CHAPTER-9

DIFFERENTIAL EQUATIONS

02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the differential equation representing the family of curves $y = ae^{2x} + 5$ constant.	2
2.	Form the differential equation of the family of hyperbolas having foci On x-axis and center at origin..	2
3.	Form the differential equation representing the family of curves y = a sin (x + b), where a ,b are arbitrary constant.	2
4.	Find the solution of $dy/dx = 2^{y^-}$.	2
5.	Find a particular solution satisfying the given condition (X+y)dy + (x-y)dx=0 Y=1 when x=1	2
6.	Given that $dy/dx = e^{-2y}$ and y = 0 when x = 5 Find the value of x when y = 3	2
7.	Solve the differential equation dy/dx + 2xy = y	2
8.	Find the general solution of $dy/dx + ay = e^{mx}$	2
9.	Verify that the function y = xsinx is a solution of the differential equation $x\frac{dy}{dx} = y + x\sqrt{x^2 - y^2}$.	2
10.	Find the general solution of the differential equation y logy dx – x dy = 0.	2
11.	Show that the differential equation $(x - y)\frac{dy}{dx} = (x + 2y)$ is homogeneous and solve it.	2
12.	Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$	2
13.	Find the general solution of the differential equation $(x+y)\frac{dy}{dx} = 1$	2
14.	It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r% per annum. Based on the above information, answer the following questions Find the value of dP/dt	2
15.	If P_0 be the initial principal, then find the solution of differential equation formed in given situation.	2
16.	Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\boldsymbol{dy/dx} = k(\mathbf{50} - \mathbf{y})$ where x denotes the number of weeks and y the number of children who have been given the drops.	

	Based on the above information, answer the following questions	
17.	Find the solution of the differential equation $\boldsymbol{dy/dx}$=k($\mathbf{50}-\mathbf{y}$) ?	2
18.	Find the value of c in the particular solution given that y(0)=0 and k=0.049 .	2
19.	Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.	2
20.	Solve that the differential equation $\frac{dy}{dx} + y = cos\ x\ - sin\ x$.	2
21.	If $y = 5e^{7x} + 6e^{-7x}$, then show that $\frac{d^2y}{dx^2} = 49y$.	2
22.	If $y = -Acos\ (3x) + Bsin\ (3x)$, then show that $\frac{d^2y}{dx^2} = -9y$.	2
23.	Solve the differential equation $cos\left(\frac{dy}{dx}\right) = a, a \in R$	2
24.	Find the general solution of the differential equation $$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$	2
25.	Find the difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$	2
26.	Find the solution of $\frac{dy}{dx}$=2^{y-x}	2
27.	Find the sum of the order and the degree of the differential equation $$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = sin\ y$$	2
28.	Find the sum of the order and the degree of the differential equation $$y''' + 2y'' + y' = 0$$	2
29.	Find the solution of the differential equation Log($\frac{dy}{dx}$) = ax + by	2
30.	Find the solution of the differential equation $\frac{dy}{dx}$=x+$\frac{y}{x}$satisfying the condition y(1) = 1.	2
31.	Solve the differential equation (e^x +1)ydy = e^x(y + 1)dx.	2
32.	Solve $\frac{dy}{dx}$+ 2xy = y	2
33.	Find the particular solution of the differential equation $\frac{dy}{dx}$= ytanx,when y(0)=1	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	dy/dx=2y-10	
2.	Equation of a hyperbola is given by $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ldots\ldots\ldots\ldots(1)$$ Differentiating both the sides, we get $$\frac{2x}{a^2}-\frac{2yy}{b^2}=0$$ By solving this equation we get $$xy\left(\frac{d^2y}{dx^2}\right)+\ xy^2-yy'=0$$	
3.	Y=asin(x+b) Differentiating w.r.t. x, we get, $$\frac{dy}{dx}=(x+b)\,.1$$ Again differentiating and by solving we get $\frac{d^2y}{dx^2}+\ y=0$	
4.	2^{-x}-2^{-y}=k	
5.	log(x^2+y^2)+2$\frac{y}{x}=\ \frac{\pi}{2}\ log2$	
6.	(e^6+9)/2	
7.	y=-$ce^{(x-x2)}$	
8.	(a+m)ye^{mx}+e^{-ax}	
9.	Given y=xsinx Then $\frac{dy}{dx}$=xcosx+sinx LHS=x$\frac{dy}{dx}$=x(xcosx+sinx). RHS= y +x$\sqrt{x^2-y^2}$=xsinx +x$\sqrt{x^2-x^2sin^2x}$ = x(xcosx+sinx) Therefore y=x.sinx is a solution of x$\frac{dy}{dx}$= y +x$\sqrt{x^2-y^2}$.	2
10.	given y logydx – xdy = 0. We get $\frac{dx}{x}$=$\frac{dy}{ylogy}$ Integrate it, we get General solution y =e^{cx}	2
11.	This is of the form $\frac{dy}{dx}$=g$\left(\frac{y}{x}\right)$. Put y=vx ,then $\frac{dy}{dx}$=v+x$\frac{dv}{dx}$ Gives $\left(\frac{v-1}{v^2+v+1}\right)dv$=$\frac{-dx}{x}$ Integrate it, we get $\frac{1}{2}$log(v^2+v+1)+ $\frac{1}{2}$logx^2=$\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)$+c Put v=$\frac{y}{x}$,we get	2

