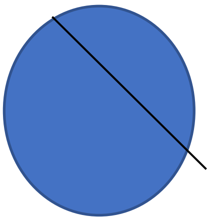
CHAPTER-8
APPLICATION OF INTEGRALS
04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	A mirror in the shape of an ellipse $\frac{x2}{9} + \frac{y2}{4}$ =1 was hanging on the wall. Arun and his sister were playing with ball inside the house, even their mother refused to do so. All of a sudden, ball hit the mirror and got scratch in the shape of line represented by $\frac{x}{3} + \frac{y}{2}$= 1.  Based on the above information, answer the following question. a) Points of intersection of ellipse and the scratch are i) (0,2), (3,0) ii) (2,0), (0,3) iii) (2,3), (0,0) iv) (0,3), (3,0) b) The area of the smaller region bounded by the mirror and scratch is i) $3(\frac{\pi}{2}+1)$ sq. unit ii) $(\frac{\pi}{2}+1)$ sq. unit iii) $(\frac{\pi}{2} -1)$ sq. unit iv) $3(\frac{\pi}{2} -1)$ sq. unit c) The value of the integration $\int_{-1}^{0} (x + 1)dx$ is i) $\frac{1}{2}$ ii) $\frac{2}{3}$ iii) $\frac{3}{4}$ iv) $\frac{1}{3}$ d) If the mirror is replaced by a circular mirror $x^2+y^2=1$ the new area of the mirror is i) 2π ii) π iii) $\pi/4$ iv) $1/\pi$	4
2.	Pratik cut pizza with a knife .the shape of pizza is represented by the equation $x^2 + y^2 = 4$ and the sharpe edge of the knife represented by the straightline $x=\sqrt{3}\, y$.	4



a) The point of intersection of the edge of knife (line) and the pizza shown in the figure are

i) $(1,\sqrt{3}), (-1,-\sqrt{3})$

ii) $(\sqrt{3},1), (-\sqrt{3},-1)$

iii) $(\sqrt{2},0),(0,\sqrt{3})$

iv) $(-\sqrt{3},1), (1,-\sqrt{3})$

b) Value of the area of the region bounded by circular pizza and edge of knife in 1st quadrant is

i) $\pi/2$ sq. unit

ii) $\pi/3$ sq. unit

iii) $\pi/5$ sq. unit

iv) πsq. Unit

c) Area of each slice of pizza when cut in to 4 pieces is

i) π. sq. Unit

ii) $\pi/2$ sq. unit

iii) 3π sq. unit

iv) 2π sq. unit

d) Area of whole pizza is

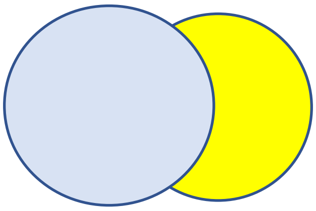
i) 3π sq. unit

ii) 2π sq. unit

iii) 5π sq. unit

iv) 4π sq. unit

3. In a partial solar eclipse when the moon and sun look overlapped as shown in the figure. The equation of the image of moon represented by the equation $(x-1)^2 + y^2 = 1$ and the image of sun is represented by the equation $x^2 + y^2 = 1$ — 4



a) The moon and sun meet each other at

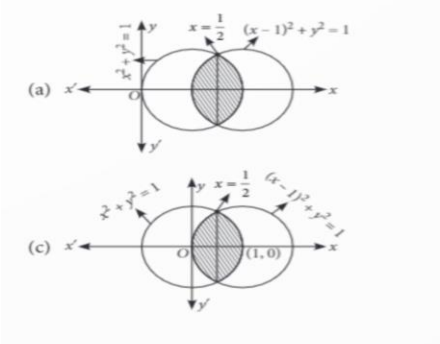
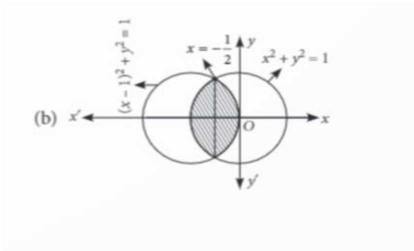
i) 1

