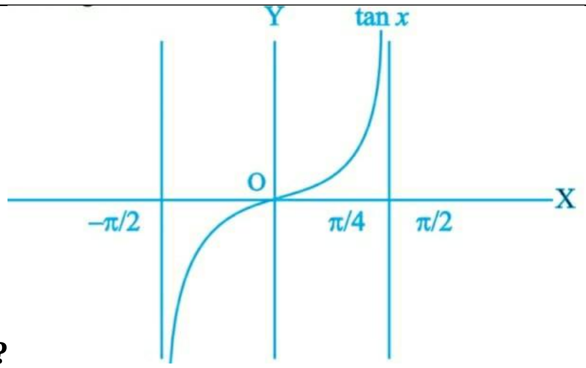
CHAPTER-2

INVERSE T FUNCTION

CLASS-XII

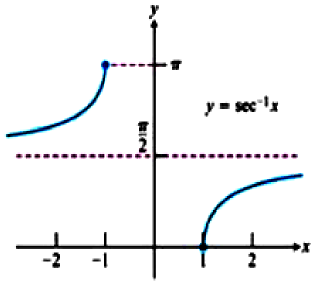
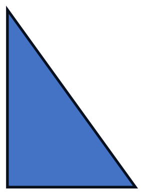
02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	$\cot^{-1} x = \cos^{-1}(-1) - \operatorname{cosec}^{-1}(2/\sqrt{3})$ Based on the above information find $\tan^{-1}(\frac{1}{x})$ using the principal value of inverse trigonometric function. Show your work.	2
2.	Find the value of $\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$.	2
3.	If $\alpha = \tan^{-1}(\tan 5\pi/4)$ and $\beta = \tan^{-1}(-\tan 2\pi/3)$, then establish a relation between α and β.	2
4.	Compute the value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$.	2
5.	If $\cos(\tan^{-1} x) = \sin(\cot^{-1}\frac{3}{4})$ then solve for x.	2
6.	Let x, y, z $\epsilon[-1,1]$ be such that $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ find the values of $x^{2020}+y^{2021}+z^{2022}$	2
7.	Draw the graph of $\sec^{-1}x$ and write the domain of $\sec^{-1}(2x+1)$	2
8.	Evaluate the following question : $\tan^{-1}(\frac{cosx}{1+sinx}), \frac{-\pi}{2} < x < \frac{\pi}{2}$	2
9.	In a right angled triangle PQR b, p, h denote the base, perpendicular and hypotenuse respectively and $<$QPR=Θ.Express $\sin^{-1}(\frac{5}{13})$ in terms of other five trigonometric functions.	2
10.	Prove that $\tan(\cot^{-1}x)=\cot(\tan^{-1}x)$ state with the reason whether the equality is valid for all the values of x	2
11.	What is the principal value of $\sin^{-1}(-2)$	2
12.	**Find the value of cot ($\tan^{-1} \alpha + \cot^{-1} \alpha$)**	2
13.	Write the principal value of $\tan^{-1}[\sin(-\pi/2)$	2
14.	Find x ,$\sin^{-1}\frac{1}{3} + \cos^{-1} x = \frac{\pi}{2}$	2
15.	If $\sin(\sin^{-1}15 + \cos^{-1}x) = 1$, then find the value of x.	2
16.	Find the value of $\tan^{-1}\left[2cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1} 1$	2
17.	Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(cos\frac{3\pi}{4}\right) + \tan^{-1} 1$	2
18.	Find the domain of $y = \sin^{-1}(x^2 - 4)$	2
19.	Find the value of $\sin^{-1}\left[\cos\frac{33\pi}{5}\right]$.	2
20.	Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1} x, \frac{1}{\sqrt{2}} \le x \le 1$.	2
21.	Simplify, $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$.	2
22.	Find the value of $\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$	2
23.	Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$	2
24.	Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$	2

