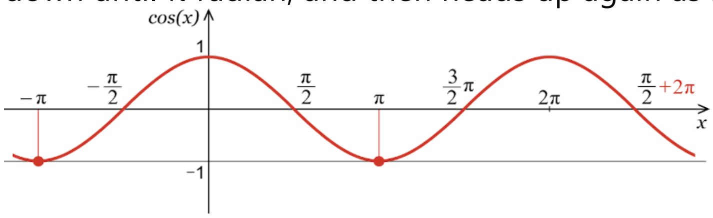
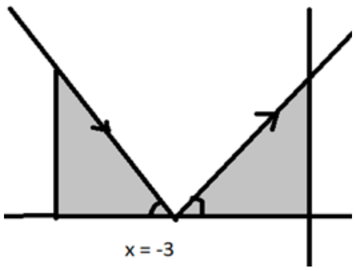
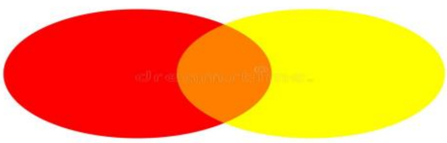
CHAPTER-8
APPLICATION OF INTEGRALS
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the area of Δ ABC, the coordinates of whose vertices are A (2, 5), B(4, 7) and C(6, 2) by using integration.	3
2.	If y= 2 sin x + sin 2x for $0 \le x \le 2\pi$ find the area enclosed by the curve and the x-axis.	3
3.	Find the area of the region bounded by the ellipse $\frac{y2}{16} + \frac{x2}{25}$ =1.	3
4.	Find the area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and $x - axis$.	3
5.	Find the area of the region bounded by the curve $y = x^2$ and $y = 16$.	3
6.	Find the area under the curve $y = x^2$ and the lines $x = -1, x = 2$ and $x - axis$	3
7.	Find the area bounded by the curve y = cos x, x-axis and the ordinates x = -5π/6 and x = π	3
8.	Find the area of larger portion of the circle $x^2 + y^2 = 4$ cut off by the line x=1	3
9.	If the area of the region enclosed by the parabola $y^2 = 4ax$ and the line y = mx is 3/8, then find a relation between a and m.	3
10.	In a classroom, the teacher explains the properties of a particular curve by saying that this particular curve has beautiful ups and downs. It starts at 1 and heads down until π radian, and then heads up again as shown in the figure  Then find the area enclosed by the curve, x = -π and x = π.	3
11.	A ray is reflected according to the below given diagram.  If both the mentioned angles and shaded regions are equal then find the graph of the curve and area of the shaded region.	3
12.	Find the area enclosed by the circle $x^2 + y^2 = 2$.	3
13.	Rishika made two chapattis and place one upon the other as shown in the figure. One of the chapatti representes the equation $(x-2)^2 + y^2 = 4$, while other chapatti represents the equation $x^2 + y^2 = 4$ 	3

Based on the above information, answer the following questions.

(i) Find the centre and of the circle of equation $(x-2)^2 + y^2 = 4$,

(a) C=(2,0), r =2 (b) C=(0,0),r =2

(b) C=(2,0) , r=1 (d) C= (0.2), r=2

(ii) Both the chapattis meet each other at

(a) $(1,\sqrt{3}),(1,-\sqrt{3})$

(b) $(1,\sqrt{3}),(1,-3)$

(c) $(1,3),(1,-3)$

(d) $(1,\sqrt{2}),(1,-\sqrt{2})$

(iii) Area bounded by two chapattis is

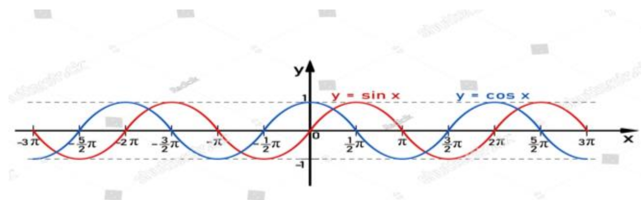
(a) $\frac{8\pi}{3} - \sqrt{3}\ \ sq.units$

(b) $\frac{8\pi}{5} - 2\sqrt{3}\ \ sq.units$

(c) $\frac{8\pi}{3} - 2\ \ sq.units$

(d) $\frac{8\pi}{3} - 2\sqrt{3}\ \ sq.units$

14. In a classroom teacher explain the properties of a particular curve by saying that this particular curve has beautiful up and downs. It starts at 1 and heads down until $\pi\ radian$, and then heads up again and closely related to sine function and both follow each, other exactly $\frac{\pi}{2}\ radian$ apart as shown in figure. 3



Based on the above information ,answer the following questions.

