CHAPTER-10

VECTORS

03 MARKS TYPE QUESTIONS

| Q. NO | QUESTION | MARK |
|---|---|---|
| 1. | The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ. | **3** |
| 2. | If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$,then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ | **3** |
| 3. | If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $\lvert \vec{a} \rvert = 3$, $\lvert \vec{b} \rvert = 5$, $\lvert \vec{c} \rvert = 7$ find the angle between $\vec{a}$ and $\vec{b}$ | **3** |
| 4. | If $\vec{a} = 3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-4\hat{k}$ , then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$. | 3 |
| 5. | If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, show that the magnitude of their difference is $\sqrt{3}$. | 3 |
| 6. | Using vector show that the points A(-2,3,5), B(7,0,-1), C(-3,-2,-5) and D(3,4,7) are such that AB and CD intersect at P(1,2,3). | 3 |
| 7. | $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ , $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$. Find a unit vector parallel to $\vec{a} - \vec{b}$ | 3 |
| 8. | Find the area of a parallelogram whose adjacent sides are given by vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ , $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$. | 3 |
| 9. | $p\hat{\imath} - 5\hat{\jmath} + 6\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} - q\hat{k}$ are collinear, find $p, q$ | 3 |
| 10. | Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be given as $a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ , $b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ , $c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ . then show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$. | 3 |
| 11. | If the position vectors of the vertices of a triangle are $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ $and$ $3\hat{\imath} + \hat{\jmath} + 2\hat{k}$, show that the triangle is an equilateral triangle. | 3 |
| 12. | If vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\vec{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $and$ $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of $\lambda$. | 3 |
| 13. | If vectors vectors $\vec{a}$ =$2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ , $\vec{b}$= -$\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c}3\hat{\imath} + \hat{\jmath}$= are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ ,then find the value of $\lambda$. | 3 |
| 14. | For any vector $\vec{a}$ ,show that $\vec{a} = (\vec{a} \cdot \hat{\imath})\,\hat{\imath} + (\vec{a} \cdot \hat{\jmath})\,\hat{\jmath} + (\vec{a} \cdot \hat{k})\,\hat{k}$ | 3 |
| 15. | Using vectors find the area of triangle ABC with vertices A(1,2,3),B(2,-1,4) and C(4,5,-1). | 3 |

# ANSWERS:

| Q. NO | ANSWER | MARKS |
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| 1. | **Ans:** $\vec{a} = 2\hat{i}+4\hat{j}-5\hat{k}$<br>$\vec{b}=\lambda\hat{i}+\hat{j}+3\hat{k}$<br>$\vec{a}+\vec{b}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$<br>**Unit vector along**<br>$\vec{a}+\vec{b}=\frac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert}$<br>$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}}$<br>$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+40}}$<br>$ATQ\ \vec{c}.(\vec{a}+\vec{b})=1$<br>$(\hat{i}+\hat{j}+\hat{k}).\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{(2+\lambda)^2+40}\right)=1$<br>$\frac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}}=1$<br>$2+\lambda+4=\sqrt{(2+\lambda)^2+40}$<br>*sq.both site*<br>$\lambda^2+36+12\lambda=(2+\lambda)^2+40$<br>$\lambda=1$ | **3** |
| 2. | **Ans:** $\lvert\vec{a}\rvert=1, \lvert\vec{b}\rvert=1, \lvert\vec{c}\rvert=1,$<br>$\vec{a}+\vec{b}+\vec{c}=0 \quad (Given)$<br>$\vec{a}.(\vec{a}+\vec{b}+\vec{c})$<br>$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$<br>$(\vec{a})^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$<br>$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$<br><br>$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1-------(i)$<br>*similiorly*<br>$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1-------(ii)$<br>*again*<br>$\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1-------(iii)$<br>*adding(i),(ii)and(iii)*<br>$2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-3 \qquad [\vec{a}.\vec{b}=\vec{b}.\vec{a}$<br>$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-3/2$ | **3** |

