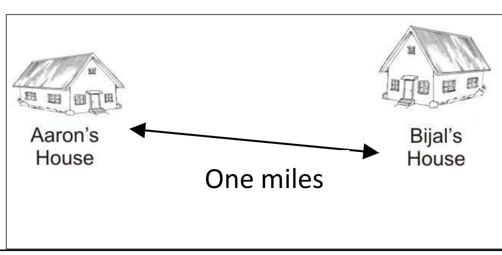
CLASS-XII
CHAPTER-01
RELATION AND FUNCTION
03 MARKS TYPE QUESTIONS

Q. No.	QUESTION	MARK
1	If R and S are equivalence relations on a set A, then prove that $R \cap S$ is also an equivalence relation.	3
2	Let S be the relation in real number R defined by $(a, b)\ S\ (c, d)\ if\ ad = bc$ for $a, b, c, d \epsilon R$ (set of real numbers). Prove that S is an equivalence relation.	3
3	Classify the function $f(x) = 2^x + 2^{\|x\|}$ as injection, surjection or bijections. Justify your answer.	3
4	Show that the relation R in the set {1, 2, 3} given by R= {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.	3
5	Let A={1,2} . Find all one-to-one function from A to A.	3
6	Prove the function f:R→R given by f(x)=cosx for all x∈R, is neither injective nor surjective.	3
7	An electrician charges a base fee of Rs. 70 plus Rs. 50 for each hour of work. Create a table that shows the amount of the electrician charges for 1, 2, 3, & 4 hours of work. Let x represent the number of hours and y represent the amount charged for x hours. Is the relation a function?	3
8	Jimmy has to fill up his car with gasoline to drive to and from work next week. If gas costs $2.79 per gallon, and his car holds a maximum of 28 gallons, what is the domain and range of the function?	3
9	"x lives within one mile of y" – Show that it is reflexive, symmetric, but not transitive relation.  	3
10	Show that the relations S in the set of real numbers defined as S={ (a,b): a,b ϵ R and a$\leq b^3$} is neither reflexive nor symmetric nor transitive	3
11	Let A=**R**-{$\frac{2}{3}$},show that the function f in set A defined by $f(x) = \frac{4x-3}{6x-4}$ $\forall x \in A$, is one-one and onto	3

12	Show that the relation R defined on the **set N×N** by (a,b) R (c,d) ⇒ $a^2 + d^2 = b^2 + c^2 \ \forall\ a, b, c, d \ \in \boldsymbol{N}$ is an equivalence relation	3
13	Let A = R-{3} ,B = R-{1} . If f:A → B be defined by $f(x) = \frac{x-2}{x-3} \ \ \forall\ x \ \in A.$ Then show that f is bijective.	3
14	Let N denotes the set of all natural numbers and R be the relation on N X N defined by (a,b) R (c,d) if ad(b+c) = bc (a+d). Show that R is an equivalence relation.	3
15	Check whether the relation R in R defined by R ={(a,b^3) : a≤ b^3) is reflexive ,symmetric or transitive.	3
16	Show that the relation R in the set of real numbers, defined as R = {(a, b): a ≤ b^2} is neither reflexive nor symmetric nor transitive.	3
17	Let A = R−{3} and B = R−{1}. Consider the function *f* : A→B defined by *f*(x) = $\frac{x-2}{x-3}$. Show that *f* is one-one and onto.	3
18	Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) : a, b ∈ Z, and (a−b) is divisible by 5}. Prove that R is an equivalence relation.	3

