Chapter 1

RELATIONS & FUNCTIONS

Be Happy. It's Maths time now!

RECAPITULATION & RELATIONS

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. TYPES OF INTERVALS

a) **Open interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying strictly between a and b is called an *open interval*. It is denoted by $]a, b[$ or (a, b) i.e., $\{x \in \mathrm{R} : a < x < b\}$.

b) **Closed interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes both a and b as well is known as a *closed interval*. It is denoted by $[a, b]$ i.e., $\{x \in \mathrm{R} : a \le x \le b\}$.

c) **Open Closed interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it excludes a and includes only b is known as an *open closed interval*. It is denoted by $]a, b]$ or $(a, b]$ i.e., $\{x \in \mathrm{R} : a < x \le b\}$.

d) **Closed Open interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes only a and excludes b is known as a *closed open interval*. It is denoted by $[a, b[$ or $[a, b)$ i.e., $\{x \in \mathrm{R} : a \le x < b\}$.

🕮 RELATIONS

02. Defining the Relation: A relation R, from a non-empty set A to another non-empty set B is mathematically defined as an arbitrary subset of $\mathrm{A} \times \mathrm{B}$. Equivalently, any subset of $\mathrm{A} \times \mathrm{B}$ is a relation from A to B.

Thus, R is a relation from A to B $\Leftrightarrow \mathrm{R} \subseteq \mathrm{A} \times \mathrm{B}$

$$\Leftrightarrow \mathrm{R} \subseteq \{(a, b) : a \in \mathrm{A},\ b \in \mathrm{B}\}.$$

Illustrations:

a) Let $\mathrm{A} = \{1, 2, 4\}, \mathrm{B} = \{4, 6\}$. Let $\mathrm{R} = \{(1,4),(1,6),(2,4),(2,6),(4,6)\}$. Here $\mathrm{R} \subseteq \mathrm{A} \times \mathrm{B}$ and therefore R is a relation from A to B.

b) Let $\mathrm{A} = \{1, 2, 3\}, \mathrm{B} = \{2, 3, 5, 7\}$. Let $\mathrm{R} = \{(2,3),(3,5),(5,7)\}$. Here $\mathrm{R} \not\subset \mathrm{A} \times \mathrm{B}$ and therefore R is not a relation from A to B. Since $(5,7) \in \mathrm{R}$ but $(5,7) \notin \mathrm{A} \times \mathrm{B}$.

c) Let $\mathrm{A} = \{-1, 1, 2\}, \mathrm{B} = \{1, 4, 9, 10\}$. Let $a \mathrm{R} b$ means $a^2 = b$ then, $\mathrm{R} = \{(-1,1),(1,1),(2,4)\}$.

☞ **Note the followings :**

- *A relation from A to B is also called a relation from A into B.*
- $(a, b) \in R$ *is also written as* aRb *(read as* ***a is R related to b****).*
- *Let A and B be two non-empty finite sets having p and q elements respectively. Then* $n(A \times B) = n(A).n(B) = pq$. *Then total number of subsets of* $A \times B = 2^{pq}$. *Since each subset of* $A \times B$ *is a relation from A to B, therefore* ***total number of relations from A to B is given as*** 2^{pq}.

03. DOMAIN & RANGE OF A RELATION

a) Domain of a relation: Let R be a relation from A to B. The domain of relation R is the set of all those elements $a \in \mathrm{A}$ such that $(a,b) \in \mathrm{R}$ for some $b \in \mathrm{B}$. Domain of R is precisely written as $Dom.(R)$ symbolically.

Thus, $\text{Dom.(R)} = \{a \in \mathrm{A} : (a,b) \in \mathrm{R} \text{ for some } b \in \mathrm{B}\}$.

That is, the domain of R is **the set of first component of all the ordered pairs which belong to R.**

b) Range of a relation: Let R be a relation from A to B. The range of relation R is the set of all those elements $b \in \mathrm{B}$ such that $(a,b) \in \mathrm{R}$ for some $a \in \mathrm{A}$.

Thus, $\text{Range of R} = \{b \in \mathrm{B} : (a,b) \in \mathrm{R} \text{ for some } a \in \mathrm{A}\}$.

That is, the range of R is the set of second components of all the ordered pairs which belong to R.

c) Codomain of a relation: Let R be a relation from A to B. Then B is called the codomain of the relation R. So we can observe that codomain of a relation R from A into B is the set B as a whole.

Illustrations:

a) *Let* $A = \{1,2,3,7\}, B = \{3,6\}$. *Let aRb means* $a < b$.

Then we have $R = \{(1,3),(1,6),(2,3),(2,6),(3,6)\}$.

Here $Dom.(R) = \{1,2,3\}$, *Range of* $R = \{3,6\}$, *Codomain of* $R = B = \{3,6\}$.

b) *Let* $A = \{1,2,3\}$, $B = \{2,4,6,8\}$. *Let* $R_1 = \{(1,2),(2,4),(3,6)\}$, $R_2 = \{(2,4),(2,6),(3,8),(1,6)\}$. *Then both* R_1 *and* R_2 *are relations from A to B because* $R_1 \subseteq A \times B$, $R_2 \subseteq A \times B$. *Here* $Dom.(R_1) = \{1,2,3\}$, *Range of* $R_1 = \{2,4,6\}$;

$Dom.(R_2) = \{2,3,1\}$, *Range of* $R_2 = \{4,6,8\}$.

04. TYPES OF RELATIONS FROM ONE SET TO ANOTHER SET

a) Empty relation: A relation R from A to B is called an empty relation or a void relation from A to B if $\mathrm{R} = \phi$.

For example, *let* $A = \{2,4,6\}, B = \{7,11\}$.

Let $R = \{(a,b) : a \in A, b \in B \text{ and } a - b \text{ is even}\}$. *Here R is an empty relation.*

b) Universal relation: A relation R from A to B is said to be the universal relation if $\mathrm{R} = \mathrm{A} \times \mathrm{B}$.

For example, *let* $A = \{1,2\}, B = \{1,3\}$. *Let* $R = \{(1,1),(1,3),(2,1),(2,3)\}$. *Here* $R = A \times B$, *so relation R is a universal relation.*

05. RELATION ON A SET & ITS VARIOUS TYPES

A relation R from a non-empty set A into itself is called a relation on A. In other words if A is a non-empty set, then a subset of $\mathrm{A} \times \mathrm{A} = \mathrm{A}^2$ is called a relation on A.

Illustrations:

Let $A = \{1,2,3\}$ *and* $R = \{(3,1),(3,2),(2,1)\}$. *Here R is relation on set A.*

<u>NOTE</u> **If A be a finite set having *n* elements then, number of relations on set A is $2^{n \times n}$ *i.e.,* 2^{n^2}.**

a) Empty relation: A relation R on a set A is said to be empty relation or a void relation if $\mathrm{R} = \phi$. In other words, a relation R in a set A is empty relation, if no element of A is related to any element of A, i.e., $\mathrm{R} = \phi \subset \mathrm{A} \times \mathrm{A}$.

For example, *let* $A = \{1,3\}$, $R = \{(a,b) : a \in A, b \in A \text{ and } a + b \text{ is odd}\}$. *Here R contains no element, therefore it is an empty relation on set A.*

b) **Universal relation:** A relation R on a set A is said to be the universal relation on A if $\mathrm{R}=\mathrm{A}\times\mathrm{A}$ i.e., $\mathrm{R}=\mathrm{A}^2$. In other words, a relation R in a set A is universal relation, if each element of A is related to every element of A, i.e., $\mathrm{R}=\mathrm{A}\times\mathrm{A}$.

For example, *let* $A=\{1,2\}$. *Let* $R=\{(1,1),(1,2),(2,1),(2,2)\}$. *Here* $R=A\times A$, *so relation R is universal relation on A.*

<u>NOTE</u> The void relation *i.e.,* ϕ and universal relation *i.e.,* $\mathrm{A}\times\mathrm{A}$ on A are respectively the *smallest* and *largest* relations defined on the set A. Also these are sometimes called *Trivial Relations*. And, any other relation is called a *non-trivial relation*.

❖ **The relations $\mathrm{R}=\phi$ and $\mathrm{R}=\mathrm{A}\times\mathrm{A}$ are two *extreme relations*.**

c) **Identity relation:** A relation R on a set A is said to be the identity relation on A if $\mathrm{R}=\{(a,b): a\in\mathrm{A}, b\in\mathrm{A} \text{ and } a=b\}$.

Thus identity relation $\mathrm{R}=\{(a,a):\ \forall\ a\in\mathrm{A}\}$.

The identity relation on set A is also denoted by $\mathrm{I_A}$.

For example, *let* $A=\{1,2,3,4\}$. *Then* $\mathrm{I_A}=\{(1,1),(2,2),(3,3),(4,4)\}$. *But the relation given by* $R=\{(1,1),(2,2),(1,3),(4,4)\}$ *is not an identity relation because element 1 is related to elements 1 and 3.*

<u>NOTE</u> In an identity relation on A every element of A should be related to itself only.

d) **Reflexive relation:** A relation R on a set A is said to be reflexive if $a\,\mathrm{R}\,a\ \forall a\in\mathrm{A}$ i.e., $(a,a)\in\mathrm{R}\ \forall a\in\mathrm{A}$.

For example, *let* $A=\{1,2,3\}$, *and* R_1, R_2, R_3 *be the relations given as* $R_1=\{(1,1),(2,2),(3,3)\}$, $R_2=\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3)\}$ *and* $R_3=\{(2,2),(2,3),(3,2),(1,1)\}$. *Here* R_1 *and* R_2 *are reflexive relations on A but* R_3 *is not reflexive as* $3\in A$ *but* $(3,3)\notin R_3$.

<u>NOTE</u> The identity relation is always a reflexive relation but the opposite may or may not be true. As shown in the example above, $\mathbf{R_1}$ is both identity as well as reflexive relation on A but $\mathbf{R_2}$ is only reflexive relation on A.

e) **Symmetric relation:** A relation R defined on a set A is symmetric if $(a,b)\in\mathrm{R} \Rightarrow (b,a)\in\mathrm{R}\ \forall a,b\in\mathrm{A}$ i.e., $a\,\mathrm{R}\,b\Rightarrow b\,\mathrm{R}\,a$ (i.e., whenever $a\mathrm{R}b$ then, $b\mathrm{R}a$).

