $1:\sqrt{2}$

Ans. (a) 1 : 2
Let side of square = x units

∴ Diagonal of the square = $\sqrt{2}$ x units
Diameter of the incircle = x units

Diameter of the circumcircle = $\sqrt{2}$ x units

$$\therefore \frac{\text{Area of incircle}}{\text{Area of circumcircle}} = \frac{\pi\left(\frac{x}{2}\right)^2}{\pi\left(\frac{\sqrt{2}x}{2}\right)^2} = \frac{1}{2}.$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.
Reason (R): If the angle between two radii of a circle is 130°, then the angle between the tangents at the end points of radii at their point of intersection is 50°.
Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is 77/6 cm^2.

Reason (R): The length of an arc of a sector of angle q and radius r is given by $l = \frac{\theta}{360^0} \times 2\pi r$

Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A)

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If (1 + cos A) (1 – cos A) = 3/4 , find the value of tan A.
Ans: (1 + cos A) (1 – cos A) = 3/4
⇒ $1 - \cos^2 A = 3/4 \Rightarrow \cos^2 A = 1 - 3/4 = 1/4 \Rightarrow \cos A = \pm 1/2$
Also, $1 - \cos^2 A = 3/4 \Rightarrow \sin^2 A = 3/4 \Rightarrow \sin A = \pm\sqrt{3}/2$
⇒ $\tan A = \sin A / \cos A = \pm\sqrt{3}$

12. A rope by which a cow is tethered is increased from 16mto 23m. How much additional ground does it have now to graze?
Ans: Given : length of rope (r) = 16 m
Increased length of rope (R) = 23 m
Hence the additional area cow can graze = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

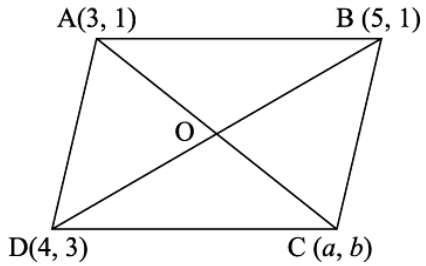
$$= \frac{22}{7}(23^2 - 16^2) = \frac{22}{7}(529 - 256)$$

$$= \frac{22}{7} \times 273 = 858 m^2$$

13. Find A and B, if sin (A + 2B) = $\sqrt{3}/2$ and cos (A + B) = 1/2.
Ans: Given : sin (A + 2B) = sin 60°
⇒ A + 2B = 60° ...(i)
cos (A + B) = cos 60°

$\Rightarrow$ A + B = 60° ...(ii)
Subtracting equation (i) and (ii), we get B = 0°
Putting the value of B in equation (ii), we get,
A = 60° – 0° = 60°
So, A = 60° and B = 0°.

14. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are vertices of a parallelogram ABCD. Find the values of a and b.
Ans:
ABCD is a parallelogram.



Since, the diagonals of a parallelogram bisect each other.

$$\therefore \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{4+5}{2}, \frac{3+1}{2}\right)$$

$$\Rightarrow \frac{3+a}{2} = \frac{9}{2} \quad \Rightarrow 3 + a = 9$$

$$\Rightarrow a = 6$$

$$\text{and } \frac{1+b}{2} = \frac{4}{2} \Rightarrow 1 + b = 4 \Rightarrow b = 3$$

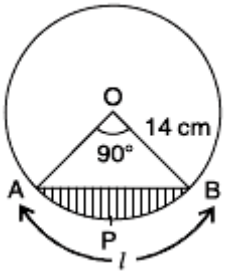
Hence, $a = 6$ and $b = 3$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. [Use $\pi = \frac{22}{7}$]

Ans: l = APB = 22 cm



$$\frac{\theta}{360^0} \times 2\pi r = 22 \Rightarrow \frac{\theta}{360^0} \times 2 \times \frac{22}{7} \times 14 = 22$$

$$\Rightarrow \theta = 360^0 \times \frac{7}{22} \times 22 \times \frac{1}{2 \times 14} = \frac{360^0}{4} = 90^0$$

Area of the sector = $\frac{lr}{2} = \frac{22 \times 14}{2}$ = 154 cm^2

Area of the triangle AOB = $\frac{1}{2}$ × OA × OB = $\frac{1}{2}$ × 14 × 14 = 98 cm^2

Area of the segment = (154 – 98) cm^2 = 56 cm^2

16. Prove that: $\frac{1}{\cos ec\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\cos ec\theta + \cot\theta}$

Ans.

$$\text{LHS} = \frac{1}{\text{cosec}\,\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{(\text{cosec}\,\theta + \cot\theta)}{(\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)} - \text{cosec}\,\theta$$

$$= \frac{\text{cosec}\,\theta + \cot\theta}{\text{cosec}^2\,\theta - \cot^2\theta} - \text{cosec}\,\theta = \text{cosec}\,\theta + \cot\theta - \text{cosec}\,\theta = \text{cosec}\,\theta - (\text{cosec}\,\theta - \cot\theta)$$

$$= \frac{1}{\sin\theta} - \frac{(\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)}{\text{cosec}\,\theta + \cot\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\text{cosec}^2\theta - \cot^2\theta}{\text{cosec}\,\theta + \cot\theta} = \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\,\theta + \cot\theta} = \text{RHS}$$

OR

Prove that: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$

And: LHS = $\frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ (Dividing numerator and denominator by cosθ)

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta + 1 - \sec\theta}$$

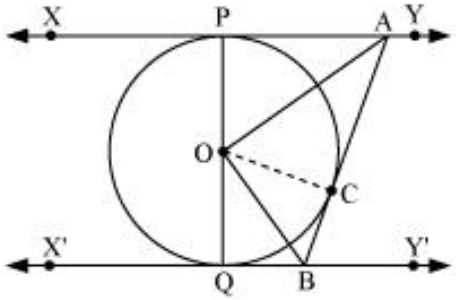
$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta + 1 - \sec\theta}$$

$$= \sec\theta + \tan\theta = \text{RHS}$$

17. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the center

Ans: Given: XY and X'Y' are two parallel tangents to the circle wth centre O and AB is the tangent at the point C, which intersects XY at A and X'Y' at B.



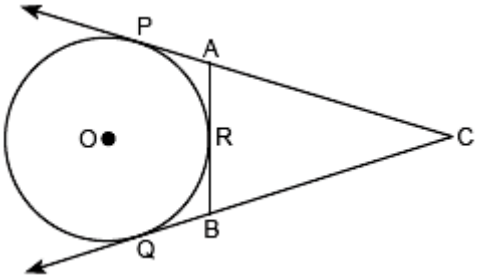
In ΔOAP and ΔOAC
AP = AC (Tangents from to same point A)
PO = OC (Radii of the same circle)
OA = OA (Common side)
so, ΔOAP = ΔOAC (SSS congruence criterion)
∴ ∠AOP =∠AOC = ∠1 (CPCT)
Similarly, ∠BOQ =∠BOC = ∠2
Now, POQ is a diameter of the circle.
Hence, it is a straight line.
∴ ∠1 + ∠1 + ∠2 + ∠2 = 180°
2(∠1 + ∠2) = 180°
∴ ∠1 + ∠2 = 90°
∴ ∠AOB = 90°.

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (3)

(b) In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR. (2)

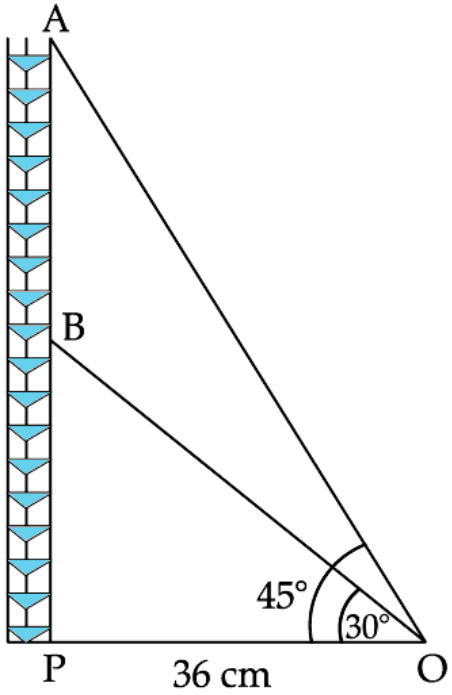


Ans. (a) Given, To Prove, Construction and Figure – 1½ marks
Correct Proof – 1½ marks
(b) Since CP = CQ = 11cm [Length of the two tangents from same external point]
CQ = CB + BQ
But BQ = BR
Therefore, 11 = 7 + BR $\Rightarrow$ BR = 4 cm

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

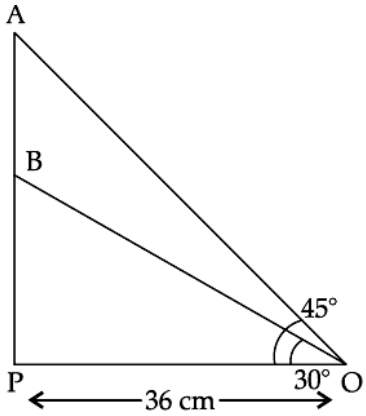


Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.
Based on the above information, answer the following questions:
(i) Find the length of the wire from the point O to the top of section B.
(ii) Find the distance AB.

OR

Find the area of ΔOPB.
(iii) Find the height of the Section A from the base of the tower.

Ans: (i) In ΔBPO, $\cos\theta = \frac{B}{H} \Rightarrow \cos 30^0 = \frac{OP}{OB}$



$\Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{OB} \Rightarrow OB = \frac{72}{\sqrt{3}} = 24\sqrt{3}cm$

Thus, the length of wire from O to top of Section B = $24\sqrt{3}$ cm .