(i) Name the curve, about which teacher explained in the classroom.

(a) cosine (b) sine

(c) tangent (d) cotangent

(ii)Area of curve explained in the passage from 0 to$\frac{\pi}{2}$ is

(a) $\frac{1}{3}\ sq\ units$

(b) $\frac{1}{2}\ sq\ units$

(c) $1\ \ sq\ units$

(d) $2\ sq\ units$

(iii) Area of curve discussed in classroom from $\frac{\pi}{2}\ to \frac{3\pi}{2}$ is

(a)$\frac{7}{2}\ sq\ units$

(b) $\frac{9}{2}\ sq\ units$

(c) $\frac{11}{2}\ sq\ units$

(d) $\frac{13}{2}\ sq\ units$

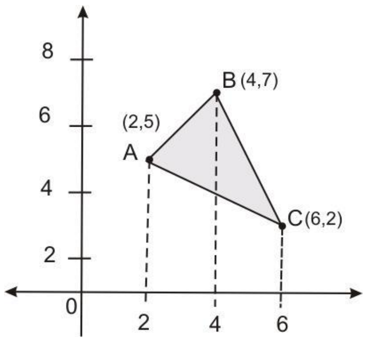
15. In geometry we have learn formulae to calculate areas of various geometrical figures including triangles,rectangles, trapezium and circle. Such formula is fundamental in the application of Mathematics to many real-life problems.The formula of geometry allow us to calculate area of many simple figure .However, they are inadequate for calculating the areas 3

	enclosed by curves. For that we need concept of integral calculus. (i) The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ is$ (a)$\pi b\ sq.units$ (b) $\pi a\ sq.units$ (c) $\pi\ sq.units$ (d) $\pi ab\ sq.units$ (ii) The area enclosed by the circle $x^2+y^2=a^2\ is$ (a)πa^2 (b) π (c) a^2 (d) a (iii) The area of the region bounded by the curve $y=x^2\ and\ the\ line\ y=4$ (a) 32 (b) 32/3 (c) 3 (d) 23	

ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>

Vertices of the given triangle are A(2,5) B(4,7) and C(6,2)
Equation of AB
$y-5 = \frac{7-5}{4-2}(x-2)$
⇨ $y - 5 = x-2$
⇨ $y = x+3$



The equation of side BC,

$$(y-7) = \frac{2-7}{6-4}(x-4)$$

$$(y-7) = \frac{-5}{2}(x-4)$$

$$2y - 14 = -5x + 20$$

$$2y = -5x + 34$$

$$y = \frac{1}{2}(-5x+34) \qquad -(2)$$

The equation of side AC,

$$(y-5) = \frac{2-5}{6-2}(x-2)$$

$$(y-5) = \frac{-3}{4}(x-2)$$

$$4y - 20 = -3x + 6$$

$$4y = -3x + 26$$

</td><td></td></tr>
</table>

$$y = \frac{1}{4}(-3x + 26) \qquad -(3)$$

$\therefore$ Area of ΔABC

$$= \int_2^4 y_{AB}\ dx + \int_4^6 y_{BC}\ dx - \int_2^6 y_{AC}\ dx$$

$$= \int_2^4 (x+3)\ dx + \int_4^6 \frac{-1}{2}(5x - 34)\ dx - \int_2^6 \frac{-1}{4}(3x - 26)\ dx$$

$$= 12 + \frac{1}{2}(18) - \frac{1}{4}(56) - 12 + 9 - 14 = 7 \text{ sq units}$$

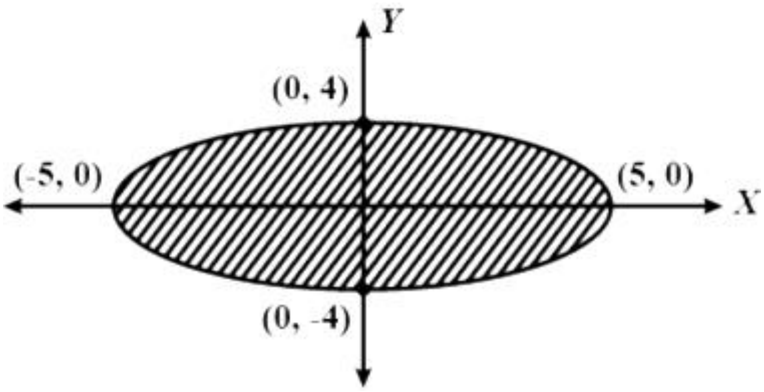
2.

