CHAPTER-7
INTEGRALS
04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find : $\int \frac{dx}{sinx+sinsin\,2x}$	4
2.	Find: $\int_0^{\frac{\pi}{4}} \frac{dx}{cos^3x\sqrt{2sin2x}}$	4
3.	The given integral $\int f(x)\ dx$ can be transformed into another form by changing the independent variable x to t by substitution x=g(t) Consider $I = \int f(x)$, put $x = g(t) \Rightarrow \frac{dx}{dt} = g'(t)$ $\Rightarrow dx = g'(t)dt \Rightarrow I = \int f(x) = \int f(g(t))\,g'(t)dt$ This change of variable formula is one of the important tools available to us in the name of integration by substitution. Based on the above information, answer the following questions: 1. Find the value of $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$ 2. . Find the value of $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ 3. Find the value of $\int \frac{\sin x}{(1+\cos x)^2}dx$ 4.Find the value of $\int \frac{\log x}{x}dx$	4
4.	There are many practical applications of Definite Integration. Definite integrals can be used to determine the mass of an object if its density function is known. We can also find work by integrating a force function, and the force exerted on an object submerged in a liquid. The most important application of Definite Integration is finding the area under the curve. Let f be a continuous function defined on the closed interval [a,b] and F be an antiderivative of f then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$ It is very useful because it gives us a method of calculating the definite integral more easily. There is no need to keep integration constant C because if we consider F(x) + C instead of F(x). $\int_a^b f(x)dx = [F(x)+C]_a^b = F(b) + C - F(a) - C$ $= F(b) - F(a)$ Based on the above information, answer the following questions: 1. Find the value of $\int_2^3 x^2dx$	4

	2. Find the value of $\int_1^{\sqrt{3}} \frac{1}{1+x^2}dx$ 3. Find the value of $\int_{-1}^{1}(x+1)dx$ 4. Find the value of $\int_2^3 \frac{1}{x}dx$	
5.	Three students in a group, studying the concept of the partial fraction, but they were confused while solving the question they did not have any idea about how to start the solution. One of the students tells them for the integration by partial fraction, first we must check we are dealing with polynomial and degree of numerator is less than the degree of denominator and proceed for partial fraction, if not, divide numerator by denominator and write it as $\frac{Numerator}{Denominator} = Quotient + \frac{Remainder}{Denominator}$ Based on the above information, answer the following: (i) If the function is $f(x) = \frac{2}{(1-x)(1+x^2)}$, write the partial fraction of the given function in constant term A, B and C. (ii) Find the value of the constant taken in the numerator of the factor (1-x), while reducing f(x) into partial fractions. (iii) Find the value of both the constants taken in the numerator of the factor (1+x^2), while reducing f(x) into partial fractions. Or Find the integration of the function f(x)	4
6.	If u and v are two functions of x, then $\int uvdx = u\int vdx - \int \left\{\frac{du}{dx}\int vdx\right\}dx$ i.e, the integral of the product of two functions = first function x Integral of second - Integral of (derivative of the first x integral of second). Here the choice of the first function is important. We can use the order ILATE, where I= Inverse trigonometric functions L = Logarithmic functions, A= Algebraic functions T = trigonometric functions, E = exponential functions If the integrand contains only one function, we take that function as the first function and 1 as the second function. Based on the above information, answer the following: (i) I = $\int x\,dx$, which functions should be taken as first and second functions (ii) How to evaluate the integral $\int \frac{x}{1+x^2}dx$ Write the integral as given in (i)	4
7.	$Find\ the\ value\ of \int \frac{x}{x-\sqrt{x^2-1}}dx$	4
8.	find the value of $\int (\sqrt{3}sinx + cosx)^{-1}dx$	4
9.	Let the definite integral be defined by the formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$. For more accurate result for $c \in (a,b)$, we can use $$\int_a^b f(x)dx = \frac{b-a}{2}\left(f(a)+f(b)+2f(c)\right)$$ where $c = \frac{a+b}{2}$. Then (i)Evaluate with more accuracy as stated: $\int_0^{\frac{\pi}{2}} sinxdx$	4