ANSWER
CHAPTER-01
RELATION AND FUNCTION
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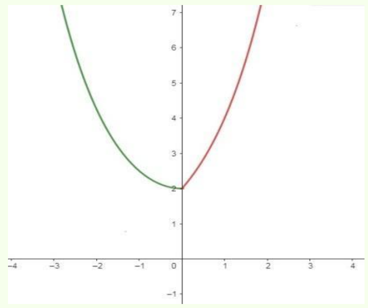
Q.No	**ANSWERS**	**Mark**
1	Given that R and S are reflexive, symmetric and transitive. <u>Reflexivity</u>: Let $a \in A$. So, $(a,a) \in R\ \ and\ \ (a,a) \in S$ (as R and S are reflexive) $\Rightarrow (a,a) \in R \cap S$. So, $R \cap S$ is reflexive. <u>Symmetricity</u>: Let $a, b \in A$ and $(a,b) \in R \cap S$ $\Rightarrow (a,b) \in R\ \ and\ \ (a,b) \in S$ $\Rightarrow (b,a) \in R\ \ and\ \ (b,a) \in S$ (as R and S are symmetric) $\Rightarrow (a,b) \in R \cap S$. So, $R \cap S$ is symmetric. <u>Transitivity</u>: Let $a, b, c \in A$ and $(a,b) \in R \cap S$ and $(b,c) \in R \cup S$ $\Rightarrow (a,b) \in R\ \ and\ \ (a,b) \in S$ and $(b,c) \in R\ \ and\ \ (b,c) \in S$ $\Rightarrow (a,b) \in R$, $(b,c) \in R$ and $(a,b) \in S$, $(b,c) \in S$ $\Rightarrow (a,c) \in R$ and $(a,c) \in S$ (as R and S are transitive) $\Rightarrow (a,c) \in R \cap S$. So, $R \cap S$ is transitive. Therefore, $R \cap S$ is an equivalence relation.	3
2	<u>Reflexivity</u>: Let $a, b \in R$ and $(a,b) \in R \times R$. Since, $ab = ba$ $\Rightarrow (a,b) S (a,b)$. So, S is reflexive. <u>Symmetricity</u>: Let $a, b, c, d \in R$ and $(a,b), (c,d) \in R \times R$ $Let\ (a,b)\ S\ (c,d)$ $\Rightarrow ad = bc$ $\Rightarrow bc = ad$ $\Rightarrow cb = da$ $\Rightarrow (c,d)\ S\ (a,b)$. So, S is symmetric. <u>Transitivity</u>: Let $a, b, c, d, e, f \in R$ and $(a,b), (c,d), (e,f) \in R \times R$	3

	$Let\ (a,b)\ S\ (c,d)$ and $(c,d)\ S\ (e,f)$ $\Rightarrow ad = bc \quad and \quad cf = de$ $\Rightarrow adcf = bcde$ $\Rightarrow af = be$ $\Rightarrow (a,b)\ S\ (e,f)$ So, S is transitive. Therefore R is an equivalence relation.	
3	$f(x) = 2^x + 2^{\|x\|} = \begin{cases} 2.2^x & x \geq 0 \\ 2^x + 2^{-x} & x < 0 \end{cases}$  <u>Case-1</u>: $x \geq 0$ let $x_1, x_2 \geq 0$ and $f(x_1) = f(x_2)$ $\Rightarrow 2.2^{x_1} = 2.2^{x_2} \quad \Rightarrow x_1 = x_2$ <u>Case-2</u>: $x < 0$ Let $x_1, x_2 < 0$ and $f(x_1) = f(x_2)$ $\Rightarrow 2^{x_1} + 2^{-x_1} = 2^{x_2} + 2^{-x_2}$ $\Rightarrow 2^{x_1} - 2^{x_2} = 2^{-x_2} - 2^{-x_1} = \frac{1}{2^{x_2}} - \frac{1}{2^{x_1}}$ $\Rightarrow 2^{x_1} - 2^{x_2} = \frac{2^{x_1} - 2^{x_2}}{2^{x_2.} \times 2^{x_1}}$ $\Rightarrow (2^{x_1} - 2^{x_2})(2^{x_2.} \times 2^{x_1}) - 2^{x_1} - 2^{x_2} = 0$ $\Rightarrow (2^{x_1} - 2^{x_2})\{(2^{x_2.} \times 2^{x_1}) - 1\} = 0$ $\Rightarrow 2^{x_1} - 2^{x_2} = 0$ because $(2^{x_2.} \times 2^{x_1}) - 1 = 2^{x_1 + x_2.} - 1 \neq 0$ $\Rightarrow x_1 = x_2$ <u>Case-3</u>: let $x_1 \geq 0 \quad and \quad x_2 < 0$. Clearly, $x_1 \neq x_2$ Now, $f(x_1) = 2.2^{x_1} \geq 2$ and $f(x_2) = 2^{x_2} + 2^{-x_2} > 2$ Therefore, $f(x_1) can\ be\ equal\ to\ f(x_2) for\ some\ x_1, x_2$. So, f is a many-one function. Again since the value of $f(x) = 2^x + 2^{\|x\|} = \begin{cases} 2.2^x & x \geq 0 \\ 2^x + 2^{-x} & x < 0 \end{cases}$ for all $x \in R$ can not be negative, so f is not onto. {From the graph, it is clear that Range is $[2, \infty)$ and co-domain=R} Hence f is many-one and onto.	3
4	R={(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} **<u>Reflexive</u>** If the relation is reflexive,then (a, a)∈R for every a∈(1, 2, 3). Since (1, 1)∈R, (2, 2)∈R & (3, 3)∈R. Therefore, R is reflexive	3

