CLASS-XII

CHAPTER-01

RELATION AND FUNCTION

05 MARKS TYPE QUESTIONS

Q. No.	QUESTION	MARK
1.	Two integers a and b are said to be **congruence modulo m** if a-b is divisible by m which is as $a \equiv b(m)$. Show that the relation $a \equiv b(5)$ on the set **Z** of all integers is an equivalence relation. Also find equivalence class [2].	5
2.	Consider a function $f: R \rightarrow R$ defined by $f(x) = x^2 + 5x - 7$. Check whether f is one-one or onto or both. If not, then what will be the domain and co-domain show that f will be bijective?	5
3.	If A = R-{3} and B = R-{1}. Consider the function f: A→B defined by f(x) x-2/ x-3 for all x∈A. Then show that f is bijective.	5
4.	If Z is the set of all integers and R is the relation on Z defined as R = {(a, b):a,b ∈ Z and a – b is divisible by 5}. Prove that R is an equivalence relation	5
5.	If we throw two dices, the total number of possible outcomes is 36. Show how it is an equivalence relation.	5
6.	Let A = R -{3} and B = R – {1}. Consider the function f: A →B defined by f (x) = (x- 2)/ (x -3). Is f one-one and onto? Justify your answer.	5
7.	Prove that the relation in the set **A={**1,2,3,4,5} given by **R**={(a,b):$\|a-b\|$ is an even} is an equivalence relation	5
8.	Kendriya Vidyalaya Sangathan conducted cycle race under two different categories- Boys and Girls. There were 32 participants in all. Among all them, finally three from category -1 and two from category-2 were selected for the final race. Amit form two sets B and G with these participants form his college project. Let B={$b_1, b_2 b_3$}, and G=(g_1, g_2},where B represents the set of Boys selected and G the set of Girls selected for the final race.  (A) How many relation from B to G ? (B) Among all the possible relations from B to G, how many functions can be formed from B to G?	5

	A function $f: B \to G$ be defined by $f: B \to G$ defined by f={(b_1, g_1), (b_2, g_2),($b_3, g_1)$}.Check f is bijective or not ?	
9.	Show that the relation R on the set A={ $x \epsilon Z$,$0 \leq x \leq 12$} ,given by R = {(a,b) , |a-b|is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 i.e. equivalence class [1].	5
10.	A function f:[-4,4] →[0,4] is given by f(x) =$\sqrt{16 - x^2}$, show that f is a onto function but not one-one. Find all possible values of "a" for which f(x)=$\sqrt{7}$	5
11.	Let R be the relation in N × N $defined\ by\ (a, b)\ R\ (c, d)$. If $a + d = b + c$ for *(a, b), (c, d)* in $N \times N$. Prove that R is an equivalence relation.	5
12.	Show that $f: N \to N$ is given by $f(x) = \begin{cases} x+1, & if\ \ x\ is\ odd \\ x-1, & if\ \ x\ is\ even \end{cases}$ is bijective (both one-one and onto).	5