To find the area enclosed by the curve and the x-axis, we need to integrate the absolute value of the function y with respect to x, between the limits 0 and 2π.
The function y = 2 sin x + sin 2x is always non-negative for 0 ≤ x ≤ 2π, so we can simply integrate it as is.

A=2 (2sin x + sin 2x)dx=2∫(2sin x + sin 2x) dx

$=4 \int_0^{\pi} \sin x$ dx+$2\int_{\pi}^{2\pi} \sin 2x$ dx= 8 + 0 = 8

3.



Given the equation of the ellipse is $\frac{y2}{16} + \frac{x2}{25}$ =1

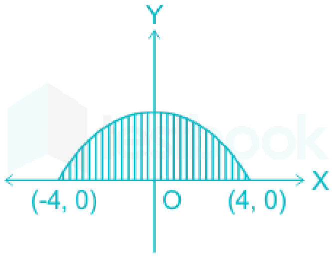
⇨ $\frac{y2}{16}$ =1 - $\frac{x2}{25}$

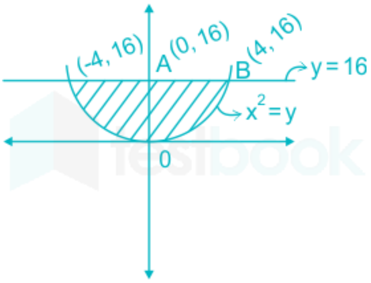
⇨ Y = $\frac{4}{5}\sqrt{25 + x^2}$

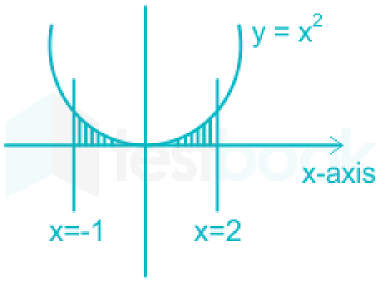
Since ellipse is symmetrical about the axes,

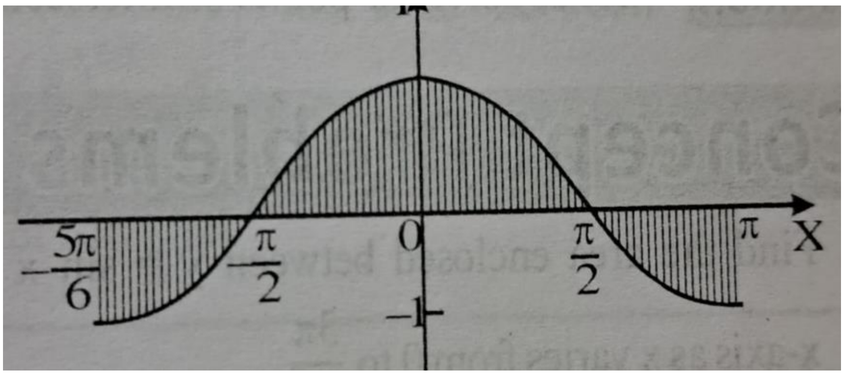
So, required area = 4* $\int_0^5 (4/5)\sqrt{25 - x^2}$ dx

= 20 π sq. units

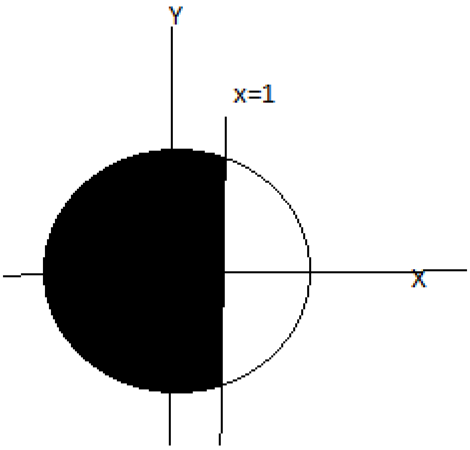
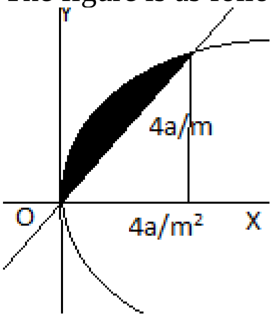
<table>
<tr><td>4.</td><td>$y = \sqrt{16 - x^2}$
At x-axis y will be 0
$0 = \sqrt{16 - x^2}$
$x = \pm 4$


