

CHAPTER-7
INTEGRALS
02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Evaluate : $\int_0^1 x^2 e^x dx$	2
2.	Find : $\int \frac{\tan^3 x}{\cos^3 x} dx$	2
3.	Find: $\int \frac{x-1}{(x-2)(x-3)} dx$	2
4.	Find: $\int_{-\frac{\pi}{4}}^0 \frac{1+\tan x}{1-\tan x} dx$	2
5.	Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx$	2
6.	Write the value of $\int \sec x (\sec x + \tan x) dx$	2
7.	Evaluate: $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$	2
8.	Evaluate: $\int \frac{dx}{5 - 8x - x^2}$	2
9.	Evaluate: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$	2
10.	Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx$	2
11.	If $f(x) = \int_0^x t \sin t dt$, then find the value of $f'(x)$	2
12.	Find $\int \frac{\sin^6 x}{\cos^8 x} dx$	2
13.	Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$	2
14.	Evaluate : $\int (\sin x) dx$	2
15.	Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx$	2

16.	Find the value of $\int \sin x \sqrt{1 - \cos 2x} \, dx$	2
17.	find the value of $\int \frac{1}{1+e^{-x}} \, dx$	2
18.	Find the value of $\int_0^{2\pi} \sin x \, dx$	2
19.	Find the value of $\int 5^{x+\sqrt{x^2+2}} \, dx$	2
20.	Find the value of $\int \sqrt{1 + \sin x} \, dx$	2
21.	Evaluate $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$	2
22.	Evaluate $\int \frac{(x+3)e^x}{(x+5)^3} \, dx$	2
23.	Evaluate $\int \operatorname{cosec}^3 x \, dx$	2
24.	Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} \, dx, \quad a > 0$	2
25.	Evaluate: $\int \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) \, dx, \quad x \in \left(-\frac{\pi}{2}, \frac{3\pi}{2} \right)$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\int_0^1 x^2 e^x dx = [x^2 e^x]_0^1 - \int_0^1 2x e^x dx$ $= [x^2 e^x - 2x e^x + 2e^x]_0^1$ $= e - 2$	2
2.	Given $I = \int \frac{\tan^3 x}{\cos^3 x} dx$ Let $\cos x = t$ So, $\sin x dx = -dt$ $I = \int \left[\frac{-1}{t^6} + \frac{1}{t^4} \right] dt = \frac{t^{-5}}{-5} + \frac{t^{-3}}{-3} + C = \frac{1}{5(\cos x)^5} - \frac{1}{3(\cos x)^3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$	2
3.	$\int \frac{x-1}{(x-2)(x-3)} dx$ Since, $\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$, on solving $A = -1$ and $B = 2$ $\Rightarrow \frac{x-1}{(x-2)(x-3)} = \frac{-1}{x-2} + \frac{2}{x-3}$ $\Rightarrow \int \frac{x-1}{(x-2)(x-3)} dx = -\int \frac{dx}{x-2} + 2 \int \frac{dx}{x-3}$ $= -\log(x-2) + 2 \log(x-3) + C$ $= -\log(x-2) + \log(x-3)^2 + C = \log \frac{(x-3)^2}{(x-2)} + C$	2
4.	$\int_{-\frac{\pi}{4}}^0 \frac{1+\tan x}{1-\tan x} dx = \int_{-\frac{\pi}{4}}^0 \tan \tan \left(\frac{\pi}{4} + x \right) dx =$ $= [\log \sec \left(\frac{\pi}{4} + x \right)]_{-\frac{\pi}{4}}^0$ $= \log \sec \frac{\pi}{4} - \log \sec \left(\frac{\pi}{4} - \frac{\pi}{4} \right) = \log(\sqrt{2}) - \log(\sec 0) = \log(\sqrt{2}) - \log 1 = \log(\sqrt{2}) = \frac{1}{2} \log 2$	2
5.	Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx$ Put $2x = t$, $\therefore dx = \frac{1}{2} dt$ $\therefore \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx = \int_2^4 \left[\frac{1}{t} - \frac{1}{t^2} \right] e^t dt = \left[\frac{1}{t} e^t \right]_2^4 = \frac{e^4}{4} - \frac{e^2}{2}$	2
6.	$I = \int \sec x (\sec x + \tan x) dx$ $= \int (\sec^2 x + \sec x \tan x) dx$ $= \int \sec^2 x dx + \int \sec x \tan x dx$ $= \tan x + \sec x + C$	2
7.	Let $I = \int \frac{x^3 - x^2 + x - 1}{x-1} dx = \int \frac{x^2(x-1) + 1(x-1)}{(x-1)} dx$ $= \int \frac{(x^2+1)(x-1)}{(x-1)} dx = \int (x^2 + 1) dx = \frac{x^3}{3} + x + c$	2
8.	$\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{5 - 2.4x - x^2 + 4^2 - 4^2}$ $= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$ $= \frac{1}{2\sqrt{21}} \log \left(\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4} \right) + c$	2
9.	$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = [\tan^{-1} x]_1^{\sqrt{3}}$ $= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$	2
10.	Use the property, $\int_{-a}^a f(x) dx = 0$, if $f(-x) = -f(x)$; $f(x)$ is a odd function $f(x) = \sin^5 x \Rightarrow f(-x) = \sin^5(-x) = -\sin^5 x = -f(x)$ so, $f(x)$ is a odd function $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx = 0$	2
11.	Differentiating both sides w.r.t to x we get $f'(x) = [tsint]_0^x$ $= x \sin x - 0 = x \sin x$	1 1

