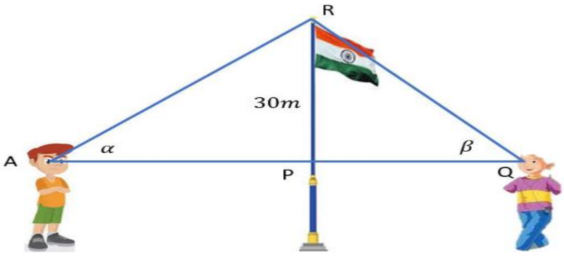
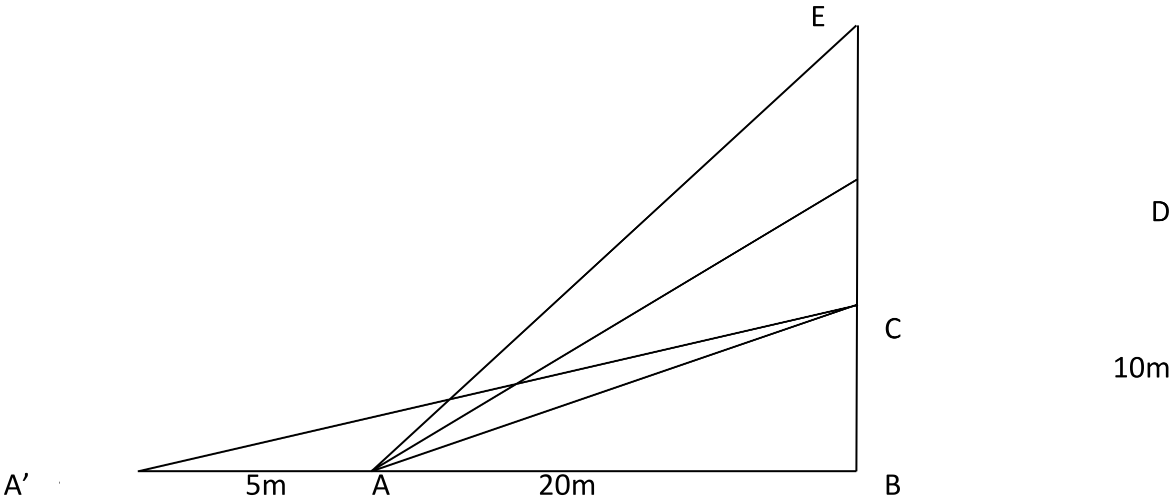
CHAPTER-2

