CHAPTER-9

DIFFERENTIAL EQUATIONS

04 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours (i)The value $\frac{\int \quad 1}{kx} dx$ (a) $log\ x\| + c$ (b)$log\ log\ \|kx\| + c$ (c)$\frac{1}{k}\ log\ log\ \|x\| + c$ (d) none (ii) If 'N' is the number of bacteria, the corresponding differential equation is ____________. (a) $\frac{dN}{dt} = kt$ (b) $\frac{dN}{dt} = kN$ (c) $\frac{dk}{dt} = N$ (d) $\frac{dk}{dN} = t$ (iii) The general solution is (a) $log\ log\ \|N\| = kt + c$ (b) $log\ log\ Nt\ \| = k + c$ (c) $log\ log\ \|N\| = t$ (d) $log\ log\ \|kt\| = N + c$ (iv)The bacteria become 10 times in ______________ hours. (a) $5log7$ (b) $\frac{5log10}{log3}$ (c) $\frac{5}{log3}$ (d) none	4
2.	 It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the produd' of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum Based on the above information, answer the following questions. (i) Find the value of $\frac{dp}{dt}$ (a)$\frac{pr}{1000}$ $(b)\ \frac{pr}{100}$ $(c)\ \frac{pr}{20}$ $(d)\ \frac{pr}{20}$ (ii). If P_0 be the initial principal, then find the solution of differential equation formed in given situation. (a)log $\frac{p}{p_0} = \frac{rt}{100}$ $(b)\ log\ log\ \left(\frac{p}{p_0}\right) = \frac{rt}{10}$ $(c)\ log\ log\ \left(\frac{p}{p_0}\right) = \frac{r}{20}$ $(d) none$ (iii)If the interest is compounded continuously at 5% per annum, in how many years will	4

	Rs.100 double itself? (a) 12.728 years (b) 14.789 years (c) 13.862 years (d) 15.872 years (iv)How much will Rs.1000 be worth at 5% interest after 10 years? (e0.5 = 1.648). (a) Rs.1648 (b) Rs 1500 (c) Rs 1664 (d) Rs 1572	
3.	 A rumour on whatsapp spreads in a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not. Also, it is given that 100 people initiate the rumour and a total of 500 people know the rumour after 2 days. (i)If yet denote the number of people who know the rumour at an instant t, then maximum value of y(t) is (a) 500 (b) 100 (c) 5000 (d) none of these (ii) $\frac{dy}{dt}$ *is proprtional to* (a)(y – 5000) (b) y(y – 500) (c) y(500 – y) (d) y(5000 – y) (iii) The value of y(0) is (a)100 (b) 500 (c) 600 (d) 200 (iv) The value of y(2) is (a)100 (b) 500 (c) 600 (d) 200	4
4.	Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $dydx$ = K (50 − y) where x denotes the number of weeks and y the number of children who have been given the drops. 1. State the order of the above given differential equation. 2. Which method of solving a differential equation can be used to solve $dydx$ = k (50 − y) a) Variable separable method b) Solving Homogeneous differential equation c) Solving Linear differential equation d) All of the above 3. The solution of the differential equation $dydx$ = k (50 − y) is given by, a) log | 50 – y| = kx + C b) - log | 50 – y| = kx + C c) log | 50 – y| = log| kx |+ C d) 50 – y = kx + C 4. The value of c in the particular solution given that y(0)=0 and k = 0.049 is : a) log 50	**4**