	General solution $\log\|x^2+xy+y^2\|=2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}\,x}\right)+c_1$	
12.	ANS:given differential equation $x\frac{dy}{dx}+2y=x^2$ Then $\frac{dy}{dx}+\frac{2}{x}y=x$ The given equ. is a L.D.E. of the type $\frac{dy}{dx}+Py=Q$,where $P=\frac{2}{x}$ and Q=x IF= $e^{\int\frac{2}{x}dx}=x^2$ solution is given by $yx^2=\int(x)(x^2)dx+c$ General solution $y=x^2/4+cx^{-2}$.	2
13.	ANS:given differential equation is $(x+y)\frac{dy}{dx}=1$ Then $\frac{dx}{dy}-x=y$ The given equ. is a L.D.E. of the type $\frac{dx}{dy}+Px=Q$,where P=-1 and Q=y IF= $e^{\int -1dy}=e^{-y}$ Solution is given by x.IF= $\int Q\times IF dy$+C $xe^{-y}=\int ye^{-y}dy+C=-ye^{-y}-e^{-y}+C$ Gives general solution $(x+y+1)=c\,e^y$.	2
14.	$\frac{dp}{dt}=\frac{Pr}{100}$	2
15.	$log\left(\frac{p}{p_0}\right)=\frac{rt}{100}$	2
16.	$-\log\|50-y\|=kx+C$	2
17.	$log\frac{1}{50}$	2
18.	$y=50(1-e^{-kx})$	2
19.	$\frac{dy}{dx}=e^{x+y}$ $\Rightarrow\frac{dy}{dx}=e^x.e^y$ $\Rightarrow\frac{dy}{e^y}=e^x dx$ $\Rightarrow e^{-y}dy=e^x dx$ $\Rightarrow\int e^{-y}dy=\int e^x dx$ $\Rightarrow -e^{-y}=e^x+c$	2
20.	$\frac{dy}{dx}+y=\cos x-\sin x\ldots(i)$ $\frac{dy}{dx}+Py=Q\ldots(ii)$ On comparison, we get $P=1, Q=\cos x-\sin x$ Integrating Factor $(\mathrm{I.F})=e^{\int p dx}=e^{\int dx}=e^x$ Hence, the $\mathrm{sol^n}$ is : $y\times\mathrm{I.F}=\int Q\times\mathrm{I.F}\,dx$ $\Rightarrow ye^x=\int e^x(\cos x-\sin x)dx$ $\Rightarrow ye^x=e^x\cos x+c$ $\left(\text{Applying}\int e^x(f(x)+f'(x))dx=e^x f(x)+c, \text{here}, f(x)=\cos x\right)$	2

21.	$y = 5e^{7x} + 6e^{-7x} \dots (i)$ Differentiating both sides w.r.t. x $\frac{dy}{dx} = 7(5e^{7x} - 6e^{-7x})$ Differentiating both sides w.r.t. x $\frac{d^2y}{dx^2} = 49(5e^{7x} + 6e^{-7x})$ $\Rightarrow \frac{d^2y}{dx^2} = 49y$ (proved)(from (i))	2
22.	$y = -A\cos(3x) + B\sin(3x) \dots (i)$ Differentiating both sides w.r.t. x $\frac{dy}{dx} = 3(A\sin(3x) + B\cos(3x))$ $\Rightarrow \frac{d^2y}{dx^2} = 9(A\cos(3x) - B\sin(3x))$ $\Rightarrow \frac{d^2y}{dx^2} = -9(-A\cos(3x) + B\sin(3x))$ $\Rightarrow \frac{d^2y}{dx^2} = -9y$ (from (i))	2
23.	$\cos\left(\frac{dy}{dx}\right) = a, a \in \mathbb{R}$ $\Rightarrow \frac{dy}{dx} = \cos^{-1} a$ $\Rightarrow dy = \cos^{-1} a.dx$ $\Rightarrow \int dy = \cos^{-1} a \int dx$ $\Rightarrow y = x\ \cos^{-1} a + c$	2
24.	$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ Separating the variable and integrating both sides, $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$ $tan^{-1}y$=$tan^{-1}x$+C, which is the general solution of given equation	2
25.	Order =2, Degree = 2, Difference of order and degree = 0	2
26.	Rewriting the equation as $\frac{dy}{dx} = \frac{2^y}{2^x}$ $\frac{dy}{2^y} = \frac{dx}{2^x}$ Integrating both sides $\int \frac{dy}{2^y} = \int \frac{dx}{2^x}$ $-\frac{2^{-y}}{\log 2} = -\frac{2^{-x}}{\log 2} + C_1$ $2^{-x} - 2^{-y} = C$ Which is the general solution of given equation.	2
27.	Order =2, Degree = 1, Sum of order and degree = 3	2
28.	Order =3, Degree = 1, Sum of order and degree = 4	2