Area of the curve $= \int_{-4}^{4} y dx$
$= \int_{-4}^{4} \sqrt{16 - x^2} dx$
$= 8\pi$ sq. unit</td><td>3</td></tr>
<tr><td>5.</td><td>Given equation of the curve are
$y = x^2$ -------(1)
$y = 16$ -------(2)
From (1) and (2)
$x = \pm 4$


Required area $= \int_{-4}^{4} y dx$
$= \int_{-4}^{4} (16 - x^2) dx$
$= 2\int_{0}^{4} (16 - x^2) dx$ $= \frac{256}{3}$ sq. units</td><td>3</td></tr>
<tr><td>6.</td><td>Given equation of the curve are
$y = x^2$ -------(1)
$x = -1$ -------(2)
$x = 2$ --------(3)


Required area $= \int_{-1}^{2} y dx$
$= \int_{-1}^{2} x^2 dx$
$= 3$ sq. units</td><td>3</td></tr>
<tr><td>7.</td><td>The graph of the function is as follows


Solving equation cos x = 0 between [-5π/6 ,π] we get that
the graph of function intersect x-axis at two points x = -π/2 and x = π/2
so, the required area is given by
$$= \int_{-5\pi/6}^{\pi} |\cos x| dx$$</td><td>3</td></tr>
</table>

	$= -\int_{-5\pi/6}^{-\pi/2} cosx\,dx + \int_{-\pi/2}^{\pi/2} cosx\,dx + \int_{\pi/2}^{\pi} cosx\,dx = 7/2$	
8.	The graph of the function cut off by line is as follows  As per figure the area of small portion is given by =area ABCA $= 2\int_1^2 y\,dx = 2\int_1^2 \sqrt{(4-x^2)}\,dx$ $= 2[\frac{x\sqrt{2^2-x^2}}{2} + \frac{2^2}{2}sin^{-1}\frac{x}{2}]_1^2$ $= \frac{4\pi-3\sqrt{3}}{3}$ So Required area is $= \pi(1)^2 - \frac{4\pi-3\sqrt{3}}{3} = \frac{3\sqrt{3}-\pi}{3}$	3
9.	The figure is as follows  Solving y^2=4ax and y = m x gives point of intersection ($4a/m^2$, 4a/m) $A = \int_0^{4a/m^2}(2\sqrt{ax} - mx)dx = [\frac{4}{3}\sqrt{ax^{\frac{3}{2}}} - \frac{mx^2}{2}]_0^{4a/m^2}$ $\frac{3}{8} = \frac{8a^2}{3m^3}$ $m^3 = a^2$	3
10.	Reqd. area = $4\int_0^{\pi/2} \cos x\,dx$ = 4 $[\sin x]_0^{\pi/2}$ =4 × 1 = 4 sq. units	3
11.	Reqd. area = $\int_{-6}^{0}\lvert x+3\rvert dx$ =$\int_{-6}^{-3}\lvert x+3\rvert dx$ + $\int_{-3}^{0}\lvert x+3\rvert dx$ = 2 $\int_{-3}^{0}(x+3)dx$ = 9 sq. units	3