For example, *let* $A=\{1,2,3\}$, $R_1=\{(1,2),(2,1)\}$, $R_2=\{(1,2),(2,1),(1,3),(3,1)\}$, $R_3=\{(2,3),(3,2),(2,2),(2,2)\}$ *i.e.* $R_3=\{(2,3),(3,2),(2,2)\}$ *and* $R_4=\{(2,3),(3,1),(1,3)\}$. *Here* R_1, R_2 *and* R_3 *are symmetric relations on A. But* R_4 *is not symmetric because* $(2,3)\in R_4$ *but* $(3,2)\notin R_4$.

f) **Transitive relation:** A relation R on a set A is transitive if $(a,b)\in\mathrm{R}$ and $(b,c)\in\mathrm{R}\Rightarrow(a,c)\in\mathrm{R}$ i.e., $a\,\mathrm{R}\,b$ and $b\,\mathrm{R}\,c\Rightarrow a\,\mathrm{R}\,c$.

For example, *let* $A=\{1,2,3\}$, $R_1=\{(1,2),(2,3),(1,3),(3,2)\}$ *and* $R_2=\{(1,3),(3,2),(1,2)\}$. *Here* R_2 *is transitive relation whereas* R_1 *is not transitive because* $(2,3)\in R_1$ *and* $(3,2)\in R_1$ *but* $(2,2)\notin R_1$.

g) **Equivalence relation:** Let A be a non-empty set, then a relation R on A is said to be an equivalence relation if

(i) R is reflexive i.e. $(a,a)\in\mathrm{R}\ \forall a\in\mathrm{A}$ i.e., $a\mathrm{R}a$.

(ii) R is symmetric i.e. $(a,b)\in\mathrm{R}\Rightarrow(b,a)\in\mathrm{R}\ \forall a,b\in\mathrm{A}$ i.e., $a\mathrm{R}b\Rightarrow b\mathrm{R}a$.

(iii) R is transitive i.e. $(a,b)\in\mathrm{R}$ and $(b,c)\in\mathrm{R}\Rightarrow(a,c)\in\mathrm{R}\ \forall a,b,c\in\mathrm{A}$ i.e., $a\mathrm{R}b$ and

$bRc \Rightarrow aRc$.

For example, *let* $A = \{1,2,3\}$, $R = \{(1,2),(1,1),(2,1),(2,2),(3,3)\}$. *Here R is reflexive, symmetric and transitive. So R is an equivalence relation on A.*

❖ ***Equivalence classes****:* Let A be an equivalence relation in a set A and let $a \in A$. Then, the set of all those elements of A which are related to a, is called equivalence class determined by a and it is denoted by $[a]$. Thus, $[a] = \{b \in A : (a, b) \in A\}$.

<u>NOTE</u> (i) Two equivalence classes are either disjoint or identical.
(ii) An equivalence relation R on a set A partitions the set into mutually disjoint equivalence classes.

An important property of an equivalence relation is that it divides the set into pair-wise disjoint subsets called **equivalence classes** whose collection is called **a partition of the set**. Note that the union of all equivalence classes gives the whole set.
e.g. Let R denotes the equivalence relation in the set Z of integers given by R = {(a, b) : 2 divides $a-b$}. Then the equivalence class [0] is [0] = $\{0, \pm 2, \pm 4, \pm 6, \ldots\}$

06. TABULAR REPRESENTATION OF A RELATION

In this form of representation of a relation R from set A to set B, elements of A and B are written in the first column and first row respectively. If $(a,b) \in R$ then we write '1' in the row containing a and column containing b and if $(a,b) \notin R$ then we write '0' in the same manner.

For example, *let* $A = \{1,2,3\}, B = \{2,5\}$ *and* $R = \{(1,2),(2,5),(3,2)\}$ *then,*

R	***2***	***5***
1	*1*	*0*
2	*0*	*1*
3	*1*	*0*

07. INVERSE RELATION

Let $R \subseteq A \times B$ be a relation from A to B. Then, the inverse relation of R, to be denoted by R^{-1}, is a relation from B to A defined by $R^{-1} = \{(b,a) : (a,b) \in R\}$.

Thus $(a,b) \in R \Leftrightarrow (b,a) \in R^{-1}\ \forall a \in A, b \in B$.

Clearly, $\text{Dom.}(R^{-1}) = \text{Range of } R$, $\text{Range of } R^{-1} = \text{Dom.}(R)$.

Also, $(R^{-1})^{-1} = R$.

For example, *let* $A = \{1,2,4\}, B = \{3,0\}$ *and let* $R = \{(1,3),(4,0),(2,3)\}$ *be a relation from A to B then,* $R^{-1} = \{(3,1),(0,4),(3,2)\}$.

Summing up all the discussion given above, here is a recap of all these for quick grasp:

01.	**a)** A relation R from A to B is an empty relation or void relation iff $R = \phi$.
	b) A relation R on a set A is an empty relation or void relation iff $R = \phi$.
02.	**a)** A relation R from A to B is a universal relation iff $R = A \times B$.
	b) A relation R on a set A is a universal relation iff $R = A \times A$.
03.	A relation R on a set A is reflexive iff $aRa, \forall a \in A$.
04.	A relation R on set A is symmetric iff whenever aRb, then bRa for all $a,b \in A$.

05.	A relation R on a set A is transitive iff whenever $a\text{R}b$ and $b\text{R}c$, then $a\text{R}c$.
06.	A relation R on A is identity relation iff $\text{R}=\{(a,a),\forall a\in\text{A}\}$ i.e., R contains only elements of the type $(a,a)\forall a\in\text{A}$ and it contains no other element.
07.	A relation R on a non-empty set A is an equivalence relation iff the following conditions are satisfied: i) **R is reflexive** i.e., for every $a\in\text{A}$, $(a,a)\in\text{R}$ i.e., $a\text{R}a$. ii) **R is symmetric** i.e., for $a,b\in\text{A}$, $a\text{R}b\Rightarrow b\text{R}a$ i.e., $(a,b)\in\text{R}\Rightarrow(b,a)\in\text{R}$. iii) **R is transitive** i.e., for all $a,b,c\in\text{A}$ we have, $a\text{R}b$ and $b\text{R}c\Rightarrow a\text{R}c$ i.e., $(a,b)\in\text{R}$ and $(b,c)\in\text{R}\Rightarrow(a,c)\in\text{R}$.

EXERCISE FOR PRACTICE

Q01. Let $\text{N}=$ set of all natural numbers and $\text{R}=\{(x,y):x+2y=0;y\in\text{N}\}$. Is R a relation on N? Give the reason in support of your answer.

Q02. Let $\text{A}=\{2,4,5\}$, $\text{B}=\{1,2,3,4,6,8\}$ and let R be a relation from A to B defined by $x\text{R}y\Leftrightarrow x$ divides y. Find the relation (in roster form), its domain and range.

Q03. Let $\text{A}=\{1,2,3,4,6\}$ and let R be a relation on A defined by $\text{R}=\{(a,b):a,b\in\text{A};a\text{ divides }b\}$. Find the relation, its domain and range.

Q04. Determine the domain and range of the relation $\text{R}=\{(x,y):y=|x-1|,x\in\text{Z and }|x|\le 3\}$.

Q05. Write the domain and range of relation $\text{R}=\{(x+1,x+5):x\in\{0,1,2,3,4,5\}\}$.

Q06. Let $\text{A}=\{1,2,3,4,5\}$ and $\text{B}=\{1,2,3,4,...,65\}$. Let R be a relation from A to B defined by $a\text{R}b$ iff *a is cube root of b*. Find R and its domain and range.

Q07. Let $\text{R}=\{(1,-1),(2,0),(3,1),(5,3)\}$. Find the inverse of R i.e., R^{-1} and its domain and range.

Q08. Let $\text{A}=\{1,2\}$. How many relations are possible on set A. List all of the relations.

Q09. Let $\text{A}=\{3,5\}$, $\text{B}=\{7,11\}$. Let $\text{R}_1=\{(a,b):a\in\text{A},b\in\text{B},\ a-b\text{ is odd}\}$ and $\text{R}_2=\{(a,b):a\in\text{A},b\in\text{B},\ a-b\text{ is even}\}$. Show that the relations R_1 and R_2 are respectively empty and universal relation from A into B.

Q10. Let $\text{A}=\{1,2,3\}$ and $\text{R}=\{(a,b):a,b\in\text{A},a\text{ divides }b\text{ and }b\text{ divides }a\}$. Show that R is an identity relation on A.

Q11. Let N be the set of all natural numbers and the relation R on N be defined by $x\text{R}y\Leftrightarrow x$ divides $y\ \forall\, x,y\in\text{N}$. Examine whether R is reflexive, symmetric or transitive.

Q12. Check whether the relation R defined in the set $\text{A}=\{1,2,3,4,5,6\}$ as $\text{R}=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive.

Q13. Show that the relation R in the set $\text{A}=\{1,2,3\}$ given by $\text{R}=\{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is reflexive but neither symmetric nor transitive. Is it equivalence relation? Why?

Q14. Check if the relation R in the set $\text{A}=\{1,2,3,...,14\}$ defined by $\text{R}=\{(x,\ y):3x-y=0\}$ is reflexive, symmetric or transitive.

Q15. Determine whether the relation R which is defined in the set $\text{A}=\{1,2,3,4,5,6\}$ as $\text{R}=\{(x,y):y\text{ is divisible by }x\}$ is reflexive, symmetric or transitive.

Q16. Show that the relation R in the set Z of integers given by $\text{R}=\{(a,b):2\text{ divides }a-b\}$ is an equivalence relation.

Q17. Show that the relation R on set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b):|a-b| \text{ is even}\}$ is equivalence relation.

Q18. Let R be a relation on the set of all lines in a plane defined by $(l_1,l_2)\in R \Leftrightarrow l_1$ is parallel to l_2. Show that R is an equivalence relation.

Q19. Prove that the relation R on the set Z of all integers defined by $(x,y)\in R \Leftrightarrow x-y$ is divisible by n, is an equivalence relation on Z.

Q20. **a)** Let A = {1, 2, 3}. Then show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is four.

b) Show that the number of equivalence relation in the set $\{1,2,3\}$ containing $(1,2)$ and $(2,1)$ is two.