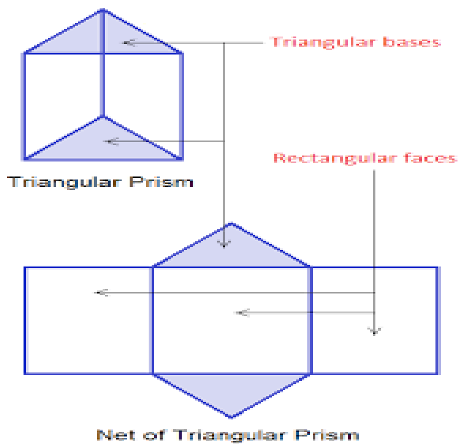
INVERSE T FUNCTION

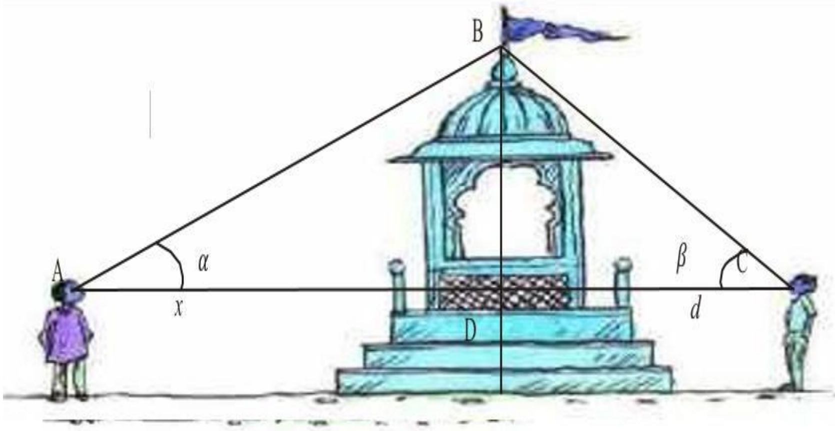
CLASS-XII

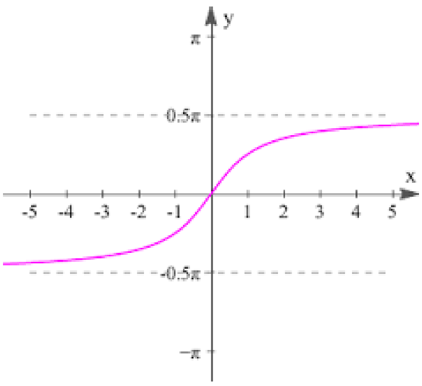
04 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Two men on either side of a flag staff of 30 metres high from the level of eye observe its top at the angles of elevation α and β respectively (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the flag staff is $30\sqrt{3}$ metres. Based on the above information answer the following questions.  a) Find ∠QAR and ∠AQR. b) Find ∠ARQ. c) Find the principal value of $\sin^{-1}\{\sin(\alpha + \frac{2\pi}{3})\}$ d) Find the principal value of $\cos^{-1}\{\cos(\beta + \frac{\pi}{3})\}$	4

2.	The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. “A”is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C,D and E. “C” is at a height of 10 metres from the ground level. For the viewer A, the angle of elevation of “D” is double the angle of elevation of “C”. The angle of elevation of “E” is triple the angle of elevation of “C” for the same viewer. Look at the figure given and based on the above information answer the following.  a) Determine $\angle CAB$ in terms of $\tan^{-1}$. b) Determine $\angle DAB$ in terms of $\tan^{-1}$. c) Determine $\angle EAB$ in terms of $\tan^{-1}$. d) A’ is another viewer standing on the same line of observation across the road. If the width of the road is 5 metres , then find the difference between $\angle CAB$ & $\angle CA'B$.	4

3.	Architect Rahut was asked to design an office building for a multinational company. the fine storied building has five pillars in the lawn, which are congruent and in the shape of triangular prisms .Two of the base angles are given to be $\tan^{-1}2$ and $\tan^{-1}3$  (i)$\tan^{-1}2+\tan^{-1}3$----------------- (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d)π (ii)the third angle is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d)π (iii)If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ then x+y+xy=---------------------- (a)1 (b)0 (c)-1 (d)none of these (iv)$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z= \frac{\pi}{2}$, then xy+yz+zx=------ (a)1 (b)0 (c)xyz (d)xy+yz+zx	4
4.	In a school project Anu was asked to construct a triangle and name it as ABC . Two angles A and B were given to be equal to $\tan^{-1}\frac{1}{2}$ and $\tan^{-1}\frac{1}{3}$ respectively;  (i)The value of sinA is ------------- (a)$\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{5}}$ (ii)Cos(A+B+C)=---------- (a)1 (b)0 (c)-1 (d)$\frac{1}{2}$ (iii)if $B=\cos^{-1}x$ then x=----- (a)$\frac{1}{\sqrt{5}}$ (b) $\frac{3}{\sqrt{10}}$ (c) $\frac{1}{\sqrt{10}}$ (d) $\frac{2}{\sqrt{5}}$ (iv)the value of A+B=-------- (a)$\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$	4
5.	Solve for X, $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}8/31$	4

