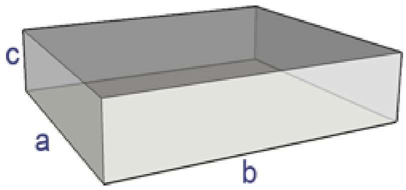
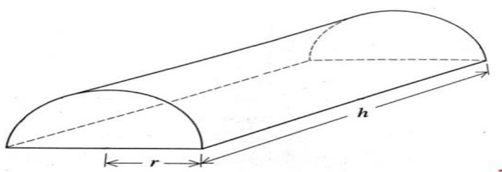
CHAPTER-6
APPLICATION OF DERIVATIVES
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Sand is pouring from a pipe at the rate of 12 cm^3/s.The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand cone increasing when the height is 4 cm ? 	3
2.	Prove that y = $\frac{4sin\theta}{(2+cos\theta)} - \theta$ is increasing function of θ in [0,$\frac{\pi}{2}$].	3
3.	A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top,by cutting off square from each corner and folding up the flaps.What should be the side of the square to be cut off so that the  volume of the box is maximum ?	3
4.	A wire is 12 meters long. If the wire is cut into two pieces and each piece is bent to form a square, find the lengths of the sides of the squares to maximize the total area.	3
5.	A kite is flying at a height of 50m above the ground. The kite string is attached to a point on the ground 100m away from the person flying the kite. The string is being let out at a rate of 2m/s. How fast is the kite rising when it is 150m away from the person? 	3
6.	A rocket is fired vertically upwards. It ascends with a velocity given by v=50t−5t^2, where t is the time in seconds and v is the velocity in m/s. Find the maximum height reached by the rocket and the time at which it occurs.	3

		
7.	Read the following passage and answer the questions given below. In a street two lamp posts are 300 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).  The combined light intensity is the sum of the two light intensities coming from both lamp posts. Find the distance of the darkest spot between the two lights ?	3
8.	Read the following passage and answer the questions given below: Some young entrepreneurs started an industry “Young achievers” for casting metal into various shapes. They put up an advertisement online stating the same and expecting order to cast method for toys, sculptures, decorative pieces and more. A group of friends wanted to make innovative toys and hence contacted the “Young achievers” to order them to cast metal into solid half cylinders with a rectangular base and semi circular ends.  (i)For the given volume V, Find the condition for the total surface area S to be minimum. (ii) Use second derivative test to prove that Surface area is minimum for given volume.	3

	OR (ii) Find the ratio h: 2r for S to be minimum.	
9.	Find the intervals in which the function f given by $f(x) = sinx + cosx$, Where $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	V = $\frac{1}{3}\pi r^2h$, Given $\frac{dV}{dt}$ = 12 cm^3/s Or, $\frac{d}{dt}(\frac{1}{3}\pi r^2h)$ = 12 Or,r = 6h Now $\frac{d}{dt}(\frac{1}{3}\pi(6h)^2h)$ = 12 Or, $\frac{dh}{dt}$ = $1/3\pi h^2$ When h= 4 cm, $\frac{dh}{dt}$ = $1/48\pi$.	3
2.	$y = \frac{4sin\theta}{(2+cos\theta)} - \theta$ $\frac{dy}{dx} = \frac{(2+cos\theta).4cos\theta-4\ sin\theta.(\theta-sin\theta)}{(2+cos\theta)^2} - 1 = \frac{cos\theta.(4-cos\theta)}{(2+cos\theta)^2}$ $\frac{dy}{dx} > 0$ for $\theta \in [\ 0,\frac{\pi}{2}\ ]$ i.e increasing function in $[\ 0,\frac{\pi}{2}\ ]$.	3
3.	Let x be the length of square V = (45-2x)(24-2x).x $\frac{dV}{dx}$ = $12x^2$- 276x +1080 = 12(x-18)(x-5) For maximum $\frac{dV}{dx}$ = 0, x = 5,18 X = 18 is not possible as breadth is 24 cm. $\frac{d^2V}{dx^2}$ at x = 5 is - 156 $<$ 0(x = 5 is the point of maximum).	3
4.	Let x be the length of one piece of wire, so the length of the other piece is 12−x. Each piece is bent into a square, so the sides of each square are $s=\frac{x}{4}$. The total area A of the two squares is $A=2s^2$. Substitute s=4x into the area equation: $A=2(\frac{x}{4})^2=\frac{x^2}{8}$ Differentiate A with respect to x, set the derivative to 0, and solve for x: $\frac{dA}{dx}=\frac{2x}{8}=\frac{x}{4}$ Setting $\frac{dA}{dx}$=0: $\frac{x}{4}$=0, which has no solution. So, there are no critical points for A and no maximum or minimum. Since the wire is 12 meters long, we must have x+(12−x)=12, which means x=6. The lengths of the sides of the squares are $s=\frac{x}{4}=\frac{6}{4}=\frac{3}{2}$ meters.	3
5.	Let x be the horizontal distance from the person to the point directly below the kite, and let h be the height of the kite above the ground. Given h=50m, $\frac{dx}{dt}$=2m/s, and x=100m, we want to find $\frac{dh}{dt}$ when x=150m. By the Pythagorean theorem, $x^2+h^2=d^2$, where d is the length of the kite string. Since the string is being let out at a constant rate, $\frac{dx}{dt}$=2m/s. Differentiate both sides of the equation with respect to time t: $2x\frac{dx}{dt}+2h\frac{dh}{dt}=2\frac{dd}{dt}$ Substitute the given values: 2(100)(2)+2(50) $\frac{dh}{dt}$=2(2) 400+100$\frac{dh}{dt}$=4 Solve for $\frac{dh}{dt}$:	3

