

CHAPTER 6

APPLICATION OF DERIVATIVES

ASSERTION – REASON QUESTION

In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R), Mark the correct choice as

- A) Both A and R are true and R is the correct explanation of A
- B) Both A and R are true but R is not the correct explanation of A
- C) A is true but R is false
- D) A is false and R is true

1	<p>Let $a, b \in R$ such that the function f given by $f(x) = \ln x + x^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = -2$.</p> <p>Assertion : f has a local maximum at $x = -1$ and at $x = 2$</p> <p>Reason: $a = \frac{1}{2}$ and $b = -\frac{1}{4}$</p>
2	<p>Assertion : The minimum value of $f(x) = x^2 - 8x + 17$ is 1</p> <p>Reason : If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum.</p>
3	<p>A function f is given by $f(x) = 2x^3 - 6x^2 + 6x + 5$</p> <p>Assertion : $x = 1$ is not a point of local maxima</p> <p>Reason : $x = 1$ is not a point of local minima</p>
4	<p>Assertion : If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then the numbers are 8 and 8.</p> <p>Reason : Let f be twice differentiable at $x = c$ such that $f'(c) = 0$ and $f''(c) > 0$ then f has a local minima and $f(c)$ is the local minimum value of f.</p>
5	<p>Assertion (A) : The function $f(x) = x^3 - 3x^2 + 6x - 100$ is strictly increasing on R</p> <p>Reason (R) : A strictly increasing functions is an injective function</p>
6	<p>Assertion (A) : The function $f(x) = \log x$ is defined for all $x \in (0, \infty)$</p> <p>Reason (R) : If $f'(x) > 0$ then $f(x)$ is strictly decreasing function.</p>
7	<p>Assertion (A) : $f(x) = a(x + \sin x)$ is an increasing function if $a \in (0, \infty)$</p> <p>Reason (R) : The given function $f(x)$ is increasing only if $a \in (0, \infty)$</p>
8	<p>Assertion (A) : The curve $y = x^2$ represents a parabola with vertex at origin.</p> <p>Reason (R) : For a curve Tangent and Normal lines are always perpendicular at the point of contact.</p>
9	<p>Assertion (A) : For the curve $y = \tan x$, the tangent and normal exists at a point $(0, 0)$.</p> <p>Reason (R) : Tangent and Normal lines are $x - y = 0$ and $x + y = 0$.</p>