6.	Prove that $\sin^{-1}(3/5) - \sin^{-1}(8/17) = \cos^{-1}(84/85)$.	4
7.	Prove that if $\frac{1}{2} \le x \le 1$, then $$\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$$	4
8.	Write the given function in simplest form $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), \frac{-\pi}{4} < x < \frac{3\pi}{4}$.	4
9.	Today in class of Mathematics, Mr. Gupta was explaining the inverse trigonometry functions. He draws the graph of the $\sin^{-1} x$ and explained that for $\sin^{-1} x$, the branch with range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is called principal value branch. Thus, $sin^{-1} : [-1,1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Based on the above information answer the following questions, (1)Find the domain of $\sin^{-1}\sqrt{x-1}$. [1] (2)Find the domain of $\sin^{-1}[x]$. [1] (3)Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$. [2]	4
10.	 Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is 40√3 meters and the distance between the first person A and the temple is 30√3 meters. Based on the above information answer the following: (i)Find $\angle CAB = \alpha$ in term of $\sin^{-1}$ (ii) Find $\angle CAB = \alpha$ in term of $\cos^{-1}$ (iii) Find $\angle BCA = \beta$ in term of $\tan^{-1}$ OR Find the domain and range of $\cos^{-1} x$	4
11.	Read the following passage and answer the following questions. In a school project Ravi was asked to construct a triangle ABC in which B and C are given by $\tan^{-1}\left(\frac{1}{2}\right)$ and $\tan^{-1}\left(\frac{1}{3}\right)$ respectively. (i) Find the value of sinB (ii) Find the value of cosC (iii) Find the value of B+C OR Find the value of cos (B+C)	4
12.	Ram and Mohan are students of class XII. One day their Mathematics teacher told them about Inverse trigonometric functions. Teacher sketch the graph of $y = \tan^{-1} x$ on the board	4

	as follows.  Based on the above information answer the following questions i) The domain of $\tan^{-1}(3x + 2)$ is a) **R** b) R^+ c) $(-\pi/2, \pi/2)$ d) $(0, \pi)$ ii) Principal value of $\tan^{-1}(-1)$ is a) $\pi/4$ b) **$-\pi/4$** c) -1 d) $3\pi/4$ **iii)**The principal value of $\tan^{-1}(\tan(-6))$ is a) -6 b) $2\pi - 6$ c) $6 - 2\pi$ d) None	
13.	In a school project manisha was asked toncinstr ucted a tringle ABC in wihich two angles B and C are given by $\tan^{-1}\left(\frac{1}{2}\right)$ and $\tan^{-1}\left(\frac{1}{3}\right)$ respective ly. (i) Find the value of sinB (ii) find the value of cosc (iii) Find the value of $B + C$ OR Find the value of $B + C$	4
14.	Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$	4
15.	Prove the following $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$	4

ANSWERS:

Q. NO	ANSWER	MARKS
1.	a)In ΔRPA, tan α = RP/AP= 30/ 30$\sqrt{3}$ = 1/$\sqrt{3}$ = tan $\pi/6$ So, α = ∠QAR= $\pi/6$ PQ = 40$\sqrt{3} - 30\sqrt{3}$= 10$\sqrt{3}$ b)In ΔRPQ, tan β =RP/PQ= 30/10$\sqrt{3}$ = $\sqrt{3}$ = tan $\pi/3$ So, β = ∠AQR= $\pi/3$ c) $\sin^{-1}\{ \sin(\alpha + \frac{2\pi}{3}) \} = \sin^{-1}\{ \sin(\pi/6+\frac{2\pi}{3}) \} = \sin^{-1}\{ \sin(5\pi/6) \}=$ $\sin^{-1}\{ \sin(\pi - \pi/6) \} = \pi/6 \epsilon [- \pi/2, \pi/2]$ d)$\cos^{-1} \{ \cos (\beta + \frac{\pi}{3})\}= \cos^{-1} \{ \cos (\pi/3 + \frac{\pi}{3})\} = \cos^{-1} \{ \cos (2\pi/3)\}$ $= 2\pi/3 \epsilon [0, \pi]$	4
2.	a)In ΔABC , tan∠CAB = BC/AB= 10/20= ½ So, ∠CAB= $\tan^{-1}(1/2)$ b) ∠DAB= 2 ∠CAB= 2 $\tan^{-1}(1/2) = \tan^{-1}(\frac{2.\frac{1}{2}}{1-(\frac{1}{2})^2}) = \tan^{-1}(4/3)$ c) ∠EAB= 3 ∠CAB= 3 $\tan^{-1}(1/2) = \tan^{-1}(\frac{\frac{3.1}{2}-(\frac{1}{2})^2}{1-3(\frac{1}{2})^2}) = \tan^{-1}(11/2)$ d) In ΔA'BC , tan A' = BC/A'B = 10/25 = 2/5 So, A' = $\tan^{-1}(2/5)$ Now, ∠CAB - ∠CA'B = $\tan^{-1}(1/2) - \tan^{-1}(2/5) = \tan^{-1} (\frac{\frac{1}{2}-\frac{2}{5}}{1+\frac{1}{2}.\frac{2}{5}})=\tan^{-1}(1/12)$	4
3.	Solution: (i)$\tan^{-1}2+\tan^{-1}3=\tan^{-1}(\frac{2+3}{1-2*3})$ $=\tan^{-1}(\frac{5}{1-6})$ $=\tan^{-1}((\frac{5}{-5})$ $=\tan^{-1}(-1)$ $=\pi - \frac{\pi}{4}$ $= \frac{3\pi}{4}$ (ii)let the third angle be x Since all three angles are in a triangle Sum of angles=180° Sum of angles=π tan-12+tan-13+x=π $\frac{3\pi}{4}$ +x= π x= π- $\frac{3\pi}{4}$ x =$\frac{\pi}{4}$ (iii)given that $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$	4

	$\tan^{-1}(\frac{x+y}{1-xy}) = \frac{\pi}{4}$ $\frac{x+y}{1-xy} = tan\frac{\pi}{4}$ $\frac{x+y}{1-xy} = 1$ x+y=1-xy x+y+xy=1 (iii)given that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ $\tan^{-1}x+\tan^{-1}y=\pi-\tan^{-1}z$ $\tan^{-1}x+\tan^{-1}y = \tan^{-1}0- \tan^{-1}z$ $\tan^{-1}(\frac{x+y}{1-xy})=\tan^{-1}(-z)$ $\frac{x+y}{1-xy} = -z$ ⇒x+y=- z + xyz x+y+z=xyz (iv)given that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{2} - \tan^{-1}z$ $\tan^{-1}x+\tan^{-1}y=\cot^{-1}z$ $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{1}{z}$ $\tan^{-1}(\frac{x+y}{1-xy}) = \tan^{-1}\frac{1}{z}$ $\frac{x+y}{1-xy} = \frac{1}{z}$ xz+yz=1-xy xy+xz+yz=1	
4.	Solution: (i)given $A=\tan^{-1}\frac{1}{2}$ $\tan A=\frac{1}{2}$ now we know that $1+\tan^2A=\sec^2A$ $1+\tan^2A=\frac{1}{cos2A}$ $1+\tan^2A=\frac{1}{1-sin2A}$ Putting $\tan A=\frac{1}{2}$ $1+(\frac{1}{2})^2=\frac{1}{1-sin2A}$ $1+\frac{1}{4}=\frac{1}{1-sin2A}$ $\Rightarrow\frac{5}{4}=\frac{1}{1-sin2A}$ $1-\sin^2A=\frac{4}{5}$ $\Rightarrow 1-\frac{4}{5}=\sin^2A$ $\Rightarrow\frac{1}{5}=\sin^2A$ $\sin^2A=\frac{1}{5}$ $\sin A=\frac{1}{\sqrt{5}}$ (ii) since ABC is a triangle By angle sum property of triangle $A+B+C=180^0$ Thus , Cos(A+B+C)=cos180°=-1 (iii) Given B = tan-1(⅓) ⇒ tan B = ⅓	4

