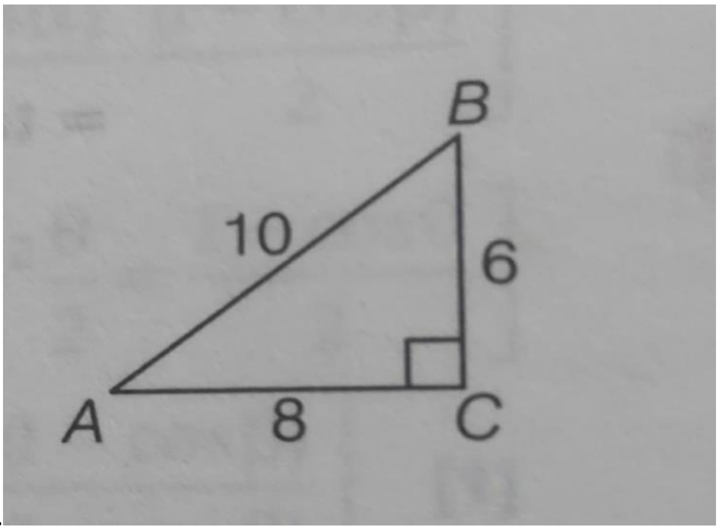
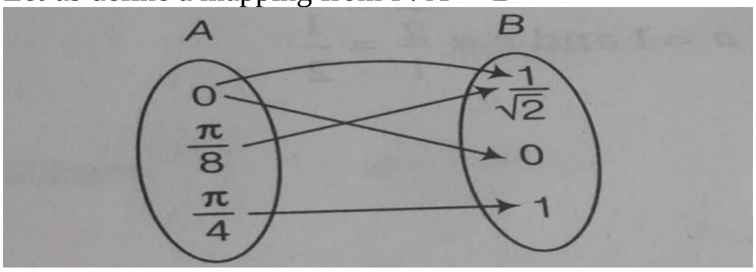
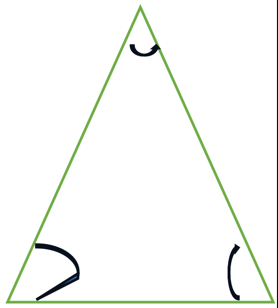
CHAPTER-2
INVERSE TROGONOMETRIC FUNCTION
CLASS-XII
03 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$ in $[\pi/2, \pi]$	3
2.	Express tan($\cos^{-1}$ x) in terms of x only and hence evaluate tan ($\cos^{-1}\frac{8}{17}$).	3
3.	Is tan ($\cot^{-1}$ x) = cot ($\tan^{-1}$ x)? Justify your answer.	3
4.	$\tan^{-1}x + \tan^{-1}y = \pi/4$; $xy < 1$, then write the value of $x + y + xy$.	3
5.	write the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$.	3
6.	Write the value of tan (2 $\tan^{-1}$1/5)	3
7.	$\tan^{-1}x + \tan^{-1}y = \pi/4$; $xy < 1$, then write the value of $x + y + xy$.	3
8.	write the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$.	3
9.	Write the value of tan (2 $\tan^{-1}$1/5)	3
10.	Evaluate $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}0$	3
11.	Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$, in the simplest form.	3
12.	Write in simplest form $\tan^{-1}\left(\frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\right), 0 < x < \pi.$	3
13.	Simplify $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, \lvert x\rvert < a$.	3
14.	Simplify $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, where, $x \in \left(0, \frac{\pi}{4}\right)$	3
15.	Prove that, $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$	3
16.	Prove the following: $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$	3
17.	Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x.$	3
18.	Prove that : $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	3
19.	Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], \lvert x\rvert < 1, y > 0$ and xy<1	3
20.	Write the following functions in simplest form $\tan^{-1}\left(\frac{\sqrt{a-x}}{\sqrt{a+x}}\right)$	3
21.	Express $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$, $x < \pi$ in the simplest form.	3
22.	If $\alpha = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$, $x \geq 0$, then find the smallest interval in which α lies.	3
23.	Solve for x : $\cos(2\sin^{-1}x) = \frac{1}{9}$	3

24.	Evaluate: $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$	**3**
25.	A right angled triangle ABC is given here. With the help of inverse trigonometric function, prove that  < A + < B + < C = 180^0.	**3**
26.	Let us define a mapping from f : A → B  Such that f(x) = sin 2x. Is the inverse function exists?.If so, find the inverse, domain and range of f (x).	**3**