(ii) AB = AP – BP

In ΔBPO, $\tan\theta = \frac{P}{B} \Rightarrow \tan 30^0 = \frac{BP}{OP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36} \Rightarrow BP = \frac{36}{\sqrt{3}} = 12\sqrt{3}cm$

In ΔAPO, $\tan 45^0 = \frac{AP}{OP} \Rightarrow 1 = \frac{AP}{36} \Rightarrow AP = 36cm$

Distance AB = $36 - 12\sqrt{3}$

= 36 – 20.78 = 15.22 cm (approx)

OR

Area of ΔOPB = $\frac{1}{2} \times$ Base × height

= $\frac{1}{2} \times 36 \times 12\sqrt{3} = 216\sqrt{3}$ cm^2

= 374.12 cm^2 (approx)

(iii) Height of Section A from base of tower = AP

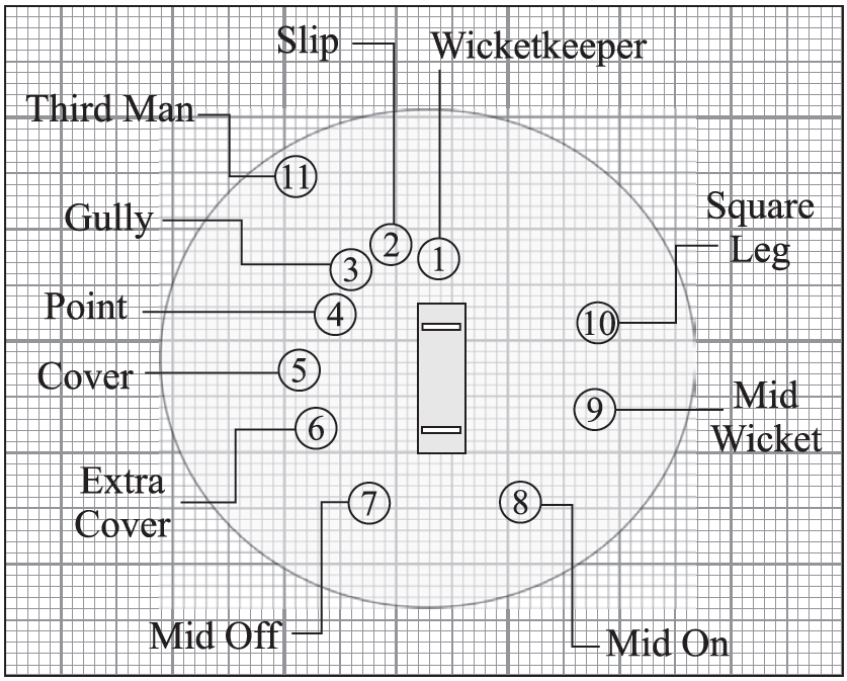
In ΔAPO, $\tan 45^0 = \frac{AP}{OP} \Rightarrow 1 = \frac{AP}{36} \Rightarrow AP = 36cm$

20. In the sport of cricket the Captain sets the field according to a plan. He instructs the players to take a position at a particular place. There are two reasons to set a cricket field—to take wickets and to stop runs being scored.

The following graph shows the position of players during a cricket match.

(i) Find the coordinate of the point on y-axis which are equidistant from the points representing the players at Cover P(2, –5) and Mid-wicket Q(–2, 9)

(ii) Find the ratio in which x-axis divides the line segment joining the points Extra Cover S(3, –3) and Fine Leg (–2, 7).



Ans: (i) Let A (0, y) be any point on the y-axis.

Since A (0, y) is equidistant from P (2, –5) and Q (–2, 9)

So AP = AQ $\Rightarrow$ $AP^2 = AQ^2$

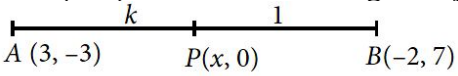
$\Rightarrow (2)^2 + (y + 5)^2 = (2)^2 + (y - 9)^2 \Rightarrow y^2 + 10y + 25 = y^2 - 18y + 81$

$\Rightarrow 28y = 81 - 25 \Rightarrow 28y = 56$

$\Rightarrow y = 28/56 = 2$

So, the point is (0, 2)

(ii) Let point P(x, 0) divides the line segment joining the points A and B in the ratio k : 1



Using section formula,

Coordinates of P are $\left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$

y-coordinate of $P = \frac{7k-3}{k+1} = 0$

$\Rightarrow \quad 7k = 3 \quad \Rightarrow \quad k = \frac{3}{7}$

Hence, the point P divides the line segment in the ratio 3 : 7.

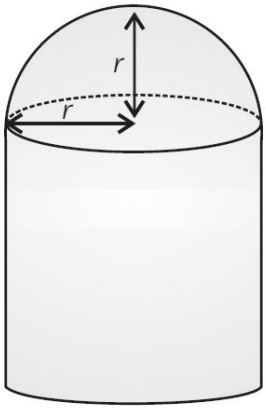
PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32
PRACTICE PAPER 14 (2024-25)
CHAPTER 12 SURFACE AREAS AND VOLUMES

SUBJECT: MATHEMATICS STANDARD **MAX. MARKS : 40**
CLASS : X **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. A solid figure made up of a right circular cylinder and a hemisphere of equal radius (r cm) has been shown. The total surface area of the solid is equal to the surface area of a sphere with twice the radius of this solid.



Which of the following gives the height of the cylinder in the above solid?
(a) 6r cm (b) 6.5r cm (c) 7r cm (d) 17.5r cm

2. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?
(a) k^3 (b) $2k^3$ (c) $3k^3$ (d) $6k^3$

3. Volumes of two spheres are in the ratio 27 : 64. The ratio of their surface areas is:
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

4. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is:
(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

5. Two cubes each with 6 cm edge are joined end to end. The surface area of the resulting cuboid is:
(a) 180 cm^2 (b) 360 cm^2 (c) 300 cm^2 (d) 260 cm^2

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by
(a) 3 (b) 4 (c) 5 (d) 6

7. A rectangular block 6 cm × 12 cm × 15 cm is cut into exact number of equal cubes. The least possible number of cubes will be
(a) 6 (b) 11 (c) 33 (d) 40

8. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.
(a) 1 : 3 (b) 1 : √3 (c) √3 : 1 (d) 3 : 1

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If two identical solid cube of side 7 cm are joined end to end. Then the total surface area of the resulting cuboid is 490 cm^2.
Reason (R): Total surface area of cuboid = lb + bh + hl

10. Assertion (A): The radii of two cones are in the ratio 2 : 3 and their volumes in the ratio 1 : 3. Then the ratio of their heights is 3 : 2.
Reason (R): Volume of the cone = $\frac{1}{3}\pi r^2 h$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that 1/8 space of the cube remains unfilled. Find the number of marbles that the cube can accommodate.

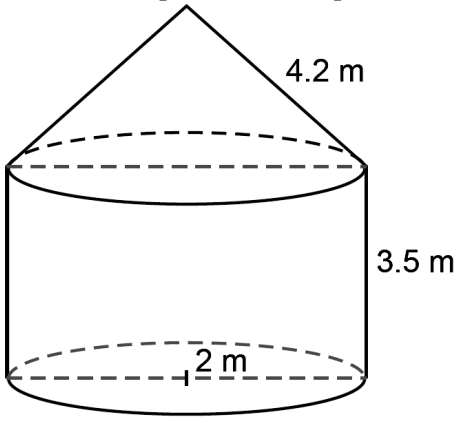
12. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. [Use π = 22/7]

13. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. [Use π = 22/7]

OR

A medicine-capsule is in the shape of a cylinder of radius 0.25 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. What is the total surface area of the capsule?

14. In the below figure, a tent is in the shape of a cylinder surmounted by a conical top. The cylindrical part is 3.5 m high and conical part has slant height 4.2 m. Both the parts have same radius 2 m. Find the area of the canvas used to make the tent. [Use π = 22/7]

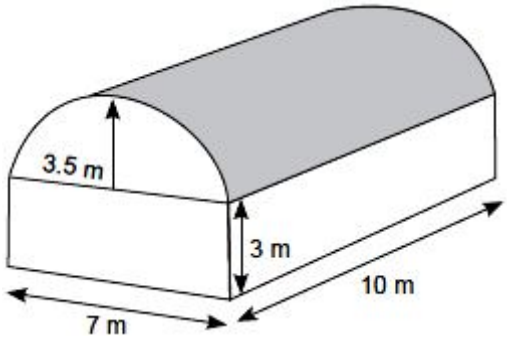


SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of the cone.

16. A godown building is in the form as shown in the figure.



The vertical cross section parallel to the width side of the building is a rectangle of dimensions 7 m × 3 m, mounted by semicircle of radius 3.5 m. The inner measurements of the cuboidal portion of the building are 10 m × 7 m × 3 m. Find the interior surface excluding the floor.

17. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

SECTION – D

Questions 18 carry 5 marks.

18. There are two identical solid cubical boxes of side 7 cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find :
(a) the ratio of the total surface area of the two new solid formed.
(b) volume of each new solid formed.

OR

A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [Use $\pi = 3.14$].

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical upto a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.

Based on the above, answer the following questions:

(i) Find slant height of the conical part. (1)

(ii) Determine the floor area of the tent. (1)

(iii) (a) Find area of the cloth used for making tent. (2)

OR

(b) Find total volume of air inside an empty tent.

20. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery. Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreigners with a variety of tea-sets, crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also refered as ‘‘The Ceramic Town’’.

One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja.

II I

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following questions :

(a) The internal radius of hemispherical bowl (filled completely with water) in I is 9 cm and radius and height of cylindrical jar in II is 1.5 cm and 4 cm respectively. If the hemispherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required ? (2)

(b) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar ? (2)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32

PRACTICE PAPER 14 (2024-25)

CHAPTER 12 SURFACE AREAS AND VOLUMES

(ANSWERS)

SUBJECT: MATHEMATICS STANDARD **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

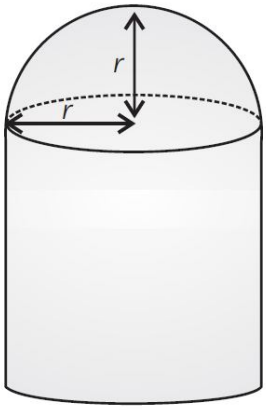
(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. A solid figure made up of a right circular cylinder and a hemisphere of equal radius (r cm) has been shown. The total surface area of the solid is equal to the surface area of a sphere with twice the radius of this solid.