Q21. Show that the relation R on the set $A=\{x\in Z: 0\le x\le 12\}$, given by $R=\{(a,b):|a-b| \text{ is a multiple of } 4\}$ is an equivalence relation.

Hence find the set of all elements related to 1 in R.

Q22. Let R be the relation in the set N given by $R=\{(a,b): a=b-2,\ b>6\}$. Which of the following is true?

a) $(2,4)\in R$ b) $(3,8)\in R$ c) $(6,8)\in R$ d) $(8,7)\in R$.

Q23. If R_1 and R_2 are equivalence relations in a set A, show that $R_1\cap R_2$ is also an equivalence relation.

Q24. Let T be the set of all triangles in a plane with R a relation in T given by R = {(T_1, T_2) : T_1 is congruent to T_2}. Show that R is an equivalence relation.

OR Let T be the set of all triangles in a plane with R as relation in T given by $R=\{(T_1,T_2): T_1\cong T_2\}$. Show that R is an equivalence relation.

Q25. Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is perpendicular to L_2}. Show that R is symmetric but neither reflexive nor transitive.

Q26. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if $xv = yu$. Show that R is an equivalence relation.

Q27. Show that the relation R on the set R of real nos., defined as $R=\{(a,b): a\le b^2\}$ is neither reflexive nor symmetric nor transitive.

Q28. Let A = {1, 2, 3}. Find the number of equivalence relations containing (1, 2).

Q29. Show that the relation R in the set $A=\{x\in Z: 0\le x\le 12\}$ is an equivalence relation where R = { (a, b) : a = b}. Hence find the set of all elements related to 1 in R.

Q30. Let Z be the set of all integers and R be a relation on Z defined as $R=\{(a,b): a,b\in Z \text{ and } (a-b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

Q31. Show that the relation R in the set A = {1,2,3,4,5} given by R = {(a, b): $|a - b|$ is even} is an equivalence relation. Show that all the elements of {1,3,5} are related to each other and all the elements of {2,4} are related to each other. But no element of {1,3,5} is related to any element of {2,4}.

Q32. Let A = {1, 2, 3}. Find the number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive.

Q33. Let A = {1, 2, 3, …, 9} and R be the relation in $A\times A$ defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in $A\times A$. Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

Q34. (a) If R = {(x, y) : x + 2y = 8} is a relation on N, write the range of R.

(b) Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Write the equivalence class [0].

(c) Write the smallest equivalence relation R on set A = {1, 2, 3}.

(d) Let R = {(a, a^3) : a is a prime number less than 5} be a relation. Find the range of R.

Q35. Determine whether the relation R defined on the set R of all real numbers as $R = \{(a, b) : a, b \in R$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive.

Q36. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)R(c,d)$ if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

FUNCTIONS & ITS VARIOUS TYPES

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. CONSTANT & TYPES OF VARIABLES

a) **Constant:** A constant is a symbol which retains the same value throughout a set of operations. So, a symbol which denotes a particular number is a constant. Constants are usually denoted by the symbols ***a, b, c, k, l, m,…*** etc.

b) **Variable:** It is a symbol which takes a number of values i.e., it can take any arbitrary values over the interval on which it has been defined. For example if x is a variable over R (set of real numbers) then we mean that x can denote any arbitrary real number. Variables are usually denoted by the symbols ***x, y, z, u, v,…*** etc.

c) **Independent variable:** That variable which can take an arbitrary value from a given set is termed as an independent variable.

d) **Dependent variable:** That variable whose value depends on the independent variable is called a dependent variable.

02. Defining A Function: Consider A and B be two non- empty sets then, a rule *f* which associates **each element of A with a unique element of B** is called a *function* or the *mapping from A to B* or *f maps A to B*. If *f* is a mapping from *A* to *B* then, we write $f : A \to B$ which is read as *'f is a mapping from A to B'* or *'f is a function from A to B'*.

If *f* associates $a \in A$ to $b \in B$, then we say that '***b* is the image of the element *a* under the function *f*'** or '***b* is the *f* - image of *a***' or '**the value of *f* at *a***' and denote it by $f(a)$ and we write $b = f(a)$. The element *a* is called the **pre-image** or **inverse-image of *b***.

Thus for a function from A to B,

(i) A and B should be non-empty.

(ii) Each element of A should have image in B.

(iii) No element of A should have more than one image in B.

(iv) If A and B have respectively *m* and *n* number of elements then the **number of functions defined from A to B is n^m**.

03. Domain, Co-domain & Range of a function

The **set A is called the domain** of the function *f* and the **set B is called the co- domain**. The set of the images of all the elements of A under the function *f* is called the **range of the function *f*** and is denoted as $f(A)$.

Thus range of the function *f* is $f(A) = \{f(x) : x \in A\}$.

Clearly $f(A) \subseteq B$.

☞ **Note the followings :**

(i) It is necessary that every f-image is in B; but there may be some elements in B which are not the *f*-images of any element of A i.e., whose pre-image under *f* is not in A.

(ii) Two or more elements of A may have same image in B.

(iii) $f : x \to y$ means that under the function *f* from A to B, an element *x* of A has image *y* in B.

(iv) Usually we denote the function *f* by writing $y = f(x)$ and read it as '***y* is a function of *x***'.

POINTS TO REMEMBER FOR FINDING THE DOMAIN & RANGE

Domain: If a function is expressed in the form $y = f(x)$, then domain of f means **set of all those real values of x for which y is real (i.e., y is well - defined)**.

❖ Remember the following points:

(i) *Negative number should not occur under the square root (even root) i.e., expression under the square root sign must be always* ≥ 0.

(ii) *Denominator should never be zero.*

(iii) *For* $\log_b a$ *to be defined* $a > 0$, $b > 0$ *and* $b \neq 1$. *Also note that* $\log_b 1$ *is equal to zero i.e.* 0.

Range: If a function is expressed in the form $y = f(x)$, then range of f means **set of all possible real values of y corresponding to every value of x in its domain**.

❖ Remember the following points:

(i) *Firstly find the domain of the given function.*

(ii) *If the domain does not contain an interval, then find the values of y putting these values of x from the domain. The set of all these values of y obtained will be the range.*

(iii) *If domain is the set of all real numbers R or set of all real numbers except a few points, then express x in terms of y and from this find the real values of y for which x is real and belongs to the domain.*

04. Function as a special type of relation: A relation f from a set A to another set B is said be a function (or mapping) from A to B if with every element (say x) of A, the relation f relates a unique element (say y) of B. This y is called f-image of x. Also x is called pre-image of y under f.

05. Difference between relation and function: A relation from a set A to another set B is any subset of $A \times B$; while a function f from A to B is a subset of $A \times B$ satisfying following conditions:

(i) For every $x \in A$, there exists $y \in B$ such that $(x, y) \in f$

(ii) If $(x, y) \in f$ and $(x, z) \in f$ then, $y = z$.

Sl. No.	Function	Relation
01.	Each element of A must be related to some element of B.	There may be some element of A which are not related to any element of B.
02.	An element of A should not be related to more than one element of B.	An element of A may be related to more than one elements of B.

06. Real valued function of a real variable: If the domain and range of a function f are subsets of R (the set of real numbers), then f is said to be **a real valued function of a real variable** or **a real function**.

07. Some important real functions and their domain & range

FUNCTION	REPRESENTATION	DOMAIN	RANGE
a) Identity function	$I(x) = x \ \forall x \in R$	R	R
b) Modulus function or Absolute value function	$f(x) = \lvert x \rvert = \begin{cases} x & if \ x \geq 0 \\ -x & if \ x < 0 \end{cases}$	R	$[0, \infty)$
c) Greatest integer function or Integral function or Step function	$f(x) = [x]$ or $f(x) = \lfloor x \rfloor \ \forall x \in R$	R	Z
d) Smallest integer function	$f(x) = \lceil x \rceil \ \forall x \in R$	R	Z
e) Signum function	$f(x) = \begin{cases} \frac{\lvert x \rvert}{x} & if \ x \neq 0 \\ 0 & if \ x = 0 \end{cases}$ i.e., $f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$	R	$\{-1, 0, 1\}$

f) Exponential function	$f(x)=a^x \ \forall a\neq 1, a>0$	R	$(0,\infty)$
g) Logarithmic function	$f(x)=\log_a x \ \forall a\neq 1, a>0$ and $x>0$	$(0,\infty)$	R

08. TYPES OF FUNCTIONS

a) One-one function (Injective function or Injection): A function $f:\mathrm{A}\to\mathrm{B}$ is one- one function or injective function if distinct elements of A have distinct images in B.

Thus, $f:\mathrm{A}\to\mathrm{B}$ is one-one $\Leftrightarrow f(a)=f(b)\Rightarrow a=b \ \forall a,b\in\mathrm{A}$
$\Leftrightarrow a\neq b\Rightarrow f(a)\neq f(b) \ \forall a,b\in\mathrm{A}$.

❖ If A and B are two sets having m and n elements respectively such that $m\leq n$, then **total number of one-one functions** from set A to set B is ${}^n\mathrm{C}_m\times m!$ *i.e.*, ${}^n\mathrm{P}_m$.

❖ If n(A) = n then the number of injective functions defined from A onto itself is $n!$.

<u>ALGORITHM TO CHECK THE INJECTIVITY OF A FUNCTION</u>

STEP1- Take any two arbitrary elements *a, b* in the domain of *f*.

STEP2- Put $f(a)=f(b)$.

STEP3- Solve $f(a)=f(b)$. If it gives $a=b$ only, then *f* is a one-one function.

b) Onto function (Surjective function or Surjection): A function $f:\mathrm{A}\to\mathrm{B}$ is onto function or a surjective function if every element of B is the *f*-image of some element of A. That implies $f(\mathrm{A})=\mathrm{B}$ or range of *f* is the co-domain of *f*.

Thus, $f:\mathrm{A}\to\mathrm{B}$ is onto $\Leftrightarrow f(\mathrm{A})=\mathrm{B}$ *i.e.*, range of f = co-domain of f.

<u>ALGORITHM TO CHECK THE SURJECTIVITY OF A FUNCTION</u>

STEP1- Take an element $b\in\mathrm{B}$.

STEP2- Put $f(x)=b$.

STEP3- Solve the equation $f(x)=b$ for *x* and obtain *x* in terms of *b*. Let $x=g(b)$.

