CHAPTER-7

INTEGRALS

05 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	For a function f(x) , if f (-x) = f(x) , then f is an even function and if f(-x) = - f(x) , then f is an odd function .Again ,we have $\int_{-a}^{a} f(x)dx = \{2\int_{0}^{a} f(x)dx\ , if\ f(x) is\ even\ 0\ , if\ f(x) is\ odd$, On the above information answer the following questions , i)f(x) = x^2sinx is an a)even (ii) odd (iii) neither even nor odd (iv) none of these ii)$\int_{-\pi}^{\pi} f(x)dx\ is$ equal to a)$\frac{\pi}{4}$ (b) 2π (c)$\frac{\pi}{2}$ (d) 0 iii) f(x) xsinx , then $\int_{-\pi}^{\pi} f(x)dx\ is$ a) π (b) 2π (c) 3π (d) 4π (iv) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \lvert sinx \rvert dx\ is$ equal to a) 0 (b) 1 (c) 2 (d) 3	5
2.	For any function we have ,$\int_{a}^{b} f(x)dx$ = $\int_{a}^{c_1} f(x)dx$ +$\int_{c_1}^{c_2} f(x)dx$ +…….+ $\int_{c_n}^{b} f(x)dx$, where a < c_1<c_2<…<$c_n < b$, Based on the above information , answer the following questions i)$\int_{0}^{1} \lvert 3x-2 \rvert$dx a)$\frac{15}{18}$ (b) $\frac{1}{2}$ (iii) $\frac{7}{3}$ (d) $\frac{11}{2}$ ii)$\int_{0}^{\pi} \lvert cosx \rvert dx$ a) 1 (b) 0 (c)2 (d)3 (iii)$\int_{0}^{2} [x]$dx a)0 (b) 1 (c) 2 (d) 3 (iv) $\int_{-1}^{1} e^{\lvert x \rvert}$dx a)e (b) 3(e-1) (c) 2(e-1) (d) 4	5
3.	Evaluate: $\int_{0}^{\pi} \frac{x.Sin\ x}{1+Cos^2 x} dx$	5
4.	Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$	5
5.	Evaluate : $\int_{0}^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x+cos^4x} dx$	5
6.	Find the value of $\int \frac{x+sinx}{1+cosx} dx$	5
7.	Find the value of $\int \frac{1}{(2-3cos2x)} dx$	5
8.	Find the value of $\int e^x \frac{x^2+1}{(x+1)^2} dx$	5
9.	Prove that $\int_{0}^{a} sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$	5
10.	Evaluate $\int_{0}^{\pi} \frac{xdx}{a^2cos^2x+b^2sin^2x}$	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	(i) (b)odd (ii) d) 0 (iii) even function , (b) 2π (iv) (c) 2	5
2.	i) a)$\frac{15}{18} = \frac{5}{6}$ (ii) (c)2 (iii) (b) 1 [$\int_0^1 0\,dx + \int_1^2$ 1dx] (iv) $\|x\| = \{x, x \geq 0 \; -x, x < 0$ $= \int_{-1}^{0} e^{-x}dx + \int_0^1 e^{x}dx = [\frac{e^{-x}}{-1}]_{-1}^{0} + [e^x]_0^1$ =−1+ e +e -1= 2e-2 = 2(e-1)	5
3.	Let $I = \int_0^{\pi} \frac{x.\operatorname{Sin} x}{1+\operatorname{Cos}^2 x} dx$ ……………..(1) $I = \int_0^{\pi} \frac{(\pi-x).\operatorname{Sin}(\pi-x)}{1+\operatorname{Cos}^2(\pi-x)} dx = \int_0^{\pi} \frac{(\pi-x).\operatorname{Sin} x}{1+\operatorname{Cos}^2 x} dx$ ………………………..(2) Add (1) and (2) $2I = \pi \int_0^{\pi} \frac{\operatorname{Sin} x}{1+\operatorname{Cos}^2 x} dx$ Let $\cos x = t \Rightarrow -\sin x\, dx = dt$ When $x = 0 \Rightarrow t = \cos 0 = 1$ and $x = \pi \Rightarrow t = \cos \pi = -1$ $2I = -\pi \int_1^{-1} \frac{dt}{1+t^2} = \int_{-1}^{1} \frac{dt}{1+t^2} = [\tan^{-1} x]_{-1}^{1}$ $2I = \pi[\tan^{-1}(1) - \tan^{-1}(-1)] = \pi\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{\pi^2}{2}$ $I = \frac{\pi^2}{4}$	5
4.	$I = \int \frac{2x}{(x^2+1)(x^2+2)^2} dx$ Let $x^2 = t \Rightarrow 2xdx = dt$ $\Rightarrow I = \int \frac{dt}{(t+1)(t+2)^2}$ $\frac{1}{(t+1)(t+2)^2} = \frac{A}{t+1} + \frac{B}{t+2} + \frac{C}{(t+2)^2}$ $1 = A(t + 2)^2 + B(t + 1)(t + 2) + C(t + 1)$ $\Rightarrow 1 = A(t^2 + 4 + 4t) + B(t^2 + 2t + t + 2) + C(t + 1)$ $\Rightarrow 1 = A(t^2 + 4t + 4) + B(t^2 + 3t + 2) + C(t + 1)$ $\Rightarrow 1 = t^2 (A + B) + t(4A + 3B + C) + 4A + 2B + C$ On comparing the coefficients of 2, and the constant term from both sides, we get A + B = 0 4A + 3B + C = 0 ……. (ii) and 4A + 2B + C = 1 …….. (iii) From Eq. (1), A = – B Put the value of A in Eqs. (ii) and (iii), we get – 4B + 3B + C = 0 ⇒ – B + C = 0 ⇒ B – C = 0 ……. (iv) and – 4B + 2B + C = 1 ⇒ – 2B + C = 1	5