25.	Find the value of $\tan(2\tan^{-1}\frac{1}{5})$	2
26.	Find the principal value of $\tan^{-1}1 + \cos^{-1}(-\frac{1}{2})$	2
27.	If $\sin^{-1}x + \sin^{-1}y = 2\pi/3$, then find the value of $\cos^{-1}x + \cos^{-1}y$	2
28.	Find the domain of $\sin^{-1}(x^2 - 4)$	2
29.	Find the value of $\sin^{-1}(\cos 33\pi/5)$	2
30.	Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$	2
31.	Write the domain and range (principal value branch) of the the following function $f(x) = \cos^{-1}x$	2
32.	Write the domain and range of $\tan^{-1}x$	2
33.	Find the value of $\tan^{-1}[2\cos(2\sin^{-1}\frac{1}{2})] + \tan^{-1}1$	2
34.	Evaluate $\sin^{-1}(\sin\frac{3\pi}{4}) + \cos^{-1}(\cos\frac{3\pi}{4}) + \tan^{-1}(1)$	2
35.	Draw the graph of $f(x) = \sin^{-1}x$, $x \in [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$ Also write the range of f(x).	2
36.	Express $\tan^{-1}(\frac{\cos x}{1+\sin x})$, $\frac{-3\pi}{2} < x < \frac{\pi}{2}$, in simplest form.	2
37.	 **Which is greater than 1 and $\tan^{-1}1$?**	2
38.	**Find the value of** $\tan^{-1}[\tan(\frac{5\pi}{6})] + \cos^{-1}[\cos(\frac{13\pi}{6})]$.	2
39.	**Express the expression in the simplest form** $\tan^{-1}[\frac{x}{a+\sqrt{a^2-x^2}}]$	2
40.	Evaluate $\sin[\frac{\pi}{3} + \sin^{-1}(-\frac{1}{2})]$.	2
41.	**Give one real example which does not satisfy the property of inverse function.**	2

ANSWERS:

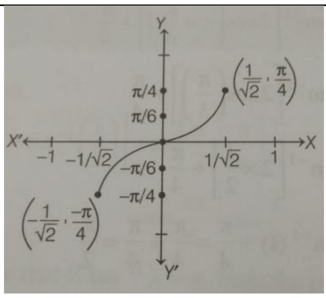
Q. NO	ANSWER	MARKS
1.	$\cot^{-1} x = \cos^{-1}(-1) - \text{cosec}^{-1} (2/\sqrt{3})$ or, $\cot^{-1} x = \pi - \pi/3 = 2\pi/3$ or, $\tan^{-1}(1/x) = 2\pi/3$	2
2.	$\tan^2 (\sec^{-1} 2) + \cot^2 (\text{cosec}^{-1} 3)$ $= \sec^2(\sec^{-1} 2) - 1 + \text{cosec}^2(\text{cosec}^{-1} 3) - 1$ $= \{\sec(\sec^{-1} 2)\}^2 + \{\text{cosec}(\text{cosec}^{-1} 3)\}^2 - 2$ $=(2)^2 + (3)^2 - 2 = 11$	2
3.	$\alpha = \tan^{-1} (\tan 5\pi/4)$ and $\beta = \tan^{-1} (-\tan 2\pi/3)$ $= \tan^{-1} [\tan(\pi + \pi/4)]$ $= \tan^{-1} (-\tan(\pi - \pi/3))$ $= \tan^{-1} [\tan \pi/4]$ $= \tan^{-1} (\tan \pi/3)$ $= \pi/4$ $= \pi/3$ $\pi = 4\alpha$ $\pi = 3\beta$ Therefore, $4\alpha = 3\beta$	2
4.	$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$ $= \tan^{-1} 1 + \tan^{-1} (\frac{2+3}{1-2.3})$ $= \tan^{-1} 1 + \tan^{-1}(-1)$ $= \pi/4 + 3\pi/4 = \pi$	2
5.	$\cos (\tan^{-1} x) = \sin (\cot^{-1} \frac{3}{4})$ Let $\tan^{-1} x = \alpha$ Or, $x = \tan\alpha$ Or, $\cos\alpha = \frac{1}{\sqrt{1+x^2}}$ Or, $\alpha = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$ Similarly let $\cot^{-1} \frac{3}{4} = \beta$ Cot β =3/4 Sin β= 4/5 Equating both the sides we get $\frac{1}{\sqrt{1+x^2}} = 4/5$ Squaring both sides $16(1 + x^2) = 25$ $X = \pm$ ¾	2
6.	Solution: For any xϵ[-1,1],the maximum value of $\sin^{-1}$x is, $\frac{\pi}{2}$ and it attains the value at x=1. $\therefore \sin^{-1}x \leq \frac{\pi}{2}, sin - 1y \leq \frac{\pi}{2}, sin - 1z \leq \frac{\pi}{2}$ for all x, y, zϵ [-1,1] $=\sin^{-1}x+\sin^{-1}y+\sin^{-1}z \leq \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$ for all x, y, zϵ [-1,1] $= \sin^{-1}x+\sin^{-1}y+\sin^{-1}z \leq \frac{3\pi}{2}$ for all x, y, zϵ [-1,1]	2