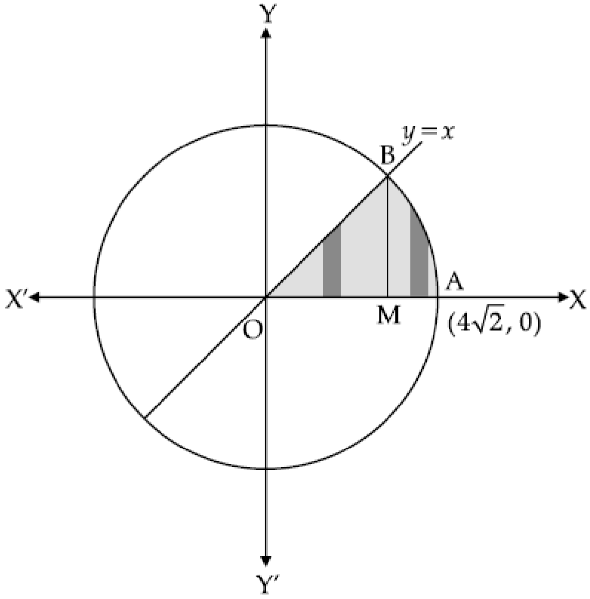
ii) ½

iii) 1/3

iv) ¼

b)

	  (d) None of these c) Value of $\int_{1/2}^{1} \sqrt{(1-x^2)}\, dx$ is i) $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$ ii) $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$ iii) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$ iv) $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$ d) Area of hidden portion of the lower circle is i) $(\frac{2\pi}{3} + \frac{\sqrt{3}}{2})$ sq. unit ii) $(\frac{\pi}{3} + \frac{\sqrt{3}}{8})$ sq. unit iii) $(\frac{\pi}{3} - \frac{\sqrt{3}}{8})$ sq. unit iv) $(\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$ sq. unit	
4.	Read the following text and answer the following questions on the basis of the same: In the figure O (0, 0) is the centre of the circle. The line y = x meets the circle in the first quadrant at the point B.	4



(i) The equation of the circle is __________.

(a) $x^2 + y^2 = 4\sqrt{2}$ (b) $x^2 + y^2 = 16$

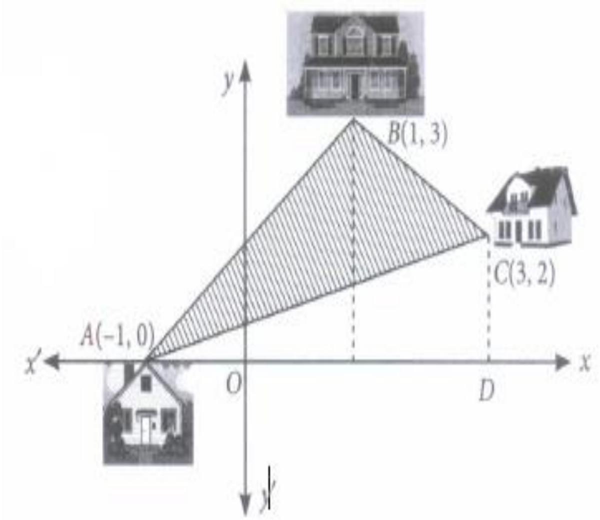
(c) $x^2 + y^2 = 32$ (d) None of these

(ii) The co-ordinates of B are __________.

(a) (1,1) (b) (2,2) (c) ($4\sqrt{2}, 4\sqrt{2}$) (d) (4,4)

(iii) Find the area of the shaded region.

5.



Location of three houses of a society is represented by the points A(-1, 0), B(1, 3) and C(3, 2) as shown in figure. Based on the above information, answer the following question

(i) Equation of line AB is

(a) $y = \frac{3}{2}(x+1)$ (b) $y = \frac{3}{2}(x-1)$

(c) $y = \frac{1}{2}(x+1)$ (d) $y = \frac{1}{2}(x-1)$

(ii) Equation of line BC is

(a) $y = \frac{1}{2}x - \frac{7}{2}$ (b) $y = \frac{3}{2}x - \frac{7}{2}$

(c) $y = \frac{-1}{2}x + \frac{7}{2}$ (d) $y = \frac{3}{2}x + \frac{7}{2}$

(iii) Equation of line AC is

(a) $2y = x + 1$ (b) $2y = x - 1$

(c) $y = 2x + 1$ (d) $y = 2x - 1$

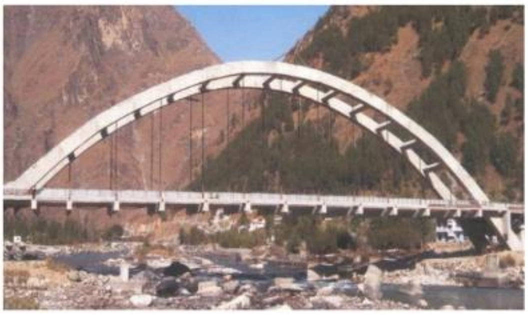
(iv) Find ar(ΔACD)

