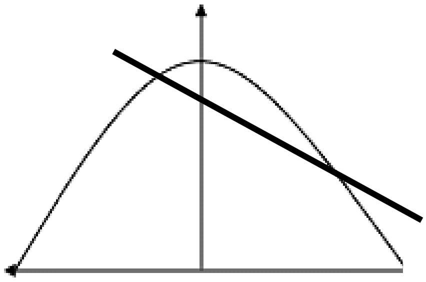
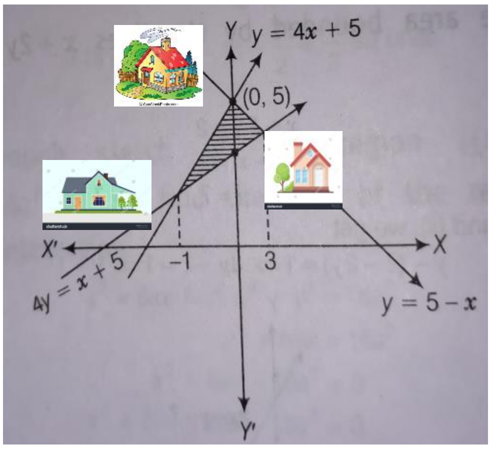
CHAPTER-8
APPLICATION OF INTEGRALS
02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the area bounded by the curve y= x |x|, x-axis and x= -1 and x=1.	2
2.	Find the area bounded by the lines |x|+|y|= 1.	2
3.	Find the area bounded by the curves y = x^2 and the line y=4.	2
4.	Find the area of the curve $y = sinx$ between 0 and π.	2
5.	Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the $x - axis$ in the first quadrant.	2
6.	Find the area between the curves $y = x$and $y = x^2$.	2
7.	Write the formula of $\int \sqrt{a^2 - x^2}\, dx$	2
8.	Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.	2
9.	Write the Geometric significance of the integral $\int_a^b f(x)$dx.	2
10.	Using integration, Find the area of the region bounded by the line 2y= -x+8, X-axis and the lines X=2 andx=4.	2
11.	Find the area bounded by the curvey^2= 2y-xand Y axis.	2
12.	Find the area of the region bounded by the curve x^2=4y and the straight line x=4y-2.	2
13.	Find the area of the region bounded by the curve X axis and y = 2x-x^2.	2
14.	Using integration find the area of the region bounded by the line 2y = -x+8 , x-axis and the line x = 2 and x = 4.	2
15.	Using integration find the area of the region bounded between the line x = 4 and the parabola $y^2 = 4x$.	2
16.	Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	2
17.	Find the area of the region bounded by the curve $y^2 = x$ and the line x = 1, x = 4 and the x- axis.	2
18.	Find the area of the region bounded by the curve parabola $y = x^2$ *and the line* $y = \|x\|$.	2
19.	Find the area bounded between y = $\sin^{-1}$x and y-axis between y = 0 and y = π/2.	2
20.	If the area bounded by the curve y = 3x, x-axis and between the ordinates x = 1 and x = b is 12 sq..units, then find the value of b.	2
21.	If the area bounded by the parabola y^2 = 16x and the line x=a is 128/3 sq. units, then find the value of a.	2
22.	Using integration check whether given statement is true or false Statement: The region under the curve y = √(1 – x^2) on the interval [-1,1] has area A = π/2,	2
23.	Find the area of the region bounded by the $y = \|x - 5\|$ and ordinates x=0 and x=1.	2
24.	Using integration, find the area of the region bounded by: y=mx (m > 0, x= 1, x= 2 and the x-axis).	2
25.	Sketch the region bounded by the lines 2x+y = 8, y = 2, y =4 and the y-axis. Hence, obtain its area, using integration.	2
26.	Find the area bounded by y = x^2, the x-axis and the lines x = 1 and x = -1.	2
27.	Find the area bounded by the curve y = x^3 , x = -2 and x = 1.	2

