





	Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is 40v3 metres and the distance between the first person A and the temple is 30v3 meters. \angle CAB = α =	
7.	 D.sin⁻¹(V3/2) The municipal corporation of a city is planning to fix hoarding boards at the face of a building on the road of a busy market for awareness on keeping the city clean. Anuj, Bala and Dilip are the three engineers who are working on this project. Hoardings are placed at C,D and E on the wall. C is the height of 10 meters from the ground level D and E are above it. 	
	E D C 10 m 5 m A 20 m B	5
	A is a point which is 20 meters away from the foot of the building. From A, the angle of elevation of D is doubled the angle of elevation of C. Also, from A the angle of elevation of E is triple the angle of elevation of C. Look at the figure and based on the above information answer the following: (i) If the measure of angle CAB = $\tan^{-1} x$, Find the value of x. (ii) If the measure of angle EAB= $\tan^{-1} z$, find the value of z. (iii) Point P is 5 meters behind A. Then find the difference between angle CAB and CPB. (iv) Give the domain and range of $\tan^{-1} x$	
8.	Two men on either side of a tower of 30 meters high observe its top at the angles of elevation α and β respectively. The distance between the two men is $40\sqrt{3}$ and the distance	5

	between the first person A and the tower is $30\sqrt{3}$ meters.				
	A Ta	I	B		
	T		Den 1		
	Based on the above (i) If angle (ii) If angle (iii) Find the) Give the doma	e information answe e $CAB = \alpha = \sin^{-1} y$, e $CAB = \alpha = \cos^{-1} y$, e measure of angle A in and range of $\cos^{-1} y$	The following: x, find the value of x. find the value of y. A. $y^{1}y$		
9.	Shriya is preparing maths and importa	g for her board exam nt facts related to ea	s. So, she decided to prepare cha ch chapter. For the chapter inve	art of formulas of rse trigonometry she	
	has prepared the fo	ollowing table to rem	nember the principal branch valu	ies of inverse	
	Functions	Domain	Range (Principal Value Branches)		
	$y = \sin^{-1} x$	[-1, 1]	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$		
	$y = \cos^{-1} x$	[-1, 1]	[0, π]		
	$y = \operatorname{cosec}^{-1} x$	R – (–1,1)	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right] - \{0\}$		5
	$y = \sec^{-1} x$	R – (–1, 1)	$[0, \pi] - \{\frac{\pi}{2}\}$		5
	$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$		
	$y = \cot^{-1} x$	R	(0, π)		
	Based on the above (1) Find the pr (2) Find the pr (3) Solve the fe	e information, answe incipal value of csc ⁻ incipal value of sec ⁻ ollowing equation, $\tan^{-1}\left(\frac{x+1}{x-1}\right)$ -	er the following questions, ${}^{-1}(-1)$. [1] ${}^{-1}(-2)$. [1] [3] $+ \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$		
10.	If $\cos^{-1}\frac{x}{a} + \cos^{-1}$	$\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2}$	$\frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha.$		5
11.	Prove that, $\tan\left(\frac{\pi}{4}\right)$	$+\frac{1}{2}\cos^{-1}\frac{a}{2}$ + tan	$\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$		5
1		2 b b	4 2 <i>D a</i>		

			5
13.	Prove that	$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \ -\frac{1}{\sqrt{2}} \le x \le 1$	
			5
14.	Read th	e following passage and answer the following questions:	5
	In a sch	ool project Manish was asked to construct a triangle ABC in which two	
	angles E	B and C are given by $\tan^{-1}(\frac{1}{2})$ and $\tan^{-1}(\frac{1}{3})$ respectively.	
	(i)	Find the value of sin B.	
	(ii)	Find the value of cos C.	
	(iii)	Find the value of B + C	
	(iv)	Find the value of cos (B+C)	
	(v)	Write the formula for tan ⁻¹ A + tan ⁻¹ B	
15.	Read th	e following passage and answer the following questions:	5
	Two me	n on either side of a temple 15 metre high observe its top at the	
	angles c	of elevation $\propto and~eta$ respectively (as shown in the figure). The	
	distance	e between the two men is 20 $\sqrt{3}$ m and the distance between the first	
	person	A and the temple is $15\sqrt{3}$.	
	(i)	Find \propto in terms of sin ⁻¹ ,	
	(ii)	Find \propto in terms of cos ⁻¹	
	(iii)	Find eta in terms of tan-1	
	(iv)	Write the domain and range of cos ⁻¹	
	(v)	Find $m \angle ABC$	
16.	Prove that	at $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, 0 < x < 1$	5
17.	Find the	greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.	5

ANSWERS:

Q. NO	ANSWER	MARKS
1.	i) Domain of tan ⁻¹ x is R or (- ∞ , ∞).	5
	ii) The principal value branch of cosec ⁻¹ x is [- $\pi/2$,0) \cup (0, $\pi/2$]	
	or $[-\pi/2, \pi/2] - \{0\}$	
	iii) Another two branches of sin ⁻¹ x other than principal value	
	branch are [$\pi/2$,3 $\pi/2$] , [3 $\pi/2$,5 $\pi/2$]	
	iv) If x < 0, then $\tan^{-1}x + \tan^{-1}\frac{1}{x} = \tan^{-1}x + \cot^{-1}x - \pi = \frac{\pi}{2} - \pi = -1$	
	$\pi/2$	
	v) To find the domain of sin ⁻¹ ($2x - 3$), we can write	
	$-1 \le 2x - 3 \le 1 \Rightarrow -1 + 3 \le 2x \le 1 + 3 \Rightarrow 2 \le 2x \le 4$	
	$\Rightarrow 1 \le x \le 2$	
	So, $x \in [1,2]$	
2.	i)As we know $3\pi < 10 < 7 \pi/2$	5
	Now, x = sin ⁻¹ (sin 10)= sin ⁻¹ (sin π – 10)= sin ⁻¹ (sin 3 π – 10)	
	But 0> 3 π - 10 > - $\pi/2$	
	i.e. $x = 3 \pi - 10$	
	ii) $\frac{1-\cos x}{1+\cos x} = \tan^2(x/2), x/2 \in (-\pi/2, \pi/2)$	
	$y = \tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\tan \frac{x}{2} \right)$	
	$x > 0 \Rightarrow y = x/2$ and $x < 0 \Rightarrow y = -x/2$	
	iii)Let $\tan^{-1} x = \theta \implies x = \tan \theta$	
	$\theta \in (-\pi/2, \pi/2)$	
	Sin θ = cos θ . tan θ = x / sec θ = x / $\sqrt{1 + x^2}$	
	iv) Let $\operatorname{cosec}^{-1} x = \theta \Rightarrow x = \operatorname{cosec} \theta \Rightarrow \sin \theta = 1/x$	
	$\theta \in [-\pi/2, \pi/2] - \{0\}$	
	$\int \cos \theta = \sqrt{1 - \frac{1}{x^2 - 1}}$	
	$\cos \theta = \sqrt{1 - \frac{1}{x^2} - \frac{1}{ x }}$	
	v) No, $\sin^{-1} x \neq (\sin x)^{-1}$	
3.	solution:	5
	$(1) \sum_{n=0}^{RC} ABC$	
	$A = \frac{10}{AB}$	
	$\tan A = \frac{1}{20}$	
	$\tan A = \frac{1}{2}$	
	$$	
	$<$ CAB=tan ⁻¹ $(\frac{1}{2})$	
	(ii)given that	
	$=2\times \tan^{-1} \frac{1}{1}$	
	$\frac{1}{2}$	
	Using Zian x-lan $\frac{1-x^2}{1-x^2}$	

	$=\tan^{-1}(\frac{2\times\frac{1}{2}}{2})$	
	$(1-(\frac{1}{2})^2)'$	
	$=\tan^{-1}(\frac{1}{1-\frac{1}{4}})$	
	$=\tan^{-1}(\frac{1}{3})$	
	$\overline{4}$	
	$\tan^{-1}\frac{1}{3}$	
	(iii)given that	
	$< EAB=2 \times < CAB$	
	$\tan^{-1}\frac{1}{2}$	
	Using $3\tan^{-1} x = \tan^{-1} \frac{3x - x^{3}}{1 - 3x^{2}}$	
	$3*\frac{1-(\frac{1}{2})3}{2}$	
	$=\tan^{-1}(\frac{2}{1-3*(1/2)2})$	
	$=\tan^{-1}\frac{(3/2)-(1/8)}{1-(3/4)}$	
	$\frac{3^{34-1}}{(3^{3}+1)}$	
	$=\operatorname{Lan}^{-}\left(-\frac{4}{4}\right)$	
	$=\tan^{-1}(\frac{\frac{11}{8}}{\frac{8}{3}})=\tan^{-1}(\{11/8\} * \{4/1\})=\tan^{-1}\frac{11}{12}$	
	$(\frac{1}{2})^{\prime}$ $(\frac{1}{2})^{\prime$	
	Tan A'=10/25=2/5	
	Angle A'=tan ⁻¹ (2/5)	
	Angle C'AB=tan ⁻¹ (2/5)	
	Now, we need to find the difference between angle CAB and angle CA'B	
	Using tan-1x-tan-1y=tan ⁻¹ $\left(\frac{x-y}{1+xy}\right)$	
	$= \tan^{-1} \frac{(1/2) - (2/5)}{1 + ((1/2) * (2/5))}$	
	$=\tan^{-1}\left(\frac{(5-4)}{(2+5)}\right)$	
	(1+(1/5))'	
	$= \tan^{-1} \left(\frac{1}{(6/5)} \right)$	
	$= \tan^{-1}((1/10)^*(5/6))$	
	(iv)since x is not defined at x= $(-\pi/2)$ and x= $(\pi/2)$	
	Range of tan ⁻¹ excludes= $(-\pi/2)$ and $(-\pi/2)$	
	Domain of tan ⁻¹ x is real numbers	
4.	Solution:	5
	I) In triangle A B D . BD	
	$\tan \alpha = \frac{AD}{AD}$	
	$\tan \alpha = \frac{30}{30\sqrt{3}}$	
	$\tan \alpha = \frac{1}{\sqrt{3}}$	
	Hence $\alpha = 30^{\circ} = \frac{\pi}{6}$	
	Thus, $\sin\alpha = \sin 30^\circ = \frac{1}{2}$	
	So, $\sin \alpha = \frac{1}{2}$	
	$\alpha = \sin^{-1}(\frac{1}{2})^2$	
	ii) Now. $\alpha = 30^{\circ} = \frac{\pi}{2}$	
	Thus.	
	$\cos\alpha = \cos 30^\circ = \sqrt{3/2}$	
	Hence $\cos \alpha = \sqrt{\frac{\sqrt{3}}{3/2}}$	