Which of the following gives the height of the cylinder in the above solid?

(a) 6r cm (b) 6.5r cm (c) 7r cm (d) 17.5r cm

Ans. (c) 7r cm

Total surface area of solid = CSA of cylinder + CSA of hemisphere = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Now, according to the question, TSA of solid = Surface area of sphere

With twice the radius of the solid

Taking radius of sphere, R = 2r [Given]

So, TSA of solid = Surface area of sphere

$\Rightarrow 2\pi r(h + r) = 4\pi R^2 \Rightarrow 2\pi r(h + r) = 4\pi(2r)^2 \Rightarrow 2\pi rh + 2\pi r^2 = 16\pi r^2$

$\Rightarrow 2\pi rh = 14\pi r^2 \Rightarrow 2h = 14r \Rightarrow h = 7r$ cm

2. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?

(a) k^3 (b) $2k^3$ (c) $3k^3$ (d) $6k^3$

Ans. (b) $2k^3$

Length of resulting cuboid, l = k + k = 2k

Height of resulting cuboid, h = k

Breadth of resulting cuboid, b = k

Volume of cuboid = $l \times b \times h = 2k \times k \times k = 2k^3$

3. Volumes of two spheres are in the ratio 27 : 64. The ratio of their surface areas is:

(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

Ans. (c) 9 : 16

Let the radius of two spheres be r_1 and r^2.

Given, the ratio of the volume of two spheres = 27 : 64

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{27}{64} \Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$$

Let the surface areas of the two spheres be S1 and S2.

$$\therefore \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

4. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is:

(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

Ans. (a) 2 : 1

Since, the radius of cone and cylinder are equal i.e., r(say).

Also, curved surface area of cone = curved surface area of cylinder

i.e., $\pi r l = 2\pi rh$ [Given]

$\Rightarrow l/h = 2/1 = 2 : 1$

5. Two cubes each with 6 cm edge are joined end to end. The surface area of the resulting cuboid is:

(a) 180 cm² (b) 360 cm² (c) 300 cm² (d) 260 cm²

Ans. (b) 360 cm²

When two cubes with side length of 6 cm are joined end to end, they form a cuboid. The resulting cuboid has different dimensions.

∴ Surface area of the resulting cuboid

A = 2lb + 2lh + 2bh

= 2 (12) 6 + 2 (12) (6) + 2(6) (6)

= 144 + 144 + 72

= 360 cm²

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by

(a) 3 (b) 4 (c) 5 (d) 6

Ans. (a) 3

Volume of sphere

= Volume of water in cylindrical vessel

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\Rightarrow \frac{4}{3} \times 9 \times 9 \times 9 = 18 \times 18 \times h \qquad \left[\because r = \frac{d}{2}\right]$$

$$\Rightarrow \frac{4 \times 9 \times 9 \times 9}{3 \times 18 \times 18} = h \Rightarrow h = 3 \text{ cm}$$

Water level rises by 3 cm

7. A rectangular block 6 cm × 12 cm × 15 cm is cut into exact number of equal cubes. The least possible number of cubes will be

(a) 6 (b) 11 (c) 33 (d) 40

Ans. (d) 40

Volume of rectangular block = 6 × 12 × 15 = 1080 cm^3

Side of largest cube = HCF of 6, 12, 15 = 3

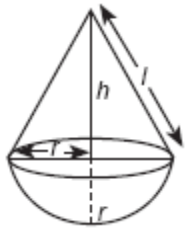
$\therefore$ Volume of 1 cube = $3^3 = 27\ cm^3$

Number of cubes = $\frac{6\times12\times15}{27} = 40$

8. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

(a) 1 : 3 (b) 1 : $\sqrt{3}$ (c) $\sqrt{3}$: 1 (d) 3 : 1

Ans: (b) 1 : $\sqrt{3}$



Let radius of the base be r

Heigth of conical part = h

Slant height of conical part = l

$\therefore\ l = \sqrt{h^2 + r^2}$ ...(i)

ATQ, $2\pi r^2 = \pi rl \Rightarrow l = 2r$

$\therefore$ Equation (i) becomes $2r = \sqrt{h^2 + r^2}$

$\Rightarrow 4r^2 = h^2 + r^2$

$\Rightarrow h^2 = 3r^2 \Rightarrow \frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If two identical solid cube of side 7 cm are joined end to end. Then the total surface area of the resulting cuboid is 490 cm^2.

Reason (R): Total surface area of cuboid = lb + bh + hl

Ans. (c) Assertion (A) is true but reason (R) is false.

When cubes are joined end to end, it forms a cuboid.

Here, $l = 2 \times 7 = 14$ cm, b = 7 cm and h = 7 cm

Total surface area of cuboid = 2(lb + bh + hl) = 2 (14 × 7 + 7 × 7 + 7 × 14) = 490 cm^2

10. Assertion (A): The radii of two cones are in the ratio 2 : 3 and their volumes in the ratio 1 : 3. Then the ratio of their heights is 3 : 2.

Reason (R): Volume of the cone = $\frac{1}{3}\pi r^2 h$.

Ans. (d) Assertion (A) is false but reason (R) is true.

Ratio of volume = $\frac{\frac{1}{3}\pi(2x)^2 h_1}{\frac{1}{3}\pi(3x)^2 h_2} \Rightarrow \frac{1}{3} = \frac{4}{9}\frac{h_1}{h_2} \Rightarrow \frac{h_1}{h_2} = \frac{3}{4}$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that 1/8 space of the cube remains unfilled. Find the number of marbles that the cube can accommodate.

Ans. According to the question,

Volume of cube = $22^3 = 10648 cm^3$

Volume of cube that remains unfilled = $1/8 \times 10648 = 1331 cm^3$

volume occupied by spherical marbles =10648−1331=9317cm^3
Radius of the spherical marble = 0.5/2=0.25cm=1/4cm
Volume of 1 spherical marble = 4/3×22/7 × (1/4)^3=11/168cm^3
Numbers of spherical marbles, n = 9317 × (11/168) =142296

12. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. [Use π = 22/7]
Ans. We have, in a right circular cylinder
Height = Radius ⇒ h = r
and volume of cylinder = $25\frac{1}{7}$ cm^3

$$\Rightarrow \quad \pi r^2 h = \frac{176}{7}$$

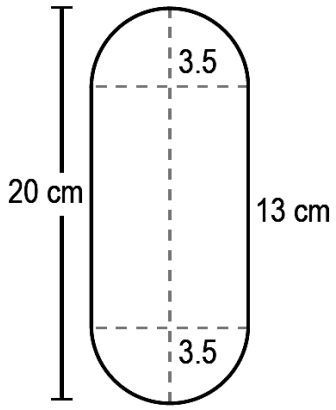
$$\Rightarrow \quad \pi \times h^2 \times h = \frac{176}{7} \quad (\because h = r)$$

$$\Rightarrow \quad \frac{22}{7} \times h^3 = \frac{176}{7}$$

$$\Rightarrow \quad h^3 = \frac{176}{22} = 8 \quad \Rightarrow \quad h^3 = (2)^3 \quad \Rightarrow \quad h = 2 \text{ cm}$$

∴ Height of the cylinder = 2 cm

13. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. [Use π = 22/7]
Ans. Height of cylinder = 20 – 7 = 13 cm.



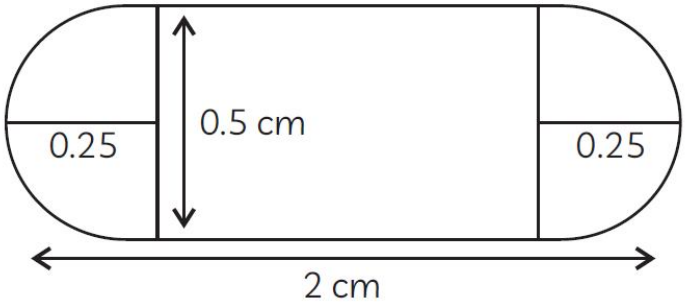
Total Volume of the solid = Volume of Cylinder + Volume of two hemisphere

$$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \pi r^2 \left(h + \frac{4}{3} r \right) = \frac{22}{7} \times \left(\frac{7}{2} \right)^2 \left[13 + \frac{4}{3} . \frac{7}{2} \right]$$

$$= \frac{22}{7} \times \frac{49}{4} \left(13 + \frac{14}{3} \right) = 11 \times \frac{7}{2} \left(\frac{39 + 14}{3} \right) = \frac{77}{2} \left(\frac{53}{3} \right) = 680.17 cm^3$$

OR

A medicine-capsule is in the shape of a cylinder of radius 0.25 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. What is the total surface area of the capsule?
Ans. Given that Radius of cylinder = Radius of hemisphere = 0.25 cm
Total length of capsule = 2 cm

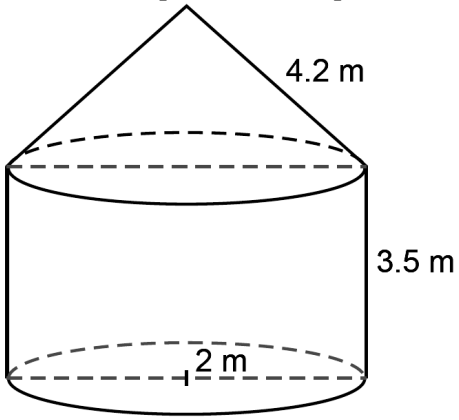


Here, the length of cylindrical part of capsule, h
= length of capsule – radius of both hemispheres = 2 – 2 × 0.25 = 1.5
Total surface area of capsule = CSA of cylindrical part + 2 x CSA of hemisphere
= 2πrh + 2(2πr^2) = 2πr (h + 2r)

$$= 2 \times \frac{22}{7} \times 0.25(1.5 + 2 \times 0.25) = 2 \times \frac{22}{7} \times \frac{1}{4} \times 2 = \frac{22}{7} = 3.14 m^2$$

Therefore, the TSA of the capsule is 3.14 cm2

14. In the below figure, a tent is in the shape of a cylinder surmounted by a conical top. The cylindrical part is 3.5 m high and conical part has slant height 4.2 m. Both the parts have same radius 2 m. Find the area of the canvas used to make the tent. [Use $\pi = 22/7$]



Ans. We have, Area of the canvas used to make the tent
= CSA of cylinder + CSA of conical part

$= 2\pi rh + \pi rl = \pi r(2h + l) = \frac{22}{7} \times 2(2 \times 3.5 + 4.2) = \frac{44}{7} \times 11.2$

$= 70.4\ m^2$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of the cone.

