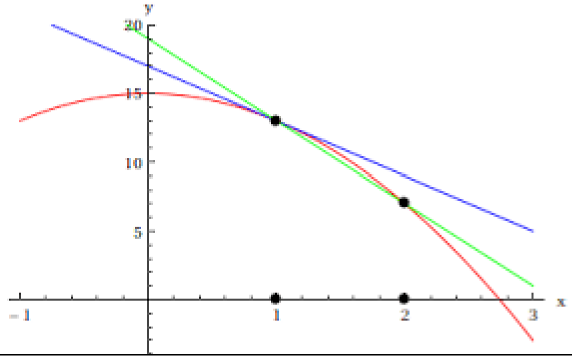
CHAPTER-9

DIFFERENTIAL EQUATIONS

03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	For each of the given differential equation, find a particular solution satisfying the given condition: dy/dx = y tan x ; y = 1 when x = 0	3
2.	Form the differential equation of the family of parabolas having vertex at origin and axis along positive Y-axis.	3
3.	Write the solution of the differential equation $\frac{dy}{dx} = 2^{-y}$.	3
4.	Solve the differential equation $dy/dx + 1 = e^{x+y}$	**3**
5.	Solve: ydx – xdy = x^2 ydx	**3**
6.	Q3 Solve the differential equation $dy/dx = 1 + x + y^2 + xy^2$, when y = 0, x = 0.	**3**
7.	Find the equation of the curve passing through the point (2, 0) whose differential equation is $x(x^2 – 1)\frac{dy}{dx} =1$	3
8.	Show that the given differential equation is homogeneous $x^2dy+(xy+y^2)dx=0$ and find its particular solution, given that, x= 1 when y = 1	3
9.	Find the particular solution satisfying the given condition: $\frac{dy}{dx}$ +2ytanx= sinx; y=0 when x=$\frac{\pi}{3}$.	3
10.	Find the equation of curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point.	3
11.	Find the particular solution of the differential equation $2ye^{y/x}dx + (y - 2xe^{\frac{x}{y}})$dy=0 given that x=0 when y=1	3
12.	Find the equation of a curve passing through the point (0,2), given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.	3
13.	Solve the differential equation $(y + 3x^2)\frac{dx}{dy} = x$.	3
14.	Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 0$ when $x = 1$.	3
15.	Solve the differential equation $(1 + x^2)dy + 2xydx = cot\ x\ dx$.	3
16.	Verify that the function $y = a\cos x + b\sin x$, where $a , b \in \boldsymbol{R}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$	3
17.	Find the general solution of the differential equation $$\frac{dy}{dx} = e^{x+y}$$	3
18.	Find the equation of a curve passing through the point (-2,3), given that the	3

<table>
<tr><td></td><td>slope of the tangent to the curve at any point (x,y) is $\frac{2x}{y^2}$
</td><td></td></tr>
<tr><td>19.</td><td>Find the particular solution of the differential equation
X dx -ye$^y\sqrt{1 + x^2}$dy=,given that y=1,when x=0</td><td>3</td></tr>
<tr><td>20.</td><td>For the differential equation given below , find a particular solution satisfying the given condition $(x+1)\frac{dy}{dx} = 2e^{-y} + 1$; y =0 when x =0.</td><td>3</td></tr>
<tr><td>21.</td><td>Solve the differential equation $(1+x^2)\frac{dy}{dx}+2xy -4x^2 = 0$
Subject to the initial condition y(0) = 0.</td><td>3</td></tr>
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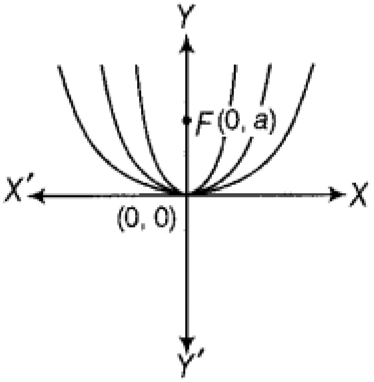
ANSWERS:

<table>
<tr><th>Q. NO</th><th>ANSWER</th><th>MARKS</th></tr>
<tr><td>1.</td><td>Differentiating equation we get
$$\frac{dy}{dx} = ytanx$$
$$\frac{dy}{y} = tanxdx$$
Integrating both side and by solving we get ycosx=x
Putting x=0 and y = 1 we get
$$1 \times \ cos0 = \ c$$
Putting c=1 we get $yosx = 1$
This is the required solution.</td><td></td></tr>
<tr><td>2.</td><td>We know that, equation of parabola having vertex at origin and axis along positive Y-axis is
$x^2 = day$, where a is the parameter. …(i)


on differentiating equation (1) w.r.t x we get $2x = 4ay'$
$$4a = \frac{2x}{y'} \dots\dots\dots\dots (ii)$$
On substituting the value of 4a from Eq. (ii) to Eq. (i), we get
$$x^2 = \frac{2x}{y'} y$$
$$=> xy' - 2y = 0, which\ is\ required\ solution.$$</td><td></td></tr>
<tr><td>3.</td><td>Given differential equation is $\frac{dy}{dx} = 2^{-y}$
On separating the variables, we get
$$2^y = dx$$
On integrating both sides, we get
$\int 2^y dy = \int \mathrm{dx}$
$\Rightarrow \frac{2^y}{log2} = x + c_1$
$\Rightarrow 2^y = xlog2 + c_1 log2$
$\Rightarrow 2^y = xlog2 + c(c = c_1 log2)$</td><td></td></tr>
<tr><td>4.</td><td>$(x-c)e^{(x+y)} +1 = 0$</td><td></td></tr>
<tr><td>5.</td><td>$y=kxe^{-x2}/2$</td><td></td></tr>
<tr><td>6.</td><td>$y=tan(x+x^2/2)$</td><td></td></tr>
</table>