	Hence $\cos \alpha = \sqrt{3/2}$	
	$\alpha = \cos^{-1}(\sqrt{3}/2)$	
	iii) In triangle ABD	
	$\tan\beta = \frac{BD}{AD}$	
	$\tan\beta = \frac{\overline{30}}{10\sqrt{3}}$	
	$\tan\beta = \frac{3}{\sqrt{2}}$	
	$\tan\beta = \sqrt{3}$	
	$\beta = \tan^{-1}(\sqrt{3})$	
	Also, $\beta = 60^{\circ} = \frac{\pi}{3}$	
	$(x) = (x - 20)^2 + (x - 20)^2$	
	IV) SINCE $\alpha = 30^{\circ}$ and $\beta = 60^{\circ}$	
	By angle sum property	
	$\alpha + \beta$ +Triangle ABC =180°	
	$30^\circ + 60^\circ + traingle ABC = 180^\circ$	
	Triangle ABC = 90°	
	V) Since $\cos r$ is defined at $r=0$ and π	
	Domen of $\cos^{-1} x$ includes -1 and 1	
	Range of $\cos^{-1} x$ also includes 0 and π	
5.	Answer:	5
	(i) b	
	(11) C	
	(111) d (iv) a	
6.	Ans= (b)	5
7.	(i) $\frac{1}{2}$ (ii) $\frac{11}{21}$ (iii) $\frac{1}{8}$ (iv) $\left(\frac{-\pi}{2}, 2\right)$	5
	1	_
8.	(i) $\frac{1}{2}$	5
	(ii) $\frac{\sqrt{3}}{2}$	
	(iii) $\frac{\pi}{2}$	
	(iv) $\begin{bmatrix} 2 \\ -1.1 \end{bmatrix}, \begin{bmatrix} 0, \pi \end{bmatrix}$	
9.	(1)Let, $\csc^{-1}(-1) = y$	5
	Then, $-1 = \csc y$	
	Since principal value branch of $\csc^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	
	And $\csc\left(-\frac{\pi}{2}\right) = -1$ and $-\frac{\pi}{2}\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	
	So, principal value of $\csc^{-1}(-1) = -\frac{\pi}{2}$	
	(2)Let $\sec^{-1}(-2) = y$	
	Since principal value branch of sec ⁻¹ $x = [0, \pi] - \left\{\frac{\pi}{2}\right\}$	
	And $\sec \frac{2\pi}{3} = -2$ and $\frac{2\pi}{3} \in [0, \pi]$	
	So, principal value of sec ⁻¹ (-2) = $\frac{2\pi}{3}$	
	$(3)\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$	

	$\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$	
17.	$(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ $(\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$	5
	$\frac{\pi^2}{4} - 2\sin^{-1}x(\frac{\pi}{2} - \sin^{-1}x)$	
	$\frac{\pi}{4} - \pi \sin^{-1}x + 2(\sin^{-1}x)^2$	
	$2\left[(\sin^{-1}x - \frac{\pi}{4})^2 + \frac{\pi^2}{16}\right]$	
	Now, $\pi \in \operatorname{simple} \in \pi$	
	$\frac{-\frac{1}{2} \le \sin^{-1}x \le \frac{\pi}{2}}{-\frac{3\pi}{2} \le \sin^{-1}x - \frac{\pi}{2} \le \frac{\pi}{2}}$	
	$ \begin{array}{l} 4 = -\frac{1}{4} \\ 0 \leq (\sin^{-1} x - \frac{\pi}{4})^2 \leq \frac{9\pi^2}{4} \end{array} $	
	$\frac{\pi^2}{16} \le (\sin^{-1}x - \frac{\pi}{4})^2 + \frac{\pi^2}{16} \le \frac{5\pi^2}{8}$	
	$\frac{\pi^2}{8} \le 2(\sin^{-1}x - \frac{\pi}{4})^2 + \frac{\pi^2}{16} \le \frac{5\pi^2}{4}$	
	Greatest value = $\frac{5\pi^2}{4}$	
	Least value = $\frac{\pi^2}{8}$	