Ans. Length of the arc = $\frac{\theta}{360^0} \times 2\pi r = \frac{1}{3} \times 2\pi \times 12 = 8\pi$ = circumference of the base of the cone

Let radius of cone be r

$\Rightarrow 2 \times \pi \times r = 8\pi \Rightarrow r = 4$ cm

$r = 4$ cm, $l = 12$ cm

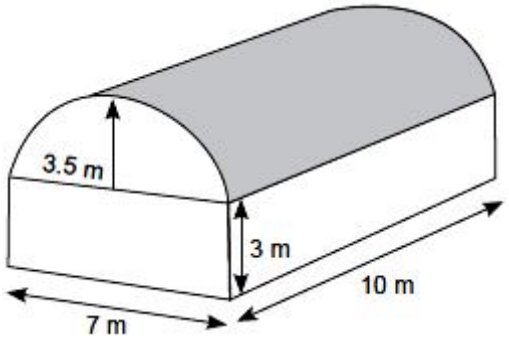
$\Rightarrow h^2 = l^2 - r^2 = 12^2 - 4^2 = 144 - 16$

$\Rightarrow h^2 = 128 \Rightarrow h = \sqrt{128} = 8\sqrt{2}$ cm

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 8\sqrt{2}$

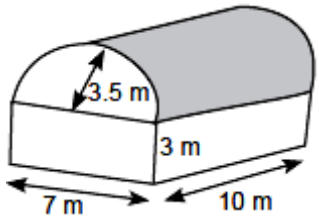
$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 8 \times 1.414\ cm^3 = 189.61\ cm^3$

16. A godown building is in the form as shown in the figure.



The vertical cross section parallel to the width side of the building is a rectangle of dimensions 7 m × 3 m, mounted by semicircle of radius 3.5 m. The inner measurements of the cuboidal portion of the building are 10 m × 7 m × 3 m. Find the interior surface excluding the floor.

Ans. Interior surface of godown = area of four walls + 2 × area of semicircles + curved area of cylindrical roof.



$= 2 \times 3\,(7+10) + 2 \times \frac{\pi(3.5)^2}{2} + \frac{2\pi(3.5)\times 10}{2}$

$= 102 + 38.5 + 110 = 250.5\ m^2$

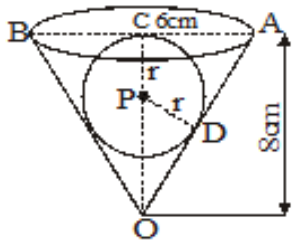
17. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of water overflows?

Ans. Radius of the conical vessel, R=AC=6cm

Height of the conical vessel, h=OC=8cm

Radius of the sphere, PD=PC=r

∴ PC = PD = r



AC = AD = 6 cm [Since, lengths of two tangents from an external point to a circle are equal]

△OCA & △OPD are right triangle.

[∵ Tangent and radius are perpendicular to each other]

$OA=\sqrt{OC^2 + AC^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm

$OP^2 = OD^2 + PD^2$

OD = OA – AD = 10 – 6 = 4 cm

$\Rightarrow OP = OC - PC = 8 - r$

$\Rightarrow (8-r)^2 = 4^2 + r^2$

$\Rightarrow 64 - 16r + r^2 = 16 + r^2$

$\Rightarrow 16r = 48 \Rightarrow r = 3$ cm

Volume of water overflows = Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times(3)^3 = 36\pi\ cm^3$

Original volume of water = volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 6^2 \times 8 = 96\pi\ cm^3$

∴ Fraction of water overflows

= Volume of water overflows / Original volume of water = 36π/96π = 3/8

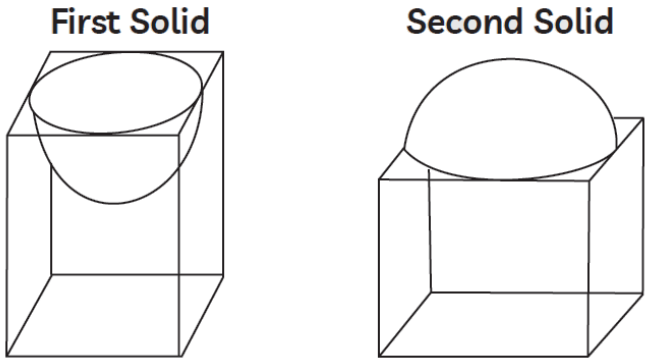
SECTION – D

Questions 18 carry 5 marks.

18. There are two identical solid cubical boxes of side 7 cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find :

(a) the ratio of the total surface area of the two new solid formed.

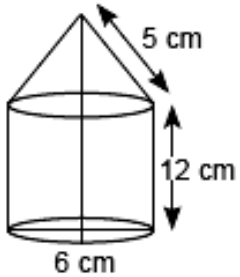
(b) volume of each new solid formed.
Ans.



(a) SA for first new solid (S_1) = $6a^2 + 2\pi r^2 - \pi r^2$
= $6 \times 7 \times 7 + 2\pi \times (3.5)^2 - \pi \times (3.5)^2$
= 294 + 77 – 38.5
= 332.5 cm^2
SA for second new solid (S_2) = $6a^2 + 2\pi r^2 - \pi r^2$
= $6 \times 7 \times 7 + 2\pi \times (3.5)^2 - \pi \times (3.5)^2$
= 294 + 77 – 38.5
= 332.5 cm^2
So, $S_1 : S_2 = 1:1$
(b) Volume for first new solid (V_1) = $7 \times 7 \times 7 - 2/3\ \pi \times (3.5)^3$
= 343 − 539/6
= 1519/6 cm^3
Volume for second new solid (V_2)
= $7 \times 7 \times 7 + 2/3\ \pi \times 3.5^3$
= 343 + 539/6
= 2597/6 cm^3

OR

A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [Use $\pi = 3.14$].
Ans. Here, r = 3 cm, h = 12 cm and l = 5 cm.



Total surface area of rocket
= C.S.A. of cone + C.S.A. of cylinder + area of base
= $\pi rl + 2\ \pi rh + \pi r^2 = \pi r(l + 2h + r)$
= $3.14 \times 3(5 + 24 + 3)$
= $9.42 \times 32 = 301.44\ cm^2$.
Volume of the rocket = $V_{Cone} + V_{Cylinder}$
= $\frac{1}{3}\pi r^2 h' + \pi r^2 h = \pi r^2\left(\frac{1}{3}h' + h\right)$
l = 5 cm, r = 3 cm, $l^2 = r^2 + h'^2$
$\Rightarrow 25 = 9 + h'^2$
$\Rightarrow 16 = h'^2 \Rightarrow h'$ = 4 cm

Volume of the rocket = $3.14\times3^2\left(\frac{1}{3}\times4+12\right)$

= $3.14\times9\left(\frac{4+36}{3}\right)$ = $3.14\times9\left(\frac{40}{3}\right)$ = 376.8 cm^3.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical upto a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.

Based on the above, answer the following questions:

(i) Find slant height of the conical part. (1)

(ii) Determine the floor area of the tent. (1)

(iii) (a) Find area of the cloth used for making tent. (2)

OR

(b) Find total volume of air inside an empty tent.

Ans. Given, Cylindrical height = 8 m, Diameter of base = 28 m

Total height of tent = 18.5 m

(i) Radius = 14 m

(Slant height)² = (Height)² + (Radius)²

⇒ $l^2 = (10.5)^2 + (14)^2 = 110.25 + 196$

⇒ $l^2 = 306.25$

⇒ l = 17.5 m

(ii) Floor Area of Tent is = πr^2

= 22/7 ×14 ×14

= 22 × 7 × 14

⇒ Area = 616 m^2

(iii) (a) Area of cloth used for making tent = $2\pi rh \times \pi rl$

= $2\pi r[h + l] = 2 \times 22/7 \times 14[8 + 17.5]$

= 2 × 22 × 2[25.5]

= 88 × 25.5

= 2244 m^2

OR

(b) Total volume Inside the Test

= $\pi r^2 h+\frac{1}{3}\pi r^2 h' = \pi r^2\left(h+\frac{1}{3}h'\right)$

= $\frac{22}{7}\times14\times14\left(8+\frac{1}{3}\times10.5\right)$

= 22 × 2 × 14(8 + 3.5)

= 616 × 11.5 = 7084 cm^3

20. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery. Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreigners with a variety of tea-sets, crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also refered as ‘‘The Ceramic Town’’.

One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip. Following are the few pottery objects of Khurja.

II I

Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following questions :

(a) The internal radius of hemispherical bowl (filled completely with water) in I is 9 cm and radius and height of cylindrical jar in II is 1.5 cm and 4 cm respectively. If the hemispherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required ? (2)

(b) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar ? (2)

Ans. (a) Given, radius by hemispherical bowl, $r_1 = 9$ cm

radius of cylindrical jar, $r_2 = 1.5$ cm

height of cylindrical jar, $h_2 = 4$ cm

Now, Volume of hemispherical bowl = $\frac{2}{3}\pi r_1^3 = \frac{2}{3}\pi(9)^3$

and Volume of cylindrical jar = $\pi r_2^2 h_2 = \pi(1.5)^2 \times 4$

Required number of cylindrical jar = Volume of hemispherical bowl / Volume of cylindrical jar

$$= \frac{\frac{2}{3}\pi(9)^3}{\pi(1.5)^2 \times 4} = \frac{2\times 9\times 9\times 9}{3\times 1.5\times 1.5\times 4} = 54$$

Hence, 54 cylindrical jars are required.