	Symmetric To check whether symmetric or not, If (a, b)∈R, then (b, a)∈R. Here (1, 2)∈R, but (2, 1)∉R. Therefore, R is not symmetric. Transitive To check whether transitive or not, If (a,b)∈R & (b,c)∈R, then (a,c)∈R. Here, (1, 2)∈R and (2, 3)∈R but (1, 3)∉R. Therefore, R is not transitive. Hence, R is reflexive but neither symmetric nor transitive	
5	A={1,2} one- one function from A to A. f(x)=y for every unique x, y should be unique. f(x)→one f(A)={1,1 | 2,2} f(A)={1,2 | 2,1}	3
6	Injectivity : We know that f(0)=cos0=1 and f(2π)=cos2π=1. So, different elements in R may have the same image. Hence, f is not an injection. Surjectivity : Since the values of cosx lie between −1 and 1, it follows that the range of f(x) is not equal to its co−domain. So, f is not a surjection.	3
7	The table can be drawn as:	3

X (hour)	Y	Base Fee	Total
1	50	70	120
2	100	70	170
3	150	70	220
4	200	70	270

	The relation is a function such that Y= x * 50 Total = Y + 70 Also, Total = (x*50) + 70, where x number of hours We can see that each x element has only one y-element as well as total number associated with it. Hence, the relation is a function, since a function is a special type of relation where each input has exactly one output, and the output can be traced back to its input.	
8	The number of gallons of gas purchased will go on the x-axis and the costs of the gasoline goes on the y-axis. Because the least amount of gas he can purchase is 0 gallons which is $0 then part of the function is 0≤x. The most amount of money he can spend on gas is $78.12 which is the full 28 gallons. i.e., 2.79 * 28 gallons = 78.12 This adds to the function making it 0≤x≤28. Then to complete the function because each gallon of gas cost $2.79 and x represents the amount of gas bought the equation is y=2.79x and 0≤x≤28. Hence, **The domain is [0,28] and the range is [0,78.12]**	3
9	Perfectly valid as well: * Any person lives within a mile of themselves (namely zero distance), so it is reflexive. * If one person lives within a mile of another, that person consequently lives within a mile of the first, so it's symmetric. * It is not ensured that if Paul lives within a mile of John, who lives within a mile of Martha, that Paul is within a mile of Martha. For instance, if Paul and Martha are two miles apart, and John is exactly between the two, we see a lack of transitivity. Hence, the given relation is reflexive, symmetric but not transitive.	3
10	**S**={ (a,b): a,b ∈ R and a≤ b^3} **Refexive:- as** $\frac{1}{2} \leq (\frac{1}{2})^3$ where $\frac{1}{2} \in R$, is not true $(\frac{1}{2}, \frac{1}{2}) \notin$ S Thus S is not reflexive…… **(1)** **Symmetric:- as** $-2 \leq (3)^3$ where (-2,30 ∈ S is true but $\mathbf{3} \leq (-2)^3$ is not true. i.e. (-2,3) ∈ S but (3,-2) ∉ S **(1)** Therefore ,S is not symmetric . **Transitive:- As** $3 \leq (\frac{3}{2})^3$ and $\frac{3}{2} \leq (\frac{4}{3})^3$ where $3, \frac{3}{2}, \frac{4}{3}$ ∈ S are true but $3 \leq (\frac{4}{3})^3$ is not true i.e. $(3, \frac{3}{2})$ ∈ S and $(\frac{3}{2}, \frac{4}{3})$ ∈ S but $(3, \frac{4}{3}) \notin$ S. therefore S is not transitive **(1)** **Hence is neither reflexive nor symmetric nor transitive**	3