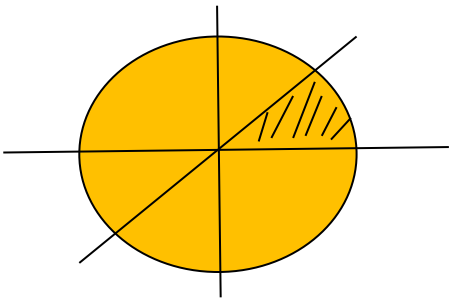
<table>
<tr><td>28.</td><td>Find the area of the region bounded by the parabola $y^2 = 8x$ and the line x = 2.</td><td>2</td></tr>
<tr><td>29.</td><td>Reshma draw a beautiful painting in which she draw mountains, trees, birds, river, houses etc. His little brother come across the painting and cut one of the mountain by drawing a straight line. Based on the above information find the area bounded by mountain and straight line . The equation of mountain is $y = -x^2$ and equation of straight line is $x + y + 2 = 0$


</td><td>2</td></tr>
<tr><td>30.</td><td>Find the area bounded by the curve $y^2 = 9x \; and \; y = 3x$.</td><td>2</td></tr>
<tr><td>31.</td><td>Location of the three houses of a society is represented by points A(0,5), B(3,2) and C (1,1). Find the area bounded by these three houses and the equqtion of line represented by house A, B, C are $y = 4x + 5,$

$$y = 5 - x, and \; 4y = x + 5.$$



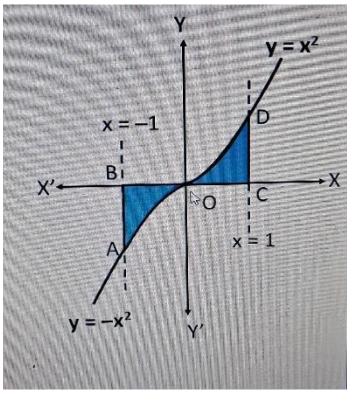
</td><td>2</td></tr>
<tr><td>32.</td><td>A circular Pizza is cut into 8 equal pieces with the help of knife then find the area of region bounded by each pieces of pizza if the equation of pizza and knife is represented by $x^2 + y^2 = 32 \; and \; y = x$ respectively.


</td><td>2</td></tr>
<tr><td>33.</td><td>Consider the following curve and find the area under the curve $y = 2\sqrt{x}$ included between the line x=0 and x=1 is</td><td>2</td></tr>
</table>

ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>

We know Y= x |x|
Y= {x^2 $\quad if\ x > 0$ $-x^2$ $\quad if\ x < 0$



Area required = Area ABO + Area DCO

Area ABO = $\int_{-1}^{0} y\,dx$ = $\int_{-1}^{0} -x^2\,dx$ = -(1/3) since area is always positive so area ABO is 1/3
Area DCO = $\int_{-1}^{0} y\,dx$ = $\int_{-1}^{0} .x^2\,dx$ = (1/3)
So, required area is 1/3 + 1/3 = 2/3.

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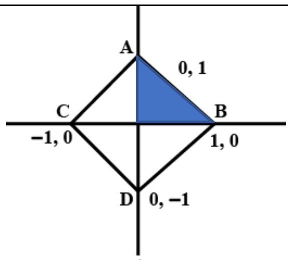
<tr><td>2.</td><td>



Area ABO = $\int_{-1}^{0} y\,dx$ where the shaded part having the oblique line equation be x + y = 1 so, y = 1 – x
Therefore Area ABO = $\int_{-1}^{0} (1 - x)\,dx$
= 1/2
So, required area is 4 * Area of AOB = 4*(1/2) = 2sq. unit

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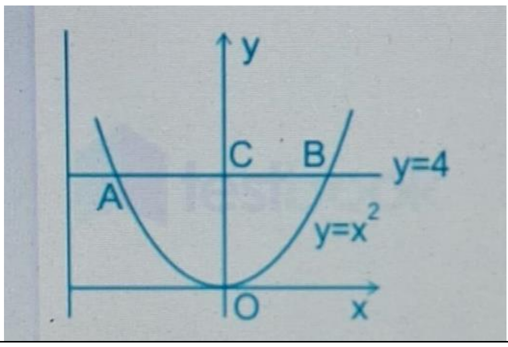
<tr><td>3.</td><td>