(a) 4sq. units (b) 8sq. units

(c)12sq. units (d) 2sq. units

4

6.	 Rajendra, a farmer had two sons and two daughters. He decided to divide his property among his sons and daughters .So he wrote a “WILL” about distribution of his property. According to his “WILL”, he desired to give 3/5 th of the property to his sons in equal proportion, 1/3 rd to his daughters in equal proportion and rest to a charitable trust. After his death his “WILL” was opened and read out by the Advocate in the presence of all villagers. He stated in his WILL that my agriculture field is in the shape of triangle with vertices A(2,5), B(4,7) and C(6,2)and all will find the solution following questions based on the field. Those who will find the solution, will be given the stated share of my property (i) Find the equations of each side of triangular field. (ii) Find the area of field using integration.	4
7.	 Construction of airport is multi disciplinary project and in it involves the pooling of various engineering disciplines, agencies, experts, contractors, executives and the end users. Before entering into the real case studies of construction of runways and application of supply chain management technique it is essential to frame a construction plan and a map. The map shows parabolic entry curvatures in which distance between the legs of entry curvature is 60 feet and height of entry curvature is 15 feet. Based on the following information answer the following questions- (i) Find the equation of parabolic curvature. (ii) Find the area within the entry curvature	4
8.	The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.	4

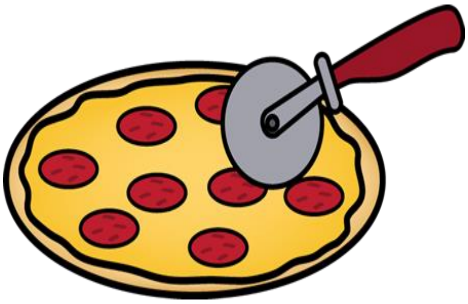


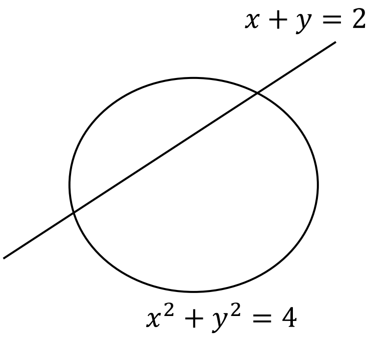
Find the equation of the parabolic curve and area covered by the bridge and arch on the bridge.

9. Find the area of the region bounded by the curves $x^2 = y$, y=x+2 and x-axis, using integration. — 4

10. Rita was celebrating her birthday with her friends. She order a cake .She cut the cake with a knife. — 4

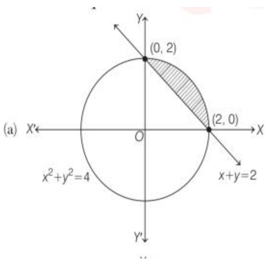
Cake was circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represented by a straight line given by $x + y = 2$

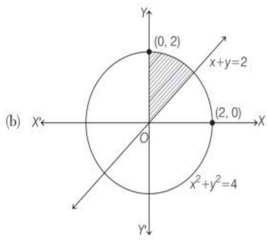


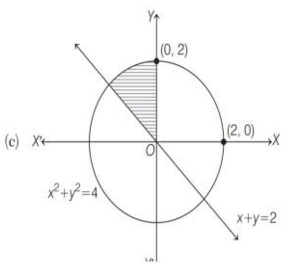


Based on the above information ,answer the following questions

(i)The points of intersection of the edge of knife (line) and cake shown in the figure is (are)
(a) $(0,2)$ and $(2,0)$
(b) $(0,1)$ and $(1,0)$
(c)$(1,2)$ and $(2,1)$
(d)$(3,1)$ and $(1,3)$
(ii) Which one of the following shaded portion represents the smaller area bounded by pizza and edge of knife in first quadrant

(a)


(b)


