

CHAPTER-5
CONTINUITY & DIFFERENTIABILITY
05 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	For what value of k is the following function continuous at $-\frac{\pi}{6}$? $f(x) = \begin{cases} \frac{x + \cos \cos x}{x + \frac{\pi}{6}}, & \text{if } x \neq -\frac{\pi}{6} \\ k, & \text{if } x = -\frac{\pi}{6} \end{cases}$	5
2.	Find $\frac{dy}{dx}$, if $y = (\tan \tan x)^{\cot x} + (\cot \cot x)^{\tan x}$	5
3.	Find $\frac{dy}{dx}$, if $y = e^{(\sin \theta)^2} \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right)$.	5
4.	$y = x^{x^{x^{x^{\dots \infty}}}}$ then prove that $x \cdot \frac{dy}{dx} = \frac{y^2}{1 - y \log x}$	5
5.	Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1 \\ bx + 2, & \text{if } x > 1 \end{cases}$ is differentiable at $x = 1$	
6.	If $x = \sin t$ and $y = \sin pt$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$	
7.	Find $\frac{dy}{dx}$, if $y = e^{\sin^2 x} \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$.	
8.	If $x \cos(a+y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence, show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$.	
9.	For what value of k is the following function continuous at $-\frac{\pi}{6}$? $f(x) = \begin{cases} \frac{\sqrt{3} \sin x + \cos x}{x + \frac{\pi}{6}}, & \text{if } x \neq -\frac{\pi}{6} \\ k, & \text{if } x = -\frac{\pi}{6} \end{cases}$	5
10.	Find $\frac{dy}{dx}$, if $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$	5
11.	If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$	5
12.	If $y = \log(x + \sqrt{x^2 + a^2})$, then show that $(x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\frac{2 \left(\frac{\sqrt{3}}{2} \sin \sin x + \cos \cos x \cdot \frac{1}{2} \right)}{\left(x + \frac{\pi}{6} \right)} = \frac{2 \left(\cos \cos \frac{\pi}{6} \sin \sin x + \cos \cos x \sin \frac{\pi}{6} \right)}{\left(x + \frac{\pi}{6} \right)}$ $= \frac{2 \sin \left(x + \frac{\pi}{6} \right)}{\left(x + \frac{\pi}{6} \right)} = 2 \Rightarrow k = 2$	5
2.	$\frac{dy}{dx} = (\tan \tan x)^{\cot x} \operatorname{cosec}^2 x (1 - \log \log \tan x)$ $+ (\cot \cot x)^{\tan x} \sec^2 x (\log \log \cot \cot x - 1)$	5
3.	$e^{(\sin \theta)^2} \cos^{-1} x \left[\sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]$	5
4.	Correct proof should be there without step missing	5
5.	a = 3, b = 5	5
6.	$dx/dt = \cos t, dy/dt = p \cos pt$ $dy/dx = p \cos pt / \cos t$ $\cos t \frac{dy}{dx} = p \cos pt$ $\cos t \frac{d^2 y}{dx^2} + \frac{dy}{dx} (-\sin t) \frac{dt}{dx} = -p^2 \sin pt \frac{dt}{dx}$ $\cos^2 t \frac{d^2 y}{dx^2} + \frac{dy}{dx} (-\sin t) = -p^2 \sin pt$ $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$	5
7.	Putting $x = \cos 2\theta$ in $\left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$, we get $2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ $\text{i.e., } 2 \tan^{-1} \sqrt{\frac{2 \sin^2 \theta}{2 \cos^2 \theta}}$ $= 2 \tan^{-1}(\tan \theta)$ $= 2\theta = \cos^{-1} x$ Hence, $y = e^{\sin^2 x} \cos^{-1} x$ or $\log y = \sin^2 x + \log(\cos^{-1} x)$ or $\frac{1}{y} \times \frac{dy}{dx} = 2 \sin x \cos x + \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}}$ $= \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}}$ or $\frac{dy}{dx} = e^{\sin^2 x} \cos^{-1} x \left[\sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]$	5