We have y = x^2 and y = 4
Let AB represent the line y=4



</td><td>2</td></tr>
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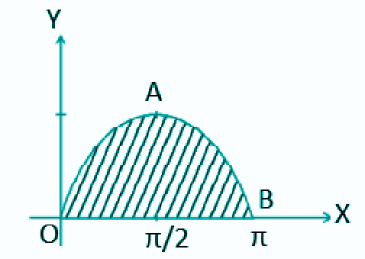
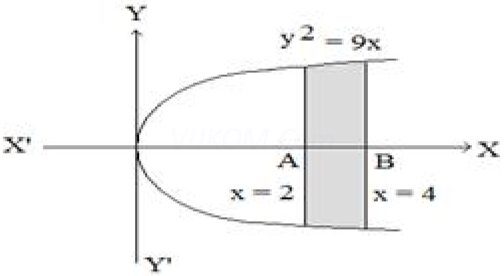
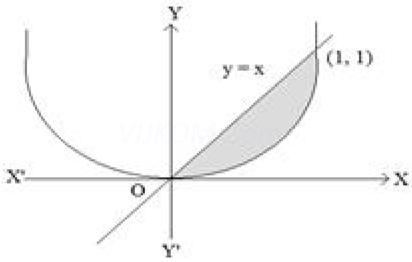
	Let AOB represent $y = x^2$ i.e $x = \pm\sqrt{y}$ Since BOCB is in the 1st quadrant , we use only positive value of $\sqrt{y}$ Area of AOBA= $2 * \int_0^4 \sqrt{y}\, dy$ = (32/3) sq. unit	
4.	 $y = sinx$ Area of OAB=$\int_0^\pi ydx$ =$\int_0^\pi sindx$ = 2 sq. units	2
5.	$y^2 = 9x$, $x = 2$, $x = 4$ and the $x - axis$ in the first quadrant  Required area=$\int_2^4 ydx$ =$\int_2^4 \sqrt{9x}dx$ =$\int_2^4 3\sqrt{x}dx$ =16-4$\sqrt{2}$ sq. units	2
6.	$y = x$ $y = x^2$ On solving x = 0, 1 $Area = \int_0^1 \left(x - x^2\right) dx$ $= \frac{1}{6} sq\ unit.$ 	2
7.	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$	2
8.	Given eq. of the lines are $y = 2x + 1$ -------(1) $y = 3x + 1$ ------(2) $x = 4$ -----(3)	2

<table>
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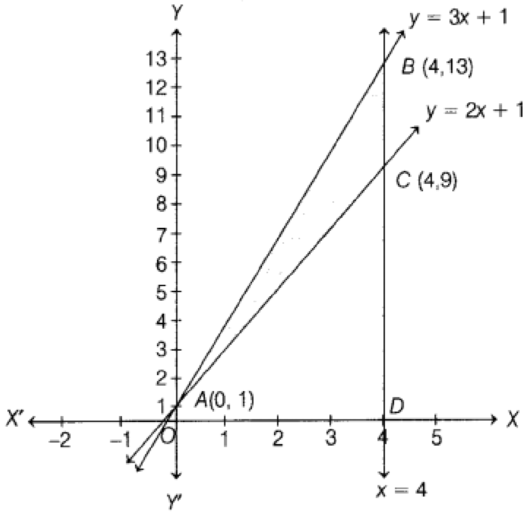
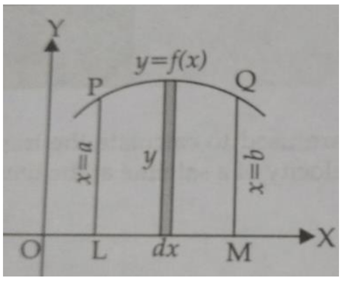
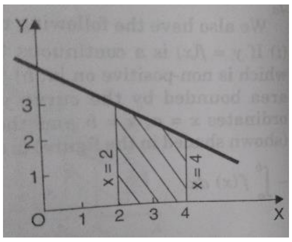


By solving these equations we get the vertices of triangle as A(0, 1), B(4, 13) and C(4, 9).
∴ Required area = Area (OABDO) – area (OACDO)
$=\int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$
= 8sq. units
</td><td></td></tr>
<tr><td>9.</td><td>
Let there be an arbitrary strip of height y and width dx.
Area of elementary strip dA = ydx,where y=f(x).total are A of the region between X-axis ordinates x=a, x=b and the curve y=f(x)
Sum of the areas of elementary strips across PQML
A=$\int_a^b ydx = \int_a^b f(X)$ dx



