## **CHAPTER 4**

## DETERMINANTS

## **CASE BASED QUESTIONS**

Manjit wants to donate a rectangular plot of land for a school in his village. When he wasasked to give dimensions of the plot, he told that if its length is decreased by 50 m andbreadth is increased by 50m, then its area will remain same, but if length is decreased by10mand breadth is decreasedby20m, then itsareawill decreaseby5300m<sup>2</sup>



Basedontheinformationgivenabove, answerthe following questions: i) The value of x (length of rectangular field) is

150 m b)400 m c) 200m d) 320 m

- ii) Thevalueofy(breadthof rectangularfield)is
- 150 m b)200 m c) 430 m d) 350 m
- iii) How much is the area of rectangular field?
- a)60000 sq m b)30000 sq m c) 3000 sq m d) 30000 m
- iv) The equations in terms of x and y are
- a. x+y =50, 3x-y = 550
  - b.x-y=50,2x+y=550
  - c.x+y= 50, 2x+ y=550

d.x+y= 50,2x+y=550

2. Three shopkeepers Ujjwal, Lohith, and Kundan are using polythene bags, handmade bags and newspaper's envelope as carry bags. It is found that the shopkeepers Ujjwal, Lohith, and Kundan are using (20, 30, 40), (30, 40, 20), and (40, 20, 30) polythene bags, handmade bags, and newspapers envelopes respectively. They spent ₹250, ₹270, and ₹200 on these carry bags respectively. Let the cost of polythene bag, handmade bag and newspaper envelope costs are x,y and z respectively.

i) What is the Linear equation representing amount spent by Lohith on carry bags?

a.20x +30y+40z=250

b.30x +40y+20z=270

c.40x +20y+30z=270

d. 250x +270y+200z=0

ii) Whatisthecostof one newspaper bag?

a. ₹1 b.₹2 c.₹3 d.₹5

iii) Find the total amount spent by Ujjwal for

handmade bags?

a.100 b.200 c.150 d.250.

3. Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award Rs.x each, Rs. y each and Rs. z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of Rs.2200. School B wants to spend Rs.3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is Rs.1200.



Using the concept of matrices and determinants, answer the following questions.

- (i) What is the award money for Honesty?
- (a) Rs.350 (b) Rs.300 (c) Rs.500 (d)Rs.400
- (ii) What is the award money for Punctuality?
- (a) Rs.300 (b) Rs.280 (c) Rs.450 (d) Rs.500
- (iii) What is the award money for Hard work?
- (a) Rs 500 (b) Rs.400 (c) 0 (d) none of these

4. Gaurav purchased 5 pens, 3 bags and 1 instrument box and pays Rs. 16. From the same shop, Dheeraj purchased 2 pens, 1 bag and 3 instrument boxes and pays Rs.19, while Ankur purchased 1 pen, 2 bags and 4 instrument boxes and pays Rs.25.



Using the concept of matrices and determinants, answer the following questions.

- (i) The cost of one pen is
- (a) Rs.2 (b) Rs.5 (c) Rs.1(d) Rs.3
- (ii) What is the cost of one pen and one bag?
- (a) Rs.3 (b) Rs.5 (c) Rs.7(d) Rs.8
- (iii) What is the cost of one pen and one instrument box?

- (a) Rs.7 (b) Rs.6 (c) Rs.8(d) Rs.9
- (iv) Which of the following is correct?
- (a) Determinant is a square matrix. (b) Determinant is a number associated to a matrix
- (c) Determinant is a number associated to a square matrix (d) All of the above
- (v) From the matrix equation AB = AC, it can be concluded that B = C provided
- (a) A is singular (b) A is non-singular (c) A is symmetric (d) A is square

5. A total amount of Rs. 7000 is deposited in three different saving banks accounts a with annual interest rates of 5%,8% and 81/2% respectively. The total annual interest from these accounts is Rs 550, Equal amounts have been deposited in the 5% and 8% savings accounts.