STEP4- If for all values of $b\in\mathrm{B}$, the values of *x* obtained from $x=g(b)$ are in A, then *f* is onto. If there are some $b\in\mathrm{B}$ for which values of *x*, given by $x=g(b)$, is not in A. Then *f* is not onto.

c) One-one onto function (Bijective function or Bijection): A function $f:\mathrm{A}\to\mathrm{B}$ is said to be one-one onto or bijective if it is both one-one and onto i.e., if the distinct elements of A have distinct images in B and each element of B is the image of some element of A.

❖ Also note that a **bijective function is also called a one-to-one function or one-to-one correspondence.**

❖ If $f:\mathrm{A}\to\mathrm{B}$ is a function such that,
i) f is one-one $\Rightarrow n(\mathrm{A})\leq n(\mathrm{B})$.
ii) f is onto $\Rightarrow n(B)\leq n(\mathrm{A})$.
iii) f is one-one onto $\Rightarrow n(\mathrm{A})=n(\mathrm{B})$.

❖ For an ordinary finite set A, a one-one function $f:\mathrm{A}\to\mathrm{A}$ is necessarily onto and an onto function $f:\mathrm{A}\to\mathrm{A}$ is necessarily one-one for every finite set A.

d) Identity Function: The function $I_A:\mathrm{A}\to\mathrm{A}$; $I_\mathrm{A}(x)=x\,\forall x\in\mathrm{A}$ is called an identity function on A.

<u>NOTE</u> Domain$\left(I_\mathrm{A}\right)=\mathrm{A}$ and Range$\left(I_\mathrm{A}\right)=\mathrm{A}$.

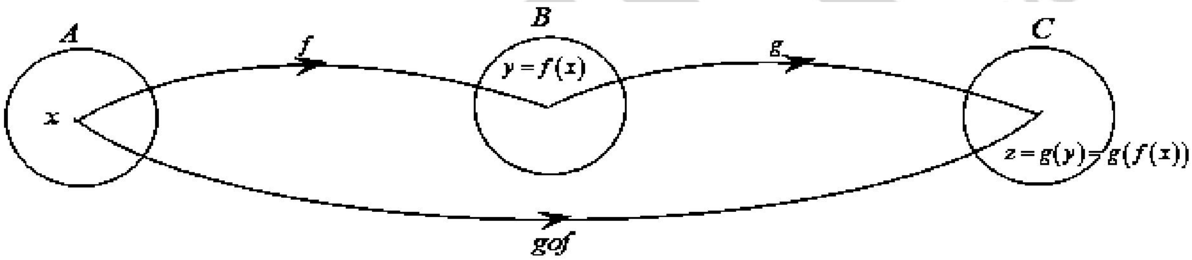
e) Equal Functions: Two function *f* and *g* having the same domain *D* are said to be equal if $f(x)=g(x)$ for all $x\in D$.

09. COMPOSITION OF FUNCTIONS

Let $f : A \to B$ and $g : B \to C$ be any two functions. Then f maps an element $x \in A$ to an element $f(x) = y \in B$; and this y is mapped by g to an element $z \in C$. Thus, $z = g(y) = g\left(f(x)\right)$. Therefore, we have a rule which associates with each $x \in A$, a unique element $z = g\left(f(x)\right)$ of C. This rule is therefore a mapping from A to C. We denote this mapping by gof (read as ***g composition f***) and call it the '**composite mapping of *f* and *g***'.

Definition: *If $f : A \to B$ and $g : B \to C$ be any two mappings (functions), then the composite mapping gof of f and g is defined by $gof : A \to C$ such that $\left(gof\right)(x) = g\left(f(x)\right)$ for all $x \in A$.*

- The composite of two functions is also called the **resultant of two functions** or **the function of a function**.
- Note that for the composite function *gof* to exist, it is essential that range of *f* must be a subset of the domain of *g*.
- Observe that the order of events occur from the right to left i.e., *gof* reads composite of *f* and *g* and it means that we have to first apply *f* and then, follow it up with *g*.
- $Dom.(gof) = Dom.(f)$ and $Co\text{-}dom.(gof) = Co\text{-}dom.(g)$.
- Remember that gof is well defined only if $Co\text{-}dom.(f) = Dom.(g)$.
- If gof is defined then, it is not necessary that fog is also defined.
- If gof is one-one then *f* is also one-one function. Similarly, if gof is onto then *g* is onto function.



10. INVERSE OF A FUNCTION

Let $f : A \to B$ be a bijection. Then a function $g : B \to A$ which associates each element $y \in B$ to a unique element $x \in A$ such that $f(x) = y$ is called the inverse of *f* i.e., $f(x) = y \Leftrightarrow g(y) = x$.

The inverse of *f* is generally denoted by f^{-1}.

Thus, if $f : A \to B$ is a bijection, then a function $f^{-1} : B \to A$ is such that $f(x) = y \Leftrightarrow f^{-1}(y) = x$.

ALGORITHM TO FIND THE INVERSE OF A FUNCTION

STEP1- Put $f(x) = y$ where $y \in B$ and $x \in A$.

STEP2- Solve $f\left(x\right) = y$ to obtain x in terms of y.

STEP3- Replace x by $f^{-1}(y)$ in the relation obtained in STEP2.

STEP4- In order to get the required inverse of f i.e. $f^{-1}(x)$, replace y by x in the expression obtained in STEP3 i.e. in the expression $f^{-1}(y)$.

EXERCISE FOR PRACTICE

Q01. Check whether $f : R \to R$ given as $f\left(x\right) = x^3 + 2$ for all $x \in R$ is one- one or not.

Q02. Discuss the surjectivity of $f : Z \to Z$ given as $f\left(x\right) = 3x + 2 \quad \forall x \in Z$.

Q03. Prove that $f : Q \to Q$ given by $f\left(x\right) = 2x - 3 \quad \forall x \in Q$ is a bijection.

Q04. Show that $f : N \to N$ given by $f\left(x\right) = 2x$ is one- one but not onto.

Q05. Show that $f : R \to R$ given by $f\left(x\right) = x^2$ is neither one- one nor onto.

Q06. For the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=2x$, prove that the function f is one- one and onto both. Is it a bijection?

Q07. Show that $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=ax+b$ where $a,b\in\mathbb{R},\ a\neq 0$ is a bijection.

Q08. Show that $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=5x^3+4$ is bijective.

Q09. Let $A=\mathbb{R}-\{2\}$, $B=\mathbb{R}-\{1\}$. If $f:A\to B$ is a mapping defined by $f(x)=\dfrac{x-1}{x-2}$ then, show that f is bijection.

Q10. If $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=x^2$ and $g:\mathbb{R}\to\mathbb{R}$ be another function defined by $g(x)=2x+1$ then, find *fog* and *gof.* Also deduce that $fog\neq gof$.

Q11. Find *fog* and *gof* if $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ are given by $f(x)=|x|$ and $g(x)=|5x-2|$.

Q12. If $f:\mathbb{R}\to\mathbb{R}$; $f(x)=\sin x$ and $g:\mathbb{R}\to\mathbb{R}$; $g(x)=x^2$, find *gof* and *fog.*

Q13. **a)** If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=x^2-3x+2$, find $f(f(x))$.

b) If $f:\mathbb{R}\to\mathbb{R}$ is given by $f(x)=(3-x^3)^{1/3}$, find $f(f(x))$.

Q14. If $f,g:\mathbb{R}\to\mathbb{R}$ are defined respectively by $f(x)=x^2+3x+1$, $g(x)=2x-3$, then find:

a) *fog* b) *gof* c) *fof* d) *gog* .

Q15. Let $f,g:\mathbb{R}\to\mathbb{R}$ be two functions such that $fog(x)=\sin x^2$ and $gof(x)=\sin^2 x$. Find the functions $f(x)$ and $g(x)$.

Q16. **a)** If $f,g:\mathbb{R}\to\mathbb{R}$ are defined by $f(x)=x^2+1$ and $g(x)=\sin x$ then, find *fog* and *gof.*

b) If the mapping *f* and *g* are given by $f=\{(1,2),(3,5),(4,1)\}$ and $g=\{(2,3),(5,1),(1,3)\}$, find *fog*.

Q17. If $f(x)=e^x$ and $g(x)=\log_e x,\ x>0$ then, find *fog* and *gof.* Check whether $fog=gof$?

Q18. If $f(x)=\sqrt{x},\ x\geq 0$ and $g(x)=x^2-1$ are two real functions then, find *fog* and *gof.* Is $fog=gof$?

Q19. Show that the function $f:\mathbb{N}\to\mathbb{N}$ given by $f(1)=f(2)=1$ and $f(x)=x-1$ for all $x>2$ is onto but not one-one.

Q20. **a)** Show that $f:\mathbb{N}\to\mathbb{N}$ given by $f(x)=\begin{cases}x+1, & if\ \ x\ is\ odd\\ x-1, & if\ \ x\ is\ even\end{cases}$ is both one- one and onto.

b) Show that $f:\mathbb{N}\cup\{0\}\to\mathbb{N}\cup\{0\}$ given by $f(n)=\begin{cases}n+1, & if\ \ n\ is\ even\\ n-1, & if\ \ n\ is\ odd\end{cases}$ is bijection. Also show that $f^{-1}=f$.

c) Let $f:W\to W$, be defined as $f(x)=x-1$, if x is odd and $f(x)=x+1$, if x is even. Show that *f* is invertible. Find the inverse of *f*, where W is the set of all whole numbers.

Q21. **a)** Show that an onto function $f:\{1,2,3\}\to\{1,2,3\}$ is always one-one.

b) Show that a one-one function $f:\{1,2,3\}\to\{1,2,3\}$ must be onto.

Q22. Show that the signum function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}1, & if\ \ x>0\\ 0, & if\ \ x=0\\ -1, & if\ \ x<0\end{cases}$ is neither one-one nor onto.

Q23. Let $A=\mathbb{R}-\{3\}$ and $B=\mathbb{R}-\{1\}$, consider the function $f:A\to B$ defined by $f(x)=\dfrac{x-2}{x-3}$. Is f one-one and onto? Give reasons.