</td><td>2</td></tr>
<tr><td>10.</td><td>
We have 2y=-x+4
⇨ x+2y=4
⇨ x/4+y/2=1
Required area is $\int_2^4 y$dx=$\int_2^4 (-\frac{x}{2}+4)$dx
=$(-\frac{x^2}{4}+4x)|_2^4$=5 sq. units



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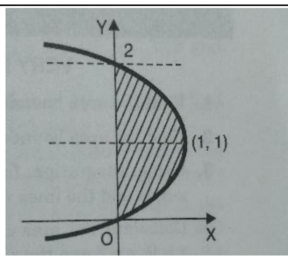
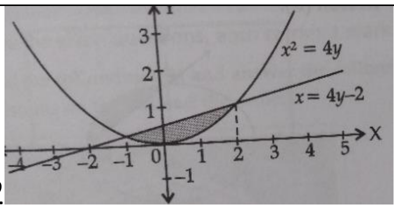
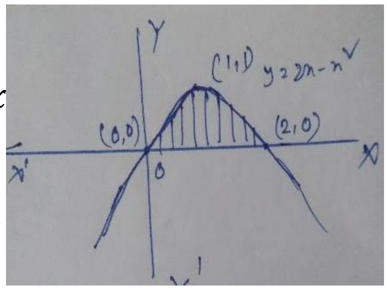
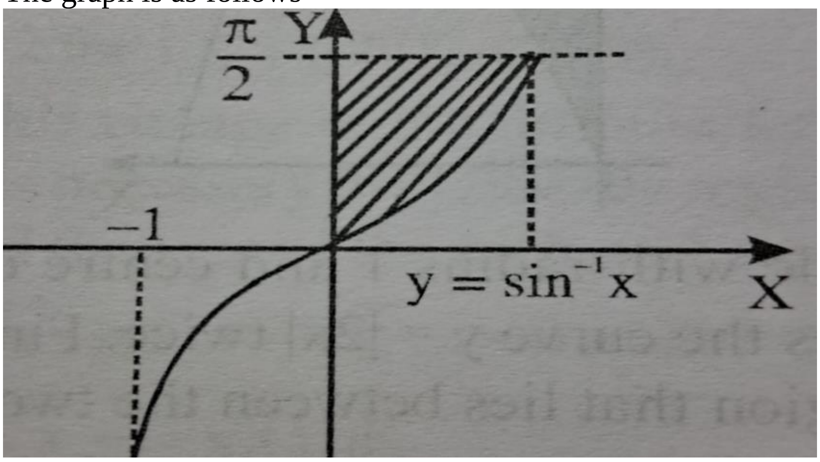
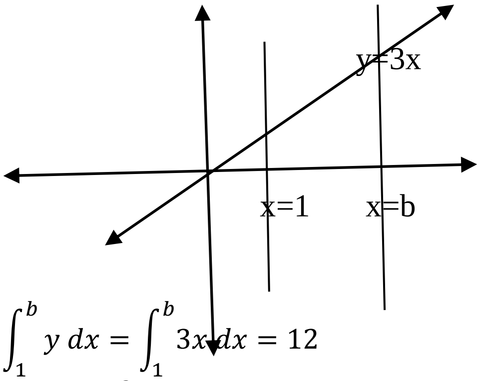
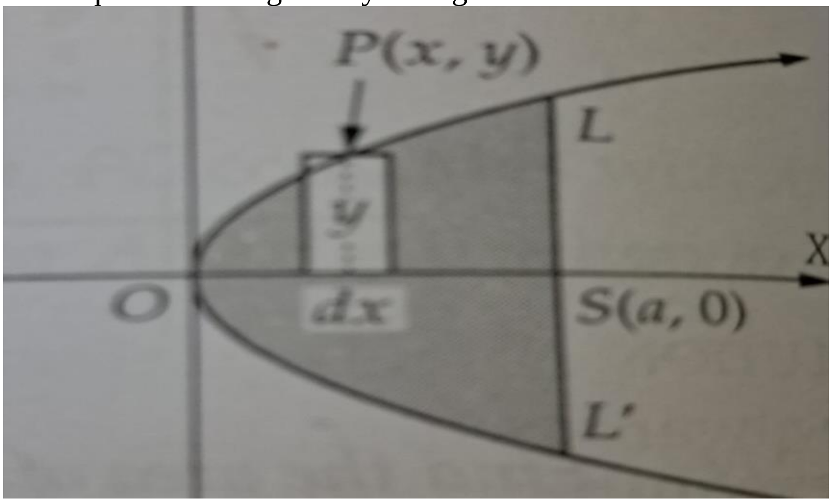
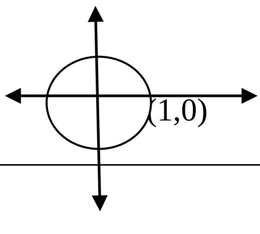
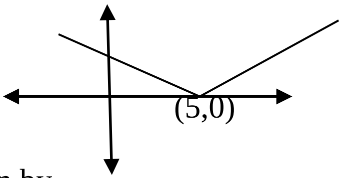
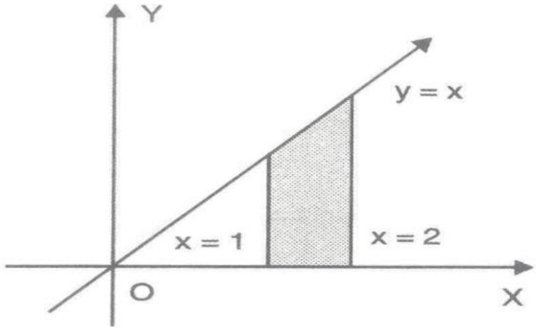
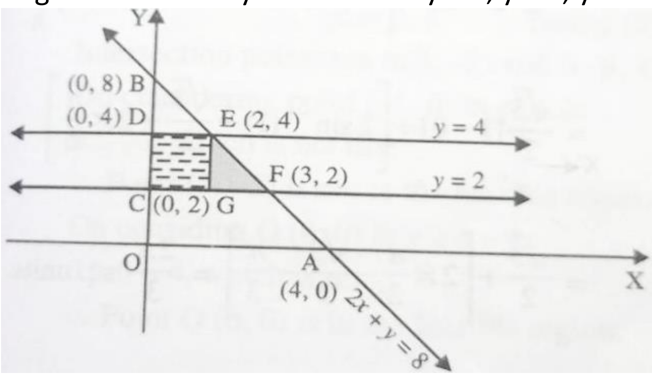
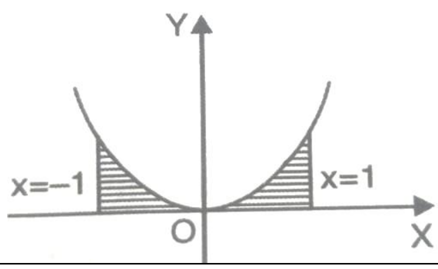
<tr><td>11.</td><td>
We have y^2=2y-x
(y-1)^2=-(x-1)
When x=0 then y=0,2
Required area is =$\int_0^2 x$dy=$\int_0^2 (2y-y^2)$ dy
=$(y^2 - y^3/3)\ |_0^2$=4/3 sq. units.