7.	Given differential equation is $x(x^2 - 1)\frac{dy}{dx}=1$ Then $dy=\frac{dx}{x(x^2-1)}$ Integrate it, we get $y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right)+C$ Substituting y=0 and x=2 we get $c= -\frac{1}{2}log\frac{3}{4}$ Particular solution $y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}log\frac{3}{4}$	3
8.	ANS:given differential equation is $x^2dy+(xy+y^2)dx=0$ This is of the form $\frac{dy}{dx}=g\left(\frac{y}{x}\right)$. Put y=vx ,then $\frac{dy}{dx}=v+x\frac{dv}{dx}$ Gives $\left(\frac{1}{v^2+2v}\right)dv=\frac{-dx}{x}$ Integrate it, we get $\log\left\|\left(\frac{v}{v+2}\right)x^2\right\|=\log c^2$ Put $v=\frac{y}{x}$,we get $\frac{x^2y}{2x+y}=A$ When x=1 ,y=1 we get Particular solution $y+2x=3x^2y$.	3
9.	ANS:given $\frac{dy}{dx}$ +2ytanx= sinx The given equ. is a L.D.E. of the type $\frac{dy}{dx}$+Py=Q ,where P=2tanx and Q=sinx IF= $e^{\int 2tanxdx}=\sec^2x$, General solution is given by y.IF=$\int Q\times IFdx+c$ y. $\sec^2x=\int sinx.\sec^2xdx+c$=secx +c put y=0 and $x=\frac{\pi}{3}$ we get c=-2 Particular solution $y= \cos x-2\cos^2x$.	3
10.	X+y+1=e^x	3
11.	$2e^{\frac{x}{y}}+log\|y\|=2$	3
12.	y =4-x-2e^x	3
13.	$(y+3x^2)\frac{dx}{dy}=x$ $\Rightarrow\frac{dy}{dx}=\frac{y+3x^2}{x}$ $\Rightarrow\frac{dy}{dx}-\frac{y}{x}=3x\ldots(i)$ $\frac{dy}{dx}+Py=Q\ldots(ii)$ On comparison, we get $P=-\frac{1}{x}, Q=3x$ Integrating Factor $(\mathrm{I.F})=e^{\int pdx}=e^{\int-\frac{dx}{x}}=e^{-\log\|x\|}=e^{\log\left\|\frac{1}{x}\right\|}=\frac{1}{x}$ Hence, the sol^n is : $y\times\mathrm{I.F}=\int Q\times\mathrm{I.F}\,dx$ $\Rightarrow\frac{y}{x}=\int 3x.\frac{1}{x}dx$ $\Rightarrow\frac{y}{x}=\int 3dx$ $\Rightarrow\frac{y}{x}=3x+c$	

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$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$\Rightarrow \frac{dy}{dx} = (1 + x^2) + y^2(1 + x^2)$

$\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$

$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$

$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)dx$

$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c$

It is given that $y = 0$ when $x = 1$

$\therefore 0 = 1 + \frac{1}{3} + c$

$\Rightarrow c = -\frac{4}{3}$

Hence, the complete $\mathrm{sol}^{\mathrm{n}}$ is :

$$\tan^{-1} y = x + \frac{x^3}{3} - \frac{4}{3}$$

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$(1 + x^2)dy + 2xydx = \cot x\, dx$

$\Rightarrow (1 + x^2)\frac{dy}{dx} + 2xy = \cot x$

$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{\cot x}{1 + x^2} \ldots (i)$

$\frac{dy}{dx} + Py = Q \ldots (ii)$

On comparison, we get

$P = \frac{2x}{1 + x^2}, Q = \frac{\cot x}{1 + x^2}$

Integrating Factor (I. F) $= e^{\int pdx} = e^{\int \frac{2xdx}{1+x^2}} = e^{\log|1+x^2|} = e^{\log|1+x^2|}$

$= 1 + x^2$

Hence, the $\mathrm{sol}^{\mathrm{n}}$ is :

$y \times \text{I. F} = \int Q \times \text{I. F}\, dx$

$\Rightarrow y(1 + x^2) = \int \frac{\cot x}{(1 + x^2)}.(1 + x^2)dx$

$\Rightarrow y(1 + x^2) = \int \cot x\, dx$

$$\Rightarrow y(1 + x^2) = \log|\sin x| + c$$

</td><td></td></tr>
<tr><td>16.</td><td>

$$y = \text{a} \cos x + b \sin x$$

On differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = -a \sin x + b \cos x$$

$$\frac{d^2y}{dx^2} = -a \cos x - b \sin x$$

Now we get $\frac{d^2y}{dx^2} + y = -a \cos x - b \sin x + \text{a} \cos x + b \sin x = 0$

Therefore, the given function is a solution of the given differential equation.

