CHAPTER 6

APPLICATION OF DERIVATIVE

CASE STUDY QUESTIONS

Q. 1

Read the passage given below and answer the following questions.

To become fit and fine every person do some physical work or exercises. One morning two friends Mohan and Ahmed went for a morning walk in a park. They have decided to choose two different parabolic paths whose equations are $y^2 = 4x$(Mohan) and $x^2 = 4y$(Ahmed) respectively for their walk.

(i) The point at which both the paths meet is

(A) (0, 4)

(B) (4, 0)

(C) (0. 0) and (4, 4)

(D) they never meet.

(ii) At the point (0, 0) the tangent to the curve $y^2 = 4x$ is

(A) parallel to the x-axis.

(B) parallel to the y-axis.

(C) parallel to the line y = 0.

(D) perpendicular to the line x = 0.

(iii) At the point (0, 0) the tangent to the curve $x^2 = 4y$ is

(A) parallel to the x-axis

(B) parallel to the y-axis

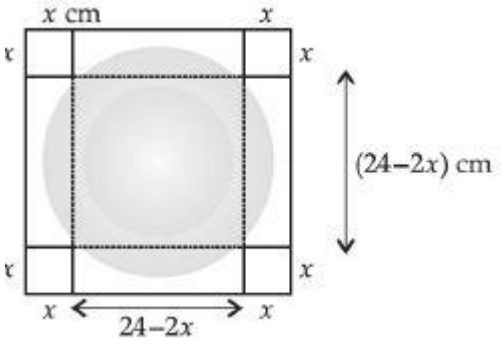
(C) perpendicular to the line y = 0.

(D) Parallel to the line x = 0.

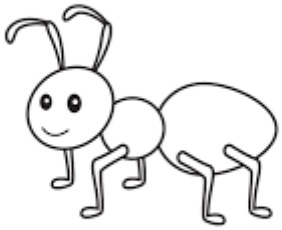
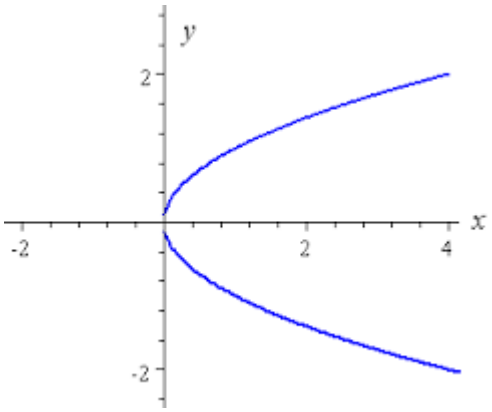
(iv) At the point (0, 0) the angle between the tangents to the curve $x^2 = 4y$ and $y^2 = 4x$ is

(A) 0

	(B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$ (v) At the point (4, 4) the slope of the tangent to the curve $x^2 = 4y$ is (A) 0 (B) 2 (C) -2 (D) ½
QUE 2	Read the passage given below and answer the following questions. Let $P(x) = 4x^3 – 6x^2 – 72x + 30$ is the total profit function of a company, where x is the production of the company.   (i) What is the value of P’(x)? (A) $12 x^2 - 12 x + 72$ (B)$12 x^2 + 12 x - 72$ (C) $x^2 - x - 6$ (D) $12 x^2 -12x - 72$ (ii) Determine the critical points of the profit function? (A) -2, -3 (B) -2, 3 (C) 2, -3 (D) 2, 3 (iii) Check in which interval the profit is strictly increasing . (A) $(– \infty, – 2] \cup (3, \infty)$ (B) $(– \infty, – 2] \cap [3, \infty)$ (C) $(– \infty, – 2) \cup (3, \infty)$ (D) $(– \infty, – 2) \cup (3, \infty)$ (iv) Check in which interval the profit is strictly decreasing . (A) [-2, 3}