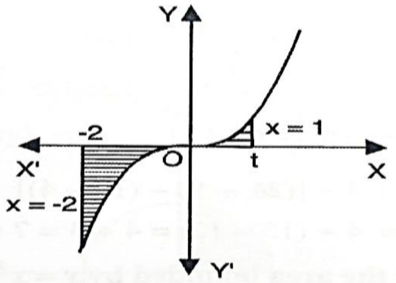
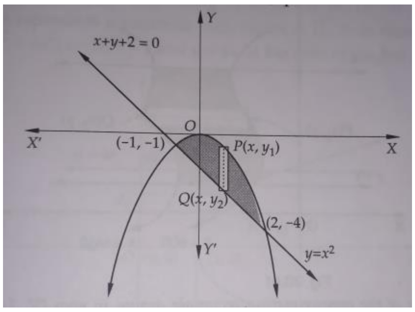
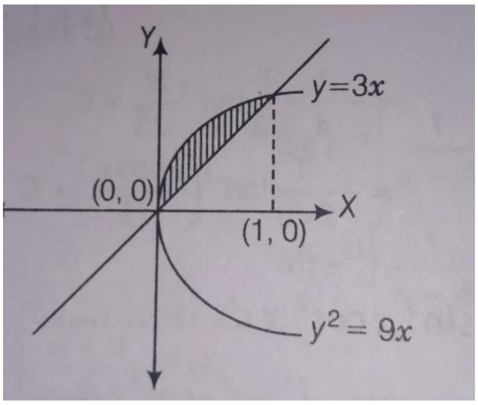
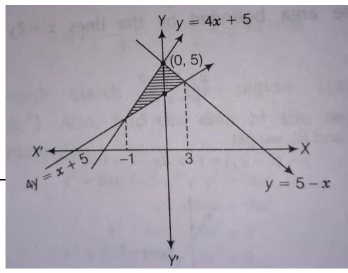