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17.	Given differential equation can be rewritten as $\frac{dy}{dx} = e^x . e^y$ $$e^{-y}dy = e^x dx$$ On integrating both sides, we get $$\int e^{-y}dy = \int e^x dx$$ $$-e^{-y} = e^x + C_1$$ $$e^x + e^{-y} = C$$	3
18.	We know that the slope of the tangent to the curve is given by $\frac{dy}{dx}$ Hence, as per condition $\frac{dy}{dx} = \frac{2x}{y^2}$ The above equation can be rewritten as $$y^2dy = 2x\ dx$$ On integrating both sides, we get $$\int y^2dy = \int 2x\ dx$$ $$\frac{y^3}{3} = x^2 + C$$ Substituting x=-2 and y=3 we get C=5 Hence the equation of the required curve is $\frac{y^3}{3} = x^2 + 5 \Rightarrow$ y= $(3x^2 + 15)^{\frac{1}{3}}$	3
19.	Given that, xdx – ye$^y\sqrt{1+x}^2$ dy = 0 =>xdx =ye$^y\sqrt{1+x}^2$dy =>$\frac{x}{\sqrt{1+x\text{^}2}}$dx = ye^{y}dy Integrating both sides, we get $\int x/\sqrt{1+x}^2$dx = $\int y$ ·e^{y}dy =>$\frac{1}{2}\int \frac{2x}{\sqrt{1+x\text{^}2}}$dx = [y·$\int e\text{^}y$dy -$\int(d\ /dy$(y)·$\int e\text{^}2$dy)dy] Let I$_1$ = $\int 2x/\sqrt{1+x}^2$dx Putting 1 + x^2 = t => 2x dx = dt [on differentiating] ∴ I$_1$ = $\int \frac{dt}{t^{1/2}} = \int t^{-1}/2$dt = $\frac{t\text{^}-1/2+1}{-\frac{1}{2}+1} = \frac{t^1/2}{\frac{1}{2}}$ = 2t$^{\text{^}1/2}$ = 2(1+x^2)$^{\text{^}1/2}$ Now, $\frac{1}{2}$·2(1+x^2)$^{\text{^}1/2}$ =y·e^y – e^y + C => (1+x^2)$^{\text{^}1/2}$ = e^y(y-1) + C When x = 0, then y = 1 ∴ (1+0)$^{\text{^}1/2}$ = e^1(1-1) + C => C = 1 So, required solution is given by (1+x^2)$^{\text{^}1/2}$ = e^y(y-1) + 1	3
20.	Given, differential equation is (x+1)$\frac{dy}{dx}$ = 2e^{-y} + 1 => (x+1)$\frac{dy}{dx}$ =$\frac{2+e\text{^}y}{e\text{^}y}$	3

	=>$\frac{e^\wedge y}{e^y+2}$dy = $\frac{dx}{x+1}$ On integrating both sides,we get $\int \frac{e^y}{e^{y+2}}$dy = $\int \frac{dx}{x} + 1$ => log(e^y + 2) = log(x+1) + logC => log(e^y + 2) = logC(x+1) …(i) =>e^y + 2 = C(x+1) Also given y = 0, when x = 0 On putting x = 0 and y = 0 in Eq.(i) we get e^0 + 2 = C(0 + 1) => C = 1 + 2 =3 On putting C in Eq. (i) we get =>e^y + 2 = 3(x+1) =>e^y = 3x +3 -2 =>e^y = 3x + 1 => y = log(3x + 1)	
21.	Given, differential equation is $(1+ x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ =>$\frac{dy}{dx} + \frac{2x}{1+x^\wedge 2}y = \frac{4x^\wedge 2}{1+x^\wedge 2}$ Which is the equation of the form $\frac{dy}{dx}$ + Py = Q Where P = $\frac{2x}{1+x^\wedge 2}$ and Q = $\frac{4x^\wedge 2}{1+x^\wedge 2}$ Now, IF = e$\int \frac{2x}{1} + x^2$ = $e^{\log(1+x^\wedge 2)}$ = 1 + x^2 The general solution is $Y(1+x^2) = \int (1 + x^2) \frac{4x^\wedge 2}{(1+x^2)}dx + C$ => $(1+ x^2)y = \int 4x^\wedge 2dx + C$ => $(1+ x^2)y = \frac{4x^\wedge 3}{3} + C$ => $y = \frac{4x^\wedge 3}{3(1+x^2)} + C(1+x^2)^{-1}$ …(i) Now, y(0) = 0 => $0 = \frac{4\cdot 0^\wedge 3}{3(1+0^2)} + C(1+0^2)^{-1}$ => C = 0 Put the value of C in Eq. (i), we get Y =$\frac{4x^\wedge 3}{3(1+x^2)}$, which is the required solution.	3