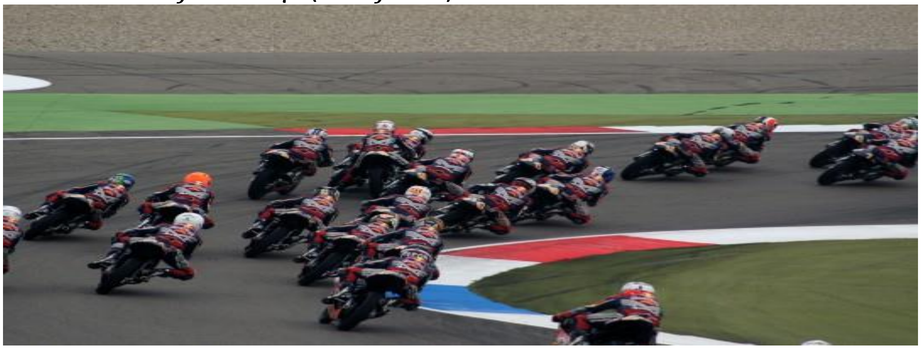
CHAPTER-11
THREE DIMENSIONAL GEOMETRY
04 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find the distance between the point P(6,5,9) and the plane determined by the points A(3,-1,2) ,B(5,2,4) and C(-1,-1,6)	4 MARKS
2.	Find the coordinates of the point where the line through the points A(3,4,1) and B(5,1,6) crosses the XY-plane.	4
3.	The equation of motion of a missile are x = 2t, y = 3t, z = t, where the time 't' is given in seconds and distance is measured in kilometers. Based on it, answer the following question; (i) What is the path of the missile? (a) Straight line (b) Parabola (c) Circle (d) Ellipse (ii) Which of the following points lie on the path of missile? (a) (1, 2, 3) (b) (2, 3, 1) (c) (4, 1, -2) (d) (1, -2, 3) (iii) At what distance will the missile be in 10 seconds from the starting point (0, 0, 0)? (a) $10\sqrt{14}$ km (b) $20\sqrt{14}$ km (c) $10\sqrt{7}$ km (d) $20\sqrt{14}$ km (iv) The position of missile at a certain instant of time is (2,-8, 15) then what will be height of the missile from the ground if ground is considered as xy-plane? (a) 2 km (b) 8 km (c) 15 km (d) 7 km	4
4.	In a class, teacher asks students what they know about space or three dimensional system, he asks students some basic questions. Help students to answer the following; (i) What is the equation of x-axis in space? (a) x = 0, y = 0 (b) y = 0, z = 0 (c) x = 0 (d) none of these (ii) What are direction ratios of y-axis? (a) 0,0,1 (c) 0,1,0 (b) 1,0,0 (d) 1,0,1 (iii) DC of a line are < m, m, m >, then (a) m > 0 (c) m < 1 (b) m < 0 (d) $m = \frac{1}{\sqrt{3}}$ or $\frac{-1}{\sqrt{3}}$ (iv) Which of the following statement is correct? (a) Direction ratios of a line are equal to its direction cosines. (b) Direction ratios of two perpendicular lines are proportional. (c) Direction ratios of two parallel lines are proportional. (d) All of these are correct.	4
5.	The equation of motion of a rocket are x = 4t , y = -4t , z = t, where the time t is given in the seconds and the distance is measured in kilometers.	4

	 (i)Find the points lie on the path of the rocket at t = 5 s. (ii) Find the distance of the rocket from the starting point (0,0,0) in 5 seconds .	
6.	Read the following text and answer the question on the basis of the same. A motor cycle race was organized in a town , where the maximum speed limit was set by the organizers . No participant are allowed to cross the specified speed limit, but two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} + 2k + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$  (i) Find the cartesian equation of the line along which motorcycle B is running. (ii) Find the shortest distance between the lines.	4
7.	Find the shortest distance between the lines l_1 & l_2 whose vector equations are given by : $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.	4
8.	Find the distance between the lines l_1 & l_2 whose vector equations are given by : $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$.	4
9.	Find the shortest distance between the following two lines $r \rightarrow= (1+\lambda)\ \hat{\imath} + (2-\lambda)\hat{\jmath} + (\lambda+1)\hat{k};$ $r \rightarrow = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$	4
10.	Find the shortest distance between the lines whose vector equations are $r \rightarrow= (1-t)\ \hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k};$ and	4