ANSWER
CHAPTER-01
RELATION AND FUNCTION
05 MARKS TYPE QUESTIONS

Q.No	ANSWERS	Mark
1.	Reflexive: For every integer x, $x - x = 0$ is divisible by 5. So $x \equiv x(mod\ 5)$. Therefore the relation **congruence modulo 5** is reflexive. Symmetric: Let $x \equiv y\ (mod\ 5)$ then $x - y$ is visible by 5. Let $x - y = 5k$, Then $y - x = -5k$ which is also divisible by 5. Hence $y \equiv x\ (mod\ 5)$. Therefore the relation **congruence modulo 5** is symmetric Transitive: Assume that $x \equiv y\ (mod\ 5)$ and $y \equiv z\ (mod\ 5)$. $\Rightarrow x - y = 5k$ and $y - z = 5l$. $\Rightarrow x - z = (x - y) + (y - z) = 5(k + l)$ is also divisible by 5. Hence $x \equiv z\ (mod\ 5)$. Therefore the relation **congruence modulo 5** is symmetric As congruence modulo 5 is reflexive, symmetric and transitive, so it is an equivalence relation.	5
2.	$f(x) = x^2 + 5x - 7$. Check whether f is one-one or onto or both. If not, then what will be the domain and co-domain show that f will be bijective? For Injective: let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ $\Rightarrow {x_1}^2 + 5x_1 - 7 = {x_2}^2 + 5x_2 - 7$ $\Rightarrow {x_1}^2 - {x_2}^2 + 5x_1 - 5x_2 = 0$ $\Rightarrow (x_1 - x_2)(x_1 - x_2 + 5) = 0$ $\Rightarrow (x_1 - x_2) = 0\ or\ (x_1 + x_2 + 5) = 0$ $\Rightarrow x_1 = x_2 \quad or \quad x_1 = x_2 - 5$ There fore f is not one-one. To be one-one $x_1 + x_2 + 5$ should not be zero. It will happen only when $x_1, x_2 \in [0, \infty)$. So to be injective, Domain must be $[0, \infty)$. For Surjective: Let $x^2 + 5x - 7 = y$ $\Rightarrow x^2 + 5x - 7 - y = 0$ $\Rightarrow x = \frac{-5 \pm \sqrt{25+4(y+7)}}{2}$ To be onto $x = \frac{-5 \pm \sqrt{25+4(y+7)}}{2} \geq 0$ $\Rightarrow \sqrt{25 + 4(y + 7)} \geq 5$ $\Rightarrow y \geq -7$ Range = $[-7, \infty) \neq$ Co-domain. So f is not onto. Therefore, f will be surjective if Co-domain=Range= $[-7, \infty)$ So, f will be bijective if domain is $[0, \infty)$ and Co-domain is $[-7, \infty)$.	5

3.	Given, function is f: A→B, where A= R-{3} and B=R-{1}, such that f(x)= x-2/x-3. For One-one Let f(x1) = f(x2), for all x1, x2∈ A ⇒x2-2/ x1-3= x2-2/ x2-3 ⇒(x1 - 2) (x2-3) = (x1-2)(x? - 3) ⇒ x1x2 - 3x1 -2x2 + 6 = x1x2 - 3x1 - 2x2 +6 ⇒- 3x12x2 = - 3x1 - 2x2 - 3 (x1 - x2)+2 (x1 - x2) = 0 - (x1 - x2) = 0 Or, x1 - x2= 0 This implies, x1 = x2. Since, f(x1) = f(x2) ⇒ x1 = x2, for all x1, x2∈ A. So, f(x) is a one-one function. Onto To show f(x) is onto, we show that range of f(x) and its codomain are same. Now, let. y =x-2 /x - 3 or, xy-3y=x-2 ⇒xy - x = 3y - 2 ⇒x(y-1) = 3y - 2 ⇒x=3y-2/y-1 ......Eqn (1) Since, x ∈R−{3}, for all y ∈ R− {1}, the range of f(x)=R−{1}.	5

	Also, the given codomain of f(x) = R-{1} Therefore, Range = Codomain. Hence, f(x) is an onto function. Therefore, f(x) is a bijective function.	
4.	The given relation is R = {(a, b): a, b ∈ Z and a − b is divisible by 5}. To prove R is an equivalence relation, we have to prove R is reflexive, symmetric and transitive. Reflexive: As for any x ∈ Z, we have x − x = 0, which is divisible by 5. ⇒ (x − x) is divisible by 5. ⇒ (x, x) ∈ R, V x ∈ Z Therefore, R is reflexive. Symmetric: Let (x, y) ∈ R, where x, y ∈ Z. ⇒ (x − y) is divisible by 5. [by definition of R] ⇒ x − y = 5A for some A ∈ Z. ⇒ y − x = 5(−A) ⇒ (y − x) is also divisible by 5. ⇒ (y, x) ∈ R Therefore, R is symmetric. Transitive: Let (x, y) ∈ R,where x, y ∈ Z. ⇒ (x − y) is divisible by 5. ⇒ x − y = 5Afor some A ∈ Z Again, let (y, z) ∈ R, where y, z ∈ Z. ⇒ (y − 1) is divisible by 5. ⇒ y − z = 5B for some B ∈ Z. Now, (x – y) + (y – 2) = 5A + 5B ⇒ x – z = 5(A + B) ⇒ (x – z) is divisible by 5 for some (A + B) ∈ Z ⇒ (x, z) ∈ R Therefore, R is transitive. Thus, R is reflexive, symmetric and transitive. Hence, it is an equivalence relation	5
5.	If we note down all the outcomes of throwing two dices, we get the following possible relations:	5