Q24. Show that if $f:\mathbb{R}-\left\{\dfrac{7}{5}\right\}\to\mathbb{R}-\left\{\dfrac{3}{5}\right\}$ is defined by $f(x)=\dfrac{3x+4}{5x-7}$ & $g:\mathbb{R}-\left\{\dfrac{3}{5}\right\}\to\mathbb{R}-\left\{\dfrac{7}{5}\right\}$

is defined by $g(x)=\dfrac{7x+4}{5x-3}$ then, $fog=I_A$ & $gof=I_B$, where $A=R-\left\{\dfrac{3}{5}\right\}$, $B=R-\left\{\dfrac{7}{5}\right\}$; $I_A(x)=x\ \forall x\in A$, $I_B(x)=x\ \forall x\in B$ are called the **identity functions** on the sets A and B respectively.

Q25. Show that if $f:A\to B$ and $g:B\to C$ are one-one then, $gof:A\to C$ is also one-one.

Q26. Show that if $f:A\to B$ and $g:B\to C$ are onto then, $gof:A\to C$ is also onto.

Q27. Let $f:\{2,3,4,5\}\to\{3,4,5,9\}$ and $g:\{3,4,5,9\}\to\{7,11,15\}$ be the functions which are defined as $f(2)=3$, $f(3)=4$, $f(4)=f(5)=5$ and $g(3)=g(4)=7$ and $g(5)=g(9)=11$. Find gof.

Q28. Find gof and fog if $f:R\to R$ and $g:R\to R$ are given by $f(x)=\cos x$ and $g(x)=3x^2$. Show that gof and fog are not the same.

Q29. Let $f:N\to Y$ be a function defined as $f(x)=4x+3$ where, $Y=\{y\in N: y=4x+3$ for some $x\in N\}$. Show that f is invertible. Find the inverse of f.

Q30. Let $Y=\{n^2: n\in N\}\subset N$. Consider $f:N\to Y$ as $f(n)=n^2$. Show that f is invertible and if the inverse exists, find it.

Q31. Let $f:N\to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f:N\to S$ where, S is the range of f, is invertible. Find the inverse of f.

Q32. Consider $f:N\to N$, $g:N\to N$ and $h:N\to R$ defined as $f(x)=2x$, $g(y)=3y+4$ and $h(z)=\sin z$ for all $x,y,z\in N$. Show that $ho(gof)=(hog)of$.

Q33. If $f(x)=\dfrac{4x+3}{6x-4}$, $x\neq\dfrac{2}{3}$, show that $fof(x)=x$ for all $x\neq\dfrac{2}{3}$. Write the expression for f^{-1}.

Q34. Consider $f:R_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible such that $f^{-1}(y)=\dfrac{\sqrt{y+6}-1}{3}$.

Q35. Let $f:R\to R$ be given by $f(x)=\dfrac{x^2+4x+30}{x^2-8x+18}$. Is f a one- one function?

Q36. Given $f(x)=[x]$ and $g(x)=|x|$, find the followings:

a) $(gof)(5/3)-(fog)(5/3)$ b) $(gof)(-5/3)-(fog)(-5/3)$ c) $(f+2g)(-1)$.

Q37. If $f(x)=\dfrac{1}{2x+1}$, $x\neq-\dfrac{1}{2}$ then, show that $f(f(x))=\dfrac{2x+1}{2x+3}$, $x\neq-\dfrac{1}{2},-\dfrac{3}{2}$.

Q38. If $f:R\to R$ is a bijection given by $f(x)=x^3+3$ then, find $f^{-1}(x)$.

Q39. Check whether $f:R-\{-1\}\to R-\{1\}$ defined by $f(x)=\dfrac{x}{x+1}$ is invertible. If it is invertible then, find f^{-1}.

Q40. If $A=\{0,1,2,3\}, B=\{7,9,11,13\}$ and a rule f from A to B is defined by $f(x)=2x+7\ \forall x\in A$, then prove that f is one-one and onto.

Q41. Let $A=\{1,2,3\}, B=\{2,4,6\}$. If $f:A\to B$ is a function defined as $f(1)=2$, $f(2)=4$, $f(3)=6$. Write down f^{-1} as a set of ordered pairs.

Q42. Let $f:R\to R$, $f(x)=ax+b$ for all $x\in R$. Find the constants a and b such that $fof=I_R$.

Q43. Find inverse of $f:R_0\to R_0$ defined as $f(x)=1/x$ for all $x\in R_0=R-\{0\}$, if it exists.

Q44. Let $A=\{1,2,3\}, B=\{4,5,6,7\}$ and let $f=\{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that f is a one-one function.

Q45. Let $f=\{(1,2),(2,3),(4,5)\}$, $g=\{(2,3),(3,5),(5,2)\}$. Find *fog* and *gof* whichever is possible.

Q46. If the function $f:\mathrm{R}\to(0,2)$ defined by $f(x)=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}+1$ is invertible then, find $f^{-1}(x)$.

Q47. Consider a function $f:[0,\pi/2]\to\mathrm{R}$ given by $f(x)=\sin x$ and $g:[0,\pi/2]\to\mathrm{R}$ given by $g(x)=\cos x$. Show that f and g are one-one, but $f+g$ is not one- one.

Q48. If $f(x)=x^2+1$ and $g(x)=1/(x-1)$ then, find $(fog)(x)$ and $(gof)(x)$.

Q49. Let $f:\mathrm{R}\to\mathrm{R}$ be defined as $f(x)=\cos(5x+2)$. Is f invertible? Justify your answer.

Q50. Find the number of all one-one functions from set A = {1, 2, 3} to itself.

Q51. **a)** If X and Y are two sets having 2 and 3 elements respectively then, find the number of functions from X to Y.

b) If A = {1, 2, 3} and B = {a, b}, write the total number of function from A to B.

c) If A = {a, b, c} and B = {–2, –1, 0, 1, 2}, write the total number of one-one functions defined from A to B.

d) Write the total number of one-one functions from {1, 2, 3, 4} to {a, b, c}.

e) Find the number of all onto functions from the set {1, 2, 3, ..., n} to itself.

Q52. What is the range of the function $f(x)=\dfrac{|x-1|}{(x-1)}$?

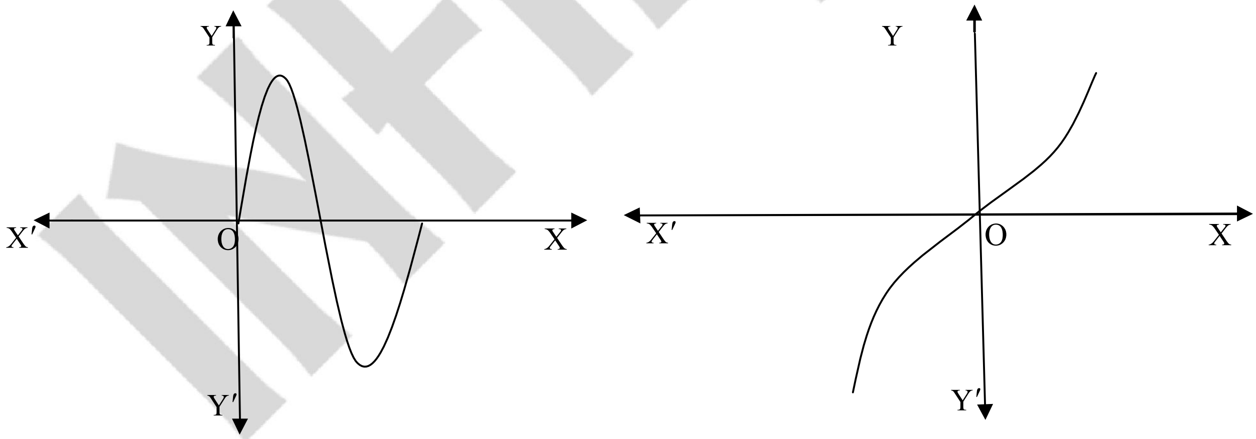
Q53. **(a)** If the function $f:[1,\infty)\to[1,\infty)$ defined by $f(x)=2^{x(x-1)}$ is invertible then, find $f^{-1}(x)$.

(b) If the function $f:\mathrm{R}\to(-1,1)$ defined by $f(x)=\dfrac{10^x-10^{-x}}{10^x+10^{-x}}$ is invertible then, find $f^{-1}(x)$.

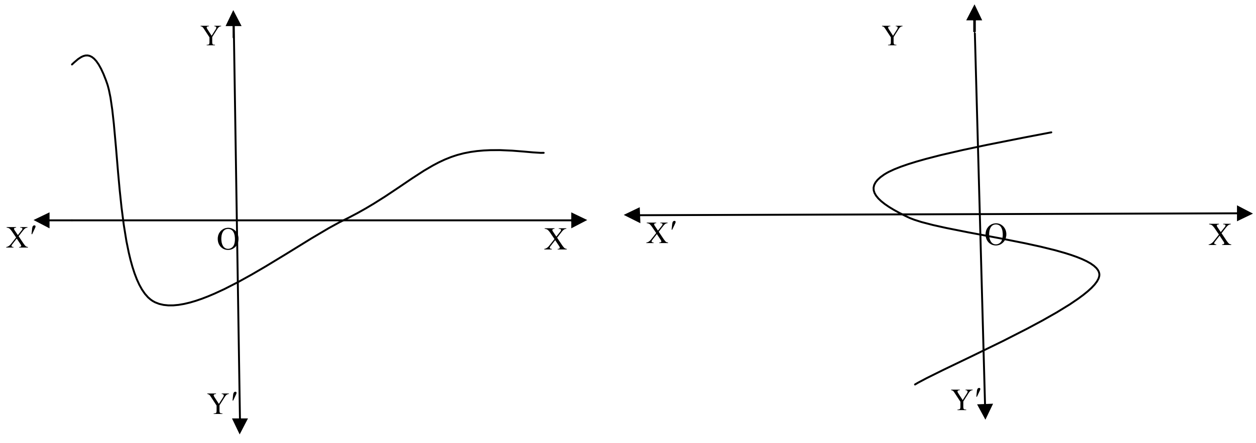
Q54. Find $f^{-1}(x)$, if $f(x)=[4-(x-7)^3]^{1/4}$.

Q55. Which one graph of the Fig.2 represents a one-one function?

Q56. Which one graph of the Fig.1 represents a function?



(a) Fig.1 (b)



(a) Fig.2 (b)

Q57. Let $f : \mathrm{N} \to \mathrm{N}$ be defined as $f(n) = \begin{cases} \dfrac{n+1}{2}, \text{ when } n \text{ is odd} \\ \dfrac{n}{2}, \text{ when } n \text{ is even} \end{cases}$ for all $n \in \mathrm{N}$.