(c)


(d) None of the above

(iii) Area of each piece of cake, when rita cut the cake into 4 equal pieces is

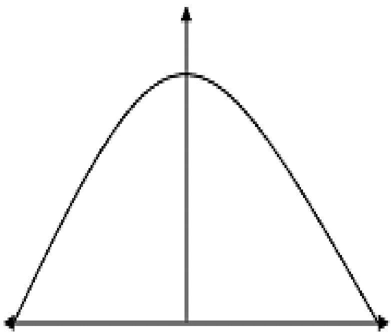
(a) $\frac{\pi}{2}\ sq\ units$
(b) $\pi\ sq\ units$
(c) $\frac{\pi}{3}\ sq\ units$
(a) $2\pi\ sq\ units$

(iv) Area of whole cake is

(a) $4\pi\ sq\ unit$
(b) $3\pi\ sq\ unit$
(c) $\pi\ sq\ unit$
(a) $\frac{\pi}{2}\ sq\ unit$

11. The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge, is 10 feet above the road at the middle of the bridge as seen in the figure. 4

Based on the information given above, answer the following questions:



(i) The equation of the parabola designed on the bridge is

(a) $x^2 = 250y$
(b) $x^2 = -250y$
(c) $y^2 = 250x$
(d) $y^2 = -250x$

(ii) The value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$

(a) $\frac{1000}{3}$
(b) $\frac{250}{3}$
(c) 1200
(d) 0

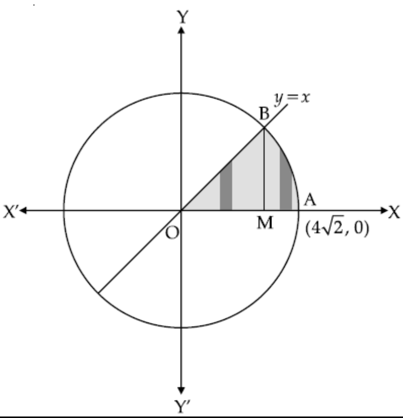
(iii) The integrand of the integral $\int_{-50}^{50} x^2 dx$ is __________ function.

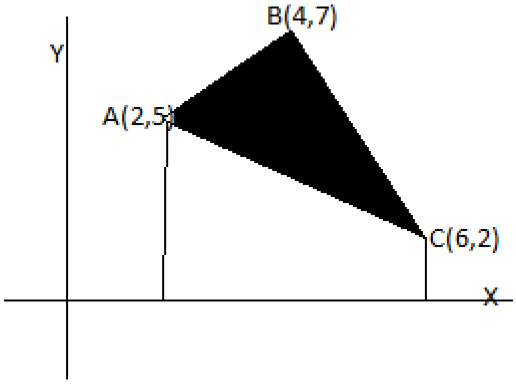
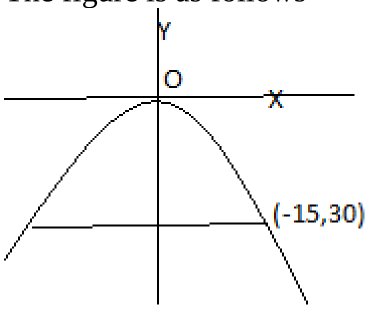
(a) Even
(b) Odd
(c) Neither odd nor even
(d) None

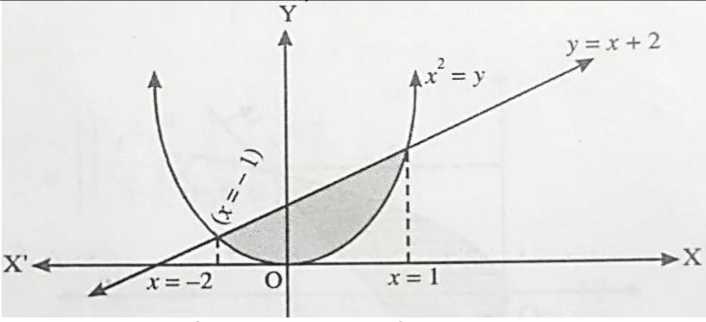
(iv) The area formed by the curve $x^2 = 250y$, x-axis,y=0 and y=10 is

	(a) $\frac{1000\sqrt{2}}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2\,000}{3}$ (d) 0	

ANSWERS:

Q. NO	ANSWER	MARKS
1.	a) i b) iv c) i d) ii	4
2.	a) ii b) ii c) i d) iv	4
3.	a) ii b) iii c) iii d) iv	4
4.	(i) (c) (ii) (d) (iii) Given curve $y = x$-----(1) $x^2 + y^2 = 32$ --------(2)  Required area = Area of OABO = ar(ΔOBM)+ar(MABM) $= \frac{1}{2} \times OM \times BM + \int_{4}^{4\sqrt{2}} \sqrt{32 - x^2 dx}$ = 8+4π-8 = 4π	4
5.	(i) a (ii) c (iii) a (iv) a	4
6.	Figure of field is as follows	4

	 The equation of sides can be obtained using result $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$ Equation of AB: x-y+3=0 Equation of BC: 5x+2y=34 Equation of AC: 3x+4y-26=0 Area is given by $$A = \int_2^4 (x+3)dx + \int_4^6 \frac{34-5x}{2}dx - \int_2^6 \frac{26-3x}{4}dx$$ $$A = 7\ sq\ units$$	
7.	The figure is as follows  (i) Let equation of entry curvature $x^2 = -4ay$ Point (-15,30) satisfies the equation , so $(30)^2 = -(4a).(-15)$ gives a=15 So, the equation of entry curvature is $x^2 = -60y$ (ii) Area under entry curvature is given by $$A = 2\int_0^{30} \lvert y \rvert\, dx = 2\int_0^{30} \frac{x^2}{60}\, dx = 2/60[\frac{x^3}{3}]_0^{30} = 300\ sq\ units$$	4
8.	Equation of the curve is $X^2 = -250y$ Reqd. area = $\int_0^{10} x\, dy$ = $\int_0^{10} \sqrt{250\, y}\ dy$ = $\frac{1000}{3}$	4
9.	$x^2 = y$ ....(1) y = x+ 2 ....(2) Solving eq. (1) and (2) $x^2 = x + 2$ x = -1, 2	4

	 Reqd. area = $\int_{-1}^{2}(x+2)dx$ - $\int_{-1}^{2}x^2dx$ = 3 + $\frac{3}{2}$ = $\frac{9}{2}$ sq. units	
10.	(i) (a) $Given\ , x^2+y^2=4\ \ldots..(1)\ and\ x+y=2 \ldots\ldots(2)$ Put the value of y from Eq. (2) in Eq. (1), we get $x^2+(2-x)^2=4$ $\Rightarrow x^2+4+x^2-4x=4$ $\Rightarrow 2x^2-4x=0$ $2x(x-2)=0$ $\Rightarrow x=0,2$ $when\ \ x=2\ \Rightarrow y=2$ $when\ \ x=2 \Rightarrow y=0$ ∴Required points of intersection are (0,2) and (2,0) (ii) (a) (iii) (b) Given equation of circle is $x^2+y^2=4$ $(x-0)^2+(y-0)^2=(2)^2$ ∴ Radius of the circle is 2 units ∴Area of one forth cake =$\frac{1}{4}\pi(2)^2=\pi sq\ units$ (iv) (a) Area of whole cake = $\pi(2)^2=4\pi\ sq\ units$	4
11.	(i) –(b)$x^2=4ay$ $(x,y)=(50,-10)$ = $50^2=4a(-10)$ = $2500=-40a$ = $a=\frac{2500}{-40}$ = $a=-\frac{250}{4}$.∴ $x^2=4\times\left(-\frac{250}{4}\right)y \Rightarrow x^2=-250y$ (ii) –(a) $\int_{-50}^{50}\frac{x^2}{250}dx \ldots\ldots..(even\ function)$ =2 $\int_{0}^{50}\frac{x^2}{250}dx$ = 2 × $\left[\frac{x^3}{250\times3}\right]_0^{50}$ = 2 × $\left(\frac{125000}{750}-0\right)$ =$\frac{1000}{3}$ (iii) –(a) even (iv) –(c) ∴Required area = 2 $\int_0^{10}xdx$	4

	$= 2\int_0^{10} \sqrt{250y}dy$ $=10\sqrt{10}\left(\frac{2}{3}y^{\frac{3}{2}}\right)_0^{10}$ $=\frac{20\sqrt{10}}{3} \times 10\sqrt{10}$ $= \frac{2000}{3}$	