CHAPTER-10

VECTORS

02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\|\vec{a}\| = 3$, $\|\vec{b}\| = 4$, $\|\vec{c}\| = 5$ and each one of them being perpendicular to the sum of the other two find $\|\vec{a} + \vec{b} + \vec{c}\|$	**2**
2.	If $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b} = -\hat{i}+\hat{k}$, $\vec{c} = 2\hat{j}-\hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a} + \vec{b})$ and $(\vec{b} + \vec{c})$	**2**
3.	Show that the points A($-2\hat{i}+3\hat{j}+5\hat{k}$), B ($\hat{i}+2\hat{j}+3\hat{k}$), C ($7\hat{i}-\hat{k}$) are collinear.	**2**
4.	If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$, $\vec{c} = 3\hat{i}+\hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.	**2**
5.	If $\vec{p}$ and $\vec{q}$ are the unit vectors forming an angle of 300 , find the area of the parallelogram having $\vec{a} = \vec{p} + 2\vec{q}$ and $\vec{b} = 2\vec{p} + \vec{q}$ as its diagonals.	**2**
6.	Find the direction ratios and direction cosines of the vector $\vec{a} = (5\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$.	2
7.	Write the value of p for $\vec{a} = 3\hat{\imath}+2\hat{\jmath}+9\hat{k}$ and $\vec{b} = \hat{\imath}+p\hat{\jmath}+3\hat{k}$ are parallel vectors.	2
8.	Find $\vec{a}.(\vec{b}\times\vec{c})$ if $\vec{a} = 2\hat{\imath}+\hat{\jmath}+3\hat{k}$, $\vec{b} = -\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c} = 3\hat{\imath}+\hat{\jmath}+2\hat{k}$.	2
9.	If $\vec{a} = x\hat{\imath}+2\hat{\jmath}-z\hat{k}$ and $\vec{b} = 3\hat{\imath}-y\hat{\jmath}+\hat{k}$ are two equal vectors, then write the value of y^x+5z.	2
10.	Find a unit vector parallel to the sum of the vectors ($\hat{\imath}+\hat{\jmath}+\hat{k}$) and ($2\hat{\imath}-3\hat{\jmath}+5\hat{k}$).	2
11.	$If\ \ \vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{k}$, then find the value of $\hat{a} - \hat{b} + 2\hat{c}$.	2
12.	The sum of two unit vectors is a unit vector. Show that the value of their difference is $\sqrt{3}$.	2
13.	Find a vector in the direction of $5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has magnitude 8.	2
14.	Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the axes OX, OY, OZ	2
15.	If $\|\vec{a}\| = 10, \|\vec{b}\| = 1$ and $\|\vec{a}.\vec{b}\| = 6$, then find $\|\vec{a} \times \vec{b}\|$	2
16.	Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.	2
17.	Prove that the points A,B and C with position vectors $\vec{a}$, $\vec{b}\ and\ \vec{c}$ respectively are collinear if and only if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$.	2
18.	Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$	2
19.	If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.	2
20.	Show that the points A,B and C with position vectors $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}\ and\ \vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ respectively from the vertices of a right angled triangle.	2
21.	For what value of a, the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear?	2
22.	Find unit vector perpendicular to both the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$.	2
23.	If $\vec{a} = 2$, $\vec{b} = \sqrt{3}$ and $\vec{a}\ .\vec{b} = \sqrt{3}$ find the angle between $\vec{a}$ and $\vec{b}$.	2