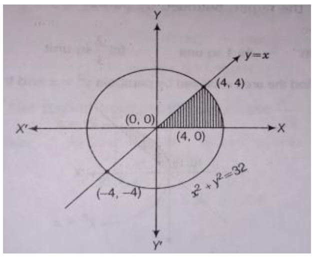
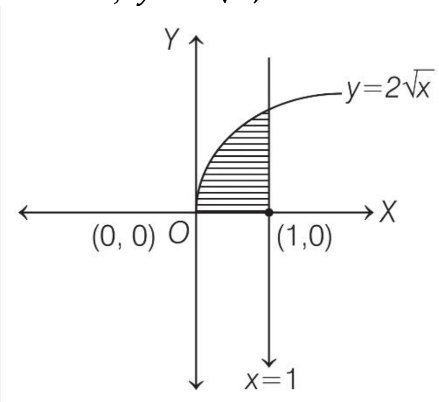
</td><td>2</td></tr>
<tr><td>12.</td><td>We have x^2=4y and the line x=4y-2</td><td>2</td></tr>
</table>

	 Then x=4y-2 ⇨x-4y=-2 ⇨x/(-2)+y/(1/2)=1 $x^2 = x + 2$ X=-1, 2 The parabola and the line intersect at the point (-1,1/4) and (2,1) ∴ The required area is $=\int_{-1}^{2} y_1 dx - \int_{-1}^{2} y_2 dx$ $=\int_{-1}^{2}(\frac{x+2}{2} - x^2/4)dx$ $=\frac{1}{4}(\frac{x^2}{2} + 2x - \frac{x^3}{3}) \|_{-1}^{2}$ $=\frac{9}{8}$ sq.units	
13.	We have y=2x-x^2 ⇨$(x-1)^2$=-(y-1) ∴ the required area is $=\int_0^2 y dx$ $=\int_0^2 (2x - x^2)\, dx$ $=(2\frac{x^2}{2} - \frac{x^3}{3})\|_0^2$ $=\frac{4}{3}$sq. units  	2
14.	A=$\int_2^4 y dx = \int_2^4 \left(\frac{-x+8}{2}\right) dx = 5 square\ unit$	2
15.	A=$2\int_0^4 y dx = 2\int_0^4 \sqrt{16x}\ dx = 8\int_0^4 \sqrt{x}\, dx = \frac{128}{3}$square units	2
16.	A= $4\int_0^a y dx = 4\int_0^a b/a \sqrt{a^2 - x^2} dx = \pi ab$	2
17.	A= $\int_1^4 y dx = \int_1^4 \sqrt{x} dx = 14/3\ square\ units$	2
18.	A2 $\int_0^1 (x - x^2) dx - \frac{1}{3}\ square\ units$	2
19.	The graph is as follows  The required area is given by	2

	$$= \int_0^{\pi/2} siny\, dy = [-cosy]_0^{\frac{\pi}{2}} = 1$$	
20.	The graph is as follows  The required area is given by $$= \int_1^b y\, dx = \int_1^b 3x\, dx = 12$$ $$\frac{3b^2}{2} = \frac{27}{2}$$ b = 3	2
21.	The required area is given by the figure  So, The required area is given by $$A = 2\int_0^a y\, dx = 2\int_0^a \sqrt{16x}\ dx$$ $$\frac{128}{3} = 8\int_0^a x^{1/2}\, dx = \frac{16}{3}\ a^{3/2}$$ a = 4	2
22.	The region is given by the figure  	2

