

CHAPTER-4
DETERMINANTS
02 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	If W is an imaginary cube root of unity, find the value of $\begin{vmatrix} W^2 & W & 1 \\ 1 & W^2 & W \\ W & 1 & W^2 \end{vmatrix}$	2
2.	If $\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix}$ Write the cofactors of a_{21} , a_{22} , a_{31} , a_{33}	2
3.	Prove that $\begin{vmatrix} x+5 & x & x \\ x & x+5 & x \\ x & x & x+5 \end{vmatrix} = 75x^2 + 125x$	2
4.	If A is an invertible matrix of order 2, then det is equal (A^{-1}) to	2
5.	If $\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{vmatrix}$ Write the cofactors of a_{21} , a_{22} , a_{31} , a_{33}	2
6.	If A is a symmetric matrix and B is skew-symmetric matrix such that $A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $ 2A $.	2
7.	If A is any matrix such that $A^5 = I$, then find the value of $ A^{-1} $.	2
8.	Evaluate the determinant $\begin{vmatrix} \log_4 9 & \log_3 8 \\ \log_4 3 & \log_3 512 \end{vmatrix}$.	2
9.	If A is a square matrix of order 3 and $ A = 4$, then find $ adj.(2A) $	2
10.	If A and B are invertible matrices of order 3 such that $ A = 2$ and $ (AB)^{-1} = \frac{-1}{6}$, then find $ B $.	2
11.	If the Matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, find $ adjA $ without computing $adj A$.	2
12.	If the matrix $\begin{bmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{bmatrix}$ is singular, find x.	2
13.	If $A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = A^2$	2
14.	Find the value(s) of k if the area of the triangle with vertices (-2,0), (0,4) and (0,k) is 4 square unit.	2
15.	If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $A^{-1} = kA$, then find the value of k.	2
16.	Barun visited three places Kolkata, Bhubaneswar and Bangalore with his younger Karan. He observed on map that the three places make a straight line. Karan wrote these places as points	2

	$(2x, x + 3), (0, x)$ and $(x + 3, x + 6)$.  Find the coordinates of Kolkata and Bangalore	
17.	A square matrix A is invertible if A is non singular. If $A = \begin{bmatrix} 2 & p & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of p so that A^{-1} exists.	2
18.	The place of Peace and reconciliation, also known as the pyramid of peace and Accord is a 62- meter high pyramid in Mursultan, the capital of Kazakistan, that serves as anon-demonstrational national spiritual centre and an event house. It has 25 equal smaller equilateral triangles as shown in figure.  If the vertices of one triangle are $(0, 0), (3, \sqrt{3})$ and $(3, -\sqrt{3})$, then find the of the triangle. Also find the area of one face of the Pyramid	2
19.	It is well known that for a square matrix, $AA^{-1} = A^{-1}A = I$ and $AI = IA = A$. Now find the matrix P satisfying the matrix equation $P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$,	2
20.	If $A(3,4), B(7,2), C(x, y)$ are collinear, then write the equation of the line passing through A, B, C.	2
21.	Find the Value of $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ \sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{vmatrix}$	2
22.	Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$	2
23.	Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ Find A^{-1}.	2
24.	Evaluate the product AB where	2

