

CHAPTER-5
CONTINUITY & DIFFERENTIABILITY
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	If the $f(x)$ given by $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 15ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, find the values of a and b .	3
2.	Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$, Determine the value of a so that $f(x)$ is continuous at $x = 0$	3
3.	If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2) \frac{dy}{dx} + y = 0$.	3
4.	If $(x - a)^2 + (x - b)^2 = c^2$ for some $c > 0$. Prove that $\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant,	3
5.	Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$	3
6.	$y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}$, find $\frac{dy}{dx}$	3
7.	Find the points of discontinuity, if any, of the function, $f(x) = \begin{cases} x + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x > 3 \end{cases}$	3
8.	If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$	3
9.	Discuss the differentiability of $f(x) = x x $ at $x = 0$	3
10.	For what value of k is the following function continuous at $x = -\frac{\pi}{6}$? $f(x) = \begin{cases} \frac{\sqrt{3} \sin x + \cos x}{x + \frac{\pi}{6}}, & x \neq -\frac{\pi}{6} \\ k, & x = -\frac{\pi}{6} \end{cases}$	3
11.	Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x = 1$.	3
12.	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$ prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.	3

13.	If the $f(x)$ given by $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, find the values of a and b.	3
14.	Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$, Determine the value of a so that $f(x)$ is continuous at $x = 0$	3
15.	If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2) \frac{dy}{dx} + y = 0$.	3
16.	If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	3
17.	A potter made a vessel, where the shape of the pot is based on $f(x) = x - 3 + x - 2$, where $f(x)$ represents the height of the pot. when $x > 4$ what will be the height in terms of x? And if the potter is trying to make a pot using the function $f(x) = [x]$, will he get a pot or not? why?	3
18.	If the function $f(x)$ given by $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, then find the area of the sign board having base 'b' and height 'a'	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$ $5a - 2b = 3a + b = 11 \Rightarrow a = 3 \text{ and } b = 2$	3
2.	$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{1 - \cos \cos 4x}{x^2} = \lim_{x \rightarrow 0^-} \frac{2 \sin^2 2x}{x^2} = 2 \times 4 \times \lim_{x \rightarrow 0^-} \left(\frac{\sin \sin 2x}{2x} \right)^2 = 8$ $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} \times \frac{\sqrt{16 + \sqrt{x}} + 4}{\sqrt{16 + \sqrt{x}} + 4} = \lim_{x \rightarrow 0^+} \sqrt{16 + \sqrt{x}} + 4 = 4 + 4 = 8$ $\Rightarrow a = 8$	3
3.	$\frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{1+x}{1-x}} \times \frac{d}{dx} \left(\frac{1-x}{1+x} \right) \Rightarrow \frac{dy}{dx}$ $= - \sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2} [\text{Multiplying by } (1-x^2)]$	3
4.	Correct proof should be there without step missing	3
5.	$\frac{dy}{dx} = \frac{\log \tan y + y \tan x}{\log \cos x - x \sec y \csc y}$	3
6.	$(\log x)^{\cos x} \left\{ -\sin x \log(\log x) + \frac{\cos x}{x \log x} \right\} + \frac{4x}{(1+x^2)^2}$	3
7.	Discontinuous at x = 3	3
8.	$x = \frac{\sin y}{\sin(a+y)}$ $\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$ $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	3
9.	LHD = RHD = 0, f(x) is differentiable at x = 0	3
10.	$\lim_{x \rightarrow \pi/6} f(x) = \lim_{x \rightarrow \pi/6} \frac{\sqrt{3} \sin x + \cos x}{x + \pi/6}$ $\lim_{x \rightarrow \pi/6} f(x) = \lim_{x \rightarrow \pi/6} \frac{2 \sin(x + \frac{\pi}{6})}{x + \pi/6} = 2$ $f(\pi/6) = k$ $k = 2$	3
11.	$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h} = \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0$ $\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \lim_{h \rightarrow 0} \frac{(0-1)}{-h} = \infty$ Since LHD $\neq$ RHD Therefore f(x) is not differentiable at x=1	3

12.	$x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ Squaring to get: $x^2(1+y) = y^2(1+x)$ Simplifying to get: $(x-y)(x+y+xy) = 0$ As, $x \neq y \therefore y = -\frac{x}{1+x}$ $\frac{dy}{dx} = \frac{-1(1+x) - (-x) \cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$	3
13.	$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$ $5a - 2b = 3a + b = 11 \Rightarrow a = 3 \text{ and } b = 2$	3
14.	$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{1 - \cos 4x}{x^2} = \lim_{x \rightarrow 0^-} \frac{2 \sin^2 2x}{x^2} = 2 \times 4 \times \lim_{x \rightarrow 0^-} \left(\frac{\sin 2x}{2x} \right)^2 = 8$ $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} \times \frac{\sqrt{16 + \sqrt{x}} + 4}{\sqrt{16 + \sqrt{x}} + 4} = \lim_{x \rightarrow 0^+} \sqrt{16 + \sqrt{x}} + 4 = 4 + 4 = 8$ $\Rightarrow a = 8$	3
15.	$\frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{1+x}{1-x}} \times \frac{d}{dx} \left(\frac{1-x}{1+x} \right) \Rightarrow \frac{dy}{dx}$ $= -\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2} [\text{Multiplying by } (1-x^2)]$	3
16.	Given, $\sin y = x \sin(a+y)$ $x = \frac{\sin y}{\sin(a+y)}$ On differentiating both side w.r.t x, we get $\Rightarrow \frac{dx}{dy} = \frac{\frac{d}{dy}(\sin y) \sin(a+y) - \frac{d}{dx}(\sin(a+y)) \sin y}{\sin^2(a+y)}$ $\Rightarrow \frac{dx}{dy} = \frac{(\cos y) \sin(a+y) - (\cos(a+y)) \sin y}{\sin^2(a+y)}$ $\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$ $\Rightarrow \frac{dx}{dy} = \frac{\sin(a)}{\sin^2(a+y)}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ Hence proved.	3
17.	The given function can be written as $f(x) = \begin{cases} 5-2x & , \text{if } x < 2 \\ 1 & , \text{if } 2 \leq x < 3 \\ 2x-5 & , \text{if } x \geq 3 \end{cases}$ when $x > 4$, $f(x) = 2x - 5$ no the potter will not get a pot because $f(x) = [x]$ is not a continuous function as it is a greatest integer function so it is discontinuous at every integral point	3

18.	The given function is $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ given that $f(x)$ is continuous at $x = 1$ therefore, L.H.L = R.H.L = $f(x)$ Now, L.H.L = $\lim_{x \rightarrow 1^-} f(x)$ $= \lim_{x \rightarrow 1^-} f(5ax - 2b)$ $= 5a - 2b$ R.H.L = $\lim_{x \rightarrow 1^+} (3ax + b)$ $= 3a + b$ also, given that $f(1) = 11$ now, L.H.L = R.H.L = $F(1) = 11$ $3a + b = 11$ $5a - 2b = 11$ on solving we get $a = 3$ and $b = 2$ now area of sign board = $\frac{1}{2}bh$ $= \frac{1}{2} * 2 * 3$ $= 3 \text{ sq. unit}$	3
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