	(B) [-2, 3) (C) (– 2, 3). (D) (-2, 3] (v) What is the value of P''(x)? (A) 24x - 12 (B) 24x + 12 (C) -24x + 12 (D) -24x - 12
QUE 3	An open box is to be made out of a piece of cardboard measuring 24 cm x 24 cm by cutting of equal squares from the corners and turning up the sides. Based on this information answer all the following Questions. 
	(i)The volume V(x) of the open box is $a)\ 4x^3 - 96x^2 + 576x$ b) $4x^3 + 96x^2 + 576x$ c) $2x^3 - 48x^2 + 288x$ d) $2x^3 + 48x^2 + 288x$
	(ii)The value of $\frac{dV}{dx}$ is a) $12(x^2 + 16x - 48)$ b) $12(x^2 - 16x + 48)$ c) $6(x^2 + 8x - 24)$ d) $6(x^2 - 8x + 24)$
	(iii)The value of $\frac{d^2V}{dx^2}$ is a) $24(x + 8)$ b) $12(x - 4)$ c) $24(x - 8)$ d) $12(x + 4)$
	(iv)For what value of the height, the volume of the open box is maximum a) 3 cm b) 9 cm c) 1 cm d) 4cm
	(v)The volume is minimum if a) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} = 0$ b) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$ c) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} > 0$ d) none of these
QUE 4	Scientist want to know the Oil- Reserves in sea so they travel over the sea along the curve $f(x) = (x+1)^3 (x-3)^3$ by an aeroplane . A student of class XII discuss the characteristic of the curve.

	 Answer the questions (Q16- Q19) on the basis of the information given above
	(i)The first order derivative of the given function is (a) $3(x+1)^2(x-3)^2$ (b)$6(x+1)^2(x-3)^2(x-1)$ (c) 2(x-1) (d) None of these
	(ii)The critical point of the given function are (A) -1,3,2 (b) 1,3,-2 (c) 1,2 (d) None of these
	(iii)The interval in which the given function is strictly increasing is (A) (1,3)U(3,∞) (b) $(-\infty, -1)U(-1,1)$ (c) (1,3)U(-1,∞) (d) None of these
	(iv)The interval in which the given function is decreasing is (a) (1,3)U(3,∞) (b) $(-\infty, -1)U(-1,1)$ (c) (1,3)U(-1,∞) (d) None of these
Que 5	Now a days Chinese and Indian troops are engaged in Aggressive melee , face-off skirmishes at locations near the disputed Pangong lake in Ladakh. One day a Helicopter of Enemy is Flying along the Curve represented by y=$\sqrt{5x-3}$ - 2 .Indian soldiers are posted at a point P (2,3) . and try to hit it . But unfortunately bullet fire goes in straight line direction by just touching the helicopter . On the basis of information answer (Q20 – Q23)
	The point on which tangents to the curve is perpendicular to the X axis. (a) (0, 3/5) (b) (3/5,0) (c) (5/3, 0) (d) None of these
	The equation of the tangents to the curve which are parallel to the y Axis. (a) 3x+ 5 =0 (b) 3x- 5 =0 (c) 5x-3 =0 (d) None of these
	The equation of the path travelled by the bullet Fire. (a) 5x-2$\sqrt{7}$ y = 15 - 4$\sqrt{7}$. (b) 2$\sqrt{7}$ x -5y = 15 - 4$\sqrt{7}$ (c) 2x – 5y = 15 (d) None of these
	The slope of the Normal to the curve at x= 3/2 .

	(a) 5/2 (b) -5/2 (c) -2/5 (d) None of these
Que no 6	Rahul sketches the path of an ant which was moving in his study table. The path of the ant was almost like the parabola $y^2 = x$.Help Rahul to find the answers to the following questions-   a) At what point on the curve tangent makes an angle $\frac{\pi}{4}$ with x-axis? b) Is there any point on the curve where tangent to the curve is parallel to x-axis? c)At what point on the curve, tangents to the curve are vertical? d)Tangent at (4,2) meet y-axis at B. What are co-ordinates of B? e) What is the equation of normal at (0,0)?
Que 7	 Rahul's Mathematics teacher was explaining the topic Increasing and decreasing functions in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which a function will be increasing or decreasing. He

took examples of different functions to make it more clear to the students. He then took the function $f(x) = (x-1)^3(x-2)^2$ and ask the students to answer the following questions. With Rahul, you can also test your knowledge by answering the questions

a) Find the stationary points on the curve.

b) what can you say about the point x=1?

c)Find the intervals where the function is increasing?