11	**Given that** $f(x) = \frac{4x-3}{6x-4}\ \forall\ x \in A$ **To show that f is One-One** **Let** $f(x_1) = f(x_2)$ Then $(\frac{4x_1+3}{6x_1-4}) = \frac{4x_2+3}{6x_2-4}$ on solving this …. **($\frac{1}{2}$)** we get $x_1 = x_2$ **(1)** **To show that f is Onto** **Let y** ϵ B so y= $f(x)$ …. ($\frac{1}{2}$) Or y=$\frac{4x-3}{6x-4}$ solve for x we get x=$\frac{4y+3}{6y-4}$ = $g(y)$ ⇒ f is Onto function. **(1)**	3
12	**For Reflexive Relation** **Let** (a,b) ∈ N×N Then $since\ \ a^2 + b^2$=$a^2 + b^2$ (a,b) R (a,b) Hence R is reflexive relation **(1)** **Symmetric:- Let** (a,b),(c,d) ∈ N×N be such that (a,b) R (c,d) ⇒ $a^2 + d^2$=$b^2 + c^2$ ⇒ $c^2 + b^2 = d^2 + a^2$ ⇒ (c,d) R (a,b) Hence R is symmetric relation **(1)** **Transitive:-** Let (a,b), (c,d), (e,f) ∈ N×N be such that (a,b) R (c,d) and (c,d) R (e,f) ⇒ $a^2 + d^2$=$b^2 + c^2$…………..(1) And ⇒ $c^2 + f^2$=$d^2 + e^2$………………(2) Adding equation.(1) and equation (2) ⇒ $a^2 + d^2$+ $c^2 + f^2$=$b^2 + c^2$+ $d^2 + e^2$ ⇒ $a^2 + f^2$=$b^2 + e^2$ ⇒ (a,b) R (e,f) Hence R is transitive relation **(1)** Since R is reflexive, symmetric and transitive Hence R is equivalence relation	3
13	**For injectivity :** $f(x_1) = f(x_2)$ therefore $\frac{x1-2}{x1-3} = \frac{\boldsymbol{x2-2}}{\boldsymbol{x2-3}}$ (x1-2)(x2-3) =(x1-3)(x2-2) X1.x2 -3x1 -2x2+6 = x1.x2 -3x2-2x1 +6 then x1 =x2 , f(x) is injective. **For Surjectivity:** $\boldsymbol{y} = \frac{\boldsymbol{x-2}}{\boldsymbol{x-3}}$; x-2 = xy – 3y ; $\boldsymbol{x} = \frac{\boldsymbol{2-3y}}{\boldsymbol{1-y}} \in A\ for\ every\ value\ of\ B$ So f is surjective Hence f is bijective.	3
14	**Reflexivity:** b+a =a+b , for a,b ∈ N ab=ba , for a,b ∈ N ab(b+a) =ba(a+b) , for a,b ∈ N (a,b) R (b,a) , R is reflexive. **Symmetric:**	3