State whether the function f is bijective. Justify your answer.

Q58. If the function $f : \mathrm{R} \to \mathrm{R}$ be given by $f(x) = x^2 + 2$ and $g : \mathrm{R} \to \mathrm{R}$ be given by $g(x) = \dfrac{x}{x-1}, x \neq 1$, find *fog* and *gof* and hence find *fog* (2) and *gof* (–3).

Q59. If *f*, *g* : R → R be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, for all $x \in \mathrm{R}$. Then find the value of *fog* and *gof*.

Q60. Consider $f : \mathrm{R}_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{54+5y} - 3}{5}$.

Q61. Prove that the function $f : \mathrm{N} \to \mathrm{N}$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Q62. Consider $f : \mathrm{R}_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that *f* is invertible with the inverse f^{-1} of *f* given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

TEN YEARS OF RELATIONS AND FUNCTIONS

✪ Note that **BINARY OPERATION has been Deleted** for session 2019-2020.

☞ Note that the Questions marked with (#) have been deleted for session 2019-20.

VERY SHORT ANSWER TYPE QUESTIONS – I ◘ 1 Mark

Q01. Write the smallest equivalence relation R on set A = {1, 2, 3}. **[SP 2014**

Q02. If $f : \mathrm{R} \to \mathrm{R}$ defined as $f(x) = \dfrac{2x-7}{4}$ is an invertible function, write $f^{-1}(x)$. **[Compt.'12**

#Q03. The binary operation $* : \mathrm{R} \times \mathrm{R} \to \mathrm{R}$ is defined as $a*b = 2a + b$. Find (2*3)*4. **[2012 AI**

#Q04. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z. **[2012 F**

#Q05. Let * be a binary operation on N given by a*b = LCM(a,b) $\forall$ a, b $\in$ N. Find 5*7. **[2012 D**

Q06. Find *fog* (x), if $f(x) = |x|$ and $g(x) = |5x - 2|$. **[2011 F**

Q07. If $f : \mathrm{R} \to \mathrm{R}$ is defined by $f(x) = 3x + 2$, define $f[f(x)]$. **[2011 F, '10 C**

Q08. Write *fog*, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$. **[2011 F**

Q09. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let *f* = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether *f* is one-one or not? **[2011 AI**

Q10. State the reason for the relation R in the set {1, 2, 3} given by $R = \{(1,2),(2,1)\}$ not to be transitive. **[2011 D**

Q11. What is the range of the function $f(x) = \dfrac{|x-1|}{(x-1)}$? **[2010 D**

Q12. If $f : R \to R$ is given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, determine $f(f(x))$. **[2010 AI**

Q13. If *f* is an invertible function, defined as $f(x) = \dfrac{3x-4}{5}$, write $f^{-1}(x)$. **[2010 F**

#Q14. If the binary operation * defined on Q, is defined as a * b = 2a + b – ab for all a, b$\in$Q, find the value of 3*4. **[2009 F**

#Q15. Let * be a binary operation on N given by a * b = HCF of a and b where a, b$\in$N. Write the value of 22*4. **[2009 AI**

#Q16. If binary operation * on the set of integers Z, is defined by a * b = a + 3b^2, then find the value of 2*4. **[2009 D**

Q17. If *f* (x) = x + 7 and *g* (x) = x – 7, x$\in$R, find *fog* (7). **[2008 D**

#Q18. Let * : $R \times R \to R$ given by (a, b) $\to$ a + 4b^2 is a binary operation. Compute (–5) * (2*0). **[Compt. 2014 AI**

Q19. Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Write the equivalence class [0]. **[Compt. 2014 D**

Q20. Let *f* : {1, 3, 4} $\to$ {1, 2, 5} and g : {1, 2, 5} $\to$ {1, 3} given by *f* = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down *gof*. **[Compt. 2014 AI**

Q21. State the reason why the relation $R = \{(a,b) : a \le b^2\}$ on the set **R** of real numbers is not reflexive. **[SP 2017**

#Q22. If * is a binary operation on the set **R** of real numbers defined by a * b = a + b – 2, then find the identity element for the binary operation *. **[SP 2017**

Q23. Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 3x^2 - 5$, $g(x) = \dfrac{x}{x^2+1}$. Find gof. **[SP '17**

Q24. Let A = {1, 2, 3, 4}. Let R be the equivalence relation on $A \times A$ defined by (a, b) R (c, d) iff $a + d = b + c$. Find the equivalence class [(1, 3)]. **[SP 2018**

#Q25. Determine whether the binary operation * on the set N of natural numbers defined by a*b = 2^{ab} is associative or not. **[SP 2018**

#Q26. If a * b denotes the larger of 'a' and 'b' and if $a \text{ o } b = (a * b) + 3$, then write the value of (5) o (10), where * and o are binary operations. **[Delhi 2018**

VERY SHORT ANSWER TYPE QUESTIONS – II ◘ 2 Marks

Q01. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all? Justify your answer. **[SP 2017**

SHORT ANSWER TYPE QUESTIONS ◘ 4 Marks

#Q01. Let S be the set of all rational numbers except 1 and * be defined on S by a*b = a + b – ab, $\forall a, b \in S$. Prove that :

a) * is a binary on S.

b) * is commutative as well as associative. Also find the identity element of *. **[SP 2014**

Q02. Prove that the function $f: \mathrm{N} \to \mathrm{N}$, defined by $f(x) = x^2 + x + 1$ is one-one function but not onto. **[SP 2013**

Q03. Let $\mathrm{A} = \mathrm{R} - \{2\}$ and $\mathrm{B} = \mathrm{R} - \{1\}$. If $f: \mathrm{A} \to \mathrm{B}$ is a function defined by $f(\mathrm{x}) = \dfrac{\mathrm{x}-1}{\mathrm{x}-2}$, show that f is one-one and onto. Hence find f^{-1}. **[Compt. 2013**

Q04. Let A = R – {2/3} and B = R – {2/3}. If $f: \mathrm{A} \to \mathrm{B}$ and $f(\mathrm{x}) = \dfrac{2\mathrm{x}-1}{3\mathrm{x}-2}$, then prove that the function f is one-one and onto. **[Compt. 2013**

Q05. Show that the function f in $\mathrm{A} = \mathrm{R} - \left\{\dfrac{2}{3}\right\}$ defined as $f(x) = \dfrac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}. **[2013 D**

Q06. If $f(x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$, show that $fof(x) = x$ for all $x \neq \dfrac{2}{3}$. What is the inverse of f? **['12 F**

Q07. Let $f: \mathrm{N} \to \mathrm{N}$ be defined as $f(n) = \begin{cases} \dfrac{n+1}{2}, \text{ when } n \text{ is odd} \\ \dfrac{n}{2}, \text{ when } n \text{ is even} \end{cases}$ for all $n \in \mathrm{N}$.

State whether the function f is bijective. Justify your answer. **[Compt. 2012, 09 AI**

Q08. Show that $f: \mathrm{N} \to \mathrm{N}$, given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one-one and onto. **['12 AI**

#Q09. Consider the binary operations *: R × R→R and o: R × R→R defined as $\mathrm{a} * \mathrm{b} = |\mathrm{a} - \mathrm{b}|$ and $\mathrm{a}\ o\ \mathrm{b} = \mathrm{a}$ for all $\mathrm{a}, \mathrm{b} \in \mathrm{R}$. Show that '*' is commutative but not associative, '*o*' is associative but not commutative. **[2012 AI**

Q10. Let $\mathrm{A} = \mathbb{R} - \{3\}$ and $\mathrm{B} = \mathbb{R} - \{1\}$. Consider the function $f: \mathrm{A} \to \mathrm{B}$ defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$.

Show that f is one-one and onto and hence find f^{-1}. **[2012 D**

OR Let $\mathrm{A} = \mathrm{R} - \{3\}$, $\mathrm{B} = \mathrm{R} - \{1\}$. Let $f: \mathrm{A} \to \mathrm{B}$ be defined by $f(\mathrm{x}) = \left(\dfrac{\mathrm{x}-2}{\mathrm{x}-3}\right)$, for all $\mathrm{x} \in \mathrm{A}$.

Then show that f is bijective. Hence find $f^{-1}(\mathrm{x})$. **[C'14 D**

Q11. Let $f: \mathrm{R} \to \mathrm{R}$ be defined as f (x) = 10x + 7. Find the function $g: \mathrm{R} \to \mathrm{R}$ such that $gof = fog = \mathrm{I_R}$. **[2011 AI**

#Q12. A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as :

$$\mathrm{a} * \mathrm{b} = \begin{cases} \mathrm{a}+\mathrm{b}, \text{ if } \mathrm{a}+\mathrm{b} < 6 \\ \mathrm{a}+\mathrm{b}-6, \text{ if } \mathrm{a}+\mathrm{b} \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is invertible with '6 – a', being the inverse of 'a'. **[2011 AI**

Q13. Consider $f: \mathrm{R}_+ \to [4, \infty]$ given by f (x) = x^2 + 4. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(\mathrm{y}) = \sqrt{\mathrm{y}-4}$, where R_+ is the set of all non-negative real nos. **[2011 AI**

#Q14. Is the binary operation defined on set N, given by $a * b = \dfrac{a+b}{2}$ for all a, b$\in$N, commutative? Is the above binary operation associative? **[2011, 08 D**

#Q15. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b = \min.\{a,b\}$. Write the operation table of the operation *. **[2011 D**

Q16. Show that the function $f : R \to R$ given by $f(x) = ax + b$, where $a, b \in R, a \neq 0$, is a bijection. **[Compt. 2010**

Q17. Consider $f : R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and $f^{-1}(y) = \left(\dfrac{\sqrt{y+6}-1}{3}\right)$. **[2010 F**

OR Consider $f : \mathbb{R}_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\dfrac{\sqrt{y+6}-1}{3}\right)$. Hence find (i) $f^{-1}(10)$ (ii) y if $f^{-1}(y) = 4/3$. **[2017 D for 6 Marks**

#Q18. Let $A = N \times N$ and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Also, find the identity element for * on A, if any. **['10 F**

Q19. Show that the relation R on the set $A = \{x \in Z : 0 \le x \le 12\}$, given by R = {(a, b) : |a – b| is multiple of 4} is an equivalence relation. **[2010 AI**

Q20. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a,b) : a, b \in Z$ and $(a-b)$ is divisible by $5\}$. Prove that R is an equivalence relation. **[2010 D**

#Q21. Let $*$ be a binary operation on Q defined by $a * b = \dfrac{3ab}{5}$. Show that the operation $*$ is commutative as well as associative. Also find its identity element, if it exists. **[2010 D**

Q22. Show that the relation S in the set R of real numbers, defined as $S = \{(a,b) : a, b \in R$ and $a \le b^3\}$ is neither reflexive, nor symmetric nor transitive. **[2010 D**

Q23. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. **[2009 D, 2015 AI for 6 Marks**

Q24. Show that the relation R on the set R of real numbers, defined as $R = \{(a,b) : a \le b^2\}$ is neither reflexive, nor symmetric nor transitive. **[2009 F**

Q25. Let T be the set of all triangles in a plane with R as relation in T given by R = {(T_1, T_2) : $T_1 \cong T_2$}. Show that R is an equivalence relation. **[2008 AI**

Q26. Let $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, for all $x \in R$. Then find fog and gof. **[Compt. 2014 AI**

#Q27. Let S be the set of all rational numbers except 1 and * be defined on S by a * b = a + b – ab, for all a, b$\in$S.