Using Rs x, Rs y, and Rs z be invested in saving accounts

Using the concept of matrices and determinants, answer the following questions.

i) Which of the following is true?
(a) x +y +z=7000 (b) 10x+16y+17z=110000 (c) x-y=0 (d)All of these
ii) The system of given equations can be written as AX=B here A=[a<sub>ij</sub>] and A<sub>ij</sub> is co-factor of a<sub>ij</sub> so A<sub>32</sub> is
(a) 7 (b) -7 (c) -1 (d) 0
iii) The value of z is

(c) 4750

(d) 1125

(b) 38000

(a) 9000

iv) If A and B be the square matrices of same order such that AB=8I=BA then which one is true

(a)  $A^{-1}=B/8$  (b)  $B^{-1}=A/8$  (c)  $A^{-1}=B$  (d) a and b both

6. Two organizations A and B want to award their selected employees on the values of Honesty, Hard work and Punctuality. The organization A wants to award  $Rs. x \ each, Rs. y$ each and Rs. z each for the three respective values to its 3, 2 and 1employees respectively with a total award money of Rs. 2000. Organization B wants to spend Rs. 3000 to award its 4, 1 and 3 employees on the respective values (by giving the same award money to the three values as organization A). The total amount of award for prizes on each value is Rs. 900.

On the basis of above information, answer the following questions.

(i) What is the award money for Honesty?

(a) *Rs*. 350(b) *Rs*. 300 (c) *Rs*. 500 (d) *Rs*. 400

(ii) What is the award money for Punctuality?

(a) *Rs*. 300(b) *Rs*. 280 (c) *Rs*. 500 (d) *Rs*. 450

(iii) What is the award money for Hard work?

(a) *Rs*. 300(b) *Rs*. 100 (c) *Rs*. 500 (d) *Rs*. 550

(iv) If a matrix B is both symmetric and skew-symmetric, then |B| is equal to

(a) 1 (b) -1 (c) 0 (d) 2

(v) If P and Q are two matrices such that PQ = Q and QP = P, then  $|Q^2|$  is equal to

(a) |Q| (b) |P| (c) 1 (d) 0

7. Three friends rahul ,ravi and rakesh went to a vegetable market to purchase vegetable . from a vegetable shop Rahul purchased 1 kg each olf potato, onion and brinjal for a total of 21rs. Ravi purchaserd 4 kg of potato , 3 kg of onion and 2kg of brinjal for 60rs. While rakesh purchased 6 kg potato , 2kg onion and 3 kg brinjal for 70rs.

Based on the above informations answer the following.

(i)If the cost of Potato, Onion and Brinjal are Rs.x, Rs.y and Rs.z per kg respectively, then algebraic representation of given situation of problem is

(A). x + y + z = 6, x + y + 3z = 11, 3x + 2y + z = 21

(B). x + y + z = 21, 4x + 3y + 2z = 60, 6x + 2y + 3z = 70

(C). 2x + 3y + z = 21, x + y + z = 60, x + 2y + z = 70

(D). x + y + z = 70, 4x + 2y + 2z = 21, 6x + 2y + 3z = 6

(ii)Matrix equation of above situation is

(A).  $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$ (B).  $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (C).  $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (D). $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (iii) If A= $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$ (A). 0 (B).1 (C).-5 (D).5 (iv)The cost of Potato, Onion and Brinjal are (A).Rs.5,Rs.8 and Rs.8 (B). Rs.4, Rs.8 and Rs.11 (C). Rs.5, Rs.11 and Rs.10 (D). Rs.5, Rs.8 and Rs.15

8. If there is a statement involving the natural number n such that (i) The statement is true for n = 1 (ii) When the statement is true for n = k (where k is some positive integer), then the statement is also true for n = k + 1. Then, the statement is true for all-natural numbers n. Also, if A is a square matrix of order n, then A2 is defined as AA. In general, Am = AA ....A (m times), where m is any positive integer.

Based on the above information, answer the following questions.