24.	If $\vec{p}$ is unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$,then find $\|\vec{x}\|$.	2
25.	Show that the points A($-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$),B($\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$), and C($7\hat{\imath} - \hat{k}$) are collinear.	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\left\|\vec{a}+\vec{b}+\vec{c}\right\|^2=\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$ $=\vec{a}.\vec{a}+\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}.\left(\vec{a}+\vec{c}\right)+\vec{c}.\vec{c}+\left(\vec{a}+\vec{b}\right)$ $=\left\|\vec{a}\right\|^2+\left\|\vec{b}\right\|^2+\left\|\vec{c}\right\|^2$ $=9+16+25$ $=50$ $\left\|\vec{a}+\vec{b}+\vec{c}\right\|=\sqrt{50}$ $=5\sqrt{2}$	**2**
2.	It is given that $\vec{a}=2\hat{i}-3\hat{j}+\hat{k},\vec{b}=-\hat{i}+\hat{k},\vec{c}=2\hat{j}-\hat{k}$ $\therefore \vec{a}+\vec{b}=\left(2\hat{i}-3\hat{j}+\hat{k}\right)+\left(-\hat{i}+\hat{k}\right)=\hat{i}-3\hat{j}+2\hat{k}$ $\vec{b}+\vec{c}=\left(-\hat{i}+\hat{k}\right)+\left(2\hat{j}-\hat{k}\right)=-\hat{i}+2\hat{j}$ We know that the area of parallelogram is $\frac{1}{2}\left\|\vec{d_1}\times\vec{d_2}\right\|$, where $\vec{d_1}$ and $\vec{d_2}$ are the diagonal vectors. Now, $\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ -1 & 2 & 0\end{vmatrix}=-4\hat{i}-2\hat{j}-\hat{k}$ $\therefore$ Area of the parallelogram having diagonals $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{b}+\vec{c}\right)$ $=\frac{1}{2}\left\|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)\right\|$ $=\frac{1}{2}\left\|-4\hat{i}-2\hat{j}-\hat{k}\right\|$ $=\frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2}$ $=\frac{\sqrt{21}}{2}$ square units Thus, the required area of the parallelogram is $\frac{\sqrt{21}}{2}$ square units.	**2**

3.	We have vector AB = (1 + 2)i + (2 - 3)j + (3 - 5)k = 3i - j - 2k vector BC = (7 - 1)i + (0 - 2)j + (-1 - 3)k = 6i - 2j - 4k vector CA = (7 + 2)i + (0 - 3)j + (-1 - 5)k = 9i - 3j - 6k Now, $\lvert\text{vector AB}\rvert^2 = 14$, $\lvert\text{vector BC}\rvert^2 = 56$, $\lvert\text{vector CA}\rvert^2 = 126$ ⇒ |vector AB| = √14, |vector BC| = 2√14, |vector CA| = 3√14 ⇒ |vector CA| = | vector AB| + |vector BC| Hence the points A, B and C are collinear.	**2**
4.	$\vec{a}+\lambda\vec{b}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$ **Ans:** $=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$ $\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0\left[\because \vec{a}+\lambda\vec{b}\perp\vec{c}\right.$ $\left[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\right].\left(3\hat{i}+\hat{j}\right)=0$ $3(2-\lambda)+(2+2\lambda)=0$ $-\lambda=-8$ $\lambda=8$	**2**
5.	$\vec{a}=\hat{p}+2\,\hat{q}$ $\vec{b}=2\,\hat{p}+\hat{q}$ $\vec{a}\times\vec{b}=(\vec{p}+2\vec{q})\times(2\vec{p}+\vec{q})$ $=2\vec{p}\times\vec{p}+\vec{p}\times\vec{q}+4\vec{q}\times\vec{p}+2\vec{q}\times\vec{q}$ $=2\,(0)+\vec{p}\times\vec{q}-4\vec{p}\times\vec{q}+2\,(0)$ $=-3\vec{p}\times\vec{q}$ Area of the parallelogram $=\frac{1}{2}\left\lvert\vec{a}\times\vec{b}\right\rvert$ $=\frac{1}{2}\lvert-3\,(\vec{p}\times\vec{q})\rvert$ $=\frac{3}{2}\lvert\vec{p}\rvert\,\lvert\vec{q}\rvert\sin 30^o$ $=\frac{3}{2}(1)\,(1)\left(\frac{1}{2}\right)(\because \vec{p}$ and $\vec{q}$ are unit vectors $)$ $=\frac{3}{4}$ sq. units	**2**