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| 3. | . **Ans:** $\vec{a}+\vec{b}+\vec{c}=0$<br>$\vec{a}+\vec{b}=-\vec{c}$<br>$(\vec{a}+\vec{b}).(-\vec{c})=-\vec{c}.(-\vec{c})$<br>$(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=\vec{c}.\vec{c}$<br>$\lvert\vec{a}\rvert^2+2\vec{a}\vec{b}+\lvert\vec{b}\rvert^2=\lvert\vec{c}\rvert^2$<br>$\vec{a}.\vec{b}=\frac{49-9-25}{2}=\frac{15}{2}$<br>$\cos\theta=\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$<br>$=\frac{1}{2}$<br>$\theta=60$ | **3** |
| 4. | $\vec{b}_1$ is parallel to $\vec{a}$<br>$\Rightarrow \vec{b}_1$ = m$\vec{a}$ for some scalar m<br>$\Rightarrow \vec{b}_1 = m(3\hat{\imath}+4\hat{\jmath}+5\hat{k}) = 3m\hat{\imath} + 4m\hat{\jmath} + 5m\hat{k}$ …..(i)<br>If $\vec{b} = \vec{b}_1+\vec{b}_2$<br>$\Rightarrow \vec{b}_2 = \vec{b}-\vec{b}_1 = (2-3m)\hat{\imath} + (1-4m)\hat{\jmath} + (-4-5m)\hat{k}$ …..(ii)<br>Also given that $\vec{b}_2$ is perpendicular to $\vec{a}$<br>$\Rightarrow \vec{b}_2.\vec{a} = 0$<br>$\Rightarrow 3(2 - 3m) + 4(1 - 4m) + 5(- 4 - 5m) = 0$<br>$\Rightarrow - 10 - 50m = 0$<br>$\Rightarrow m = -1/5$<br>Therefore,<br>$\vec{b}_1 = -\frac{1}{5}(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$<br>And $\vec{b}_2 = \frac{13}{5}\hat{\imath} + \frac{9}{5}\hat{\jmath} - 3\hat{k}$<br>$\therefore \vec{b} =(-\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath} - \hat{k}) + (\frac{13}{5}\hat{\imath} + \frac{9}{5}\hat{\jmath} - 3\hat{k}) = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ is the required expression. | 3 |
| 5. | Given $\hat{a}$ and $\hat{b}$ are unit vectors and $\hat{a}+\hat{b}$ is also a unit vector.<br>$\Rightarrow \lvert\hat{a}\rvert = 1, \lvert\hat{b}\rvert = 1$ and $\lvert\hat{a}+\hat{b}\rvert = 1$<br>We have $\lvert\hat{a}+\hat{b}\rvert^2 = \lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 + 2\hat{a}.\hat{b}$<br>$\Rightarrow 1 = 1 + 1 +2\hat{a}.\hat{b} \Rightarrow 2\hat{a}.\hat{b} = -1$<br>Also we have,<br>$\lvert\hat{a}-\hat{b}\rvert^2 = \lvert\hat{a}\rvert^2 + \lvert\hat{b}\rvert^2 - 2\hat{a}.\hat{b} \Rightarrow \lvert\hat{a}-\hat{b}\rvert^2 = 1 + 1 - (-1) = 3$<br>$\Rightarrow \lvert\hat{a}-\hat{b}\rvert = \sqrt{3}$<br>i.e., the magnitude of the difference is $\sqrt{3}$ | 3 |
| 6. | To prove P intersects $\overrightarrow{AB}$ and $\overrightarrow{CD}$, we have to show that A,P,B are collinear and C,P,D are collinear<br>$\overrightarrow{AP} = (1+2)\hat{\imath} + (2-3)\hat{\jmath} + (3-5)\hat{k} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$<br>$\overrightarrow{PB} = (7-1)\hat{\imath} + (0-2)\hat{\jmath} + (-1-3)\hat{k} = 6\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$<br>$\Rightarrow \overrightarrow{PB} = 2(3\hat{\imath} - \hat{\jmath} - 2\hat{k}) = 2AP$<br>$\Rightarrow$the vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are collinear. | 3 |