	$A = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{vmatrix}$	
25.	Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then what will be value of $\alpha + \beta$.	2
26.	If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then what will be the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$.	2
27.	Given that $A = [a_{ij}]$ is a square matrix of order 3 and $ A = -7$, then find the value of $\sum_{i=1}^3 a_{i2}A_{i2}$.	2
28.	Let A be a square matrix of order 3 such that $A(\text{adj } A) = 2I$, where I is the identity matrix. Write the value of $ \text{adj } A $.	2
29.	Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$. If $B = A + A^4$, then find the value of $\text{Det}(B)$.	2
30.	Find the value of y if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2y & 4 \\ 6 & y \end{vmatrix}$	2
31.	Find the equation of the line joining (1, 2) and (3,6) using determinants.	2
32.	Verify $A(\text{adj. } A) = (\text{adj. } A)A = (\det A) \cdot I$ for $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$	2
33.	What is the inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$?	2
34.	If we find positive integral power of a symmetric matrix then we get which type of matrix- Symmetric or Skew symmetric?	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\begin{vmatrix} w^2 & w & 1 \\ 1 & w^2 & w \\ w & 1 & w^2 \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$ $\begin{vmatrix} w^2 + w + 1 & w & 1 \\ 1 + w^2 + w & w^2 & w \\ w + 1 + w^2 & 1 & w^2 \end{vmatrix}$ So we know that $w^2 + w + 1 = 0$ $\begin{vmatrix} 0 & w & 1 \\ 0 & w^2 & w \\ 0 & 1 & w^2 \end{vmatrix} = 0$	2
2.	$\begin{vmatrix} w^2 & w & 1 \\ 1 & w^2 & w \\ w & 1 & w^2 \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$ $\begin{vmatrix} w^2 + w + 1 & w & 1 \\ 1 + w^2 + w & w^2 & w \\ w + 1 + w^2 & 1 & w^2 \end{vmatrix}$ So we know that $w^2 + w + 1 = 0$ $\begin{vmatrix} 0 & w & 1 \\ 0 & w^2 & w \\ 0 & 1 & w^2 \end{vmatrix} = 0$	2
3.	<u>L.H.S</u> $\begin{vmatrix} x+5 & x & x \\ x & x+5 & x \\ x & x & x+5 \end{vmatrix}$ $C_1 \rightarrow C_1 + C_2 + C_3$ $\begin{vmatrix} 3x+5 & x & x \\ 3x+5 & x+5 & x \\ 3x+5 & x & x+5 \end{vmatrix}$ Taking common (3x+5) from C_1 $(3x+5) \begin{vmatrix} 1 & x & x \\ 1 & x+5 & x \\ 1 & x & x+5 \end{vmatrix}$ $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ $= (3x+5) \begin{vmatrix} 1 & x & x \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix}$ Expand R_3 $(3x+5)5(5x) = 25x(3x+5)$ $= 75x^2 + 125x = \text{R.H.S}$	2

4.	A is invertible $AA^{-1} = I$ $\det(AA^{-1}) = \det(I)$ $\det A \cdot (\det A^{-1}) = \det(\sqrt{2}) [AB] = A B$ $\det A^{-1} = \frac{1}{\det A} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \text{ i.e. } I = 1 \right\}$	2
5.	$\begin{vmatrix} x+5 & x & x \\ x & x+5 & x \\ x & x & x+5 \end{vmatrix}$ $C_1 \rightarrow C_1 + C_2 + C_3$ $\begin{vmatrix} 3x+5 & x & x \\ 3x+5 & x+5 & x \\ 3x+5 & x & x+5 \end{vmatrix}$ Taking common $(3x+5)$ from C_1 $(3x+5) \begin{vmatrix} 1 & x & x \\ 1 & x+5 & x \\ 1 & x & x+5 \end{vmatrix}$ $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$ $= (3x+5) \begin{vmatrix} 1 & x & x \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix}$ Expand R_3 $(3x+5)5(5x) = 25x(3x+5)$ $= 75x^2 + 125x = \text{R.H.S}$	2
6.	$A^T = A, B^T = -B$ and $A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow (A - B)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T$ $\Rightarrow A^T - B^T = A + B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow A + B + A - B = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ $\therefore 2A = 16 - 25 = -9$	2
7.	$A^5 = I \Rightarrow A^{-1} \cdot A^5 = A^{-1} \cdot I \Rightarrow A^4 = A^{-1}$ $\Rightarrow A^{-1} = A^4 = A ^4 = 1^4 = 1$	2
8.	$\log_4 9 = \frac{\log 9}{\log 4}; \log_3 8 = \frac{\log 8}{\log 3}; \log_4 3 = \frac{\log 3}{\log 4}; \log_3 512 = \frac{\log 512}{\log 3}$ $\begin{vmatrix} \log_4 9 & \log_3 8 \\ \log_4 3 & \log_3 512 \end{vmatrix} = \log_4 9 \cdot \log_3 512 - \log_4 3 \cdot \log_3 8 = \frac{\log 9}{\log 4} \cdot \frac{\log 512}{\log 3} - \frac{\log 3}{\log 4} \cdot \frac{\log 8}{\log 3}$ $= \frac{2 \log 3}{2 \log 2} \cdot \frac{9 \log 2}{\log 3} - \frac{\log 3}{2 \log 2} \cdot \frac{3 \log 2}{\log 3} = 9 - \frac{3}{2} = \frac{15}{2}$	2
9.	$\therefore adj.A = A ^{n-1} \therefore adj.(2A) = 2A ^{3-1} = 2A ^2 = (2^3 A)^2 = (2^3 \cdot 4)^2 = 2^{10}$	2
10.	$(AB)^{-1} = \frac{-1}{6} \Rightarrow \frac{1}{ AB } = \frac{-1}{6} \Rightarrow \frac{1}{ A \cdot B } = \frac{-1}{6} \Rightarrow \frac{1}{2 \cdot B } = \frac{-1}{6} \Rightarrow \frac{1}{ B } = \frac{-1}{3}$	2