	b) log 1 50 c) 50 d) -50 5. Which of the following solutions may be used to find the number of children who have been given the polio drops? a) y = 50 – ekx b) y = 50 − e $-kx$ c) y = 50 (1 – e $-k$) d) y = 50 (ekx– 1)	
5.	A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6 ○F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4 ○F. The room in which the cat was put is always at 70 ○F. The normal temperature of the cat was 98.6 ○F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation:$dTdt$∝ (T − 70), where 70 ○F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation $dTdt$ = k(T − 70) where k is a constant of proportion, time of death is calculated. 1) 1) State the degree of the above given differential equation. 2) 2) Which method of solving a differential equation helped in calculation of the time of death? **a)** Variable separable method b) Solving Homogeneous differential equation c) Solving Linear differential equation d) all of the above 3) If the temperature was measured 2 hours after 11.30pm, will the time of death change? (Yes/No) 4) The solution of the differential equation $dTdt$ = k(T − 70) is given by, a) log | T – 70| = kt + C b) log | T – 70| = log |kt |+ C c) T – 70 = kt + C d) T – 70 = kt C 5) If t = 0 when T is 72, then the value of c is a) -2 b) 0 c) 2 d) Log 2	4
6.	In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself?	4
7.	Solve the differential equation: (xdy-ydx)y $\sin\left(\frac{y}{x}\right)$=(ydx+xdy)x $\cos\left(\frac{y}{x}\right)$.	4
8.	Find a particular solution of the differential equation $\frac{dy}{dx}+2y\, tanx = sinx\, ; y = 0\ when\ x = \frac{\pi}{3}$.	4
9.	In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years($\log_e 2$=0.6931).	4
10.	Find the particular solution of the differential equation $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$, given that $y = 1\ when\ x = 0$	4
11.	Solve the differential equation $xdy - ydx = \sqrt{x^2+y^2}dx$, given that $y = 0\ when\ x = 1$	4
12.	Find the particular solution of the differential equation	4

	$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$	
13.	In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself ? 	4
14.	A Veterinary doctor was examining a sick cat brought by a pet lover.When it was brought to the hospital, it was already dead.The pet lover wanted to find its time of death. He took the temperature of the cat at 11:30pm which wass 94.6∘ F. He took the temperature again after 1h; the temperature was lower than the first observation .It was 93.4∘ F. The room in which the cat was put is always at 70∘ F.The normal temperature of the cat is taken as 98.6∘ $F$when it was alive . The doctor estimated the time of time of death using Newton law of cooling which is governed by the differential equation :$\frac{dT}{dt} \propto (T – 70)$,*where 70*∘ $F$is the room temperature and Tis the temperature of object at time t. Substituting the two different observations of Tand t made, in the solution of the differential equation $\frac{dT}{dt}$ = k(T – 70), where k is a constant ofproportion, time of death is calculated. Answer the following questions using the above information. (i)The degree of the above given differential equation is (a)0 (b)1 (c)2 (d)3 (ii)If the temperature was measured 2h after 11:30pm, will time of death change? (a)Yes (b)No (c)Can't say (d)None of these (iii)The solution of the differential equation $\frac{dT}{dt}$= k(T- 70)is given by, (a)log|T-70| =kt + C (b)log|T-70| = log|kt| + C (c)T-70 = kt + C (d) t- 70 = kT + C (iv)If t =0 when T is 72, then the value of C is (a)-2 (b)0 (c)2 (d)log2	4
15.	Consider the differential equation $\frac{dy}{dx}$ + 2ytanx = sinx. Answer the following questions which are based on above information. (i)Find the values of Pand Q , if the given differential equation can be written in the form of $\frac{dy}{dx}$ +Py = Q. (ii)Find the integrating factor of the differential equation. (iii)Find the solution of the differential equation. (iv)If y$\left(\frac{\pi}{3}\right)$= 0,then write the relation between x and y.	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	i)(c) ii) (b). iii)(a). iv)(b)	
2.	i)(b)Here, P denotes the principal at any time t and the rate of interest be r% per annum compounded continuously, then according to the law given in the problem, we get $\frac{pr}{100}$ (ii) (a). (iii) (c). (iv) (d)	
3.	i)c)Since, size of population is 5000. Hence, Maximum value of y(t) is 5000. ii) (d) : Clearly, according to given information $$\frac{dy}{dt} = ky(5000 - y)$$,where k is the constant of proportionality. iii)(a) y(0)=100 iv)(b) y(2)=500	
4.	1) Degree is 1 2) (a) Variable separable method 3) No 4) (a) log | T –70| = kt + C 5) (d)log 2	
5.	1) Order is 1 2) (a) Variable separable method 3. (b)- log | 50– y| = kx + C 4. (b) log 1 50 5. (c) y = 50 ((1 – e^{-k})	
6.	Let P be the principal at any time t. According to the given problem, $\frac{dP}{dt}=\left(\frac{5}{100}\right)P$ $\frac{dP}{P}=\frac{dt}{20}$ Integrate it, we get General solution P=C$e^{\frac{t}{20}}$, Substituting P=1000 when t=0 we get c=1000 Let t years be the time required to double the principal. Then 2000 = 1000$e^{\frac{t}{20}}$ gives t=20$\log_e 2$	4
7.	ANS:This is of the form $\frac{dy}{dx}=g\left(\frac{y}{x}\right)$. Put y=vx ,then $\frac{dy}{dx}=v+x\frac{dv}{dx}$ Gives $\left(\frac{vsinv-cosv}{vcosv}\right) dv=2\frac{dx}{x}$ Integrate it, we get $\frac{secv}{vx^2}=c$ Put v=$\frac{y}{x}$,we get General solution sec$\left(\frac{y}{x}\right)$ =cxy	4
8.	y =$cosx - 2cos^2x$	4
9.	P=C$e^{\frac{rt}{100}}$,r=6.931%	4