	∴ $sin^{-1}x+sin^{-1}y+sin^{-1}z=\frac{3\pi}{2}$ ∴ $sin^{-1}x=\frac{\pi}{2}$, $sin^{-1}y=\frac{\pi}{2}$, $sin^{-1}z=\frac{\pi}{2}$ = x = 1, y = 1 ,z = 1 ∴$x^{2020}+y^{2021}+z^{2022}=(1)^{2021}+(1)^{2022}+(1)^{2023}=3$	
7.	Solution: The graph of sec-1x  The domain of sec-1x is(-∞,-1]∪[1,∞) Therefore, sec-1(2x+1) is meaningful,if 2x+1 ≥ 1 or , 2x+1 ≤ -1 2x ≥ 0 or , 2x≤ -2 x ≥ 0 or , x ≤ -1 x ϵ(-∞,-1]∪ [0, ∞) Hence, the domain of $sac^{-1}(2x+1)$ is (-∞,-1] ∪ [0, ∞)	2
8.	Solution: tan-1($\frac{cosx}{1+sinx}$) =tan-1{$\frac{cos2\frac{x}{2}-sin2\frac{x}{2}}{cos2\frac{x}{2}+sin2\frac{x}{2}+2sin\frac{x}{2}cos\frac{x}{2}}$} ={$\frac{(cos\frac{x}{2}-sin\frac{x}{2})(cos\frac{x}{2}+sin\frac{x}{2})}{(cos\frac{x}{2}+sin\frac{x}{2})2}$} = tan-1{$\frac{cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}}$} =tan-1{$\frac{1-tan\frac{x}{2}}{1+tan\frac{x}{2}}$} =tan-1{tan($\frac{\pi}{4}-\frac{x}{2}$)} $=\frac{\pi}{4}-\frac{x}{2}$ [$\frac{-\pi}{2}<x<\frac{\pi}{2}=\frac{-\pi}{4}<-\frac{x}{2}<\frac{\pi}{4}=0<\frac{\pi}{4}-\frac{x}{2}<\frac{\pi}{2}$]	2
9.	solution: Let b, p and h denote the base , perpendicular and hypotenuse of a right triangle PQR and let <QPR=ϴ If sin-1$\frac{5}{13}$ is to be expressed in terms of other five inverse trigonometric , then we construct a right triangle with perpendicular p=5 and hypotenuse h=13.trhe base b of this triangle is b=12. ϴ=sin-1$\frac{5}{13}$ =cos-1$\frac{12}{13}$= tan-1$\frac{5}{12}$= cot-1$\frac{12}{5}$ = sec-1$\frac{13}{12}$ =cosec-1$\frac{13}{5}$ 	2
10.	Solution: We know that tan-1x+cot-1=$\frac{\pi}{2}$ Or, cot-1x=$\frac{\pi}{2}$ - tan-1x for all xϵR tan(cot-1x)=tan($\frac{\pi}{2}$ -tan-1x) for all xϵR =cot(tan-1x) for all xϵR Clearly, the equality holds for all xϵR as tan-1x+cot-1x=$\frac{\pi}{2}$	2