11.	Here Determinant $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix} = 1(2-2) + 3(1+3) = 12$ A is no singular $\text{Adj } A = (12) * (12) = 144$	2
12.	Use the determinant expansion and expand the matrix ,find the value of x using simple equation. $x = -4/3$	2
13.	By definition of matrix b is inverse of A if $AB = I = BA$.Here we have to show that A^2 is inverse of A there it is sufficient to shoe that $A^2 A = I = AA^2$ i.e. $A^3 = I$	2
14.	The absolute value of $\frac{1}{2} \begin{bmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{bmatrix} = 4$ The absolute value of $\frac{1}{2} (-2)(4-k) = 4$ Solving above ,we get $k = 8$ and 0	2
15.	Here $ A = -19, \text{adj}A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}, A^{-1} = \frac{1}{ A } \text{adj}A$ $\therefore A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} \Rightarrow \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix} \Rightarrow k = \frac{1}{19}$	2
16.	As points are on a straight line $\therefore \begin{vmatrix} 2x & x+3 & 1 \\ 0 & x & 1 \\ x+3 & x+6 & 1 \end{vmatrix} = 0 \Rightarrow -12x + 3(x+3) = 0 \Rightarrow x = 1$ Therefore co-ordinates of Kolkata are (2,4) and Bangalore are (4,8)	2
17.	A^{-1} exist if $ A \neq 0 \Rightarrow \begin{vmatrix} 2 & p & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0 \Rightarrow p \neq \frac{-8}{5}$	2
18.	Area of a triangle $= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 3 & \sqrt{3} & 1 \\ 3 & -\sqrt{3} & 1 \end{vmatrix} = 3\sqrt{3} \text{sq. unit}$ $\therefore$ area of one of the face of the Pyramid $= 25 \times 3\sqrt{3} = 75\sqrt{3} \text{ sq. unit}$	2
19.	$\begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = -1, \therefore \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \frac{1}{-1} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ $\therefore P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$	2
20.	A(3,4) ,B(7,2) ,C (x, y) are collinear so $\frac{1}{2} \begin{vmatrix} 3 & 4 & 1 \\ 7 & 2 & 1 \\ x & y & 1 \end{vmatrix} = 0$ $4(y-4)+2(x-3)= 0$ $2x+4y =22$ is the equation of line passing through A(3,4) ,B(7,2).	1 1
21.	$\cos^2 15^\circ - \sin^2 15^\circ$ $= \cos 30^\circ$ $= \sqrt{3}/2$	1 $\frac{1}{2}$ $\frac{1}{2}$
22.	$\Delta = (x+y)(-3x+3y)-(y+z)(-3z+3y)+(z+x)(-3z+3x)=3(y^2-x^2-y^2+z^2-z^2+x^2)=0$	1 1
23.	$ A =10,$	$\frac{1}{2}$

30.	$+\sqrt{3}, -\sqrt{3}$	2
31.	$2x-y=0$	2
32.	Verification	2
33.	$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$	2
34.	symmetric	2