10.		$\frac{dy}{dx} = -\left[\frac{x + y\cos x}{1+\sin x}\right]$ $\Rightarrow \frac{dy}{dx} + \frac{\cos x\, y}{1+\sin x} = \frac{-x}{1+\sin x} \ldots (i)$ $\frac{dy}{dx} + Py = Q \ldots (ii)$ On comparison, we get $P = \frac{\cos x}{1+\sin x}, Q = \frac{-x}{1+\sin x}$ Integrating Factor (I. F) $= e^{\int p dx} = e^{\int \frac{\cos x}{1+\sin x}} = e^{\log\|1+\sin x\|} = 1 + \sin x$ Hence, the sol$^{\mathrm{n}}$ is : $y \times \mathrm{I.F} = \int Q \times \mathrm{I.F}\, dx$ $\Rightarrow y(1+\sin x) = \int \frac{-x}{(1+\sin x)}.(1+\sin x)dx$ $\Rightarrow y(1+\sin x) = -\int x\, dx$ $\Rightarrow y(1+\sin x) = -\frac{x^2}{2} + c$ It is given that $y = 1$ when $x = 0$ $\therefore 1 = 0 + c$ $\Rightarrow c = 1$ Hence, the complete sol$^{\mathrm{n}}$ is : $$y(1+\sin x) = -\frac{x^2}{2} + 1$$	4
11.		$xdy - ydx = \sqrt{x^2+y^2}dx$ $\Rightarrow xdy = \left(y + \sqrt{x^2+y^2}\right)dx$ $\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} \ldots (i)$ Let, $f(x,y) = \frac{y+\sqrt{x^2+y^2}}{x}$ $\therefore\ f(\lambda x, \lambda y) = \frac{\lambda y + \sqrt{\lambda^2 x^2 + \lambda^2 y^2}}{\lambda x} = \frac{y+\sqrt{x^2+y^2}}{x} = f(x,y)$ Hence, f is a homogeneous function of degree 0. Let, $y = vx$ Differentiating both sides w. r. t. x $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Hence, eq$^{\mathrm{n}}$(i) becomes $v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2+v^2x^2}}{x}$ $\Rightarrow v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ $\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2}$ $\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ $\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$ $\Rightarrow \log\left\|v + \sqrt{1+v^2}\right\| = \log\|x\| + c$	4

	$\Rightarrow \log\left\|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right\| = \log\|x\| + c$ It is given that $y = 0$ when $x = 1$ $\therefore 0 = 0 + c$ $\Rightarrow c = 0$ Hence, the complete sol^{n} is : $\log\left\|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right\| = \log\|x\|$	
12.	Given differential equation can be rewritten as $$\frac{dx}{dy} = \frac{x^2+y^2}{xy} = \frac{x}{y}+\frac{y}{x} = \left(\frac{x}{y}\right)+\left(\frac{1}{\frac{x}{y}}\right) = f\left(\frac{x}{y}\right) ------ (i)$$ Which is a homogeneous differential equation Now. Let $\frac{x}{y} = v \Rightarrow x = vy$ On differentiating wrt y on both sides, we get $$\frac{dx}{dy} = v + y\frac{dv}{dy} ------(ii)$$ using (ii) in (i), we get $$v + y\frac{dv}{dy} = v + \frac{1}{v}$$ $$\int v\,dv = \int \frac{dy}{y}$$ $$\frac{v^2}{2} = \log y + C$$ $$\frac{x^2}{2y^2} = \log y + C$$ y=1, when x=0 so we get C=0 Hence, particular solution of the given differential equation is $\frac{x^2}{2y^2} = \log y \quad \Rightarrow \quad x^2 = 2y^2 \log y$	4
13.	Let P be the principal at any time t. So, $\frac{dp}{dt} = \left(\frac{5}{100}\right)P \quad \Rightarrow \quad \frac{dp}{dt} = \frac{P}{20}$----------------------(i) Separating the variables $\frac{dp}{P} = \frac{dt}{20}$------------------(ii) Integrating both sides $$\log P = \frac{t}{20} + C_1$$ $$P = e^{\frac{t}{20}}.e^{C_1}$$	4