If A =  $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ , then for any positive integer n: i) a)  $A^{n} = \begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ b)  $A^{n} = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ c)  $A^{n} = \begin{bmatrix} 3n & -8n \\ 1 & -n \end{bmatrix}$ d)  $A^{n} = \begin{bmatrix} 1+3n & -4n \\ n & 1-3n \end{bmatrix}$ ii) If A =  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ , then  $|A^n|$  where  $n \in N$  is equal to: a) 2<sup>n</sup> b) 3<sup>n</sup> c) N d) 1 iii) If  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ , and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , then which of the following holds for any natural number  $n \ge 1$ . a)  $A^n = n A - (n - 1) I$ b)  $A^n = 2^{n-1}A - (n-1)I$ c)  $A^n = n A + (n - 1) I$ d)  $A^n = 2^{n-1}A + (n-1)I$ iv) Let A =  $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$  and A<sup>n</sup> =  $\begin{bmatrix} a_{ij} \end{bmatrix}_3 \times_3$  for some positive integer n, then the value of cofactor a<sub>13</sub> is: a) a<sup>n</sup> b) -a<sup>n</sup> c) 2a<sup>n</sup> d) 0 v) A is a square matrix and |A| = 2, then for any positive integer n. the value of  $|A^n|$ : a) 0 b) 2n c) 2<sup>n</sup> d) N<sup>2</sup> 9. Area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by the

determinant  $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$  Since, area is a positive quantity, so we always take the

absolute value of the determinant D. Also, the area of the triangle formed by three collinear points is zero.

Based on the above information, answer the following questions.

- i) Find the area of the triangle whose vertices are (-2, 6), (3, -6) and (1, 5).
  - a) 30 sq. unit
  - b) 35 sq. unit
  - c) 40 sq. unit
  - d) 15.5 sq. Unit
- ii) If the points (2, -3), (k, -1) and (0, 4) are collinear, then find the value of 4k.
  - a) 40
  - b)  $\frac{40}{7}$
  - c) 50
  - d) 43
- iii) If the area of a triangle ABC, with vertices A (1, 3), B (0, 0) and C (k, 0) is 3 sq. units, then a value of k is:
  - a) 2
  - b) 3
  - c) 4
  - d) 5
- iv) Using determinants, find the equation of the line joining the points A (1, 2) and B (3, 6).
  - a) y = 2x
  - b) x = 3y
  - c) y = x
  - d) 4x y = 5
- v) If  $A \equiv (11, 7)$ ,  $B \equiv (5, 5)$  and  $C \equiv (-1, 3)$ , then
  - a)  $\Delta ABC$  is scalene
  - b)  $\Delta ABC$  is equilateral
  - c) A, B and C are collinear
  - d) None of these

10. The management committee of a residential colony in Dimapur decided to award some of its member (say x) for hard work, some (say y) for helping others and some others (say z) for supervising covid 19 guidelines. The sum of all the awardees is 12. Three times the sum of awardees for supervision added to two times the number of awardees for hard work is 33. The sum of the number of awardees for hard work and supervision is twice the number of awardees for helping.

i) x + y + z = ..... (A) 3 (B) 5

- (C) 7
- (D) 12

ii) x – 2y = .....

- (A) z
- (B) —z
- (C) 2z
- (D) -2z

iii) The value of  $2x + 3y + 5z = \dots$ 

(A) 40

- (B) 43
- (C) 50
- (D) 53

iv) The value of  $x + 2y = \dots$ 

- (A) 9
- (B) 10
- (C) 11
- (D) 12

ANSWERS

1	i	В	ii	C	iii	A	iv	В	v	
2	i	A	ii	A	iii	С	iv		v	
3	i	В	ii	D	iii	С	iv		V	
4	i	A	ii	A	iii	В	iv	С	v	В
5	i	D	ii	В	iii	С	iv	D	v	
6	i	С	ii	A	iii	В	iv	С	v	А
7	i	В	ii	A	iii	С	iv	A	v	
8	i	В	ii	D	iii	A	iv	D	v	С
9	i	D	ii	В	iii	В	iv	A	v	C
10	i	D	ii	В	iii	В	iv	С	v	

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