	R = {(1,1), (2,2), (3,3), (1,2), (2,3), (3,4), ………)} Reflexive: For the relation to be reflexive (a, a) ∈ R for all a∈ *R* Since, (1,1), (2,2), (3,3), ……. ∈ R Hence, the relation is reflexive. <u>Symmetric:</u> For the relation to be symmetric (a, b) ∈ R => (b, a) ∈ R for all a, b∈ *R* In this relation, (1,2) ∈ R => (2,1) ∈ R (2,3) ∈ R => (3,2) ∈ R (3,4) ∈ R => (4,3) ∈ R . Hence, it satisfies the condition of symmetric. Hence, the function is symmetric. <u>Transitive:</u> For the relation to be transitive (a, b) ∈ R and (b, c) ∈ R (a, c) ∈ R for all a, b, c ∈ *R* In this relation, (1,2) ∈ R & (2,3) ∈ R => (1,3) ∈ R (2,3) ∈ R & (3,2) ∈ R => (2,2) ∈ R (4,5) ∈ R & (5,2) ∈ R => (4,2) ∈ R . . . Hence, it satisfies the condition of transitivity. Hence, the relation is transitive. Since, the relation R is reflexive, symmetric as well as transitive, the relation R is equivalence relation. Hence, throwing two dices is an example of equivalence relation.	
6.	Given function: $f(x) = (x-2)/(x-3)$ Checking for one-one function: $f(x_1) = (x_1 - 2)/(x_1 - 3)$ $f(x_2) = (x_2 - 2)/(x_2 - 3)$ Putting $f(x_1) = f(x_2)$ $(x_1 - 2)/(x_1 - 3) = (x_2 - 2)/(x_2 - 3)$ $(x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$ $x_1(x_2 - 3) - 2(x_2 - 3) = x_1(x_2 - 2) - 3(x_2 - 2)$ $x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$ $-3x_1 - 2x_2 = -2x_1 - 3x_2$ $3x_2 - 2x_2 = -2x_1 + 3x_1$ $x_1 = x_2$ Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$ Thus, the function f is one-one function. Checking for onto function:	5

	f (x) = (x-2)/(x-3) Let f(x) = y such that y B i.e., y ∈ R – {1} So, y = (x -2)/ (x- 3) y (x -3) = x- 2 xy -3y = x -2 xy – x = 3y-2 x (y -1) = 3y- 2 x = (3y -2) /(y-1) For y = 1, x is not defined but it is given that. y ∈ R – {1} Hence, x = (3y- 2)/ (y- 1) ∈ R - {3} Hence, f is onto.	
7.	The given relation in the set **A={**1,2,3,4,5} given by **R**={(a,b):$\|a-b\|$ is an even}. **Reflexive:-** As $\|x-x\|$=0 is even $\forall\ x \in A$ Hence R is reflexive relation (1) **Symmetric Relation:-** Let (x,y) ∈ R ⇒ $\|x-y\|$ is even (by definition of given relation ⇒ $\|y-x\|$ is also even since $\|a\| = \|-a\|\ \forall\ a \in A$ ⇒ (y,x) ∈ R $\forall\ x, y \in A$ ∴ R is symmetric relation. ($1\frac{1}{2}$) **Transitive Relation:-** Let (x,y) ∈ R and (y,z) ∈ R ⇒ $\|x-y\|$ is even (by definition of given relation ⇒ $x-y = \pm 2l$ (l is an integer)………..(1) ⇒ $\|y-z\|$ is even (by definition of given relation ⇒ $y-z = \pm 2m$(m is an integer)………(2) Add equation(1) and equation(2) $(x-y)$+$(y-z)$= $\pm 2l \pm 2m$== $\pm 2k$ is an integer . ⇒ $x-z = \pm 2k$ ⇒ $\|x-z\|$ is an even integer number . (2) ⇒ (x,z) ∈ R ∴ R is transitive relation.	5
8.	**B=**$\{b_1, b_2 b_3\}$ **G**==$\{g_1, g_2\}$ n(B)=3 n(G)=2 **since** n(B× G) = n(B)×n(G)=3× 2 (1) (A) Number of relation from B to G=2^6 (1) (B) Number of functions from B to G=$2^{n(B\times G)}$ =$[n(G)]^{n(B)} = 2^3$=8 (1) (C) f={(b_1, g_1), (b_2, g_2),($b_3, g_1)$}. Since $f(b_1) = g_1$ and $f(b_3) = g_1$ ⇒ $f(b_1) = f(b_3)$ but $(b_1) \neq (b_3)$ $As\ b_1 and\ b_1$ represents two different boys. ⇒ f is not one-one. ⇒ f is not bijective map. (2)	5