10	Assertion (A): The function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing in R Reason (R) 2: A function will be increasing in an interval if $f'(x) \geq 0$
11	Assertion (A): The function $f(x) = \log(\sin x)$ is strictly increasing $(0, \frac{\pi}{2})$ Reason (R): $\sin x$ is positive in first quadrant
12	Assertion (A): The function $f(x) = x^2 - x + 1$ is increasing strictly in the interval (0,1) Reason (R): The function has turning point at $x = \frac{1}{2}$
13	Assertion (A): The curve $16x^2 - 9y^2 = 144$ has two tangents parallel to x-axis. Reason (R): At the points where tangents drawn to a curve are parallel to x-axis, $\frac{dy}{dx} = 0$
14	Assertion (A): $f = e^x$ do not have maxima or minima in R. Reason(R): Monotonic functions have maxima or minima at endpoints of Domain.
15	Assertion (A): Minimum value of $f = 9x^2 + 12x + 2$ is -2. Reason(R): Maximum value of $f = 9x^2 + 12x + 2$ does not exist.
16	Assertion (A): The function $f = x - 3 $ does not have any critical point. Reason(R): Critical points of a function are where $f' = 0$ or f is not differentiable.
17	An open-top box is to be constructed by removing equal squares from each corner of a 3m by 8m rectangular sheet and folding up the sides. Assertion (A): Maximum volume of the box is $200/27$ cubic meters. Reason(R): Volume is largest when the square of side $2/3$ is removed from each corner
18	Assertion (A): The absolute maximum value of the function $f = (x - 1)^2 + 3$ in $[-3, 1]$ is 19 Reason(R): The absolute value of function exists only on critical point of a function in I.
19	Assertion (A): The equation of tangent to the curve $y = \sin x$ at point (0,0) is $y = x$. Reason (R): if $y = \sin x$ then $\frac{dy}{dx}$ at $x = 0$ is 1..
20	Assertion (A): The Normal to the curve $2y + x^2 = 3$ is at point (1, 1) is $x - y = 0$. Reason (R) : Slope of the Normal at (1,1) is 1.
21	Lets consider the function $f(x) = \frac{4x^2 + 1}{x}$ Assertion (A) : $f(x)$ is increasing on $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$ Reason (R) Critical points are $-\frac{1}{2}, \frac{1}{2}$.
22	The slope of the curve $2y^2 = ax^2 + b$ at (1, -1) is -1 then Assertion (A) : Value of 'a' is 2. Reason (R) : Value of 'b' is 0.
23	Assertion(A):The function $f(x)=x^3-3x^2+6x-100$ is strictly increasing on R Reason(R) :A strictly increasing function is an injective function
24	Assertion(A):The function $y = [x(x-2)]^2$ is increasing in $(0,1) \cup (2,\infty)$ Reason(R): $dy/dx=0$, when $x=0,1,2$.
25	Assertion (A):If $f(x)=a(x+\sin x)$ is increasing function if $a \in (0,\infty)$ Reason(R) :The given function $f(x)$ is increasing only if $a \in (0,\infty)$
26	Assertion (A):For the curve $y = \tan x$, the tangent and normal exists at a point (0,0). Reason(R) :tangent and normal lines are $x-y = 0$ and $x+y=0$.
27	Assertion (A):The curve $y=x^2$ represents a parabola with vertex at origin. Reason(R) :for a curve tangent and normal lines are always perpendicular at the point of contact.

28	Assertion (A): $f(x) = \sin 2x + 3$ is defined for all real values of x . Reason (R): Minimum value of $f(x)$ is 2 and Maximum value is 4.
29	Assertion (A): $f(x) = \sin(\sin x)$ is defined for all real values of x . Reason (R): Minimum and maximum values does not exist.
30	Assertion (A): $f(x) = - x+1 + 3$ is defined for all real values of x except $x = -1$. Reason (R): Maximum value of $f(x)$ is 3 and Minimum value does not exist.
31	AB is the diameter of a circle and C is any point on the circle. Assertion (A): The area of triangle ABC is maximum when it is isosceles. Reason (R): Triangle ABC is a right-angled triangle.
32	Assertion (A): A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, then length 1.782m and breadth 2.812m of the rectangle will produce the largest area of the window. Reason (R): For maximum or minimum $f'(x) = 0$.
33	The sum of the surface area (s) of a sphere of radius r and cuboid with sides $\frac{x}{3}$, x and $2x$ is a constant. Assertion: the sum of their volumes (V) is minimum when x equals three times the radius of the sphere. Reason: V is minimum when $r = \sqrt{\frac{5}{54 + 4\pi}}$.
34	Assertion: The function $y = \log(1+x) - \frac{2x}{2+x}$ is a decreasing function of x throughout its domain. Reason: The domain of the function $f(x) = \log(1+x) - \frac{2x}{2+x}$ where maximum and minimum lies is $(-1, \infty)$.
35	AB is a diameter of the circle and c is any point on the circle. Assertion: the area of $\triangle ABC$ is minimum when it is isosceles. Reason: $\triangle ABC$ is a right angled triangle.
36	A cylinder is inscribed in a sphere of radius R. Assertion: Height of the cylinder of the maximum volume is $\frac{2R}{\sqrt{3}}$ units. Reason: The maximum volume of the cylinder is $\frac{4\pi R^3}{\sqrt{3}}$ units.
37	Assertion: The altitude of the cone of maximum volume that be inscribed in a sphere of radius r is $\frac{4r}{3}$. Reason: The maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.
38	<u>Assertion</u> : Both $\sin x$ and $\cos x$ are decreasing and functions in the interval $(\frac{\pi}{2}, \pi)$.