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x)$, $[\pi/2, \pi]$ $\Rightarrow \sqrt{1+2\cos^2 x - 1} = \sqrt{2}\cos^{-1}(\cos x)$ $\Rightarrow \sqrt{2}\cos x = \sqrt{2}\cos^{-1}(\cos x)$ $\Rightarrow \cos x = x$ which is not true for any $x \in [\pi/2, \pi]$ Hence, no real solution exists in the given interval.	3
2.	Let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta$ Now $\sin\theta = \sqrt{1-x^2}$ So, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-x^2}}{x}$ Hence, $\tan(\cos^{-1}\frac{8}{17}) = \frac{\sqrt{1-(\frac{8}{17})^2}}{\frac{8}{17}} = \frac{\frac{15}{17}}{\frac{8}{17}} = 15/8$	3
3.	Let $\cot^{-1} x = \theta$, $X = \cot\theta$ $= \tan(\pi/2 - \theta)$ $\text{Tan}^{-1} x = (\pi/2 - \theta)$ So, $\tan(\cot^{-1} x) = \tan\theta = \cot(\pi/2 - \theta) = \cot(\pi/2 - \cot^{-1} x) = \cot(\text{Tan}^{-1} x)$ This equality is valid for all values of x since $\tan^{-1}x$ and $\cot^{-1} x$ are true for all $x \epsilon R$.	3
4.	We have $a_1 = a, a_2 = a + d, a_3 = a + 2d.......$ And, $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... = a_n - a_{n-1}$ Given that, $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \tan^{-1}\left(\frac{d}{1+a_3a_4}\right) + ...\right.$ $= \tan^{-1}\left[\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{a_n - a}{1+a_{n-}}\right.\right.$ $= \tan\left[\left(\tan^{-1}a_2 - \tan^{-1}a_1\right) + \left(\tan^{-1}a_3 - \tan^{-1}a_2\right) + ... + \left(\tan^{-1}\right.\right.$ $= \tan\left[\tan^{-1}a_n - \tan^{-1}a_1\right]$ $\left[Scince, \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$ $= \tan\left[\tan^{-1}\left(\frac{a_n - a_1}{1+a_1a_n}\right)\right]$ $\left[scince, \tan\left(\tan^{-1}x\right) = x\right]$ $= \frac{a_n - a_1}{1+a_1a_n}$	3

5.	Solution: The sum of three angles of triangle is π A+B+C=π $\Rightarrow \cot^{-1}3+\cot^{-1}2+C=\pi$ $\Rightarrow \cot^{-1}\frac{6-1}{3+2} + C = \pi$ { $\cot^{-1}x+\cot^{-1}y=\cot^{-1}\frac{xy-1}{x+y}$ } $\Rightarrow \cot^{-1}\frac{5}{5}+C = \pi$ $\Rightarrow \cot^{-1}1 + C = \pi$ $\Rightarrow \frac{\pi}{4} + C = \pi$ $\Rightarrow C = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ 	3
6.	Solution: The given equation is; $\text{Cos}(\tan^{-1}x)=\sin(\cot^{-1}\frac{3}{4})$ $\Rightarrow \text{Cos}(\tan^{-1}x)=\cos(\frac{\pi}{2}-\cot^{-1}\frac{3}{4})$ [$\sin\Theta=\cos(\frac{\pi}{2}-\Theta)$] $\Rightarrow \cos(\tan^{-1}x)=\cos(\tan^{-1}\frac{3}{4})$ [$\tan^{-1}x+\cot^{-1}=\frac{\pi}{2}$] $\Rightarrow \tan^{-1}x=\tan^{-1}\frac{3}{4}$ $\Rightarrow x=\frac{3}{4}$	3
7.	Given, $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}, xy<1$ We know that, $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right), xy<1$ $\therefore \quad \tan^{-1}\left(\frac{x+y}{1-xy}\right)=\frac{\pi}{4} \Rightarrow \frac{x+y}{1-xy}=\tan\frac{\pi}{4}$ $\Rightarrow \quad \frac{x+y}{1-xy}=1 \quad \left[\because \tan\frac{\pi}{4}=1\right]$ $\Rightarrow \quad x+y=1-xy$ $\therefore \quad x+y+xy=1$	3

8.	We have, $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$ $=\left[\pi-\cos^{-1}\left(\frac{1}{2}\right)\right]+2\sin^{-1}\left(\frac{1}{2}\right)$ $[\because \cos^{-1}(-x)=\pi-\cos^{-1}x;\ \forall x\in[-1,1]]$ $=\left[\pi-\cos^{-1}\left(\cos\frac{\pi}{3}\right)\right]+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)$ $\left[\because \cos\frac{\pi}{3}=\frac{1}{2}\text{ and }\sin\frac{\pi}{6}=\frac{1}{2}\right]$ $=\left[\pi-\frac{\pi}{3}\right]+2\times\frac{\pi}{6}$ $\left[\begin{array}{l}\because\ \cos^{-1}(\cos\theta)=\theta;\ \forall\theta\in[0,\pi]\\ \text{and }\sin^{-1}(\sin\theta)=\theta;\ \forall\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\end{array}\right]$ $=\frac{2\pi}{3}+\frac{\pi}{3}=\frac{2\pi+\pi}{3}=\pi$	3
9.	$\tan\left(2\tan^{-1}\frac{1}{5}\right)=\tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)\right]$ $\left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right);-1<x<1\right]$ $=\tan\left[\tan^{-1}\left(\frac{2\times 5}{24}\right)\right]=\tan\left[\tan^{-1}\left(\frac{5}{12}\right)\right]=\frac{5}{12}$ $[\because \tan(\tan^{-1}x)=x;\ \forall x\in R]$	3
10.	$3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}0$ $=3\sin^{-1}\left(\sin\frac{\pi}{4}\right)+2\cos^{-1}\left(\cos\frac{\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{2}\right)$ $=3\frac{\pi}{4}+2*\frac{\pi}{6}+\frac{\pi}{2}$ $=\frac{19\pi}{12}$	3
11.	We have $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $=\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$ $=\tan^{-1}\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$ $=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right]$ $=\frac{\pi}{4}+\frac{x}{2}$ as $\frac{-3\pi}{2}<x<\frac{\pi}{2}$	3