(b) Volume of water flow out of the jar

= Volume of conical funnel

$$= \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\times\frac{22}{7}(1.5)^2\times 4 = \frac{22\times 9}{3\times 7}$$

= 9.43 cubic cm

Therefore, water flow out of the jar is 9.43 cubic cm.

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32
PRACTICE PAPER 16 (2024-25)
CHAPTER 13 STATISTICS

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In a continuous frequency distribution with usual notations, if $l = 32.5$, $f_1 = 15$, $f_0 = 12$, $f_2 = 8$ and $h = 8$, then the mode of the data is:

(a) 32.5 (b) 33.5 (c) 33.9 (d) 34.9

2. For the following distribution:

Height (in cm)	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
No. of Students	4	11	29	40	46	51

the upper limit of the modal class is

(a) 165 (b) 160 (c) 155 (d) 150

3. Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	15	13	10	8	9	5

The upper limit of the median class in the given data is:

(a) 165 (b) 155 (c) 160 (d) 170

4. If mode of some data is 7 and their mean is also 7 then their median is

(a) 10 (b) 9 (c) 8 (d) 7

5. The mean and median of a distribution are 14 and 15, respectively. The value of the mode is:

(a) 16 (b) 17 (c) 18 (d) 13

6. If the value of each observation of a statistical data is increased by 3, then the mean of the data

(a) remains unchanged (b) increases by 3 (c) increases by 6 (d) increases by 3n

7. Consider the following distribution:

Marks Obtained	**No. of Students**
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of the class 30–40 is:

(a) 3 (b) 4 (c) 48 (d) 51

8. Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	16	12	9	7	10	6

The sum of the lower limit of the modal class and the upper limit of the median class is
(a) 310 (b) 315 (c) 320 (d) 330

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If the value of mode and median is 50.5 and 45.5 respectively, then the value of 2 mean is 86.
Reason (R): Median = (Mode + 2 Mean)

10. Assertion (A): Consider the following frequency distribution:

Class Interval	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	5	9	12	6	8

The modal class is 10 – 15.
Reason (R): The class having maximum frequency is called the modal class.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If mode of the following frequency distribution is 55 then find the value of x.

Class	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Frequency	10	7	*x*	15	10	12

12. The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal class is 8. Find the frequency of the modal class.

13. If the mean of the following frequency distribution is 62.8, then find the missing frequency x :

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	*x*	12	7	8

14. Calculate median marks of the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	Total
No. of Students	8	16	36	34	6	100

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	12	15	32	*k*	13

16. The below table shows the ages of persons who visited a museum on a certain day. Find the median age of the person visiting the museum.

Age (in years)	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
No. of persons	3	10	22	40	54	71

17. Heights of 50 students in class X of a school are recorded and following data is obtained:

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
No. of students	4	11	12	7	10	6

Find the median height of the students.

SECTION – D

Questions 18 carry 5 marks.

18. The distribution below gives the marks of 40 students of a class, if the median marks are 32.5, find the frequencies f_1 and f_2

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
No. of students	f_1	5	9	12	f_2	3	2	40

OR

The mean of the following data is 42. Find the missing frequencies x and y if the sum of frequencies is 100.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	10	x	13	y	10	14	9

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. India meteorological department observe seasonal and annual rainfall every year in different subdivisions of our country.

It helps them to compare and analyse the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018 :

Rainfall (in mm)	**Number of Sub-divisions**
200 – 400	2
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	2
1200 – 1400	3
1400 – 1600	1
1600 – 1800	1

Based on the above information, answer the following questions.

(a) Write the modal class.

(b) Find the median of the given data.

OR

Find the mean rainfall in this season.

(c) If sub-division having at least 1000 mm rainfall during monsoon season, is considered good rainfall sub-division, then how many sub-divisions had good rainfall?

20. 'Swachh Bharat Abhiyan' is a country-wide campaign initiated by our Honourable Prime Minister of India, Mr. Narendra Singh Modi in the year 2014 to eliminate open defecation, to improve solid waste management and to accelerate the efforts to achieve universal sanitisation.

As part of the 'Swachh Bharat Abhiyan', some houses of a locality in Agra decided to clean up and beautify a Primary School of their locality by planting a number of plants. They involved the school kids and the local community in doing so.

The data indicating the number of plants contributed by different houses is tabulated below:

Number of plants contributed	**Number of houses**
1 – 3	10
4 – 6	8
7 – 9	x
10 – 12	7
13 – 15	12
16 – 18	4

(a) If the mean number of plants contributed is 8.9, then how many houses contributed 7 to 9 plants? (2)

(b) What is the median class? (1)

(c) Find the median number of plants contributed. (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32
PRACTICE PAPER 16 (2024-25)
CHAPTER 13 STATISTICS

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In a continuous frequency distribution with usual notations, if $l = 32.5$, $f_1 = 15$, $f_0 = 12$, $f_2 = 8$ and $h = 8$, then the mode of the data is:

(a) 32.5 (b) 33.5 (c) 33.9 (d) 34.9

Ans. (d) 34.9

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow Mode = 32.5 + \frac{15-12}{30-12-8} \times 8 = 32.5 + \frac{3}{10} \times 8 = 32.5 + 2.5 = 34.9$$

2. For the following distribution:

Height (in cm)	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
No. of Students	4	11	29	40	46	51

the upper limit of the modal class is

(a) 165 (b) 160 (c) 155 (d) 150

Ans.

Height (in cm)	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
No. of Students	4	7	18	11	6	5

Highest frequency is 18 which belong to 145 – 150. Hence, Modal class is 145 – 150

∴ Upper limit of the modal class is 150

3. Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	15	13	10	8	9	5

The upper limit of the median class in the given data is:

(a) 165 (b) 155 (c) 160 (d) 170

Ans. (a) 165

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	15	13	10	8	9	5
cf	15	28	38	46	55	60

Here, $n = 60 \Rightarrow n/2 = 30$

Median class is 160 – 165

Hence, upper limit is 165

4. If mode of some data is 7 and their mean is also 7 then their median is

(a) 10 (b) 9 (c) 8 (d) 7

Ans. (d) 7
By Empirical Formula, 3 Median = Mode + 2 Mean
Given Mode = 7, Mean = 7
$\Rightarrow$ 3 Median = $7 + 2 \times 7$
$\Rightarrow$ 3 Median = $7 + 14 \Rightarrow$ 3 Median = $21 \Rightarrow$ Median = 7

5. The mean and median of a distribution are 14 and 15, respectively. The value of the mode is:
(a) 16 (b) 17 (c) 18 (d) 13
Ans. (b) 17
Using empirical formula we have 3 Median = Mode + 2 Mean
$\Rightarrow$ Mode = 3 Median – 2 Mean = $3(15) – 2(14) = 45 – 28 = 17$

6. If the value of each observation of a statistical data is increased by 3, then the mean of the data
(a) remains unchanged (b) increases by 3 (c) increases by 6 (d) increases by 3n
Ans. (b) increases by 3
If each value of observation is increased by 3, then mean is also increased by 3.

7. Consider the following distribution:

Marks Obtained	**No. of Students**
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of the class 30–40 is:
(a) 3 (b) 4 (c) 48 (d) 51
Ans. (a) 3

Marks Obtained	**No. of Students**
0 – 10	5
10 – 20	3
20 – 30	4
30 – 40	3
40 – 50	6
50 – 60	42

Hence the frequency of class interval 30 – 40 is 3.

8. Consider the following frequency distribution of the heights (in cm) of 60 students of a class:

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	16	12	9	7	10	6

The sum of the lower limit of the modal class and the upper limit of the median class is
(a) 310 (b) 315 (c) 320 (d) 330
Ans. (b) 315

Class	150 – 155	155 – 160	160 – 165	165 – 170	170 – 175	175 – 180
Frequency	16	12	9	7	10	6
cf	16	28	37	44	54	60

The class having the maximum frequency is the modal class.
So, the modal class is 150 – 155 and its lower limit is 150.
Also, n = 60 $\Rightarrow$ n/2 = 30
Median class is 160 – 165 whose upper limit is 165
$\therefore$ Required sum = (150 + 165) = 315

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): If the value of mode and median is 50.5 and 45.5 respectively, then the value of 2 mean is 86.
Reason (R): Median = (Mode + 2 Mean)
Ans. (c) Assertion (A) is true but reason (R) is false.
We know that, Mode = 3 Median – 2 Mean
(50.5) = 3 (45.5) – 2 Mean
2 Mean = 136.5 –50.5 = 86

10. Assertion (A): Consider the following frequency distribution:

Class Interval	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	5	9	12	6	8

The modal class is 10 – 15.
Reason (R): The class having maximum frequency is called the modal class.
Ans. (d) Assertion (A) is false but reason (R) is true.
The maximum frequency is 12, which lies in the interval 20 – 25.
So, the modal class is 20 – 25.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If mode of the following frequency distribution is 55 then find the value of x.