	∴ cos B = 3/√10 B = cos-1(3/√10) ⇒ x = 3/√10 (iv) Given A=$\tan^{-1}\frac{1}{2}$ B=tan-1$\frac{1}{3}$ Now, A+B= $\tan^{-1}\frac{1}{2}$+ $\tan^{-1}\frac{1}{3}$ Using $\tan^{-1}x+\tan^{-1}y=\tan^{-1}(\frac{x+y}{1-xy})$ $= \tan^{-1}(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1\times1}{2\times3}})$ $= \tan^{-1}(\frac{\frac{3+2}{2\times3}}{1-\frac{1}{6}})$ $=\tan^{-1}(\frac{\frac{5}{6}}{\frac{5}{6}})$ $= \tan^{-1}(1)$ $=\frac{\pi}{4}$	
5.	$\Rightarrow \tan^{-1}\left[\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right]=\tan^{-1}\frac{8}{31}$ $\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right); xy<1\right]$ $\Rightarrow \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right)=\tan^{-1}\frac{8}{31}$ $\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$ **⇒ 62x = 16 – 8x^2** **⇒ 8x^2 + 62x – 16 = 0** **⇒ 4x^2 + 31x – 8 = 0** **⇒ 4x^2 + 32x – x – 8 = 0** **⇒ 4x(x + 8) – 1 (x + 8) = 0** **⇒ (x + 8) (4x – 1) = 0** **∴ x = -8 or x = 14** **But x = – 8 gives LHS = $\tan^{-1}$ (- 7) + $\tan^{-1}$ (- 9)** **= – $\tan^{-1}$ (7) – $\tan^{-1}$ (9),** **which is negative, while RHS is positive.** **So, x = – 8 is not possible.** **Hence, x = 14 is the only solution of the given equation**.	4
6.	**Let $\sin^{-1}$ (3/5) = a and $\sin^{-1}$ (8/17) = b** **Thus, we can write sin a = 3/5 and sin b = 8/17** **Now, find the value of cos a and cos b** **To find cos a:** **Cos a = √[1 - $\sin^2$ a]** **= √[1 - $(3/5)^2$]** **= √[1 - (9/25)]**	4

	= √[(25-9)/25] = 4/5 Thus, the value of cos a = 4/5 To find cos b: Cos b= √[1 - $\sin^2$ b] = √[1 - $(8/17)^2$] = √[1 - (64/289)] = √[(289-64)/289] = 15/17 Thus, the value of cos b = 15/17 We know that cos (a- b) = cos a cos b + sin a sin b Now, substitute the values for cos a, cos b, sin a and sin b in the formula, we get: cos (a - b) = (4/5)x (15/17) + (3/5)x(8/17) cos (a - b) = (60 + 24)/(17x 5) cos (a - b) = 84/85 (a - b) = $\cos^{-1}$ (84/85) Substituting the values of a and b $\sin^{-1}$ (3/5)- $\sin^{-1}$ (8/7) = $\cos^{-1}$ (84/85) Hence proved.	
7.	Let $\cos^{-1} x = \theta$ then x= cos θ We have $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$ LHS= θ+$\cos^{-1}\left[\frac{cos\theta}{2} + \frac{\sqrt{3-3cos^2\theta}}{2}\right]$ $= \theta + \cos^{-1}\left[\frac{cos\theta}{2} + \frac{\sqrt{3}\sqrt{1-cos^2\theta}}{2}\right]$ $= \theta + \cos^{-1}\left[\frac{cos\theta}{2} + \frac{\sqrt{3}sin\theta}{2}\right]$	4