29.	. Given, $\log(\frac{dy}{dx}) = ax + by$ $\frac{dy}{dx} = e^{ax + by}$ [∵$\log_b a=c= a=b^c$] ➢ $\frac{dy}{dx} = e^{ax} \cdot e^{by} = \frac{dy}{e^{by}} = e^{ax}dx$ ➢ $e^{-by}dy = e^{ax}dx$ On integrating both sides, we get $\int e^{-by}dy = \int e^{ax}dx$ ➢ $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$ [∵$\int e^{ax + b}dx = \frac{e^{ax}+ b}{a} + C$] ➢ $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C = 0$ Which is required solution	2
30.	Given, equation can be rewritten as $\frac{Dy}{dx} = \frac{1}{x} .y =1$ Here, $P = -\frac{1}{x}$ and $Q =1$ ∵ IF = $e^{\int Pdx} = e^\wedge -\int \frac{1}{x}dx = e^{-\log x} = \frac{1}{x}$ ∵ Required solution is $Y\left(\frac{1}{x}\right) = \int \frac{1}{x}dx = \log x + C$ [∵$y \cdot IF = \int (Q \cdot IF)dx+C$] Since, y(1) =1 and C = 1 ∵ y = xlogx + x	2
31.	We have, $(e^x + 1)ydy = e^x(y + 1)dx$ On separating the variables , we get $\frac{y}{y+1}dy = \frac{e^x}{e^x+1}dx$ On integrating both sides, we get $\int \frac{y}{y+1} dy = \int \frac{e^x}{e^x+1}dx$ ➢ $\int y + 1 - \frac{1}{y} + 1dy = \int \frac{e^x}{e^x+1}dx$ ➢ $\int (1 - \frac{1}{y+1})dy = \int \frac{e^x}{e^x+1}dx$ ➢ $y - \log(y+1) = \log(e^x + 1) + C$ which is required solution.	2
32.	Given that, $xdx-ye^y\sqrt{1 + x^2}dy=0 \Rightarrow xdx = ye^y\sqrt{1 + x^2}dy$ ⇨ $\frac{x}{\sqrt{1+x^\wedge 2}}dx = ye^{dy}$ Integrating both sides,we get $\int x/\sqrt{1 + x^2}dx = \int y \cdot e^y dy$ $\Rightarrow \frac{1}{2}\int 2x/\sqrt{1 + x^2}dx = [y\cdot\int e^\wedge y dy - \int (\frac{d}{dy}(y)\cdot\int e^\wedge y dy)dy]$ Let $I_1 = \int 2x/\sqrt{1 + x^2}dx$ Putting $1 + x^2 = t \Rightarrow 2x\, dx = dt$ [on differentiating] ∵ $I_1 = \int dt/t^{1/2} = \int t^{-1/2}dt$ $= t^{-1/2+1}/-\frac{1}{2}+1 = t^{-1/2}/\frac{1}{2} = 2t^{1/2}$ $=2(1+x^2)^{½}$ Now, $½.2(1+x^2)^{½} = y\cdot e^y - e^y + C$ $\Rightarrow (1+x^2)^{1/2} = e^y(y-1) + C$ When x = 0, then y = 1	2

	∴ $(1+0)^{1/2} = e^{1}(1-1) + C$ $C = 1$ So, required solution is given by $(1+x^2)^{1/2} = e^y(y-1) + 1$	
33.	We have, $\frac{dy}{y}$ = ytanx On separating variable both sides, we get $\frac{dy}{y}$ = tanx dx On intergrating both sides,we get $\int dy/y = \int tanx\ dx$ => logy =logsecx + logC => logy = log(C secx) [∵loga+logb = logab] => y = C secx …(i) Now, it is given that x = 0 and y = 1 ∴ 1 =C sec0 => 1 =C On putting C = 1 in Eq. (i), we get Y = secx. Which is required solution.	2