Class	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
Frequency	10	7	x	15	10	12

Ans: Since the mode is 55 which belongs to 45 – 60, therefore modal class is 45 – 60
Here, $l = 45$, $f_0 = x$, $f_1 = 15$, $f_2 = 10$, $h = 15$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow 55 = 45 + \frac{15 - x}{30 - x - 10} \times 15$$

$$\Rightarrow 10 = \frac{15 - x}{20 - x} \times 15 \Rightarrow 2 = \frac{15 - x}{20 - x} \times 3 \Rightarrow 40 - 2x = 45 - 3x$$

$$\Rightarrow 30 - 2x = 45 - 40 \Rightarrow x = 5$$

12. The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal class is 8. Find the frequency of the modal class.
Ans. Here, $l = 65$, $f_0 = 6$, $f_1 = x$, $f_2 = 8$, $h = 15$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow 75 = 65 + \frac{x - 6}{2x - 6 - 8} \times 15$$

$$\Rightarrow 10 = \frac{x - 6}{2x - 14} \times 15 \Rightarrow 2 = \frac{x - 6}{2x - 14} \times 3 \Rightarrow 4x - 28 = 3x - 18$$

$$\Rightarrow 4x - 3x = 28 - 18 \Rightarrow x = 10$$

13. If the mean of the following frequency distribution is 62.8, then find the missing frequency x :

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	x	12	7	8

Ans.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	
Frequency	5	8	x	12	7	8	$x + 40$
$\boldsymbol{x}$	10	30	*50*	70	90	110	

fx	50	240	$50x$	840	630	880	$50x + 2640$

Here, $\sum f = x + 40$ and $\sum fx = 50x + 2640$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow 62.8 = \frac{50x + 2640}{x + 40} \Rightarrow 2512 + 62.8x = 50x + 2640$$

$$\Rightarrow 62.8x - 50x = 2640 - 2512 \Rightarrow 12.8x = 128 \Rightarrow x = 10$$

$\therefore$ Missing frequency, $x = 10$

14. Calculate median marks of the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	Total
No. of Students	8	16	36	34	6	100

Ans.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	Total
No. of Students	8	16	36	34	6	100
cf	8	24	60	94	100	

Here, n = 100 $\Rightarrow$ n/2 = 50

$\Rightarrow$ Median class is 20 – 30

$l = 20$, $cf = 24$, $f = 36$, $h = 10$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\Rightarrow Median = 20 + \left(\frac{50 - 24}{36} \right) \times 10 = 20 + \frac{26 \times 10}{36} = 20 + \frac{65}{9} = 20 + 7.22 = 27.22$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	12	15	32	k	13

Ans.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	12	15	32	k	13	$k + 72$
x	10	30	50	70	90	
u	–3	–2	–1	0	1	
fu	–36	–30	–32	0	13	–85

Here, $\sum f = k + 72$ and $\sum fu = -85$, $h = 20$, $a = 70$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h \right) \Rightarrow 53 = 70 + \left(\frac{-85}{k + 72} \times 20 \right) \Rightarrow -17 = \frac{-85 \times 20}{k + 72} \Rightarrow 1 = \frac{100}{k + 72}$$

$$\Rightarrow k + 72 = 100 \Rightarrow k = 100 - 72 = 28$$

16. The below table shows the ages of persons who visited a museum on a certain day. Find the median age of the person visiting the museum.

Age (in years)	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
No. of persons	3	10	22	40	54	71

Ans.

Age (in years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of persons	3	7	12	18	14	17
cf	3	10	22	40	54	71

Here, n = 71 $\Rightarrow$ n/2 = 35.5

⇒ Median class is 30 – 40

$l = 30$, $cf = 22$, $f = 18$, $h = 10$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\Rightarrow Median = 30 + \left(\frac{35.5 - 22}{18} \right) \times 10 = 30 + \frac{13.5 \times 10}{18} = 30 + \frac{135}{18} = 30 + 7.5 = 37.5$$

The median age of the person visiting the museum is 37.5 years.

17. Heights of 50 students in class X of a school are recorded and following data is obtained:

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
No. of students	4	11	12	7	10	6

Find the median height of the students.

Ans.

Height (in cm)	130 – 135	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160
No. of students	4	11	12	7	10	6
cf	4	15	27	34	44	50

Here, n = 50 ⇒ n/2 = 25

⇒ Median class is 140 – 145

$l = 140$, $cf = 15$, $f = 12$, $h = 5$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$Median = 140 + \left(\frac{25 - 15}{12} \right) \times 5 = 140 + \frac{10 \times 5}{12} = 140 + \frac{25}{6} = 140 + 4.16 = 144.16$$

∴ Median height of the students = 144.16 cm.

SECTION – D

Questions 18 carry 5 marks.

18. The distribution below gives the marks of 40 students of a class, if the median marks are 32.5, find the frequencies f_1 and f_2

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
No. of students	f_1	5	9	12	f_2	3	2	40

Ans.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
No. of students	f_1	5	9	12	f_2	3	2	40
cf	f_1	$5 + f_1$	$14 + f_1$	$26 + f_1$	$26 + f_1 + f_2$	$29 + f_1 + f_2$	$31 + f_1 + f_2$	

Here, $n = 40 \Rightarrow 31 + f_1 + f_2 = 40$

$\Rightarrow f_1 + f_2 = 9$ ... (i)

Given, median = 32.5, which lies in the class interval 30-40.

So, median class is 30-40.

∴ $l = 30$, $h = 10$, $f = 12$, $n = 40$ and c.f. of preceding class, $cf = f_1 + 14$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\Rightarrow 32.5 = 30 + \left(\frac{20 - f_1 - 14}{12} \right) \times 10 \Rightarrow 2.5 = \left(\frac{6 - f_1}{12} \right) \times 10$$

$\Rightarrow 30 = (6 - f_1) \times 10 \Rightarrow 3 = 6 - f_1 \Rightarrow f_1 = 3$

$\Rightarrow f_2 = 9 - 3 = 6$

OR

The mean of the following data is 42. Find the missing frequencies x and y if the sum of frequencies is 100.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	10	x	13	y	10	14	9

Ans.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	**Total**
Frequency	7	10	x	13	y	10	14	9	100
$\boldsymbol{x}$	5	15	25	35	45	55	65	75	
$\boldsymbol{u}$	–2	–1	0	1	2	3	4	5	
$\boldsymbol{fu}$	–14	–10	0	13	$2y$	30	56	45	$2y + 120$

Here, $\sum f = 100 = x + y + 63 \Rightarrow x + y = 37$

and $\sum fu = 2y + 120$, $h = 10$, $a = 25$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h \right) \Rightarrow 42 = 25 + \left(\frac{2y + 120}{100} \times 10 \right) \Rightarrow 17 = \frac{2y + 120}{10} \Rightarrow 170 = 2y + 120$$

$\Rightarrow 2y = 170 - 120 = 50 \Rightarrow y = 25$

$\Rightarrow x = 37 - 25 = 12$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. India meteorological department observe seasonal and annual rainfall every year in different subdivisions of our country.

It helps them to compare and analyse the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018 :

Rainfall (in mm)	**Number of Sub-divisions**
200 – 400	2
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	2
1200 – 1400	3
1400 – 1600	1
1600 – 1800	1

Based on the above information, answer the following questions.

(a) Write the modal class.

(b) Find the median of the given data.

OR

Find the mean rainfall in this season.

(c) If sub-division having at least 1000 mm rainfall during monsoon season, is considered good rainfall sub-division, then how many sub-divisions had good rainfall?

Ans.

Rainfall (in mm)	Number of Sub-divisions	*cf*
200 – 400	2	2
400 – 600	4	6
600 – 800	7	13
800 – 1000	4	17
1000 – 1200	2	19
1200 – 1400	3	22
1400 – 1600	1	23
1600 – 1800	1	24

(a) Here, maximum class frequency is 7 and class corresponding to this frequency is 600-800, so the modal class is 600 – 800.

(b) Here, n/2 = 24/2 =12

⇒ Median class is 600 – 800

∴ $l = 600$, $cf = 6$, $f = 7$, $h = 200$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$Median = 600 + \left(\frac{12-6}{7} \right) \times 200 = 600 + \frac{6 \times 200}{7} = 600 + 171.429 = 771.43(approx)$$

So, the median of the given data is 771.43

OR

Rainfall (in mm)	Number of Sub-divisions	*x*	*u*	*fu*
200 – 400	2	300	-3	-6
400 – 600	4	500	-2	-8
600 – 800	7	700	-1	-7
800 – 1000	4	900	0	0
1000 – 1200	2	1100	1	2
1200 – 1400	3	1300	2	6
1400 – 1600	1	1500	3	3
1600 – 1800	1	1700	4	4
Total	24			-6

Here, $\sum f = 24$ and $\sum fu = -6$, $h = 200$, $a = 900$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h \right) = 900 + \left(\frac{-6}{24} \times 200 \right) = 900 + (-50) = 850$$

So, mean rainfall in the season is 850 mm.

(c) Number of sub-division having good rainfall

= 2 + 3 + 1 + 1 = 7

20. 'Swachh Bharat Abhiyan' is a country-wide campaign initiated by our Honourable Prime Minister of India, Mr. Narendra Singh Modi in the year 2014 to eliminate open defecation, to improve solid waste management and to accelerate the efforts to achieve universal sanitisation.

As part of the 'Swachh Bharat Abhiyan', some houses of a locality in Agra decided to clean up and beautify a Primary School of their locality by planting a number of plants. They involved the school kids and the local community in doing so.