12.	$\int \frac{\sin^6 x}{\cos^8 x} dx = \int \tan^6 x \sec^2 x dx$ $= \int t^6 dt, \text{ where } \tan x = t \Rightarrow \sec^2 x dx = dt$ $= \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C$	1 1
13.	Let $I = \int_e^{e^2} \frac{dx}{x \log x}$, Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$ When $x = e$, $t = \log e = 1$ and when $x = e^2$, $t = 2 \log e = 2$ $\text{Let } I = \int_e^{e^2} \frac{dx}{x \log x} = \int_1^2 \frac{dt}{t} = [\log t]_1^2 = \log 2 - \log 1 = \log 2$	1 1
14.	$\int (\sin x) dx = \int \left(\cos \left(\frac{\pi}{2} - x \right) \right) dx$ $= \int \left(\frac{\pi}{2} - x \right) dx = \frac{\pi}{2} x - \frac{x^2}{2} + C$	1 1
15.	Let $f(x) = x \cos^2 x$ $\Rightarrow f(-x) = (-x) \cos^2(-x) = -x \cos^2 x = -f(x)$ $\Rightarrow f(x)$ is an odd function $\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx = 0$	1 1
16.	$\int \sin x \sqrt{1 - \cos 2x} dx$ $= \int (\sin x)(\sqrt{2} \sin x) dx$ $= \frac{\sqrt{2}}{2} \int 2(\sin x)^2 dx$ $= \frac{1}{\sqrt{2}} \int (1 - \cos 2x) dx$ $= \frac{x}{\sqrt{2}} - \frac{\sin 2x}{2\sqrt{2}} + c \text{ (Answer)}$	2
17.	$\int \frac{1}{1 + e^{-x}} dx$ $= \int \frac{e^x}{e^x + 1} dx$ $= \int \frac{d(e^x + 1)}{e^x + 1}$ $= \log \log e^x + 1 + c \text{ (Answer)}$	2
18.	$\int_0^{2\pi} \sin x dx$ $= \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} -\sin x dx$ $= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$ $= -\cos \pi + \cos 0 + \cos 2\pi - \cos \pi$	2

	$= 4(\text{answer})$	
19.	$\int 5^{x+x} \left(\frac{x^2+2}{x^2+1} \right) dx$ $= \int 5^{x+x} \left(1 + \frac{1}{x^2+1} \right) dx$ $= \int 5^u du \quad \left[\text{Taking, } u = x+x, \text{ hence } du = \left(1 + \frac{1}{1+x^2} \right) dx \right]$ $= \frac{5^u}{\log 5} + c$ $= \frac{5^{x+x}}{\log \log 5} + c \text{ (Answer)}$	2
20.	$\int \sqrt{1 + \sin x} dx$ $= \int \sqrt{\left(\sin \frac{x}{2} \right)^2 + \left(\cos \frac{x}{2} \right)^2 + 2 \cos \frac{x}{2} \sin \frac{x}{2}} dx$ $= \int \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) dx$ $= -2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c \text{ (Answer)}$	2
21.	$\int \frac{dx}{\cos x + \sqrt{3} \sin x} = \frac{1}{2} \int \frac{dx}{\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x} = \frac{1}{2} \int \frac{dx}{\sin \frac{\pi}{6} \cos x + \cos \frac{\pi}{6} \sin x} = \frac{1}{2} \int \frac{dx}{\sin \left(x + \frac{\pi}{6} \right)}$ $= \frac{1}{2} \int \operatorname{cosec} \left(x + \frac{\pi}{6} \right) dx = \frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$	2
22.	$\int \frac{(x+3)e^x}{(x+5)^3} dx = \int \frac{(x+5-2)e^x}{(x+5)^3} dx = \int e^x \left\{ \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} \right\} dx = e^x \frac{1}{(x+5)^2} + C$	2
23.	$I = \int \operatorname{cosec}^3 x dx = \int \operatorname{cosec} x \operatorname{cosec}^2 x dx =$ $= \operatorname{cosec} x \int \operatorname{cosec}^2 x dx - \int \frac{d}{dx} (\operatorname{cosec} x) \int \operatorname{cosec}^2 x dx$ $= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x dx$ $= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) dx$ $= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec} x dx$ $= -\operatorname{cosec} x \cot x - I + \int \operatorname{cosec} x dx$ $2I = -\operatorname{cosec} x \cot x + \log \left \tan \frac{x}{2} \right $ $\therefore I = -\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \log \left \tan \frac{x}{2} \right + C$	2
24.	$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx \text{ ----- (i)}$ $\text{Also } I = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}} dx = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx \text{ ---- (ii)}$ $\text{Adding } 2I = \int_{-\pi}^{\pi} \cos^2 x dx = 2 \int_0^{\pi} \cos^2 x dx = 2 \cdot 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx$ $I = 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx = 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad 0 = 2 \int_0^{\frac{\pi}{2}} dx - 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx$ $I + I = 2 \int_0^{\frac{\pi}{2}} dx = \pi \therefore I = \frac{\pi}{2}$	2
25.	$\int \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) dx, x \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$	2

	$= \int \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2} \right) dx$ $= \int \tan^{-1} \left(\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right) dx = \int \tan^{-1} \left(\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right) dx = \int \tan^{-1} \left(\tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right) dx$ $= \int \left(\frac{x}{2} + \frac{\pi}{4} \right) dx = \frac{x^2}{4} + \frac{\pi x}{4} + C$	
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