11.	$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ [∵ $\sec^{-1}(-x) = \pi - \sec^{-1}(x); \|x\| \geq 1$ $= \pi - \sec^{-1}(\sec \pi 3) = \pi - \pi 3$ [∵ $\sec \pi 3 = 2$ and $\sec^{-1}(\sec\theta) = \theta;\ \forall\ \theta \in [0, \pi] - \{\pi 2\}$] $= 2\pi 3$ which is the required principal value.	2
12.	Given that: cot ($\tan^{-1} \alpha + \cot^{-1} \alpha$) = cot ($\pi/2$) (since, $\tan^{-1} x + \cot^{-1} x = \pi/2$) = cot (180°/2) (we know that cot 90° = 0) = cot (90°) = 0 Therefore, the value of cot ($\tan^{-1} \alpha + \cot^{-1} \alpha$) is 0.	2
13.	We have, $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ $= \tan^{-1}\left[-\sin\left(\frac{\pi}{2}\right)\right]$ $\left[\because \sin^{-1}(-x) = -\sin^{-1} x,\ x \in (-1, 1)\right]$ $= \tan^{-1}(-1)$ $\left[\because \sin\left(\frac{\pi}{2}\right) = 1\right]$ $= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\because \tan\frac{\pi}{4} = 1\right]$ $= \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$ $\left[\because \tan^{-1}(\tan\theta) = \theta; \forall\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$	2
14.	$\sin^{-1}\frac{1}{3} + \cos^{-1} x = \frac{\pi}{2}$ Or $\sin^{-1}\frac{1}{3} + \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2}$ Or $\sin^{-1}\frac{1}{3} = \sin^{-1} x$ Or x=1/3	2

15.	Given, $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ $\Rightarrow \quad \sin^{-1}\frac{1}{5}+\cos^{-1}x=\sin^{-1}(1)$ $[\because \sin\theta = x \Rightarrow \theta=\sin^{-1}x]$ $\Rightarrow \quad \sin^{-1}\frac{1}{5}+\cos^{-1}x=\sin^{-1}\left(\sin\frac{\pi}{2}\right) \quad \left[\because \sin\frac{\pi}{2}=1\right]$ $\Rightarrow \quad \sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$ $\Rightarrow \quad \sin^{-1}\frac{1}{5}=\frac{\pi}{2}-\cos^{-1}x$ $\Rightarrow \quad \sin^{-1}\frac{1}{5}=\sin^{-1}x$ $\left[\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2};\, x\in[-1,1]\right]$ $\therefore \quad x=\frac{1}{5}$	2
16.	We have $\tan^{-1}\left[2cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1}1$ $=\tan^{-1}\left[2cos\left(2\sin^{-1}\left(sin\frac{\pi}{6}\right)\right)\right]+\tan^{-1}\left(tan\frac{\pi}{4}\right)$ $=\tan^{-1}\left[2cos\left(2*\frac{\pi}{6}\right)\right]+\frac{\pi}{4}$ $=\frac{\pi}{4}+\frac{\pi}{4}$ $=\frac{\pi}{2}$	2
17.	$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}\left(cos\frac{3\pi}{4}\right)+\tan^{-1}1$ $=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{4}\right)\right)+\frac{3\pi}{4}+\frac{\pi}{4}$ $=\sin^{-1}\left(\sin\frac{\pi}{4}\right)+\pi \quad =\frac{5\pi}{4}$	2
18.	We have , $y=\sin^{-1}(x^2-4)$ $-1\le x^2-4\le 1$ $-1+4\le x^2\le 1+4$ $3\le x^2\le 5$ $\sqrt{3}\le \lvert x\rvert\le\sqrt{5}$ So domain of y is $\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$	2
19.	let $y=\sin^{-1}\left[\cos\frac{33\pi}{5}\right]$ $=\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ $=\sin^{-1}\left(cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right)$ $=\sin^{-1}\left(-\sin\frac{\pi}{10}\right) \quad =-\frac{\pi}{10}$	2
20.	$\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\cos^{-1}x$ On putting $x=\cos t$ where $t=\cos^{-1}x$ L.H.S = $\sin^{-1}\left(2\cos t\sqrt{1-cos^2t}\right)$ $=\sin^{-1}(2\cos t\sin t)$ $=\sin^{-1}(\sin 2t)$	2