	=(a,b)R(c,d) ad(b+c)= bc(a+d) cb(d+a) = da(c+b) = (c,d) R (a,b) **Transitivity:** Let (a,b)R(c,d) and (c,d) R (e,f) then (a,b) R (e,f)	
15	**Reflexive** R ={(a,b^3) : a≤ b^3) Here $\frac{1}{3} \in R$ $\frac{1}{3} > \frac{1}{27}$ $(\frac{1}{3}, \frac{1}{3}) \notin R$ **Symmetric** Let $(1,2) \in R$ $1 \leq 8$ *or* $1 \leq 2^3$ but $(2,1) \in R$ and $8 \geq 1$ it is not symmetric. **Transitive:** $(10,3) \in R$ and $(3,2) \in R$ but (10,3) doesnot belongs to R Relation is not transitive.	3
16	**For reflexive**: Let a = $\frac{1}{2}$, (a, a) ϵ R ⇒ $\frac{1}{2} \leq (\frac{1}{2})^2$ ⇒ $\frac{1}{2} \leq \frac{1}{4}$, false, Hence, not reflexive. **For symmetric**: Let (-1, 2) ϵ R as $-1 \leq (2)^2$ is true. Now (2,-1) ϵ R ⇒ $2 \leq (-1)^2$ ⇒ 2≤1 is false. As (-1, 2) ϵ R ⇏ (2,-1) ϵ R, Hence, not symmetric. **For transitive**: Let (6,3),(3,2) ϵ R (6,3) ϵ R ⇒ $6 \leq (3)^2$ ⇒ 6 ≤ 9, true (3,2) ϵ R ⇒ $3 \leq (2)^2$ ⇒ 3 ≤ 4, true We have to show, (6, 2) ϵ R =>$6 \leq (2)^2$ => 6 ≤ 4, false. So, not transitive.	1 1 1
17	*Given, A = R−{3}, B = R−{1} and* $f(x) = \frac{x-2}{x-3}$. ***For one-one:*** *Let for* $x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$ $\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$ $\Rightarrow x_1 = x_2$	$1\frac{1}{2}$

	As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Hence, function is one-one.* **<u>For onto</u>:** Let y ∈ B, there exists x ∈ A such that y = *f*(x) => $y = \frac{x-2}{x-3}$ $\Rightarrow xy - 3y = x - 2$ $\Rightarrow xy - x = 3y - 2$ $\Rightarrow x(y-1) = 3y - 2$ $\Rightarrow x = \frac{3y-2}{y-1} \in A$. Hence, onto.	$1\frac{1}{2}$
18	**<u>For reflexive</u>**: *For any a ∈ Z, we have a − a = 0, which is divisible by 5.* ⇒ *(a, a) ∈ R* *Thus, (a, a) ∈ R for all a ∈ A.* *So, R is reflexive.* **<u>For symmetric</u>**: *Let (a, b) ϵ R. Then, (a, b) ϵ R* ⇒ *a−b is divisible by 5* ⇒ *−(a−b) is divisible by 5* ⇒ *b−a is divisible by 5* ⇒ *(b, a) ϵ R* *So, R is symmetric.* ***<u>For transitive</u>**: Let (a,b) ϵ R and (b,c) ϵ R, for a, b, c ϵ Z* ⇒ *(a-b) is divisible by 5 and (b-c) is divisible by 5* ⇒ *(a - b)+(b - c) = a-c is divisible by 5* ⇒ *(a, c) ϵ R* *As (a,b) ϵ R and (b,c) ϵ R ⇒ (a,c) ϵ R* *Hence, R is transitive.* *Since R is reflexive, symmetric and transitive* ⇒ *R is equivalence relation.*	1 1 1