	$= \theta + \cos^{-1}\left[cos\theta cos\frac{\pi}{3} + sin\frac{\pi}{3}sin\theta\right]$ $= \theta + \cos^{-1}\left[cos\left(\theta - \frac{\pi}{3}\right)\right]$ $= \theta + \frac{\pi}{3} - \theta \quad = \frac{\pi}{3}$	
8.	Let $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = y(\text{say})$ Taking common 'cos x' from numerator and denominator, we get $Y = \tan^{-1}\left[\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right] = \tan^{-1}\left[\frac{1-\tan x}{1+\tan x}\right]$ $= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}tanx}\right]$ $\quad \left[\tan\frac{\pi}{4} = 1\right]$ $= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$ $\quad \left[\frac{\tan A - \tan B}{1+\tan A \tan B} = \tan(A-B)\right]$ $= \frac{\pi}{4} - x, 0 < x < \pi$	4
9.	(1)Since, $0 \leq x - 1 \leq 1$ $1 \leq x \leq 2,$ Domain = $[1,2]$ (2)Since, domain of $\sin^{-1} x = [-1,1]$ So, $[x] = \begin{cases} -1, -1 \leq x < 0 \\ 0, \quad 0 \leq x < 1 \\ 1, \quad 1 \leq x < 2 \end{cases}$ So Domain of $\sin^{-1}[x] = [-1, 2)$ (3) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ $= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\left(\frac{\pi}{2}\right) = 1$	4
10.	(i) $\alpha = \sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (iii) $\beta = \tan^{-1}(\sqrt{3})$ OR Range = $[0, \pi]$	4
11.	(i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{3}{\sqrt{10}}$ (iii) $\frac{\pi}{4}$ OR $\frac{1}{\sqrt{2}}$	4
12.	**i) (a)** x is real, implies 3x + 2 is real. So domain is R (ii)(b) $\tan(-\pi/4) = -1$ implies $\tan^{-1}(-1) = -\pi/4$ (iii) **(b)**$\tan^{-1}(\tan(-6)) = \tan^{-1}(-\tan 6) = \tan^{-1}(\tan(2\pi - 6)) = 2\pi - 6$	

13.	We have, $\tan^{-1}\left(\frac{1}{2}\right) = B \Rightarrow \tan B = 1/2$ and $\tan^{-1}\left(\frac{1}{3}\right) = C \Rightarrow \tan C = 1/3$ (i) $\sin B = \frac{1}{\sqrt{5}}$ and (ii) $\cos C = \frac{3}{\sqrt{10}}$ $(iii)\ \tan(B+C) = \frac{\tan B + \tan C}{1 - \tan B \tan C} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\text{ x }\frac{1}{3}} = 1 = \tan\frac{\pi}{4} \Rightarrow B+C = \frac{\pi}{4}$ OR $\cos B = \frac{2}{\sqrt{5}}$ and $\sin C = \frac{3}{\sqrt{10}}$ $\cos(B+C) = \cos B\cos C - \sin B\sin C = \frac{2}{\sqrt{5}}\text{ x }\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}}\text{ x }\frac{3}{\sqrt{10}} = \frac{1}{\sqrt{2}}$	
14.	Let $\sin^{-1}\frac{3}{4} = x$, $\sin x = 3/4$, $\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{3}{4}$ $\tan\frac{x}{2} = \frac{4 \pm \sqrt{7}}{3}$ $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$	4
15.	Let $\cot^{-1}x = y$ $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \cos[\tan^{-1}\{\sin y\}] = \cos[\tan^{-1}\{\frac{1}{cosec\, y}\}]$ $= \cos[\tan^{-1}\{\frac{1}{\sqrt{1+\cot^2 y}}\}] = \cos[\tan^{-1}\{\frac{1}{\sqrt{1+x^2}}\}]$ let $\tan^{-1}\frac{1}{\sqrt{1+x^2}} = a$ such that $\tan a = \frac{1}{\sqrt{1+x^2}}$ $\tan^2 a = \frac{1}{1+x^2}$ $\frac{\sin^2 a}{\cos^2 a} + 1 = \frac{1}{1+x^2} + 1$ $\frac{1}{\cos^2 a} = \frac{2+x^2}{1+x^2}$ $\cos a = \sqrt{\frac{1+x^2}{2+x^2}}$	4