

CHAPTER-7

INTEGRALS

01 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	$\int \tan^{-1} \sqrt{x} \, dx$ is equal to a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$ b) $x \tan^{-1}\sqrt{x} - \sqrt{x} + C$ c) $\sqrt{x} - x \tan^{-1}\sqrt{x} + C$ (d) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$	1
2.	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$ is equal to a)-1 (b) 0 (c) 1 (d) 2	1
3.	$\int \frac{e^x(1+x)}{\cos^2(xe^x)} \, dx$ is equal to a) $\tan(xe^x) + C$ b) $\cot(xe^x) + C$ c) $\cot(e^x) + C$ d) $\tan [e^x(1+x)] + C$	1
4.	$\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to a) $\tan x + \cot x + C$ b) $(\tan \tan x + \cot x)^2 + C$ c) $\tan x - \cot x + C$ d) $(\tan \tan x - \cot x)^2 + C$	1
5.	If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} \, dx = ax + b \log 4e^x + 5e^{-x} + C$, then a) $a = \frac{-1}{8}, b = \frac{7}{8}$ b) $a = \frac{1}{8}, b = \frac{7}{8}$ c) $a = \frac{-1}{8}, b = \frac{-7}{8}$ d) $a = \frac{1}{8}, b = \frac{-7}{8}$	1
6.	$\int_0^{\frac{\pi}{8}} \tan^2(2x) \, dx$ is equal to a) $\frac{4-\pi}{8}$ b) $\frac{4+\pi}{8}$ c) $\frac{4-\pi}{4}$ d) $\frac{4-\pi}{2}$	1
7.	$\int_{-1}^1 \frac{x^3 + x + 1}{x^2 + 2 x + 1} \, dx$ is equal to a) $\log 2$ b) $2 \log 2$ c) $\frac{1}{2} \log 2$ d) $4 \log 2$	1
8.	$\int_{-2}^2 x \cos \pi x \, dx$ is equal to a) $\frac{8}{\pi}$ b) $\frac{4}{\pi}$ c) $\frac{2}{\pi}$ d) $\frac{1}{\pi}$	1
9.	$\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6}) \, dx$ is equal to a) $\frac{1}{\sqrt{3}}$ b) $-\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) $-\sqrt{3}$	1
10.	If $\frac{d}{dx} [f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to a) $a+b$ b) $\frac{ax^2}{2} + bx$ c) $\frac{ax^2}{2} + bx + C$ d) b	1
11.	If $I = \int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} \, dx$, then value of I will be. (a) $\tan x + \cos x + C$ (b) $\tan x + \operatorname{cosec} x + C$ (c) $\tan x + \cot x + C$ (d) $\tan x + \sec x + C$	1

12.	$\int \frac{dx}{1+\cos 2x}$ is equal to, (a) $\tan x + c$ (b) $\frac{1}{2} \tan x + c$ (c) $2 \tan x + c$ (d) none of these	1
13.	$\int_{-2}^2 x dx$ is equals to, (a) 0 (b) 2 (c) 4 (d) 1	1
14.	$\frac{d}{dx} \int f(x) dx$ is equals to, (a) $f'(x)$ (b) $f(x)$ (c) $f(x')$ (d) $f'(x')$	1
15.	What is the value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$	1
16.	What is the value of $\int_1^e \left(\frac{1+\log x}{x} \right) dx$ (a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) e (d) $\frac{1}{e}$	1
17.	What is the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x dx$ (a) 0 (b) 1 (c) -1 (d) 2	1
18.	Value of $\int_0^1 \left(\frac{x}{1+x} \right) dx$ is (a) $1 - \log 2$ (b) $\log 2 - 1$ (c) $1 + \log 2$ (d) $\log 2$	1
19.	Assertion (A): $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{2} \right) + c$ Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$ (a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false and R is True	1
20.	Assertion (A): $\int e^x [\sin x + \cos x] dx = e^x \sin x + c$ Reason (R): $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$	1