Prove that

(i) * is a binary operation on S.

(ii) * is commutative as well as associative. **[Compt. 2014 D**

Q28. Let R be a relation defined on the set of natural numbers N as follows :

$R = \{(x, y) : x \in N, y \in N$ and $2x + y = 24\}$. Find the domain and range of the relation R. Also, find if R is an equivalence relation or not. **[Compt. 2014 D**

Q29. Let A = {1, 2, 3, …, 9} and R be the relation in $A \times A$ defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)]. **[2014 Delhi**

LONG ANSWER TYPE QUESTIONS ◘ 6 Marks

Q01. Let N denote the set of all natural numbers and R be the relation on N×N defined by $(a,b)R(c,d)$ if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation. **[2015 Delhi**

Q02. Let $f: N \to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f: N \to S$, where S is the range of f, is invertible. Also find the inverse of f. **[2015 F**

Q03. Determine whether the relation R defined on the set R of all real numbers as $R=\{(a,b): a,b \in R$ and $a-b+\sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive. **[2015 AI**

#Q04. Let $A = R \times R$ and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Prove that * is commutative and associative. Find the identity element for * on A. Also write the inverse element of the element (3, –5) in A. **[2015 AI**

Q05. Consider $f: R_+ \to [-9, \infty)$ given by $f(x)=5x^2+6x-9$. Prove that f is invertible with $f^{-1}(y)=\dfrac{\sqrt{54+5y}-3}{5}$. **[2015 AI**

#Q06. A binary operation * is defined on the set $X=R-\{-1\}$ by $x*y=x+y+xy, \forall x,y \in X$. Check whether * is commutative and associative. Find the identity element and also find the inverse of each element of X. **[2015 AI**

#Q07. On the set {0, 1, 2, 3, 4, 5, 6}, a binary operation * is defined as : $a*b = \begin{cases} a+b, \text{ if } a+b<7 \\ a+b-7, \text{ if } a+b \geq 7 \end{cases}$.

Write the operation table of the operation * and prove that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with '7 – a' being the inverse of 'a'. **[2015 AI**

#Q08. Let $A=Q \times Q$, where Q is the set of all rational numbers, and * be a binary operation on A defined by $(a,b)*(c,d)=(ac, b+ad)$ for $(a,b),(c,d) \in A$. Then find

(i) The identity element of * in A.

(ii) Invertible elements of A, and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$. **['15 AI**

OR Let $A=Q \times Q$, where Q is the set of all rational numbers, and * be a binary operation defined on A by $(a,b)*(c,d)=(ac, b+ad)$ for all $(a,b),(c,d) \in A$. Then find

(i) the identity element in A.

(ii) the invertible element of A. **[2015 AI**

Q09. Let $f: W \to W$ be defined as $f(n)=\begin{cases} n-1, \text{ if } n \text{ is odd} \\ n+1, \text{ if } n \text{ is even} \end{cases}$.

Show that f is invertible and find the inverse of f. Here, W is the set of all whole numbers. **[2015 AI**

#Q10. Check whether the operation * defined on the set $A=R \times R$ as

$$(a,b)*(c,d)=(a+c,\ b+d)$$

is a binary operation or not, where R is the set of all real numbers. If it is a binary operation, is it commutative and associative too? Also find the identity element of *. **[2015 AI**

Q11. Let $A=\{-1,0,1,2\}$, $B=\{-4,-2,0,2\}$ and $f,\ g:A\to B$ be functions defined by $f(x)=x^2-x$, $x\in A$ and $g(x)=2\left|x-\frac{1}{2}\right|-1$, $x\in A$. Find $gof(x)$ and hence show that $f=g=gof$. **[2015 AI**

Q12. If the function $f: R\to R$ be defined by $f(x) = 2x - 3$ and $g: R\to R$ by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$. **[2015 AI**

Q13. Let $f:N\to N$ be a function defined as $f(x)=9x^2+6x-5$. Show that $f:N\to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$. **[2016 D**

Q14. If $f,g:R\to R$ be two functions defined as $f(x)=|x|+x$ and $g(x)=|x|-x$, $\forall\ x\in R$. Then find fog and gof. Hence find $fog(-3)$, $fog(5)$ and $gof(-2)$. **[2016 F**

Q15. Let $f:N\to N$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f:N\to S$ is invertible (where S is the range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$. **[2016 AI**

#Q16. Let $A=R\times R$ and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d).

Show that * is commutative and associative. Find the identity element for * on A. Also, find the inverse of every element $(a, b)\in A$. **[2016 AI**

#Q17. Show that the binary operation * on $A=R-\{-1\}$ defined as $a*b=a+b+ab$ for all $a,b\in A$ is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible. **[2016 AI**

Q18. Show that the relation R defined by $(a,b)R(c,d)\Rightarrow a+d=b+c$ on the $A\times A$, where $A=\{1,2,3,...,10\}$ is an equivalence relation.

Hence write the equivalence class [(3, 4)]; $a,b,c,d\in A$. **[2016 AI**

Q19. Show that the function $f:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\dfrac{x}{1+|x|}$, $x\in R$ is one-one and onto function. Hence find $f^{-1}(x)$. **[2016 AI**

Q20. Let $f:[0,\infty)\to \mathbf{R}$ be a function defined by $f(x)=9x^2+6x-5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse. **[2017 SP**

#Q21. Let * be a binary operation defined on $Q\times Q$ by (a, b) * (c, d) = (ac, b + ad), where Q is the set of rational numbers. Determine, whether * is commutative and associative. Find the identity element for * and the invertible elements of $Q\times Q$. **[2017 SP, 2017 AI**

Q22. Consider $f:\mathbb{R}-\left\{-\frac{4}{3}\right\}\to\mathbb{R}-\left\{\frac{4}{3}\right\}$ given by $f(x)=\dfrac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x)=2$. **[2017 AI**

Q23. Let $f:\mathbb{R}-\left\{-\frac{4}{3}\right\}\to\mathbb{R}$ be a function defined as $f(x)=\dfrac{4x}{3x+4}$. Show that, in $f:\mathbb{R}-\left\{-\frac{4}{3}\right\}\to$ Range of f, f is one-one and onto. Hence find f^{-1} in Range of $f\to\mathbb{R}-\left\{-\frac{4}{3}\right\}$. **['17 C Delhi**

#Q24. Let $A = \mathbb{R}\times\mathbb{R}$ and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A, if any. **['17 C Delhi**

#Q25. Determine whether the operation * defined below on Q is binary operation or not :

$a*b = ab+1$. If yes, check the commutative and associative properties. Check the existence of identity element and the inverse of all elements in Q. **[SP 2017**

#Q26. Discuss the commutativity and associativity of binary operation '*' defined on

$A = Q-\{1\}$ by the rule $a*b = a-b+ab$ for all $a, b \in A$. Also find the identity element of * in A and hence find the invertible elements of A. **[2017 D**

[**NOTE** that * is not binary here. Let's see how! Assume that $a, b \in A = Q-\{1\}$.

If we take $a = 0,\ b = -1$ then, $0*(-1) = 0-(-1)+0(-1) = 1 \notin A$. That is, $a*b \notin A\ \forall\ a, b \in A$.

Hence * is not binary operation.]

Q27. If the function $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x) = 2x-3$ and $g:\mathbb{R}\to\mathbb{R}$ by $g(x) = x^3+5$, then find fog and show that fog is invertible. Also find $(fog)^{-1}$, hence find $(fog)^{-1}$ (9). **[SP 2018**

#Q28. A binary operation * is defined on the set $\mathbb{R}$ of real numbers by

$a*b = \begin{cases} a, \text{ if } b = 0 \\ |a|+b, \text{ if } b \neq 0 \end{cases}$. If at least one of a and b is 0, then prove that a * b = b * a.

Check whether * is commutative. Find the identity element for *, if it exists. **[SP 2018**

Q29. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that
$R = \{(a, b) : a, b \in A, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation.
Find the set of all elements related to 1. Also write the equivalence class [2]. **[Delhi 2018**

Q30. Show that the function $f:R\to R$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall\ x \in R$ is neither one-one nor onto. Also, if $g:R\to R$ is defined as $g(x) = 2x-1$, find fog (x). **[Delhi 2018**

Q31. Let R be the relation defined in the set $A = \{1,2,3,4,5,6,7,8,9\}$ by $R = \{(x, y) : x, y \in A$, x and y are either both odd or both even$\}$. Show that R is an equivalence relation.
Write all the equivalence classes of set A. **[For Blind Candidate 2018**

#Q32. Let A be the set of all real numbers except –1 and let * be a binary operation on A defined by a * b = a + b + ab, $\forall\ a, b \in A$. Prove that (i) * is commutative and associative, and (ii) number 0 is its identity element. **[For Blind Candidate 2018**

✍...Here you can write your important notes!

Chapter 1

SOLUTIONS *Of* EXERCISE FOR PRACTICE

Based on Relation & its Types

Q02. $R = \{(2,2),(2,4),(2,6),(2,8),(4,4),(4,8)\}$; Dom.(R) $= \{2,4\}$, Range(R) $= \{2,4,6,8\}$

Q11. R is reflexive and transitive but not symmetric.
Q12. R is neither reflexive, symmetric nor transitive.
Q13. R is not an equivalence relation.
Q14. R is neither reflexive, symmetric nor transitive.
Q15. R is reflexive and transitive but not symmetric. Q21. $\{1, 5, 9\}$ Q22. $(6,8) \in R$.