The data indicating the number of plants contributed by different houses is tabulated below:

Number of plants contributed	**Number of houses**
1 – 3	10
4 – 6	8
7 – 9	x
10 – 12	7
13 – 15	12
16 – 18	4

(a) If the mean number of plants contributed is 8.9, then how many houses contributed 7 to 9 plants? (2)

(b) What is the median class? (1)

(c) Find the median number of plants contributed. (1)

Ans. (a)

Number of plants contributed	**Number of houses**	***x***	***u***	***fu***
0.5 – 3.5	10	2	-2	-20
3.5 – 6.5	8	5	-1	-8
6.5 – 9.5	x	8	0	0
9.5 – 12.5	7	11	1	7
12.5 – 15.5	12	14	2	24
15.5 – 18.5	4	17	3	12
Total	x + 41			15

Here, $\sum f = x + 41$ and $\sum fu = 15$, $h = 3$, $a = 8$

$$Mean, \bar{x} = a + \left(\frac{\sum fu}{\sum f} \times h\right) \Rightarrow 8.9 = 8 + \left(\frac{15}{x+41} \times 3\right) \Rightarrow 0.9 = \frac{45}{x+41}$$

$$\Rightarrow x + 41 = 50 \Rightarrow x = 9$$

(b)

Number of plants contributed	**Number of houses**	***cf***
0.5 – 3.5	10	10
3.5 – 6.5	8	18
6.5 – 9.5	9	27
9.5 – 12.5	7	34
12.5 – 15.5	12	46
15.5 – 18.5	4	50

Here, n/2 = 50/2 =25

⇒ Median class is 6.5 – 9.5

(c) Here, $l = 6.5$, $cf = 18$, $f = 9$, $h = 3$

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$Median = 6.5 + \left(\frac{25 - 18}{9} \right) \times 3 = 6.5 + \frac{7}{3} = 6.5 + 2.33 = 8.83$$

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32

PRACTICE PAPER 15 (2024-25)

CHAPTER 14 PROBABILITY

SUBJECT: MATHEMATICS STANDARD **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

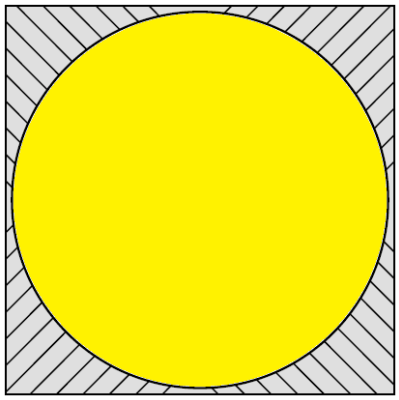
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. There is a square board of side '2a' units circumscribing a yellow circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the shaded region is:

(a) $\pi/4$ (b) $(4-\pi)/4$ (c) $(\pi-4)/4$ (d) $4/\pi$



2. If a card is drawn from a deck of cards, what is the probability of a card drawn to be a red or a black card and what can we say about that event?

(a) 1 and it is a sure event. (b) 0 and it is a sure event.

(c) 1 and it is an impossible event. (d) 0 and it is an impossible event.

3. In an MCQ test, a student guesses the correct answer x out of y times. If the probability that the student guesses the answer to be wrong is 2/3 then what is the relation between x and y

(a) y = 3x (b) x = 3y (c) 3x = 2y (d) 2x = 3y

4. If a letter is chosen at random from the letters of English alphabets, then the probability that it is a letter of the word 'MATHEMATICS' is:

(a) 4/13 (b) 9/26 (c) 5/13 (d) 11/26

5. Cards numbered 7 to 40 were put in a box. Anish selects a card at random. What is the probability that the selected card is a multiple of 7?

(a) 7/34 (b) 5/34 (c) 6/35 (d) 7/35

6. A bowl contains 3 red and 2 blue marbles.

Roohi wants to pick a red marble. Which of the following changes could she make so that the probability of picking a red marble is greater than it was before?

(i) Adding a red marble

(ii) Removing a blue marble
(iii) Adding 1 red and 1 blue marble
(a) Only (i) (b) Only (i) and (ii) (c) Only (i) and (iii) (d) All of the above

7. A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is:
(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4

8. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and the rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from houses A, B and C is:
(a) 4/23 (b) 6/23 (c) 8/23 (d) 17/23

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The probability that a leap year has 53 Sundays is 2/7.
Reason (R): The probability that a non–leap year has 53 Sundays is 5/7.

10. **Assertion (A):** The probability of getting a bad egg in a lot of 400 is 0.035. The number of good eggs in the lot is 386.
Reason (R): If the probability of an event is p, the probability of its complementary event will be 1 – p.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Cards, marked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is
(*i*) a prime number less than 10. (*ii*) a number which is a perfect square.

12. The king, queen and jack of diamonds are removed from a pack of 52 cards and then the pack is well shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (*i*) diamonds, (*ii*) a jack

13. Two different dice are tossed together. Find the probability
(*i*) that the number on each dice is even
(*ii*) that the sum of numbers appearing on two dice is 5.

14. Find the probability that a leap year should have exactly 52 tuesday.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A bag contains 12 balls out of which x are white.
(*i*) If one ball is drawn at random, what is the probability that it will be a white ball?
(*ii*) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double than that in (*i*). Find *x.*

16. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting
(i) a face card or a black card (ii) neither an ace nor a king (iii) a jack and a black card

17. Two different dice are thrown together. Find the probability that the numbers obtained:
(a) have a sum less than 7 (b) have a product less than 16 (c) is a doublet of odd numbers.

SECTION – D

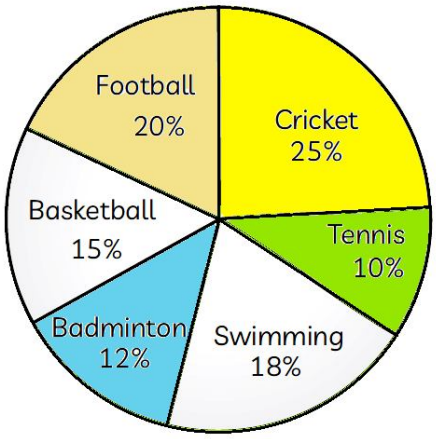
Questions 18 carry 5 marks.

18. From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is:
(a) a black queen. (b) a card of red colour. (c) a Jack of black colour. (d) a face card.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A school offers several sports to its students such as cricket, football, basketball, tennis, badminton and swimming. Based on past records, the sports teacher prepared a pie chart as shown below showing preference of students towards a particular sport.



(a) Find the probability of favourite sport being either swimming or badminton.
(b) Find the probability of favourite sport being neither football nor cricket.
(c) Find the probability of favourite sport being basketball, tennis or cricket.

20. Two friends are travelling in a bus. They were feeling bored, so they started playing a game with a pair of dice that one of them had. Each of them started rolling the pair of dice one by one, stating one condition before rolling. If the person gets the numbers according to the condition stated by him, he wins and get a score.

Based on the above information, answer the following questions.
(i) (a) First friend says, "a doublet". What is the probability of his winning? (1)
(b) Second friend says, ''sum less than 9''. What is the probability of his winning? (1)
(ii) (a) First one says, "6 will come up either time." What is the probability of his winning? (1)
(b) Second one says, "sum is an even number". What is the probability of his losing? (1)

PM SHRI KENDRIYA VIDYALAYA GACHIBOWLI,GPRA CAMPUS,HYD-32

PRACTICE PAPER 15 (2024-25)

CHAPTER 14 PROBABILITY (ANSWERS)

SUBJECT: MATHEMATICS STANDARD **MAX. MARKS : 40**

CLASS : X **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

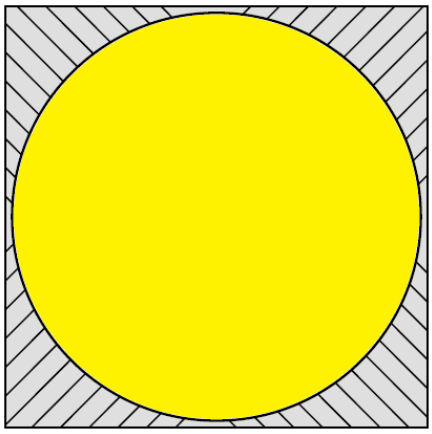
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. There is a square board of side '2a' units circumscribing a yellow circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the shaded region is:

(a) $\pi/4$ (b) $(4-\pi)/4$ (c) $(\pi-4)/4$ (d) $4/\pi$



Ans. (b) $(4-\pi)/4$

Area of square = $(2a)^2 = 4a^2$

Area of circle = $\pi r^2 = \pi a^2$

Difference = $4a^2 - \pi a^2 = a^2(4-p)$

Required probability = Favourable outcomes/Sample space

$= a^2(4-p) / 4a^2 = (4-\pi)/4$

2. If a card is drawn from a deck of cards, what is the probability of a card drawn to be a red or a black card and what can we say about that event?

(a) 1 and it is a sure event. (b) 0 and it is a sure event.

(c) 1 and it is an impossible event. (d) 0 and it is an impossible event.

Ans. (a) 1 and it is a sure event

3. In an MCQ test, a student guesses the correct answer x out of y times. If the probability that the student guesses the answer to be wrong is 2/3 then what is the relation between x and y

(a) y = 3x (b) x = 3y (c) 3x = 2y (d) 2x = 3y

Ans. (a) y = 3x

According to the given information, P(wrong) = 2/3

The probability of guessing the correct answer is the compliment of the probability of guessing wrong answer.

P(correct) = 1 – P(wrong) = 1 – (2/3)

P(correct) = 1/3
Now, the probability of guessing the correct answer P(correct) is the ratio of the number of correct (guesses) (x) to the total number of guesses (y)
P(correct) = x/y
$\therefore$ x/y = 1/3 $\Rightarrow$ 3x = y $\Rightarrow$ y = 3x

4. If a letter is chosen at random from the letters of English alphabets, then the probability that it is a letter of the word 'MATHEMATICS' is:
(a) 4/13 (b) 9/26 (c) 5/13 (d) 11/26
Ans. (a) 4/13
Total number of letters in English alphabets = 26
Unique letters in the word MATHEMATICS = {M, A, T, H, E, I, C, S}
$\Rightarrow$ Number of unique letters = 8
$\therefore$ Required probability = 8/26 = 4/13

5. Cards numbered 7 to 40 were put in a box. Anish selects a card at random. What is the probability that the selected card is a multiple of 7?
(a) 7/34 (b) 5/34 (c) 6/35 (d) 7/35
Ans. (b) 5/34
Total possible outcomes = 34
Favourable outcomes (Card is a multiple of 7) = 5 (7, 14, 21, 28, 35)
P(card being a multiple of 7) = Favourable outcomes/Total possible outcomes
= 5/34

6. A bowl contains 3 red and 2 blue marbles.
Roohi wants to pick a red marble. Which of the following changes could she make so that the probability of picking a red marble is greater than it was before?
(i) Adding a red marble
(ii) Removing a blue marble
(iii) Adding 1 red and 1 blue marble
(a) Only (i) (b) Only (i) and (ii) (c) Only (i) and (iii) (d) All of the above
Ans. (b) Only (i) and (ii)
Given, a bowl contains 3 red marbles and 2 blue marbles
Total number of outcomes = 5
P(picking a red marble) = 3/5
(i) On adding a red marble, Red marbles = 4
Blue marbles = 2
P(picking a red marble) = 4/6
(ii) On removing a blue marble, Red marbles = 3
Blue marbles = 1
P(picking a red marble) = 3/4
(iii) On adding 1 red marble and 1 blue marble, Red marbles = 4
Blue marbles = 3
P(picking a red marble) = 4/7
Thus, on adding a red marble and removing a blue marble, the probability will be greater than it was before.