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| | Since P is a common point to $\overrightarrow{AP}$ and $\overrightarrow{PB}$, the points A, P, B are collinear.<br><br>Similarly,<br>$\overrightarrow{CP}$ = (1+3) $\hat{\imath}$ + (2+2) $\hat{\jmath}$ + (3+5) $\hat{k}$ = $4\hat{\imath} + 4\hat{\jmath} + 8\hat{k}$<br>$\overrightarrow{PD}$ = (3-1) $\hat{\imath}$ + (4-2) $\hat{\jmath}$ + (7-3) $\hat{k}$ = $2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$<br>$\Rightarrow \overrightarrow{CP} = 2(2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 2\overrightarrow{PD}$<br>⇒ the vectors $\overrightarrow{CP}$ and $\overrightarrow{PD}$ are collinear<br><br>Since P is a common point to $\overrightarrow{CP}$ and $\overrightarrow{PD}$, the points C, P, D are collinear.<br>i.e., P is a common point to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ and so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect at P. | |
| 7. | $\vec{a} - \vec{b} = -2\hat{\imath} + \hat{\jmath} + 4\hat{k}$<br>$req.\ vector\ parallel\ to\ \ \vec{a} - \vec{b} = \frac{\vec{a}-\vec{b}}{\lvert \vec{a}-\vec{b}\rvert} = \frac{-2\hat{\imath}+\hat{\jmath}+4\hat{k}}{\sqrt{21}}$ | 3 |
| 8. | $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$<br><br>$\lvert \vec{a} \times \vec{b} \rvert = 15\sqrt{2}$ | 3 |
| 9. | $p\hat{\imath} - 5\hat{\jmath} + 6\hat{k} = \alpha(2\hat{\imath} - 3\hat{\jmath} - q\hat{k})$<br>$\alpha = \frac{5}{3}$ on comparing<br>$p = \frac{10}{3}, q = -\frac{18}{5}$ | 3 |
| 10. | For correct proof | 3 |
| 11. | $\lvert\overrightarrow{AB}\rvert = \sqrt{6} = \lvert\overrightarrow{BC}\rvert = \lvert\overrightarrow{CA}\rvert$ | 3 |
| 12. | $\lambda = 8.$ | 3 |
| 13. | As $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$<br>$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$<br>$((2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}) \cdot (3\hat{\imath} + \hat{\jmath}) = 0$<br>$3(2-\lambda) + (2+2\lambda) = 0$<br>$\lambda = 8$ | |
| 14. | Let $\vec{a} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$<br>$\vec{a} \cdot \hat{\imath} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{\imath}$<br>= l<br>$\vec{a} \cdot \hat{\jmath} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{\jmath}$<br>= m<br>$\vec{a} \cdot \hat{k} = (l\hat{\imath} + m\hat{\jmath} + n\hat{k}) \cdot \hat{k}$<br>= n<br>RHS= $(\vec{a} \cdot \hat{\imath})\hat{\imath} + (\vec{a} \cdot \hat{\jmath})\hat{\jmath} + (\vec{a} \cdot \hat{k})\hat{k}$<br>$= l\hat{\imath} + m\hat{\jmath} + n\hat{k}$<br>$= \vec{a}$<br>=LHS | |

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| 15. | We know that Area of triangle = $\frac{1}{2}\left\vert\overrightarrow{BC} \times \overrightarrow{BA}\right\vert$<br>$\overrightarrow{BC}$=(4-2) $\hat{\imath}$ +(5+1) $\hat{\jmath}$ + (-1-4) $\hat{k}$<br>=2 $\hat{\imath}$ + 6$\hat{\jmath}$ -5$\hat{k}$<br>$\overrightarrow{BA} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$<br>A(1,2,3)<br>( 2,-1,4) B C(4,5,-1)<br>$\overrightarrow{BC} \times \overrightarrow{BA} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & -5 \\ -1 & 3 & -1 \end{bmatrix} = 9\hat{\imath} + 7\hat{\jmath} + 12\hat{k}$<br>$\left\vert \overrightarrow{BC} \times \overrightarrow{BA} \right\vert = \sqrt{81 + 49 + 144} = \sqrt{274}$<br>Area of triangle = $\frac{1}{2}\left\vert\overrightarrow{BC} \times \overrightarrow{BA}\right\vert$<br>= $\frac{1}{2}\sqrt{274}$ sq. units | |