6.	Given that $\vec{a}=(5\hat{\imath}-3\hat{\jmath}+4\hat{k})$ For any vector $\vec{a}=a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}$ the direction ratios are represented as (a_x,a_y,a_z) The direction ratios are (5, -3, 4) $\lvert\vec{a}\rvert=\sqrt{25+9+16}=\sqrt{50}=5\sqrt{2}$ ∴ The direction cosines are = $\frac{5}{5\sqrt{2}},\frac{-3}{5\sqrt{2}},\frac{4}{5\sqrt{2}}=\frac{1}{\sqrt{2}},\frac{-3}{5\sqrt{2}},\frac{4}{5\sqrt{2}}$	2

7.	*Given that* $\vec{a} = 3\hat{\imath}+2\hat{\jmath}+9\hat{k}$ and $\vec{b} = \hat{\imath}+p\hat{\jmath}+3\hat{k}$ Since these two vectors are parallel to each other, so the angle between them is $\theta = 0$. Therefore $\vec{a}\times\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin 0 = 0$ We know that $\vec{a}\times\vec{b} = (a_2b_3-b_2a_3)\hat{\imath} + (a_3b_1-a_1b_3)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k}$ $\therefore \vec{a}\times\vec{b} = 0$ $\Rightarrow (a_2b_3-b_2a_3)\hat{\imath} + (a_3b_1-a_1b_3)\hat{\jmath} + (a_1b_2-a_2b_1)\hat{k} = 0$ $\Rightarrow \hat{\imath}\,(6-9p) + \hat{\jmath}\,(9-9) + \hat{k}\,(3p - 2) = 0$ $\Rightarrow -3\hat{\imath}\,(3p-2) + \hat{k}\,(3p - 2) = 0$ $\Rightarrow 3p-2 = 0 \Rightarrow$ Thus $p = 2/3$	2
8.	Given that $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ To find $\vec{a}.(\vec{b}\times\vec{c})$ We know that $\vec{b}\times\vec{c} = \hat{\imath}(b_2c_3-c_2b_3) + \hat{\jmath}(c_1b_3-b_1c_3) + \hat{k}(b_1c_2-c_1b_2)$ Here $a_1=2$, $a_2=1$, $a_3=3$, $b_1=-1$, $b_2=2$, $b_3=1$, $c_1=3$, $c_2=1$, $c_3=2$ $\therefore \vec{b}\times\vec{c} = \hat{\imath}\,(4-1) + \hat{\jmath}(3+2) + \hat{k}(-1-6) = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$ Therefore, $\vec{a}.(\vec{b}\times\vec{c}) = (2\hat{\imath}+\hat{\jmath}+3\hat{k}).(3i+5\hat{\jmath}-7\hat{k}) = ((2\times3) + (1\times5) + (3\times(-7)) = 6+5-21$ $= -10$	2
9.	Given that $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors. $\therefore x = 3, y = -2$ and $z = -1$ $\therefore y^x+5z = (-2)^3 + 5(-1) = -8 - 5 = -13$	2
10.	Let $\vec{a} = (\hat{\imath}+\hat{\jmath}+\hat{k})$ and $\vec{b} = (2\hat{\imath}-3\hat{\jmath}+5\hat{k})$ $\vec{a}+\vec{b} = (\hat{\imath}+\hat{\jmath}+\hat{k}) + (2\hat{\imath}-3\hat{\jmath}+5\hat{k}) = 3\hat{\imath}-2\hat{\jmath}+6\hat{k}$ The unit vector parallel to the sum of the given vectors $= \frac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert} = \frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{\sqrt{9+4+36}} = \frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{\sqrt{49}} =$ $\frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{7} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$	2
11.	$\lvert\hat{a}-\hat{b}+2\hat{c}\rvert = \sqrt{4+25+1} = \sqrt{30}$ $d.r = -\frac{2}{\sqrt{30}}.-\frac{5}{\sqrt{30}},-\frac{1}{\sqrt{30}}$	2
12.	$\lvert\vec{a}\rvert = 1, \lvert\vec{b}\rvert = 1, \lvert\vec{a}+\vec{b}\rvert = 1$ $(\lvert\vec{a}+\vec{b}\rvert)^2 + (\lvert\vec{a}-\vec{b}\rvert)^2 = 2\left\{\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2\right\} = 4$ $(\lvert\vec{a}-\vec{b}\rvert)^2 = 3$ $\lvert\vec{a}-\vec{b}\rvert = \sqrt{3}$	2