8.	Given, $x \cos(a + y) = \cos y$ or $x = \frac{\cos y}{\cos(a + y)}$ On differentiating both sides w.r.t. y, we get $\frac{dx}{dy} = \frac{\cos(a + y) \frac{d}{dy} \cos y - \cos y \frac{d}{dy} \cos(a + y)}{\cos^2(a + y)} \quad 1$ [By using quotient rule of derivative] $= \frac{\cos(a + y) \times (-\sin y) + \cos y \times \sin(a + y)}{\cos^2(a + y)} \quad 1/2$ $= \frac{\sin(a + y) \cos y - \cos(a + y) \sin y}{\cos^2(a + y)}$ or $\frac{dx}{dy} = \frac{\sin(a + y - y)}{\cos^2(a + y)} = \frac{\sin a}{\cos^2(a + y)}$ [$\because \sin A \cos B - \cos A \sin B = \sin(A - B)$] or $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a} \quad \dots(i) \quad 1/2$ Again, on differentiating both sides of Eq. (i) w.r.t. x, we get $\frac{d^2 y}{dx^2} = \frac{1}{\sin a} \frac{d}{dx} \cos^2(a + y)$ $= \frac{1}{\sin a} \times \frac{d}{dy} \cos^2(a + y) \times \frac{dy}{dx} \quad 1$ $= \frac{1}{\sin a} \times 2 \cos(a + y) [-\sin(a + y)] \times \frac{dy}{dx}$ $= -\frac{2 \sin(a + y) \cos(a + y)}{\sin a} \times \frac{dy}{dx} \quad 1$ $\therefore \frac{d^2 y}{dx^2} = \frac{-\sin 2(a + y)}{\sin a} \frac{dy}{dx}$ $\therefore \sin a \frac{d^2 y}{dx^2} + \sin 2(a + y) \frac{dy}{dx} = 0$	5

9.	$\lim_{x \rightarrow -\frac{\pi}{6}} \frac{2 \left(\frac{\sqrt{3}}{2} \sin x + \cos x \cdot \frac{1}{2} \right)}{\left(x + \frac{\pi}{6} \right)} = \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2 \left(\cos \frac{\pi}{6} \sin x + \cos x \sin \frac{\pi}{6} \right)}{\left(x + \frac{\pi}{6} \right)}$ $= \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2 \sin \left(x + \frac{\pi}{6} \right)}{\left(x + \frac{\pi}{6} \right)} = 2 \Rightarrow k = 2$	5
10.	$\frac{dy}{dx} = (\tan x)^{\cot x} \operatorname{cosec}^2 x (1 - \log \tan x) + (\cot x)^{\tan x} \sec^2 x (\log \cot x - 1)$	5
11.	Given that: $x = \cos t(3 - 2\cos^2 t)$ on differentiating w.r.t 't' we get $dx/dt = d/dt(\cos t(3 - 2\cos^2 t))$ $= \cos t(4\cos t \sin t) - \sin t(3 - 2\cos^2 t)$ $= 4\cos^2 t \sin t - 3\sin t + 2\cos^2 t \sin t$ $= 6\cos^2 t \sin t - 3\sin t \dots \dots \dots (i)$ Now, $y = \sin t(3 - 2\sin^2 t)$ on differentiating w.r.t 't' we get $dy/dt = d/dt(\sin t(3 - 2\sin^2 t))$ $= \sin t(-4\sin t \cos t) + \cos t(3 - 2\sin^2 t)$ $= -4\sin^2 t \cos t + 3\cos t - 2\sin^2 t \cos t$ $= 3\cos t - 6\sin^2 t \cos t \dots \dots \dots (ii)$ eq. (ii) / (i) $dy/dx = \frac{3\cos t - 6\sin^2 t \cos t}{6\cos^2 t \sin t - 3\sin t}$ $= \frac{3\cos t(1 - 2\sin^2 t)}{3\sin t(2\cos^2 t - 1)}$ $= \frac{3\cos t \cos 2t}{3\sin t \cos 2t}$ $= \cot t$ at $t = \frac{\pi}{4}$ $dy/dx = \cot \frac{\pi}{4}$ $= 1$	5
12.	Given, $y = \log[x + \sqrt{x^2 + a^2}]$ Differentiating both side w.r.t x, we get, $\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \frac{d}{dx} (x + \sqrt{x^2 + a^2}) \left[\because \frac{d}{dx} (\log x) = \frac{1}{x} d(x) \right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \left(1 + \frac{2x}{2\sqrt{x^2 + a^2}} \right) \left[\because \frac{d}{dx} (\sqrt{x^2 + a^2}) = \frac{2x}{2\sqrt{x^2 + a^2}} \right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \left(\frac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2}} \right)$ $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$ $\Rightarrow \frac{dy}{dx} (\sqrt{x^2 + a^2}) = 1$	5

	Again differentiating both side w.r.t x, we get $\Rightarrow (\sqrt{x^2 + a^2}) \frac{d}{dx} \left(\frac{dy}{dx} \right) + \frac{dy}{dx} \frac{d}{dx} (\sqrt{x^2 + a^2}) = \frac{d(1)}{dx}$ $\Rightarrow (\sqrt{x^2 + a^2}) \frac{d^2y}{dx^2} + \frac{2x \frac{dy}{dx}}{2 \sqrt{x^2 + a^2}} = 0 \left[\because \frac{d}{dx} (\sqrt{x^2 + a^2}) = \frac{2x}{2 \sqrt{x^2 + a^2}} \right]$ $\Rightarrow (\sqrt{x^2 + a^2}) \frac{d^2y}{dx^2} + \frac{dy}{dx} \frac{x}{\sqrt{x^2 + a^2}} = 0 \left[\text{multiplying } \sqrt{x^2 + a^2} \text{ both side} \right]$ Hence, Proved	
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