	$r \rightarrow= (s+1)\ \hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k};$	

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{3\sqrt{34}}{17}$	
2.	$(\frac{13}{5}, \frac{23}{5}, 0)$	
3.	(i) (a) (ii) (b) (iii) (a) (iv) (c)	4
4.	(i) (b) (ii) (c) (iii) (d) (iv) (c)	4
5.	(i) The equation of motion of a rocket are x = 4t , y = -4t , z = t, at t = 5 x= 20, y = -20, z = 5 so, points lie on the path = (20,-20,5) (ii) points lie on the path after 5 seconds = (20,-20,5) Distance from starting point (0,0,0) = I $\sqrt{400 + 400 + 25}$ I = I $\sqrt{825}$ I =5 $\sqrt{33}$	4
6.	(i)The line along which motorcycle B is running $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} + 2k + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ $(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) = (3+2\mu)\hat{\imath} + (3+\mu)\hat{\jmath} + (2+\mu)k$ x = $(3+2\mu)$, y =$(3+\mu)$, z = $(2+\mu)$ Or, (x-3)/2 = μ , y-3 = μ , z-2 = μ The required cartesian equation = (x-3)/2 = y-3 = z-2 (iii) $a_1 = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $a_2 = 3\hat{\imath} + 3\hat{\jmath} + 2k$ $b_1 = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $b_2 = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ $a_2 - a_1 = 3\hat{\imath} + 3\hat{\jmath} + 2\hat{k} - \hat{\imath} - 2\hat{\jmath} + \hat{k} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ $b_1 \times b_2 = \begin{bmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} = 3\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ $(a_2 - a_1).(b_1 \times b_2) = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}).(3\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$ =6 +3 – 9 = 0 So, Shortest distance between given lines = 0.	4
7.	Given equation of lines are given by :	

	$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ ………….(1) $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$………(2) Comparing (1) and (2) by the standard equations $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ respectively, we get $\overrightarrow{a_1} = \hat{\imath} + \hat{\jmath}$, $\overrightarrow{b_1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\overrightarrow{b_2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ Therefore, $\overrightarrow{a_2} - \overrightarrow{a_1} = \hat{\imath} - \hat{k}$, and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$ So, $\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\| = \sqrt{9+1+4} = \sqrt{59}$ Hence, the shortest distance between the given lines is given by $d = \left\|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\|}\right\| = \left\|\frac{(3\hat{\imath} - \hat{\jmath} - 7\hat{k}).(\hat{\imath} - \hat{k})}{\sqrt{59}}\right\| = \frac{10}{\sqrt{59}}$ units.	1 1 1 1
8.	The given equations of lines are: $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ ……..(1) $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$…….(2) Clearly, lines given in (1) and (2) are parallel. Comparing (1) and (2) by the standard equations $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\vec{b}$ respectively, we get, $\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, $\overrightarrow{a_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$. Therefore, $\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, Also, $\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1}) = (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) \times (2\hat{\imath} + \hat{\jmath} - \hat{k}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = -9\hat{\imath} + 14\hat{\jmath} - 4\hat{k}$. Hence, the distance between the given lines is given by $d = \left\|\frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{\|\vec{b}\|}\right\| = \left\|\frac{-9\hat{\imath} + 14\hat{\jmath} - 4\hat{k}}{\sqrt{4+9+36}}\right\| = \frac{\sqrt{81+196+16}}{7}$ $d = \frac{\sqrt{293}}{7}$ units.	1 1 $\frac{3}{2}$ $\frac{1}{2}$
9.	$-3\hat{\imath} + 3\hat{k}$	4
10.	$\left\|\frac{-4+12}{\sqrt{29}}\right\| = \frac{8}{\sqrt{29}}$ units	4