12.	$Let\ \tan^{-1}\left(\frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\right)=y$ $\text{Y}=\tan^{-1}\left[\sqrt{\frac{2sin^2\left(\frac{x}{2}\right)}{2cos^2\left(\frac{x}{2}\right)}}\right]$ $\text{Y}=\tan^{-1}\left[\sqrt{tan^2\left(\frac{x}{2}\right)}\right]$ $\text{Y}=\tan^{-1}\left[tan\left(\frac{x}{2}\right)\right]$ $\text{Y}=\frac{x}{2},\ x<\pi$	3
13.	$y=\tan^{-1}\frac{x}{\sqrt{a^2-x^2}},$ Let $x=a\sin\theta$ So, $y=\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2cos^2\theta}}\right)$ $=\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)\ =\theta\ =\sin^{-1}\frac{x}{a}$	3
14.	$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ $=\cot^{-1}\left(\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right)$ $=\cot^{-1}\left(\cot\frac{x}{2}\right)\ =\frac{x}{2}$	3
15.	$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$	3
16.	$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$ $\text{L.H.S.}=\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$ Let $\cot^{-1}x=\theta\quad\Rightarrow x=cot\theta$ $\text{L.H.S.}=\cos[\tan^{-1}\{\sin\theta\}]$ $==\sqrt{\frac{1+x^2}{2+x^2}}$	3
17.	$\text{L.H.S.}=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ $=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\times\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$ $=\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$ $=\ \tan^{-1}\left(\frac{1-\sqrt{1-sin^2\theta}}{sin\theta}\right)$ where $x=sin\theta$ $=\tan^{-1}\left(\tan\frac{\theta}{2}\right)$ $=\frac{\theta}{2}$ $=\frac{1}{2}\sin^{-1}x$	3

	$= \frac{1}{2}\left(\frac{\pi}{2} - \cos^{-1} x\right)$ $= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x.$	
18.	We have, $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\text{x}\frac{5}{12}}\right) = \tan^{-1}\left(\frac{56}{33}\right)$ $= \cos^{-1}\left(\frac{33}{65}\right)$	3
19.	We have , $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right] = \tan\frac{1}{2}\left[2\tan^{-1}x + 2\tan^{-1}y\right] = \tan\left[\tan^{-1}x + \tan^{-1}t\right]$ $= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right] = \frac{x+y}{1-xy}$	3
20.	We have , $\tan^{-1}\left(\frac{\sqrt{a-x}}{\sqrt{a+x}}\right)$, Putting x= a cost So $\tan^{-1}\left(\frac{\sqrt{a-a\cos t}}{\sqrt{a+a\cos t}}\right) = \tan^{-1}\left(\frac{\sqrt{2a}\sin t}{\sqrt{2a}\cos t}\right) = \tan^{-1}(\tan t) = t = \cos^{-1}\frac{x}{a}$	3
21.	$\frac{\pi}{4} - \frac{x}{2}$	3
22.	$\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{2}$	3
23.	x = $\frac{2}{3}$	3
24.	$\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$ $- \frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$ $- \frac{\pi}{12}$	3
25.	Sin A = $\frac{3}{5}$, A = $\sin^{-1}(\frac{3}{5})$ sin B = $\frac{4}{5}$, B = $\sin^{-1}(\frac{4}{5})$ $< A + < B + < C = \sin^{-1}(\frac{3}{5}) + \sin^{-1}(\frac{4}{5}) + 90^0$ Let x = $\sin^{-1}(\frac{3}{5})$, y = $\sin^{-1}(\frac{4}{5})$. Then, cos x = $\frac{4}{5}$, cos y = $\frac{3}{5}$. Sin(x + y) = sinx cosy + cosx siny $= \frac{25}{25}$ $= 1$ x + y = 90^0 $< A + < B + < C = \sin^{-1}(\frac{3}{5}) + \sin^{-1}(\frac{4}{5}) + 90^0$	3

	$= 90^0 + 90^0$ $= 180^0$	
26.	It is one one onto. So inverse exists. $2x = \sin^{-1}y$ $x = ½ \sin^{-1}y$ Domain = { $\frac{1}{\sqrt{2}}$, 0, 1 }, Range = { 0, $\frac{\pi}{8}$, $\frac{\pi}{4}$ }	3