	$\Rightarrow 28 - C = -1$ Now, from Eqs. (iv) and (y), we get $-B = 1 \Rightarrow B = -1$ $\therefore A = 1$ and $C = -1$ $\Rightarrow I = \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt - \int \frac{1}{(t+2)^2} dt$ $\Rightarrow I = \log(t+1) - \log(t+2) + \frac{1}{t+2} + c$ $\Rightarrow I = \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + c$	
5.	$Let\ I = \int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x + cos^4x} dx = \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x)sin(\frac{\pi}{2} - x)cos(\frac{\pi}{2} - x)}{sin^4(\frac{\pi}{2} - x) + cos^4(\frac{\pi}{2} - x)} dx$ $= \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x)sinxcosx}{sin^4x + cos^4x} dx \Rightarrow 2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{xsinxcosx}{sin^4x + cos^4x} dx$ $\Rightarrow I = \frac{\pi}{2 \times 4} \int_0^{\frac{\pi}{2}} \frac{2sinxcosx}{sin^4x + cos^4x} dx \Rightarrow I = \frac{\pi}{8} \int_0^{\frac{\pi}{2}} \frac{sin2x}{sin^4x + cos^4x} dx$ $\Rightarrow I = \frac{\pi}{8} \int_0^{\frac{\pi}{2}} \frac{sin2x}{(sin^2x)^2 + (1 - sin^2x)^2} dx$ Put $sin^2x = t \Rightarrow sin2xdx = dt, when\ x = 0, t = 0\ and\ when\ x = \frac{\pi}{2}, t = 1$ $I = \frac{\pi}{8} \int_0^1 \frac{dt}{t^2 + (1-t)^2} = \frac{\pi}{8} \int_0^1 \frac{dt}{2t^2 - 2t + 1}$ $I = \frac{\pi}{16} \int_0^1 \frac{dt}{(t-1/2)^2 + (\frac{1}{2})^2} = \frac{\pi}{8} [(2t-1)]_0^1 = \frac{\pi^2}{16}$	1 1 1 1 1
6.	Let $I = \int \frac{x+sinx}{1+cosx} dx = \int \frac{x + 2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}} dx$ $= \int (\frac{x}{2cos^2\frac{x}{2}} + \frac{2sin\frac{x}{2}cos\frac{x}{2}}{2cos^2\frac{x}{2}}) dx = \int \left(\frac{1}{2} xsec^2\frac{x}{2} + tan\frac{x}{2}\right) dx$ $= \frac{1}{2} [x \int sec^2\frac{x}{2} dx - \int (\frac{d}{dx}(x) \int sec^2\frac{x}{2} dx) dx] + \int tan\frac{x}{2} dx$ By doing integration by parts we get I = $xtan\frac{x}{2} + C$	1 1 1 2
7.	$\int \frac{1}{(2 - 3cos2x)} dx$ $= \int \frac{1}{2(cos\ cos\ x)^2 + 2(sin\ sin\ x)^2 - 3\{(cosx)^2 - (sin\ sin\ x)^2\}} dx$ $= \int \frac{1}{5(sin\ sin\ x)^2 - (cos\ cos\ x)^2} dx$ $= \int \frac{(secsec\ x)^2}{5(tantan\ x)^2 - 1} dx \quad ...(i)$ Taking, $\sqrt{5}\ tan\ tan\ x = u$ We get, $\sqrt{5}(sec\ sec\ x)^2 dx = du$ $(i)\ becomes \int \frac{1}{(2 - 3cos2x)} dx$ $= \int \frac{1}{\sqrt{5}(u^2 - 1)} du$	5

	$= \frac{1}{2\sqrt{5}} log\ log \mid \frac{u-1}{u+1} \mid + c$ $= \frac{1}{2\sqrt{5}} log\ log \left\| \frac{\sqrt{5}\ tan\ tan\ x - 1}{\sqrt{5}\ tan\ tan\ x + 1} \right\| + c, [\ put\ the\ value\ of\ u]$ is the required answer.	
8.	$\int e^x \frac{x^2+1}{(x+1)^2} dx$ $= \int e^x \frac{x^2+2x+1-2x}{(x+1)^2} dx$ $= \int e^x \frac{(x+1)^2-2x}{(x+1)^2} dx$ $= \int e^x dx - 2\int \frac{xe^x}{(x+1)^2} dx$ $= e^x - 2\int \frac{(x+1-1)e^x}{(x+1)^2} dx$ $= e^x - 2\int (\frac{e^x}{x+1} - \frac{e^x}{(x+1)^2}) dx \quad ...(i)$ Taking, $\frac{e^x}{x+1} = u$ We get, $\left(\frac{e^x}{x+1} - \frac{e^x}{(x+1)^2}\right) dx = du$ Now, $(i) becomes, \int e^x \frac{x^2+1}{(x+1)^2} dx = e^x - 2\int du$ $= e^x - 2u + c$ $= e^x - \frac{2e^x}{x+1} + c$ (Answer)	5
9.	$\int_0^a sin^{-1}\sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$ (Put $x = atan^2\theta\ \ and\ the\ apply\ Integration\ by\ parts$)	5
10.	$\int_0^\pi \frac{xdx}{a^2cos^2x+b^2sin^2x} (= \frac{\pi^2}{2ab})$ Apply the following properties in series: (i)$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ (ii)$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$	5