	(a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false and R is True	
21.	If $\frac{d}{dx}(f(x)) = 5x^4 - \frac{4}{x^5}$ such that $f(2) = 0$. Then $f(x)$ is (a) $x^5 + \frac{1}{x^4} - \frac{129}{8}$ (b) $x^5 + \frac{1}{x^4} + \frac{129}{8}$ (c) $x^5 + \frac{1}{x^4} - \frac{513}{16}$ $x^5 + \frac{1}{x^4} + \frac{513}{16}$	1
22.	$\int \frac{1}{\sin^2 x \cos^2 x} dx$ equals (a) $\tan x + \cot x + C$ (b) $\tan x - \cot x + C$ (c) $\tan x \cot x + C$ (d) $\tan x - \cot 2x + C$	1
23.	$\int \frac{1}{x(x^3+1)} dx$ equals (a) $\frac{1}{3} \log \log \left \frac{x^3}{x^3-1} \right + C$ (b) $\frac{1}{3} \log \log \left \frac{x^3+1}{x^3} \right + C$ (c) $\frac{1}{3} \log \log \left \frac{x^3}{x^3+1} \right + C$ (d) $\frac{1}{3} \log \log \left \frac{x^3-1}{x^3} \right + C$	1
24.	$\int \frac{5x^4 + 5^x 5^1}{x^5 + 5^x} dx$ equals (a) $5^x - x^5 + C$ (b) $5^x + x^5 + C$ (c) $(5^x - x^5)^{-1} + C$ (d) $\log(5^x + x^5) + C$	1
25.	$\int e^x \sec \sec x (1 + \tan x) dx$ equals (a) $e^x \cos x + C$ (b) $e^x \sec x + C$ (c) $e^x \sin x + C$ (d) $e^x \tan x + C$	1
26.	$\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to (a) $2(\sin x + x \cos \theta) + C$	1

	(b) $2(\sin x - x \cos \theta) + C$ (c) $2(\sin x + 2x \cos \theta) + C$ (d) $2(\sin x - 2x \cos \theta) + C$	
27.	$\int_0^{2/3} \frac{1}{4+9x^2} dx$ is equal to (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$	1
28.	$\int_{-1}^1 \frac{ x-2 }{x-2} dx, x \neq 2$ is equal to (a) 1 (b) -1 (c) 2 (d) -2	1
29.	The value of the integral $\int_0^{\frac{\pi}{2}} \log \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$ is (a) 2 (b) $\frac{3}{4}$ (c) 0 (d) -2	1
30.	$\int_{a+c}^{b+c} f(x) dx$ is equal to (a) $\int_a^b f(x-c) dx$ (b) $\int_a^b f(x+c) dx$ (c) $\int_a^b f(x) dx$ (d) $\int_{a-c}^{b-c} f(x) dx$	1
31.	Anti derivative of $\sin \sin(ax+b)$ is (a) $\cos \cos(ax+b) + c$ (b) $a \cos \cos(ax+b) + c$ (c) $-\frac{\cos \cos(ax+b)}{a} + c$ (d) $-\frac{\cos \cos(ax+b)}{b} + c$	1
32.	$\int e^{2x} dx =$ (a) $e^x + c$ (b) $\frac{e^{2x}}{2} + c$ (c) $x^2 + c$ (d) $\frac{x^3}{3} + c$	1
33.	$\int \cos \cos \frac{7\pi}{6} dx =$ (a) $\frac{7\pi}{6} x + c$ (b) $\frac{5\pi}{6} x + c$ (c) $\frac{\pi}{6} x + c$ (d) $\frac{\pi}{3} x + c$	1
34.	$\int e^{(\sin x)^2} \sin 2x dx =$	1