	$100\frac{dh}{dt}$=4−400 $100\frac{dh}{dt}$=−396 $\frac{dh}{dt}$=−396/100 $\frac{dh}{dt}$=−3.96m/s So, when the kite is 150m away from the person, the kite is rising at a rate of 3.96m/s downwards .	
6.	Given the velocity function v=50t−5t2, we want to find the maximum height reached by the rocket and the time at which it occurs. The height h of the rocket can be determined by integrating the velocity function: $h=\int v\,dt=\int (50t-5t^2)\,dt$ $h=25t^2-\frac{5}{3}t^3+C$ To find the constant C, we can use the initial condition that at t=0, h=0 $0=25(0)^2-\frac{5}{3}(0)^3+C$ $C=0$ So, the height function is $h=25t^2-\frac{5}{3}t^3$ To find the maximum height, we need to find the critical points of h by differentiating with respect to t and setting the derivative equal to 0: $\frac{dh}{dt}=50t-5t^2$ Setting $\frac{dh}{dt}$=0: 0=50t−5t2 $0=t(50-5t)$ This gives t=0 and t=10 as critical points. Evaluate the height at these critical points: $h(0)=25(0)^2-\frac{5}{3}(0)^3=0$ $h(10)=25(10)^2-\frac{5}{3}(10)^3=2500-\frac{5000}{3}=\frac{500}{3}$m So, the rocket reaches a maximum height of $\frac{500}{3}$ m at t=10 seconds.	3
7.	) We have l(x) = $\frac{1000}{x^2}+\frac{125}{(300-x)^2}$ l'(x) = $\frac{-2000}{x^3}+\frac{250}{(300-x)^3}$ l'(x) = $\frac{-6000}{x^4}+\frac{750}{(300-x)^4}$ For maxima / minima, l'(x) =0 $\frac{-2000}{x^3}+\frac{250}{(300-x)^3}=0 \Rightarrow \frac{2000}{x^3}=\frac{250}{(300-x)^3} \Rightarrow 8(300-x)^3=x^3$ Taking cube root on both sides, we get $2(300-x)=x \Rightarrow x=200$ Thus l(x) is minimum when you are at 200 feet from the strong intensity lamp post. Since, l(x) is minimum when x = 200 feet, therefore the darkest spot between the two light is at a distance of 200	3

	feet from stronger lamp post, i.e., at a distance of 300 – 200 = 100 feet from the weaker lamp post.	
8.	(i) $\frac{dS}{dr}=2\pi r-\frac{2V(\pi+2)}{\pi r^2}$ $For \min \frac{dS}{dr}=0\Rightarrow\pi^2r^3=V(\pi+2)$ (ii) $\frac{dS}{dr}=2\pi r-\frac{2V(\pi+2)}{\pi r^2}$ $\frac{d^2S}{dr^2}=2\pi+\frac{4V(\pi+2)}{\pi}.\frac{1}{r^3}$ $=2\pi+\frac{4\pi^2r^3}{\pi r^3}=6\pi>0$ OR Since $\pi^2 r^3=V(\pi+2)$ $\Rightarrow \pi^2 r^3=\frac{1}{2}\pi r^2h(\pi+2)$ $\Rightarrow h:2r=\pi:\pi+2$	3
9.	[0, π/4),(π/4, 5π/4) and(5 π/4, 2π] checking in each intervals, for Strictly increasing & Strictly decreasing	3