12.	Reqd. area = $4\int_0^{\sqrt{2}} \sqrt{2-x^2}\ dx$ $= 2\pi$ sq. units	3
13.	(i) (a) Given eq. of circle is $(x-2)^2 + y^2 = 4$, $\Rightarrow (x-2)^2 + (y-0)^2 = 2^2$, Eq. of circle $(x-h)^2 + (y-k)^2 = r^2, where\ centre\ (h,k)\ and\ radius = r$ So, by comparing above eq. we get centre (2,0) and radius = 2 (ii) (a)$(x-2)^2 + y^2 = 4 \ldots\ldots(1)$ $x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2$…….(2) From eq.(1) and (2) we get $(x-2)^2 + 4 - x^2 = 4$ $x^2 - 4x + 4 + 4 - x^2 = 4$ $-4x + 4 = 0 \Rightarrow x = 1$ On putting x=1 in $x^2 + y^2 = 4 \Rightarrow 1^2 + y^2 = 4 \Rightarrow y^2 = 3 \Rightarrow y = \pm\sqrt{3}$ Therefore point of intersections are $(1,\sqrt{3}), (1,-\sqrt{3})$ (iii) (d) Required area =$2\left(\int_0^1 y_1 dx + \int_1^2 y_2\ dx\right)$ $= 2\left(\int_0^1 \sqrt{4-x^2}dx + \int_0^1 \sqrt{4-(x-2)^2}dx\right)$ $= \left[x\sqrt{4-(x)^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2 + \left[(x-2)\sqrt{4-(x-2)^2} + 4\sin^{-1}\frac{x-2}{2}\right]_0^1$ $= 4\sin^{-1}1 - \left(\sqrt{3} + 4\times\frac{\pi}{6}\right) + \left\{-\sqrt{3} + 4\sin^{-1}\left(-\frac{1}{2}\right)\right\} - \{0 +$ $4\sin^{-1}(-1)\}$ $=4\times\frac{\pi}{2} - \left(\sqrt{3} + \frac{2\pi}{3}\right) + \left(-\sqrt{3} - \frac{4\pi}{6}\right) - \left(-\frac{4\pi}{3}\right)$ $=\frac{8\pi}{3} - 2\sqrt{3}\ sq.units$	3
14.	(i) (a) Here the teacher explained about cosine curve. (ii) (c) ∴ Required area =$\int_0^{\frac{\pi}{2}} cosxdx$ $=[sinx]_0^{\frac{\pi}{2}}$ $= sin\frac{\pi}{2} - \sin 0$ $= 1 - 0 = 1\ sq\ units$ (iii) (b) ∴ Required area =$\left\|\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} cosxdx\right\|$ $= \left\|[sinx]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right\|$ $= \left\|sin\frac{3\pi}{2} - sin\frac{\pi}{2}\right\|$ $= \|-1-1\|$ $= \|-2\| = 2\ sq\ units$	3
15.	(i) (d) The given equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ldots\ldots\ldots\ldots\ldots(1)$ Area of ellipse = 4(area of region 1st quadrant) $=4\int_0^a y\ dx$ $= \int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$ $\left[\because (1) \Rightarrow y = \pm\frac{b}{a}\sqrt{a^2-x^2}\right]$	3

(But region OABO lies in 1st quadrant , y is positive)

$$=4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$=\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=\frac{4b}{a}\left[\left\{\frac{a}{2}(0)+\frac{a^2}{2}\sin^{-1}(1)\right\}-\{0-0\}\right]$$

$$=\frac{4b}{a}\left[\frac{a^2}{2}.\frac{\pi}{2}\right]$$

$$=\pi ab\ sq\ units$$

(ii) (a) The given equation of ellipse is $x^2+y^2=a^2$ … … … … … (1)

This is a circle whose centre is (0,0) and radius ‘a’

Area of circle = 4(area of region 1st quadrant)

$$=4\int_0^a y\,dx$$

$$=\int_0^a \sqrt{a^2-x^2}dx$$

$$\left[\because (1)\Rightarrow y=\pm\sqrt{a^2-x^2}\right]$$

(But region OABO lies in 1st quadrant , y is positive)

$$=4\int_0^a \sqrt{a^2-x^2}\,dx$$

$$=4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=4\left[\left\{\frac{a}{2}(0)+\frac{a^2}{2}\sin^{-1}(1)\right\}-\{0-0\}\right]$$

$$=4\left[\frac{a^2}{2}.\frac{\pi}{2}\right]$$

$$=\pi a^2\ sq\ units$$

(iii) (b) The given curve is $y=x^2$ … . (1)

And the given line is $y=4$ … … . (2)

∴ Required area = $2\int_0^4 x\ dy$

$$=2\int_0^4 \sqrt{y}dy$$

$$=2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$=\frac{4}{3}\left[4^{\frac{3}{2}}-0\right]=\frac{4}{3}(8)=\frac{32}{3}sq\ units$$