Q28. The smallest equivalence relation containing (1, 2) is given by, $R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. Now, we are left with only four pairs i.e., (2, 3), (3, 2), (1, 3), and (3, 1). If we odd any one pair [say (2, 3)] to R_1, then for symmetry we must add (3, 2). Also, for transitivity we are required to add (1,3) and (3, 1). Hence, the only equivalence relation (bigger than R_1) is the universal relation.
This shows that the total number of equivalence relations containing (1, 2) is two.

Q29. $\{1\}$ Q32. One Q33. $[(2, 5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

Q34. (a) $\{1, 2, 3\}$ Q34. (b) $\{0, 2, 4\}$ Q34. (c) $R = \{(1, 1), (2, 2), (3, 3)\}$ Q34. (d) $\{8, 27\}$

Q35. R is reflexive, symmetric and transitive.

Based on Function & its Types

Q01. f is one-one Q02. f isn't surjective.

Q10. $(2x+1)^2,\ 2x^2+1$ Q11. $\|5x-2\|,\ |5|x|-2|$

Q12. $\sin^2 x,\ \sin x^2$ Q13.a) $x^4-6x^3+10x^2-3x$ Q13.b) x

Q14.a) $fog: R \to R,\ (fog)(x) = 4x^2-6x+1$ b) $gof: R \to R,\ (gof)(x) = 2x^2+6x-1$

c) $fof: R \to R,\ (fof)(x) = x^4+6x^3+14x^2+15x+5$ d) $gog: R \to R,\ (gog)(x) = 4x-9$

Q15. $f(x) = \sin x$ and, $g(x) = x^2$. Q16.a) $1+\sin^2 x,\ \sin(x^2+1)$

Q16.b) $fog = \{(2,5),(5,2),(1,5)\}$. Q18. $\sqrt{x^2-1},\ x-1$

Q20. (c) $f^{-1}(x) = \begin{cases} x-1, & \text{if x is odd} \\ x+1, & \text{if x is even} \end{cases}$ Q23. One-one and onto both.

Q27. $\{(2, 7), (3, 7), (4, 11), (5, 11)\}$ Q28. $3\cos^2 x,\ \cos 3x^2$

Q29. $f^{-1} = \dfrac{x-3}{4}$. Q30. $f^{-1} = \sqrt{n}$ Q31. $f^{-1}(x) = \dfrac{\sqrt{x-6}-3}{2}$.

Q33. $\dfrac{4x+3}{6x-4}$ Q35. not one-one. Q36. (a) 1 (b) 0 (c) 1

Q38. $f^{-1}(x) = (x-3)^{1/3}$ Q39. $\dfrac{x}{1-x}$ Q41. $f^{-1} = \{(2,1),(4,2),(6,3)\}$

Q42. $a=1,\ b=0$ or $a=-1,\ b \in R$. Q43. $f^{-1}(x) = x^{-1},\ x \in R_0$.

Q45. fog is not defined and gof $= \{(1,3),(2,5),(4,2)\}$ Q46. $\log\sqrt{\dfrac{y}{2-y}}$

Q49. Since f is not one-one. Therefore, f is not invertible. Q50. 3! i.e., 6.

Q51.a) 9. Q51.b) 8 Q51.c) 60 Q51.d) 0 Q51.e) n!. Q52. $\{-1, 1\}$

Q53. (a) $\dfrac{1+\sqrt{1+4\log_2 x}}{2}$ Q53. (b) $\log_{10}\sqrt{\dfrac{1+x}{1-x}}$ Q54. $7+(4-x^4)^{1/3}$

TEN YEARS ANSWERS

CHAPTER 01

(Very Short Answer I)

Q01. R = {(1, 1), (2, 2), (3, 3)} **Q02.** $\frac{(4x+7)}{2}$ **Q03.** 18

Q04. 5 **Q05.** 35 **Q06.** |5x – 2| **Q07.** 9x + 8 **Q08.** 8x

Q09. Function *f* is one-one as no two elements of A has the same *f*-images in B.

Q10. As we know that a relation R in a set A is transitive if $(a,b)\in R$ and $(b,c)\in R$ implies $(a,c)\in R\ \forall a,b,c\in A$. Here it can be observed that (1, 2), (2, 1)$\in$R but (1, 1)$\notin$R. Hence the relation R in the set {1, 2, 3} is not transitive.

Q11. {–1, 1} **Q12.** x **Q13.** $\frac{5x+4}{3}$ **Q14.** –2 **Q15.** 2

Q16. 50 **Q17.** 7 **Q19.** {0, 2, 4} **Q20.** $\{(1,3),(3,1),(4,3)\}$

Q21. As $\frac{1}{2}>\left(\frac{1}{2}\right)^2 \Rightarrow \left(\frac{1}{2},\frac{1}{2}\right)\notin R$. Hence R is not reflexive. **Q22.** $e=2$

Q23. $\text{gof}(x)=\frac{3x^2-5}{9x^4-30x^2+26}$ **Q24.** $\{(1,3),\ (2,4)\}$ **Q25.** * is not associative.

Q26. 13.

(Very Short Answer II)

Q01. Following equivalence relations can be possible in the given conditions these are,
$\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ and $\{(1,1),(2,2),(3,3),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$.
Clearly, only two equivalence relations are there.

(Short Answer)

Q01. Identity element : 0 **Q03.** $f^{-1}=\frac{2y-1}{y-1}$

Q05. $f^{-1}(x)=\frac{4x+3}{6x-4}$ **Q06.** $f^{-1}(x)=\frac{4x+3}{6x-4}$

Q07. Function *f* is not one-one, so it is not bijective (as a bijective function is necessarily both one-one as well as onto function).

Q10. $f^{-1}=\frac{3y-2}{y-1}$ **OR** $f^{-1}(x)=\frac{3x-2}{x-1}$ **Q11.** $\frac{x-7}{10}$

Q15.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Q17. **OR** (i) $f^{-1}(10)=1$ (ii) $y=19$

Q18. No identity element exists **Q21.** Identity element: 5/3

Q26. Value of $fog=\begin{cases}0, & \text{if } x\geq 0\\ -4x, & \text{if } x<0\end{cases}$ and $gof=0\ \forall x\in R$.

Q27. See CBSE Sample Paper 2014 (Get it from **theopgupta.com**)

Q28. $R=\{(1,22),(2,20),(3,18),(4,16),(5,14),(6,12),(7,10),(8,8),(9,6),(10,4),(11,2)\}$
Dom.(R) = $\{1,2,3,4,5,6,7,8,9,10,11\}$, Range = $\{2,4,6,8,10,12,14,16,18,20,22\}$

Also as $(1,22) \in R$ but $(22,1) \notin R$ $\therefore$ R is not symmetric, so R is not an equivalence relation.

Q29. $[(2, 5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$.

(Long Answer)

Q02. $f^{-1}(x) = \dfrac{\sqrt{x-6}-3}{2}$

Q03. R is reflexive, R isn't symmetric and R isn't transitive as well.

Q04. **Identity element :** (0, 0) and inverse of $(3,-5)$ is $(-3,5)$

Q06. * is commutative as well as associative; Identity element : 0, Inverse of an element : $-\dfrac{x}{1+x}$

Q07.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Q08. (i) Identity element : (1, 0)

(ii) Inverse of $(a,b) = \left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$, Inverse of elements (5, 3) is $\left(\dfrac{1}{5}, -\dfrac{3}{5}\right)$ and of $\left(\dfrac{1}{2}, 4\right)$ is $(2,-8)$

Q09. $f^{-1}(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$ i.e., $f^{-1}(n) = f(n)$

Q10. * is a binary operation, * is commutative as well as associative, identity element is (0, 0)

Q11. $gof = \{(-1,2), (0,0), (1,0), (2,2)\}$ **Q12.** $(fog)^{-1}(x) = \sqrt[3]{\dfrac{x-7}{2}}$

Q13. $f^{-1}(x) = \dfrac{\sqrt{x+6}-1}{3}$, $f^{-1}(43) = 2$ and $f^{-1}(163) = 4$.

Q14. $fog(x) = \big||x|-x\big| + |x| - x$, $gof(x) = \big||x|+x\big| - |x| - x$, $fog(-3) = 12$, $fog(5) = 0$, $gof(-2) = 0$.

Q15. $f^{-1}(x) = \dfrac{\sqrt{x-6}-3}{2}$, $f^{-1}(31) = 1$, $f^{-1}(87) = 3$.

Q16. Identity element is (0, 0), inverse of (a,b) is $(-a,-b)$.

Q17. Identity element : $e = 0$, Inverse of an element in A is $-\dfrac{a}{1+a}$.

Q18. $[(3, 4)] = \{(1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,8), (8,9), (9,10)\}$.

Q19. $f^{-1}(x) = \begin{cases} \dfrac{x}{1+x}, & \text{if } x < 0 \\ \dfrac{x}{1-x}, & \text{if } x > 0 \end{cases}$

Q20. Modified codomain of f is $[-5,\infty)$, $f^{-1} : [-5,\infty) \to [0,\infty)$, $f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$

Q21. Binary operation * is non commutative but it is associative. Identity element for binary operation * is $(1,0)$ and inverse of an element (a,b) is $\left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$.

Q22. $f^{-1} = \frac{3-4y}{3y-4}$, $f^{-1}(0) = -\frac{3}{4}$, $x = \frac{11}{10}$

Q23. $f^{-1}(x) = \frac{4x}{4-3x}$

Q24. Identity element is (0, 0).

Q25. * is binary operation on Q. Also * is commutative but it's not associative. For *, the Identity element doesn't exist. Inverse of an element : Since there is no identity element hence, there is no inverse.

Q27. $2x^3 + 7, \sqrt[3]{\frac{x-7}{2}}, 1$

Q28. * is not commutative, identity element = 0.

Q29. {1, 5, 9}, [2] = {2, 6, 10}.

Q30. $\frac{2x-1}{2(2x^2-2x+1)}$.

Q31. [1] = [3] = [5] = [7] = [9] = {1, 3, 5, 7, 9}, [2] = [4] = [6] = [8] = {2, 4, 6, 8}.