	So, the area is $$A = 4\int_0^1 y\,dx = 4\int_0^1 \sqrt{(1-x^2)}\,dx$$ $$A = \pi$$ (False)	
23.	The area is given by figure as follows  The required area is given by $$= \int_0^1 y\,dx = \int_0^1 \lvert x-5\rvert\ dx = \frac{9}{2} sq.units$$	2
24.	Required area = shaded area  $=\int_1^2 mx\,dx$ $=\frac{3}{2}$ m sq. unit	2
25.	Region bounded by the lines 2x+y = 8, y = 2, y=4 is as shown in the below figure:  Reqd. area = ar(CDEG) + ar(ΔGEF) = 2×2 + $\int_2^3 (8-2x)dx$ = 7 sq units	2
26.	 Reqd. area = $\int_{-1}^{1} x^2\,dx$ = $\frac{2}{3}$ sq. units	2
27.	We have y = x^3	2

	$\therefore$ **Reqd. area** $= \left\| \int_{-2}^{0} x^3\, dx \right\| + \int_{0}^{1} x^3\, dx$  $= \left\| \left[\frac{x^4}{4} \right]_{-2}^{0} \right\| + \left[\frac{x^4}{4} \right]_{0}^{1}$ $= \left\| \left(0 - \frac{16}{4} \right) \right\| + \left(\frac{1}{4} - 0 \right) = \frac{16}{4} + \frac{1}{4} = \frac{17}{4}.$	
28.	Reqd. area = $2\int_0^2 \sqrt{8x}\, dx$ $= \frac{8}{3}\sqrt{2}\,[2^{\frac{3}{2}} - 0]$ = 32/3 sq. units	2
29.	Required area = $\left(\int_{-1}^{2} (y_1 - y_2) dx \right)$ $= \int_{-1}^{2} -x^2 — x - 2\, dx$ $= \int_{-1}^{2} -x^2 + x + 2\, dx$ $= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{2}$ $= \left(-\frac{8}{3} + 6 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$ $= \frac{9}{2}\, sq. units$  	2
30.	We have $y^2 = 9x$ *and* $y = 3x$ $.\Rightarrow (3x)^2 = 9x$ $.\Rightarrow 9x^2 = 9x$ $.\Rightarrow 9x(x-1) = 0$ $.\Rightarrow x = 0,1$ $\therefore$ Required bounded area $= \int_0^1 \sqrt{9x}dx - \int_0^1 3x dx$ $= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 - 3\left[\frac{x^2}{2} \right]_0^1$ $= 3\left(\frac{2}{3} - 0 \right) - 3\left(\frac{1}{2} - 0 \right)$ $= 2 - \frac{3}{2}$ $= \frac{1}{2}\, sq\ units$  	2
31.	$\therefore$ Required bounded area between three houses $= \int_{-1}^{0} (4x+5)dx - \int_0^3 (5-x)dx - \frac{1}{4}\int_{-1}^{3} (x+5)dx$ $= \left[\frac{4x^2}{2} + 5 \right]_{-1}^{0} + \left[5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4}[x^2 + 5x]_{-1}^{3}$ $= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0 \right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5 \right]$  	2

	$=3+\frac{21}{2}-\frac{1}{4}.24$ $=-3+\frac{21}{2}=\frac{15}{2}\ sq\ units$	
32.	∴ Required area of each slice of pizza  $=\int_0^4 x dx+\int_4^{4\sqrt{2}}\sqrt{\left(4\sqrt{2}\right)^2-x^2}\,dx$ $=\left\|\frac{x^2}{2}\right\|_0^4+\left\|\frac{x}{2}\sqrt{\left(4\sqrt{2}\right)^2-x^2}+\frac{4\sqrt{2}}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right\|_4^{4\sqrt{2}}$ $=\frac{16}{2}+\left[\frac{4\sqrt{2}}{2}.0+16\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}}-\frac{4}{2}\sqrt{\left(4\sqrt{2}\right)^2-16}-16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$ $=8+\left[16.\frac{\pi}{2}-2.\sqrt{16}-16.\frac{\pi}{4}\right]$ $=8+[8\pi-8-4\pi]$ $=4\pi\ sq\ units$	2
33.	We have, $y=2\sqrt{x},\ x=0\ and\ x=1$  ∴ Area of shaded region = $\int_0^1 2\sqrt{x}\,dx$ $=2\left[\frac{x^{\frac{3}{2}}}{3}.2\right]_0^1=2\left(\frac{2}{3}.1-0\right)=\frac{4}{3}sq\ units$	2