	(ii)Evaluate with min accuracy as stated: $\int_0^{\frac{\pi}{2}} cosxdx$	
10.	For the integral $\int_0^{\frac{3}{2}} \lvert xcos\pi x \rvert$dx (i) Find possible c if $a = -1 \ \ and \ \ b = \frac{3}{2}$ provided $a \leq c \leq b$ (ii) Evaluate the integral.	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	I= $\int \frac{dx}{sinx+sinsin\ 2x} = \int \frac{1}{sinx+2sinx\ cosx} dx = \int \frac{1}{sinx(1+2cosx)} dx = \int \frac{sinx}{sin^2x(1+2cosx)} dx =$ $\int \frac{sinx}{(1-cos^2x)(1+2cosx)} dx$ Put cosx = t , so –sinx dx = dt $\therefore$ I = $\int \frac{-dt}{(1-t^2)(1+2t)} = -\int \frac{dt}{(1+t)(1-t)(1+2t)}$ Then write $\frac{1}{(1+t)(1-t)(1+2t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{1+2t}$, on solving A = $\frac{-1}{2}$, B = $\frac{1}{6}$, C = $\frac{4}{3}$ I = $\frac{1}{2}\log\|1+t\| + \frac{1}{6}\log\|1-t\| - \frac{4}{3x2}\log\|1+2t\| + C$ I = $\frac{1}{2}\log\|1+cosx\| + \frac{1}{6}\log\|1-cosx\| - \frac{4}{3x2}\log\|1+2cosx\| + C$	4
2.	Let I= $\int_0^{\frac{\pi}{4}} \frac{dx}{cos^3x\sqrt{2sin2x}} = \int_0^{\frac{\pi}{4}} \frac{dx}{cos^3x\sqrt{2.2\ sinxcosx}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{cos^3x\sqrt{\frac{sinx}{cosx}cos^2x}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{cos^4x\sqrt{tanx}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{sec^2x\ sec^2xdx}{\sqrt{tanx}}$ Put tanx = t $\Rightarrow sec^2xdx$= dt , x= 0 $\Rightarrow$ t= 0 and x = $\frac{\pi}{4} \Rightarrow$ t = 1 $\therefore$ I = $\frac{1}{2}\int_0^1 \frac{(1+t^2)dt}{\sqrt{t}} = \frac{1}{2}[\frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}]_0^1 + \frac{1}{2}[\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1}]_0^1 = \frac{6}{5}$	4
3.	1. $I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ Let $\tan^{-1}x = t \Rightarrow \frac{dx}{1+x^2} = dt$ $\Rightarrow I = \int e^t\, dt = e^t + c = e^{\tan^{-1}x} + c$ 2. $I = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$ Let $\sin^{-1}x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$ $\Rightarrow I = \int t\, dt = \frac{t^2}{2} + c = \frac{(\sin^{-1}x)^2}{2} + c$ 3. $I = \int \frac{\sin x}{(1+\cos x)^2} dx$ Let $1 + \cos x = t \Rightarrow -\sin x\, dx = dt$ $\Rightarrow I = \int \frac{1}{t^2} dt = \frac{-1}{t} + c = \frac{-1}{1+\cos x} + c$ 4. $I = \int \frac{\log x}{x} dx$ Let $\log x = t \Rightarrow \frac{dx}{x} = dt$ $\Rightarrow I = \int t\, dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c$	4
4.	(i) $I = \int_2^3 x^2\, dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$ $(ii) \int_1^{\sqrt{3}} \frac{1}{1+x^2} dx = [\tan^{-1}x]_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ (iii) $I = \int_{-1}^1 (x+1)dx = \left[\frac{x^2}{2} + x\right]_{-1}^1 = \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right) = 2$ (iv) $I = \int_2^3 \frac{1}{x} dx = [\log x]_2^3 = \log 3 - \log 2 = \log\left(\frac{3}{2}\right)$	4
5.	(i) $f(x) = \frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$	1

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<tr><td></td><td>(ii) $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$
$\Rightarrow \quad 2 = A(1+x^2) + (1-x)(Bx+C)$
On solving, we get, A = B = C= 1
(iii) B = 1, C = 1
Or
$\int f(x)dx = \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x+1}{1+x^2}dx$
$= -log|1-x| + \frac{1}{2} log\ log\ |1+x^2| + x + C$</td><td>1

2</td></tr>
<tr><td>6.</td><td>(i) x as first function and 1 as second function.
(ii) $\int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{2x}{1+x^2}dx = \frac{1}{2}log|1+x^2|$
(iii) I = $\int x\,dx = x\int 1dx - \int \left\{\frac{d(x)}{dx}\int 1dx\right\}dx$
$= x - \int \frac{x}{1+x^2}dx = xx - \frac{1}{2}log|1+x^2| + C$</td><td>1
1

1
1</td></tr>
<tr><td>7.</td><td>$$\int \frac{x}{x-\sqrt{x^2-1}}\,dx$$
$$= \int \frac{x(x+\sqrt{x^2-1})}{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}\,dx$$
$$= \int \frac{x^2+x\sqrt{x^2-1}}{x^2-(x^2-1)}\,dx$$
$= \int x^2\,dx + \int x\sqrt{x^2-1}\,dx$
$$= \frac{x^3}{3} + \int x\sqrt{x^2-1}\,dx \ \ldots(i)$$
$$taking, x^2-1 = u\ \ in\ 2nd\ part\ of\ (i)$$
We get, $2xdx = du$
$(i)\ becomes\ \int \frac{x}{x-\sqrt{x^2-1}}\,dx$
$= \frac{x^3}{3} + \frac{1}{2}\int u^{\frac{1}{2}}\,du$
$= \frac{x^3}{3} + \frac{1}{2} \times \frac{2}{3}u^{\frac{3}{2}} + c$
$= \frac{x^3}{3} + \frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$ (Answer)</td><td>4</td></tr>
<tr><td>8.</td><td>$$\int (\sqrt{3}sinx + cosx)^{-1}dx$$
$$= \int \frac{1}{\sqrt{3}\ sin\ sin\ x + cos\ cos\ x}dx$$
$$= \int \frac{1}{2(\frac{\sqrt{3}}{2}\ sin\ sin\ x + \frac{1}{2}\ cos\ cos\ x\ )}dx$$
$$= \int \frac{1}{2(sinx\ cos\ cos\ \frac{\pi}{6} + cos\ cos\ xsin\ \frac{\pi}{6})}\ dx$$</td><td>4</td></tr>
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	$= \frac{1}{2}\int \frac{1}{sin(x+\frac{\pi}{6})}\, dx$ $= \frac{1}{2}\int cosec\,(x+\frac{\pi}{6})\, dx$ $= \frac{1}{2}\, log\, log\, \lvert\, tan\, tan\, \left(\frac{x}{2}+\frac{\pi}{12}\right) \rvert + c$ (Answer)	
9.	(i)$\frac{\pi}{8}(1+\sqrt{2})(ii)\frac{\pi}{8}$	4
10.	(i)The value of c is $\frac{1}{2}$.(ii)$\frac{5}{2\pi} - \frac{1}{\pi^2}$	4