	(a) $(\sin \sin x)^2 + c$ (b) $e^{(\cos x)^2} + c$ (c) $e^{(\sin x)^2} + c$ (d) none of these	
35.	$\int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx =$ (a) 0 (b) π (iii) 2π (iv) $\frac{\pi}{2}$	1
36.	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 (\sin x)^4 dx =$ (a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$	1
37.	$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$ (a) $e^x + c$ (b) $\frac{e^x}{x} + c$ (c) $\frac{e^x}{x^2} + c$ (d) none of these	1
38.	$\int (e^x + 1)^2 e^x dx$ (a) $e^x + 1 + c$ (b) $(e^x + 1)^2 + c$ (c) $\frac{(e^x + 1)^3}{3} + c$ (d) $e^{2x} + c$	1
39.	$\int_0^3 [x] dx =$, where $[x]$ means the greatest integer less than or equal to x . (a) 0 (b) 3 (c) 2 (d) 1	1
40.	$\int x d(x^2 + 2)$ (a) $\frac{x^2}{2} + c$ (b) $x + c$ (c) $\frac{2x^3}{3} + c$ (d) $\frac{x^4}{4} + c$	1
41.	$\int 2^x 3^x dx$ is equal to (a) $\frac{2^x}{\ln 2} + C$ (b) $\frac{3^x}{\ln 3} + C$ (c) $\frac{2^x 3^x}{\ln 2 \ln 3} + C$ (d) $\frac{6^x}{\ln 6} + C$	1
42.	If $\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx = A\sqrt{\cot x} + K$, then the value of A is __ (a) 2 (b) 1 (c) -2 (d) -1	1
43.	The anti-derivative of $\int \frac{\sin^2 x}{\cos^4 x} dx$ is (a) a polynomial of degree 5 in $\sin x$ (b) a polynomial of degree 4 in $\tan x$ (c) a polynomial of degree 5 in $\tan x$ (d) a polynomial of degree x in $\cos x$	1
44.	$\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to (a) $\frac{1}{5x} \left(4 + \frac{1}{x^2} \right)^{-5} + C$ (c) $\frac{1}{5} \left(4 + \frac{1}{x^2} \right)^{-5} + C$ (b) $\frac{1}{10} \left(4 + \frac{1}{x} \right)^{-5} + C$ (d) $\frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5} + C$	1
45.	$\int \frac{dx}{x(x^n-1)}$ is equal to	1

	(a) $\frac{1}{n} \log \left 1 - \frac{1}{x^n} \right + C$ (c) $\frac{1}{n} \log \left \frac{x^n}{x^{n-1}} \right + C$ (b) $\frac{1}{x^n} \log \left \frac{x^n}{x^{n-1}} \right + C$ (d) $\frac{1}{x^n} \log \left \frac{x^{n-1}}{x^n} \right + C$	
46.	$\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = Px + Q \log 4e^x + 5e^{-x} + \text{Constant}$, then (a) $P = \frac{-1}{8}, Q = \frac{-7}{8}$ (c) $P = \frac{1}{8}, Q = \frac{7}{8}$ (b) $P = \frac{-1}{8}, Q = \frac{7}{8}$ (d) $P = \frac{1}{8}, Q = \frac{-7}{8}$	1
47.	The value of $\int_{-2}^3 1 - x^2 dx$ is (a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$	1
48.	If $\int_0^\pi xf(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$, then A is (a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0	1
49.	$\int_0^\pi \sin x dx$ is (a) 2 (b) 2π (c) π (d) 0	1
50.	$\int_0^2 [x^2] dx$ is (a) $2 - \sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - \sqrt{3} + 5$	1

ANSWERS:

Q. NO	ANSWER	MARKS
1.	<p>a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$</p> <p>let $I = \int \tan^{-1}\sqrt{x} dx$, put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt \Rightarrow 2t dt$</p> <p>so, $I = \int \tan^{-1}t 2t dt$</p> $= \tan^{-1}t 2\frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot 2\frac{t^2}{2} dt \quad (\text{integrating by parts})$ $= t^2 \tan^{-1}t - \int \frac{1}{1+t^2} \cdot t^2 dt$ $= t^2 \tan^{-1}t - \int \frac{1+t^2-1}{1+t^2} dt$ $= t^2 \tan^{-1}t - \int \left(1 - \frac{1}{1+t^2}\right) dt$ $= t^2 \tan^{-1}t - [t - \tan^{-1}t] = t^2 \tan^{-1}t - t + \tan^{-1}t = \tan^{-1}t (t^2 + 1) - t = (x+1)$ $\tan^{-1}\sqrt{x} - \sqrt{x} + C$	1
2.	<p>(d) 2</p> <p>We have $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx = [\tan x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \tan \frac{\pi}{4} - \tan(-\frac{\pi}{4}) = 1 + 1 = 2$</p>	1
3.	<p>a) $\tan(xe^x) + C$</p> <p>let $I = \int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$, put $xe^x = t \Rightarrow (xe^x + e^x) dx = dt$</p> $\Rightarrow e^x(x+1) dx = dt$ <p>So, $I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t dt = \tan t + C = \tan(xe^x) + C$</p>	1
4.	<p>(c) $\tan x - \cot x + C$</p> <p>$I = \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx = \tan x - \cot x + C$</p>	1
5.	<p>a) $a = \frac{-1}{8}$, b = $\frac{7}{8}$</p>	1
6.	<p>a) $\frac{4-\pi}{8}$</p> <p>$\int_0^{\frac{\pi}{8}} \tan^2(2x) dx = \int_0^{\frac{\pi}{8}} \sec^2 2x - 1 dx = [\frac{\tan 2x}{2} - x]_0^{\frac{\pi}{8}} = \frac{\tan \frac{\pi}{4}}{2} - \frac{\pi}{8} - 0 = \frac{1}{2} - \frac{\pi}{8} = \frac{4-\pi}{8}$</p>	1
7.	<p>(b) $2 \log 2$</p> <p>$I = \int_{-1}^1 \frac{x^3 + x + 1}{x^2 + 2 x + 1} dx = \int_{-1}^1 \frac{x^3}{x^2 + 2 x + 1} dx + \int_{-1}^1 \frac{ x + 1}{x^2 + 2 x + 1} dx$</p> $= 0 + 2 \int_0^1 \frac{ x + 1}{(x+1)^2} dx \quad [\text{odd function} + \text{even function}]$ $= 2 \int_0^1 \frac{x+1}{(x+1)^2} dx$ $= 2 \int_0^1 \frac{1}{x+1} dx = 2 [\log x+1]_0^1 = 2 \log 2$	1
8.	<p>a) $\frac{8}{\pi}$</p> <p>since $I = \int_{-2}^2 x \cos \pi x dx = 2 \int_0^2 x \cos \pi x dx = 2 \left\{ \int_0^{\frac{1}{2}} x \cos \pi x dx + \right.$</p>	1

	$\int_{\frac{1}{2}}^{\frac{3}{2}} x \cos \pi x dx + \int_{\frac{3}{2}}^2 x \cos \pi x dx = \frac{8}{\pi}$	
9.	a) $\frac{1}{\sqrt{3}}$ $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6}) dx = [\tan \tan(x - \frac{\pi}{6})]_0^{\frac{\pi}{6}} = \tan(\frac{\pi}{6} - \frac{\pi}{6}) - \tan(0 - \frac{\pi}{6}) = \tan 0 - \tan(-\frac{\pi}{6}) = 0 + \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$	1
10.	(b) $\frac{ax^2}{2} + bx$ Given, $\frac{d}{dx} [f(x)] = ax + b$ and $f(0) = 0$ On integrating both sides, we have $f(x) = \int (ax + b) dx = \frac{ax^2}{2} + bx + C$ $\Rightarrow f(x) = \frac{ax^2}{2} + bx + C \quad \dots \dots (i)$ Also, $f(0) = 0$, we have from (i) $f(0) = C$ $\Rightarrow 0 = C$ Putting in (i), we have $f(x) = \frac{ax^2}{2} + bx$	1
11.	(c)	1
12.	(b)	1
13.	(c)	1
14.	(b)	1
15.	(b)	1
16.	(a)	1
17.	(a)	1
18.	(a)	1
19.	(d)	1
20.	(a)	1
21.	(c)	1
22.	(b)	1
23.	(c)	1
24.	(d)	1
25.	(b)	1
26.	(a)	1
27.	(c)	1
28.	(b)	1
29.	(c)	1
30.	(b)	1
31.	(c)	1
32.	(d)	1
33.	(b)	1
34.	(c)	1
35.	(b)	1
36.	(a)	1
37.	(b)	1
38.	(c)	1

39.	(b)	1
40.	(c)	1
41.	(d) $\frac{6^x}{\ln 6} + C$	1
42.	(c) -2	1
43.	(c) a polynomial of degree 3 in $\tan x$	1
44.	(d) $\frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5} + C$	1
45.	(a) $\frac{1}{n} \log \left 1 - \frac{1}{x^n} \right + C$	1
46.	(b) $P = \frac{-1}{8}, Q = \frac{7}{8}$	1
47.	(d) $\frac{28}{3}$	1
48.	(b) π	1
49.	(a) 2	1
50.	(d) $-\sqrt{2} - \sqrt{3} + 5$	1