9.	**Reflexivity:** For a $\in A$, we have |a-a| =0, which is a multiple of 4 R is reflexive. **<u>Symmetric</u>** Let (a,b) $\in R$ |a-b| is a multiple of 4 |b-a| will also multiple of 4 (b,a) $\in R$ R is symmetric. **<u>Transitive:</u>** Let (a,b) ,(b,c) $\in R$ |a-b| is a multiple of 4 |a-b| =4ʎ , a-b= $\pm$4ʎ |b-c| is a multiple of 4 |b-c| =4μ , b-c= ± 4μ Therefore a-c= $\pm 4 \pm$ 4μ (a,c) $\in R$ R is transitive. For equivalence class: |x-1|= 0,4,8,12 X=1,5,9	5
10.	y =$\sqrt{16-x^2}$ y^2= $16-x^2$ x =$\sqrt{16-y^2}$ clearly for x to be x$\in [-4,4]$ $16-y^2 \geq 0$ (y-4) (y+4)$\leq$ 0 0 $\leq$ y $\leq$ 4 Therefore it is onto When x=4 , y=0 When x=-4 ,y=0 So it is not one one Also— F(a)= $\sqrt{7}$ $\sqrt{16-a^2}$ = $\sqrt{7}$ $16-a^2=7$ a$\in [-3,3]$	5
11.	**For Reflexive** (a, b) R (a, b) ⇒ a + b = b + a which is true since addition is commutative on N. ⇒ R is reflexive. **For Symmetric** Let (a, b) R (c, d) ⇒ a + d = b + c ⇒ b + c = a + d ⇒ c + b = d + a ⇒ (c, d) R (a, b)	1 2

	⇒ R is symmetric. **For Transitive** for (a, b), (c, d),(e, f) in N × N Let (a, b) R (c, d) and (c, d) R (e, f) ⇒ a + d = b + c and c + f = d + e ⇒ (a + d) – (d + e) = (b + c) – (c + f) ⇒ a - e = b - f ⇒ a + f = b + e ⇒ (a, b) R (e, f) ⇒ R is transitive. Hence, R is an equivalence relation.	2
12.	**One-One**: Suppose $f(x_1) = f(x_2)$. **Case 1**: When x_1 is odd and x_2 is even. In this case $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 - 1 \Rightarrow x_2 - x_1 = 2$ This is a contradiction, since the difference between an even natural number and an odd natural number can never be 2. Thus in this case $f(x_1) \neq f(x_2)$. Similarly, When x_1 is even and x_2 is odd, then $f(x_1) \neq f(x_2)$. **Case 2**: When x_1 and x_2 are both odd. In this case $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$ ∴ f is one-one. **Case 3**: When x_1 and x_2 are both even. In this case $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$ ∴ f is one-one. **Onto**: Let $y \in N$ $(codomain)$. **Case 1**: When y is odd then $y + 1$ is even. ∴ $f(\mathrm{y} + 1) = (\mathrm{y} + 1) - 1 = \mathrm{y}$ **Case 2**: When y is even then $y - 1$ is odd. ∴ $f(\mathrm{y} - 1) = (\mathrm{y} - 1) + 1 = \mathrm{y}$. Thus each $y \in N$ $(codomain)$has its pre-image in dom(f). ∴ f is onto. Hence f is both one-one and onto (bijective).	1 1 1 1 1