	<u>Reason</u> : If a differentiable function decreases in an interval (a, b) , then its derivative also decreases in (a, b) .
39	<u>Assertion</u> : The function $f(x) = \log \cos x$ is increasing function for $\left[0, \frac{\pi}{2}\right]$. <u>Reason</u> : function is strictly increasing when slope of tangent is positive and function is strictly decreasing when the slope of tangent is negative.
40	<u>Assertion</u> : If $f(x) = \log \sin x$, $x > 0$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$. <u>Reason</u> : if $f'(x) \geq 0$, then $f(x)$ is strictly increasing function. Which of the following is true :
41	<u>Assertion</u> : $x+y-3=0$ is the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(1, 2)$. <u>Reason</u> : if slope of the tangent line is zero then $\tan \theta = 0$ and so $\theta = 0$, which means the tangent line is parallel to x- axis.
42	<u>Assertion</u> : Two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ are orthogonal if $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$. <u>Reason</u> : Two curves intersect orthogonally at a point if product of their slope at that point is - 1.
43	Assertion (A) Curve $y = xe^x$ is minimum at the point $x = -1$. Reason (R) $\frac{dy}{dx} < 0$ at $x = -1$.
44	Assertion (A) The largest area $A(x)$ rectangular field which can be enclosed with 200m fencing is 2500m^2 . Reason (R) $A''(x) < 0$ at $x = 50$
45	Assertion (A) The maximum value of $f(x) = \frac{x}{4+x+x^2}$ on $[-1, 1]$ is $\frac{1}{6}$. Reason (R) $f'(x) = 0$ and $f''(x) < 0$ at $x = 1$.
46	Assertion (A) Absolute maximum and minimum value of function $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [-1, 1]$ are 18 and 0 at points $x = -1$ and $x = 0$ respectively. Reason (R) For $f(x)$, $f\left(\frac{1}{8}\right) < f(0) < f(1) < f(-1)$.
47	Assertion (A) The radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of the radius of the cone. Reason (R) If x, r and S are radius of cylinder, radius of cone and curved surface area of cylinder then $S'(x) = 0$ at $x = \frac{r}{2}$.

48	Assertion (A): - Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$, then f is one one. Reason (R): $f(x)$ neither increasing nor decreasing function.
49	Assertion (A):- the curve $x=y^2$ and $xy=k$ cut at right angle if $8k^2=1$ Reason(R):-Two curves intersect at right angles if the tangents to the curve at the point of intersection are perpendicular to each other i.e. product of their slope is -1.
50	Assertion (A): The tangent to the curve $y=x^3-x^2-x+2$ at $(1, 1)$ is parallel to the x - axis Reason (R): The slope of the tangent to the curve at $(1, 1)$ is zero
51	Assertion (A): The function $f(x) = \frac{\log x}{x}$ is increasing in the interval $(0, e)$ Reason (R):A function is increasing if $f'(x) > 0$
52	Assertion (A): The local maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x + 15, x \in \mathbb{R}$, are 19 and 15 respectively. Reason (R): <i>Point $x = 1$ is a point of local maxima and $x = 3$ is a point of local minima.</i>

ANSWERS:

1	A	11	B	21	A	31	A	41	B	51	A
2	A	12	D	22	A	32	A	42	A	52	A
3	B	13	D	23	(B)	33	A	43	C		
4	A	14	A	24	(B)	34	D	44	A		
5	B	15	B	25	(D)	35	A	45	C		
6	C	16	D	26	(A)	36	C	46	D		
7	D	17	A	27	(B)	37	B	47	B		
8	B	18	C	28	A	38	C	48	C		
9	A	19	A	29	C	39	D	49	A		
10	A	20	A	30	D	40	D	50	A		

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