	$=2t=2\cos^{-1}x = R.H.S$	
21.	$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)$ $=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{3}\right)\right]$ $=\frac{9}{4}\cos^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$	2
22.	$\tan^{-1}[2\sin(2\cos^{-1}\frac{\sqrt{3}}{2})]$ $= \tan^{-1}[2\sin(2.\frac{\pi}{6})]$ $= \tan^{-1}[2\sin\frac{\pi}{3}]$ $= \tan^{-1}\sqrt{3}$ $=\frac{\pi}{3}$	2
23.	$\tan^{-1}(-\frac{1}{\sqrt{3}})+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$ $=-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)-\tan^{-1}1$ $= -\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}$ $=-\frac{\pi}{12}$	2
24.	$\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ $=\frac{\pi}{3}-(\pi-\sec^{-1}(2))$ $=\frac{\pi}{3}-\pi+\frac{\pi}{3}$ $= -\frac{\pi}{3}$	2
25.	$\tan(2\tan^{-1}\frac{1}{5})$ $=\frac{5}{12}$	2
26.	$\tan^{-1}1+\cos^{-1}(-\frac{1}{2})$ $=\frac{\pi}{4}+\pi-\frac{\pi}{3}$ $=\frac{11\pi}{12}$	2
27.	$(\pi/2-\cos^{-1}x) + (\pi/2-\cos^{-1}x) = 2\pi/3$, implies $\cos^{-1}x+\cos^{-1}y=\pi/3$	2
28.	$-1 \le x^2-4 \le 1 \Rightarrow 3 \le x^2 \le 5 \Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	2
29.	$\sin^{-1}(\cos 33\pi/5) = \sin^{-1}(\cos(6\pi+3\pi/5)) = \sin^{-1}(\cos(3\pi/5)) = \pi/2-\cos^{-1}(\cos(3\pi/5))$ $= \pi/2-3\pi/5 = -\pi/10$	2

30.	We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$ $\sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$ $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right) = \frac{2\pi}{3}$	2
31.	since, $\cos : [0,\pi] \to [-1,1]$ is one one and onto function. so its inverse exists and is given by $\cos^{-1} : [-1,1] \to [0,\pi]$. Domain $= [-1,1]$ and Range $= [0,\pi]$	2
32.	Domain = R Range = $(\frac{-\pi}{2}, \frac{\pi}{2})$	2
33.	$\frac{\pi}{2}$	2
34.	$\frac{5\pi}{4}$	2
35.	 Range = $[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$	2
36.	$\frac{\pi}{4} + \frac{x}{2}$	2
37.	We have, $1 > \frac{\pi}{4}$ $\tan 1 > \tan\frac{\pi}{4}$ $\tan 1 > 1$ $\tan 1 > 1 > \frac{\pi}{4}$ $\tan 1 > \frac{\pi}{4}$ tan 1 > **$\tan^{-1}1$**	2
38.	$\tan^{-1}[\tan(\frac{5\pi}{6})] + \cos^{-1}[\cos(\frac{13\pi}{6})]$ $\tan^{-1}[-\tan(\frac{\pi}{6})] + \cos^{-1}[\cos(\frac{\pi}{6})]$ $\tan^{-1}[\tan(-\frac{\pi}{6})] + \frac{\pi}{6}$ $-\frac{\pi}{6} + \frac{\pi}{6}$ 0	2
39.	**$\tan^{-1}\left[\frac{x}{a+\sqrt{a^2-x^2}}\right]$** **$\tan^{-1}\left[\frac{a\sin\theta}{a+\sqrt{a^2(1-\sin^2\theta)}}\right]$**	2

	$\tan^{-1}\left[\frac{a\,sin\theta}{a(1+cos\theta)}\right]$ $\tan^{-1}\left[\tan\frac{\theta}{2}\right]$ $\frac{\theta}{2}$ $\frac{1}{2}\sin^{-1}\left[\left(\frac{x}{a}\right)\right]$	
40.	$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(-\frac{1}{2}\right)\right]$. $\sin\left[\frac{\pi}{3}-\frac{\pi}{6}\right]$. $\sin\left[\frac{\pi}{6}\right]$. $\frac{1}{2}$	2
41.	f : Parents → Children Parents = { Vivek, Sarita} Children = { Pinki, Gopal, Rajan}	2