7. A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is:
(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4
Ans. (a) 1/3
Total number of outcomes on throwing a dice twice = 36
Here, favourable outcomes = {(4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)}
$\therefore$ Number of favourable outcomes = 12

∴ Required probability = 12/36 = 1/3

8. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and the rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from houses A, B and C is:
(a) 4/23 (b) 6/23 (c) 8/23 (d) 17/23
Ans. (b) 6/23
Total no. of students = 23
No. of students from houses A, B and C = 4 + 8 + 5 = 17
∴ Remaining no. of students = 23 – 17 = 6
∴ Required probability = No. of students, not from A, B and C / Total no. of students houses
= 6/23

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The probability that a leap year has 53 Sundays is 2/7.
Reason (R): The probability that a non–leap year has 53 Sundays is 5/7.
Ans. (c) Assertion (A) is true but Reason (R) is false.

10. Assertion (A): The probability of getting a bad egg in a lot of 400 is 0.035. The number of good eggs in the lot is 386.
Reason (R): If the probability of an event is p, the probability of its complementary event will be 1 – p.
Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Cards, marked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is
(*i*) a prime number less than 10. (*ii*) a number which is a perfect square.
Ans. Total no. of cards = 46
Total no. of ways to select a card = 46
(*i*) Prime no. less than 10 in these cards are 5, 7
∴ No. of ways to select a prime no. less than 10 = 2.
∴ Probability that the number on the card is prime = 2/46 = 1/23
(*ii*) No. which is a perfect square, i.e. 9, 16, 25, 36, 49.
No. of ways to select a card with perfect square = 5.
∴ Probability = 5/46

12. The king, queen and jack of diamonds are removed from a pack of 52 cards and then the pack is well shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (*i*) diamonds, (*ii*) a jack
Ans. Total number of cards in the deck = 52
Number of cards removed = 3 [king, Queen & Jack of diamonds]
Number of cards remaining = 52 – 3 = 49
(*i*) Number of diamonds left = 13 – 3 = 10 [as 3 diamonds have been removed]
∴ Probability of drawing a diamond = 10/49
(*ii*) Number of jacks left = 4 – 1 = 3 [as jack of diamond has been removed]
Probability of drawing a jack = 3/49

13. Two different dice are tossed together. Find the probability
(*i*) that the number on each dice is even
(*ii*) that the sum of numbers appearing on two dice is 5.
Ans. Two different dice are tossed. Therefore, total outcomes are 36.
(*i*) Favourable outcomes for even number on both dice = 9, (2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)
∴ Probability of getting even number on both dice = 9/36 = 1/4
(*ii*) Favourable outcomes that the sum of the numbers appearing in two dice is 5 are (1, 4), (2, 3), (3, 2), (4, 1), i.e. 4.
∴ Probability of getting sum of numbers appearing on two dice is 5 = 4/36 = 1/9

14. Find the probability that a leap year should have exactly 52 tuesday.
Ans. Number of days in a leap year = 366, Number of weeks = 52
∴ Number of tuesdays in 52 weeks = 52
Number of days left after 52 weeks = 366 – 52 × 7 = 2.
Now, exactly 52 tuesday mean there should not be a tuesday in the remaining 2 days
Possible outcome of remaing two days
(Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) or (Sunday, Monday)
Total possible outcome = 7
Probability of not getting a Tuesday = 5/7
∴ Probability of getting exactly 52 Tuesday = 5/7

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A bag contains 12 balls out of which x are white.
(*i*) If one ball is drawn at random, what is the probability that it will be a white ball?
(*ii*) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double than that in (*i*). Find *x.*
Ans. $n(S) = 12$

(*i*) Let A be the event of drawing a white ball $n(A) = x$, $P(A) = \frac{x}{12}$

(*ii*) Number of white balls = $x + 6$
Total number of balls = 12 + 6 = 18.
Let B be the event of drawing a white ball

∴ $n(B) = x + 6$, $P(B) = \frac{x+6}{18}$

According to the question, P(B) = 2P(A)

$\Rightarrow \frac{x+6}{18} = 2 \times \frac{x}{12}$

$\Rightarrow 6x + 36 = 18x$

$\Rightarrow 12x = 36 \Rightarrow x = 3$

16. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting
(i) a face card or a black card
(ii) neither an ace nor a king
(iii) a jack and a black card
Ans. Total number of playing cards = 52
(*i*) Favourable cases for a face card or a black card are 32 (12 + 26 – 6)
∴ Probability of drawing a king or a jack = 32/52 = 8/13
(*ii*) Favourable cases for neither ace nor king card are 44 (52 cards – 4 aces – 4 king)

∴ Probability of drawing a non-ace = 44/52 = 11/13
(*iii*) Favourable cases for jack and black card are 2
∴ Probability of drawing a red card = 2/52 = 1/26

17. Two different dice are thrown together. Find the probability that the numbers obtained:
(a) have a sum less than 7
(b) have a product less than 16
(c) is a doublet of odd numbers.
Ans. The outcomes when two dice are thrown together, are:
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6),
∴ Total number of outcomes = 36
(a) Favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2) and (5, 1).
∴ Number of favourable outcomes = 15
∴ Required probability = 15/36 = 5/12
(b) Favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1) and (6, 2).
∴ Number of favourable outcomes = 25
∴ Required probability = 25/36
(c) Favourable outcomes are (1, 1), (3, 3) and (5, 5)
∴ Number of favourable outcomes = 3
∴ Required probability = 3/36 = 1/12

SECTION – D

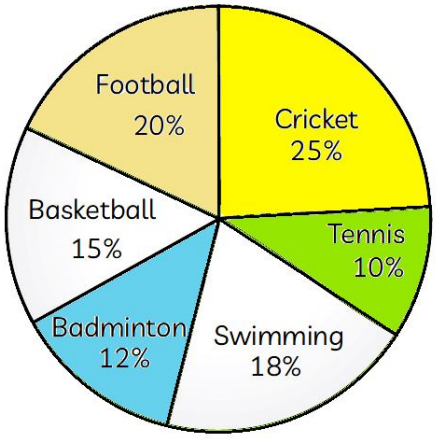
Questions 18 carry 5 marks.

18. From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is:
(a) a black queen.
(b) a card of red colour.
(c) a Jack of black colour.
(d) a face card.
Ans. Number of cards removed = (2 + 2 + 2 + 2) = 8
Total number of remaining cards = (52 – 8) = 44
Now, there are 2 jacks, 2 kings of black colour and 2 queens, 2 aces of red colour left.
(a) Number of black queens = 0
∴ P(getting a black queen) = 0/44 = 0
(b) Number of red cards = 26 – 4 = 22
∴ P(getting a red card) = 22/44 = 1/2
(c) Number of jacks of black colour = 2
∴ P(getting a black jack) = 2/44 = 1/22
(d) We know that jacks, queens and kings are face cards.
∴ Number of remaining face cards = (2 + 2 + 2) = 6
∴ P(getting a face card) = 6/44 = 3/22

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A school offers several sports to its students such as cricket, football, basketball, tennis, badminton and swimming. Based on past records, the sports teacher prepared a pie chart as shown below showing preference of students towards a particular sport.



(a) Find the probability of favourite sport being either swimming or badminton.
(b) Find the probability of favourite sport being neither football nor cricket.
(c) Find the probability of favourite sport being basketball, tennis or cricket.
Ans. (a) 18% students prefer swimming and 12% prefer badminton. Therefore, percentage of students showing preference for swimming or badminton = 30%.
Hence, probability of favourite sport being either swimming or badminton = 30/100 = 3/10.
(b) The preference for cricket is 25% and for football is 20%. Therefore, preference for either cricket or football is 25% + 20% i.e., 45%.
That means the preference for neither cricket nor football is (100 – 45)% i.e., 55%.
Hence, Probability of favourite sport being neither football nor cricket = 55/100 = 11/20
(c) 15% students prefer basketball, 10% students prefer tennis while 25% students prefer cricket.
Therefore, percentage of students showing preference for basketball, tennis or cricket = 15% + 10% + 25% = 50%
Hence, probability of favourite sport being basketball, tennis or cricket = 50/100 = ½

20. Two friends are travelling in a bus. They were feeling bored, so they started playing a game with a pair of dice that one of them had. Each of them started rolling the pair of dice one by one, stating one condition before rolling. If the person gets the numbers according to the condition stated by him, he wins and get a score.

Based on the above information, answer the following questions.
(i) (a) First friend says, "a doublet". What is the probability of his winning? (1)
(b) Second friend says, ''sum less than 9''. What is the probability of his winning? (1)
(ii) (a) First one says, "6 will come up either time." What is the probability of his winning? (1)
(b) Second one says, "sum is an even number". What is the probability of his losing? (1)
Ans. (i) (a) Number of doublets are {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} i.e., 6.
Total possible events = 36
∴ P(E) = 6/36 = 1/6
(b) Possible cases of sum less than 9 are {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 1), (3, 2), (3, 3), (3, 4), (3, 5) (4, 1), (4, 2), (4, 3), (4, 4) (5, 1), (5, 2), (5, 3) (6, 1), (6, 2)} i.e., 26.
∴ P(E) = 26/36 = 13/18
(ii) (a) Possible cases when 6 will come up either time are {(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} i.e., 11.

Number of favourable outcomes = 11.
∴ P(E) = 11/36
(b) Possible cases for which sum is an even number are {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)} i.e., 18.
∴ P(E) = 18/36 = 1/2
Probability of his losing is 1/2