	$$P = C\, e^{\frac{t}{20}}$$ Now P=1000, when t=0, we get $$P = 1000\, e^{\frac{t}{20}}$$ Let t years be the time required to double the Principal. Then $$2000 = 1000\, e^{\frac{t}{20}}$$ $\Rightarrow \quad t = 20log_e 2$	
14.	Given, differential equation can be rewritten as F(x,y) = $\frac{dy}{dx} = \frac{y}{2x - x\log(\frac{y}{x})}$ Verify F(λx, λy) = F(x,y) On putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, then given equation Becomes => v + x$\frac{\text{dv}}{\text{dx}} = \frac{vx}{2x - xlog(\frac{vx}{x})}$ => x $\frac{dv}{dx} = \frac{v}{2-logv}$ - v =>$\int 2 - logv/v(logv - 1)$dv = $\int dx/x$ On putting log v = t and $\frac{1}{v}$dv = dt, we get $\int 2 - t$/t-1dt = log|x| + C =>$\int(\frac{1}{\text{t}-1}$- - 1)dt = log|x| + C => log(t-1) – t = log|x| + C => log[log4)-1] – log4 = log|x| + C => log[(log$\frac{y}{x}$) -1] -log($\frac{y}{x}$) = logx + C => log[(log$\frac{y}{x}$) – 1] -logy = C :· log|$\frac{\frac{logy}{x} - 1}{y}$| = C	4
15.	(i) Given , differential equation is $\frac{\text{dy}}{\text{dx}}$ + 2ytanx = sinx Which is a linear differential equation of the form $\frac{\text{dy}}{\text{dx}}$ + Py = Q. Here,P = 2tanx and Q = sinx (ii) IF =$e^{\int Pdx} = e^{\int 2tanx\, dx}$ =$e^{\int tanx\, dx}$ = $e^{2\log\|\sec x\|}$ = $e^{logsec^2 x}$ = $\sec^2$ x [:·$e^{\log(x)}$ = f(x)] (iii) The solution of differential equation is given by y·$\sec^2$ x = $\int sec^2$x·sinxdx + C => y·$\sec^2$ x = $\int sinx/\cos ^2x\, dx$·sinxdx + C Put cosx = t , then -sinxdx =dt :· y·$\sec^2$x = -$\int dt/t$^2 + C => y·$\sec^2$x = -$\int t^{-2}$dt + C => y·$\sec^2$x = -1$\frac{t-^1}{(-1)}$ + C	4

$y \cdot \sec^2 x = \frac{1}{t} + C$

$y \cdot \sec^2 x = \frac{1}{cosx} + C$

$y \cdot \sec^2 x = \sec x + C$

=> $y = \frac{1}{\sec x} + \frac{C}{\sec^2 x}$ [dividing each term by $\sec^2 x$]

=> $y = \cos x + C \cdot \cos^2 x$

(iv) It given that y = 0, when $x = \frac{\pi}{3}$

$\therefore$ $0 = \cos\frac{\pi}{3} + C \cdot \cos^2\frac{\pi}{3}$

=> $0 = \frac{1}{2} + C \cdot \frac{1}{4}$ [$\because \cos\frac{\pi}{3} = \frac{1}{2}$]

=> $\frac{-1}{2} = \frac{C}{4}$ => $C = -2$

$\therefore$ The required particular solution is

=> $y = \cos x - 2\cos^2 x$