13.	$\vec{a} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}, \quad \hat{a} = \frac{5\hat{\imath}-\hat{\jmath}+2\hat{k}}{\sqrt{30}}$ $Req\ vector = 8\hat{a} = 8.\frac{5\hat{\imath}-\hat{\jmath}+2\hat{k}}{\sqrt{30}}$	2
14.	$\lvert\vec{a}\rvert = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ d.c $= (\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})$ $cos\alpha = \frac{1}{\sqrt{3}}, cos\beta = \frac{1}{\sqrt{3}}, cos\gamma = \frac{1}{\sqrt{3}}$ $\alpha = \beta = \gamma$(where α,β,γ are the inclination of $\vec{a}\ with\ OX, OY, OZ$ resp.)	2
15.	$(\lvert\vec{a}.\vec{b}\rvert)^2 + (\lvert\vec{a}\times\vec{b}\rvert)^2 = \left\{\lvert\vec{a}\rvert^2.\lvert\vec{b}\rvert^2\right\}$ $(\lvert\vec{a}\times\vec{b}\rvert)^2 = 64$	2

	$\lvert \vec{a} \times \vec{b} \rvert = 8$	
16.	$-\frac{1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{1}{\sqrt{6}}\hat{k}$	2
17.	Proving $\overrightarrow{AB} \times \overrightarrow{BC} = 0, (\vec{b} - \vec{c}) \times (\vec{c} - \vec{b}) = 0$ and proceeding further to prove.	2
18.	Expanding and solving.	2
19.	$2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$	2
20.	$\lvert\overrightarrow{AB}\rvert = \sqrt{35}, \lvert\overrightarrow{BC}\rvert = \sqrt{41}\ and\ \lvert\overrightarrow{CA}\rvert = \sqrt{6}$ and apply Pythagoras theorem.	2
21.	Let $\vec{A} = 2\,\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ $\vec{B} = a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ $\vec{A}$ and $\vec{B}$ are collinear so , $\vec{A} = \lambda\vec{B}$ $\frac{2}{a} = \frac{-3}{6} = \frac{4}{-8}$ a= -4	2
22.	Unit vector perpendicular to $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$ is $\hat{n} = \frac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert}$ … (i) $\vec{a} \times \vec{b} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = -\hat{\imath} + \hat{\jmath}$ $\lvert \vec{a} \times \vec{b} \rvert = \sqrt{1+1} = \sqrt{2}$ From equation (i) $\hat{n} = \frac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert} = \frac{1}{2}(-\hat{\imath} + \hat{\jmath})$	2
23.	We know that angle between $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\cos\theta = \frac{\sqrt{3}}{2\sqrt{3}}$ $\cos\theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$	2
24.	It is given that $\vec{p}$ is unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$ $\lvert\vec{x}\rvert^2 - \lvert\vec{p}\rvert^2 = 80$ $\lvert\vec{x}\rvert^2 = 80 + 1 = 81$ $\lvert\vec{x}\rvert = 9$	2
25.	Given points A$(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$ B$(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ C$(7\hat{\imath} - \hat{k})$ $\overrightarrow{AB}$=P.V. of B – P.V. OF A $=(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$ $=(3\hat{\imath} - \hat{\jmath} - 2\hat{k})$ $\overrightarrow{BC}$=P.V. of C – P.V. of B $=(7\hat{\imath} - \hat{k}) - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$	2

	$=(6\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$ $\overrightarrow{BC}=2\,\overrightarrow{AB}$ $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$. B is common. Hence A,B,C are collinear.	