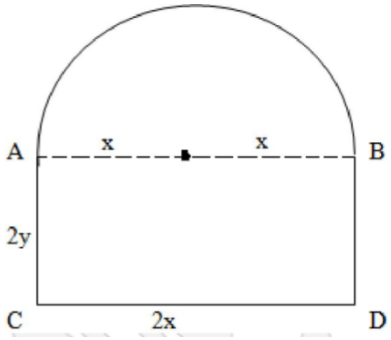
d)Find the intervals where the function is decreasing?

e) Is the function monotonic in the interval (1,2)?

Ans 1. X=1,2,8/5 2. Not a turning point 3. $(-\infty, 1) \cup (1, \frac{8}{5}) \cup (2, \infty)$4. $(\frac{8}{5}, 2)$ 5. No

Que 8

Dr. Anuradha residing in Chandigarh went to see an apartment of 3 BHK sector 31. The window of the house was in the form of a rectangle surmounted by a semicircular opening having a perimeter of the window 10 m in as shown in the figure below:



Based on the above information answer the following:

(i)Relation between x and y

a. $x + y + \frac{\pi x}{2} = 10$	b. $2x + 4y + \pi x = 10$
c. $2x + y + \pi x = 10$	d. $x + 2y - \pi x$=10

(ii) The area of the window(A) expressed as a function of x is:

a. A=$10x - 2x^2 - \frac{\pi x^2}{2}$	b. $A = 5x + \frac{x^2}{2} - \frac{\pi x^2}{2}$
c. $A = 5x - \frac{x^2}{2} + \frac{\pi x^2}{2}$	d. $A = 5x + \frac{x^2}{2} + \frac{\pi x^2}{2}$

(iii) Dr Anuradha is interested in maximizing the area of the whole window. For this to happen the value of x is:

a. $\frac{10}{\pi}$	b. $\frac{10}{2+\pi}$
c. $\frac{10}{4+\pi}$	d. $\frac{10}{1+\pi}$

(iv) For the maximum value of A, the breadth of the rectangle part of the window is:

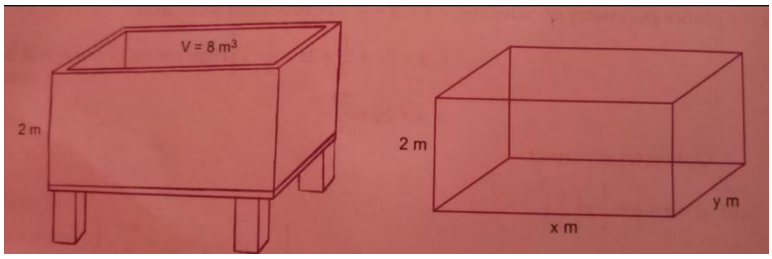
a. $\frac{10}{\pi}$	b. $\frac{10}{2+\pi}$
c. $\frac{10}{4+\pi}$	d. $\frac{10}{1+\pi}$

(v) The maximum area of the window is:

a. $\frac{20}{4+\pi}$	b. $\frac{30}{4+\pi}$
c. $\frac{40}{4+\pi}$	d. $\frac{50}{4+\pi}$

Que 9

At the request of villagers, a construction agency designs a tank with the help of an architect. The tank consists of a rectangular base with rectangular sides, open at the top so that its dept is 2 m and volume is 8 m^3 as shown below:



Based on the above information answer the following:

(i)If x and y represent the length and breadth of its rectangular base, then the relation between the variables:

a. x+ y=8	b. x .y=4
c. x+ y=4	d. x/y=4

(ii) If the construction of the tank cost Rs.70 per sq. meter for the base and Rs.45 per sq. meter for sides, the making cost 'C' expressed as a function of x is:

a. $C = 100 + 80(x + \frac{4}{x})$	b. $C = 180 + 280(x - \frac{4}{x})$
c. $C = 280 + 180(x + \frac{4}{x})$	d. $C = 180 + 280(x + \frac{4}{x})$

(iii) The owner of a construction agency is interested in minimizing the cost 'C' of the whole tank for this to happen the value of x should be

a. 4m	b. 3m
c. 1m	d. 2m

(iv) For minimum cost 'C' the value of y should be

a. 1m	b. 3m
c. 2m	d. 4m

(v) The Pradhan of the village wants to know the minimum cost. The minimum cost is

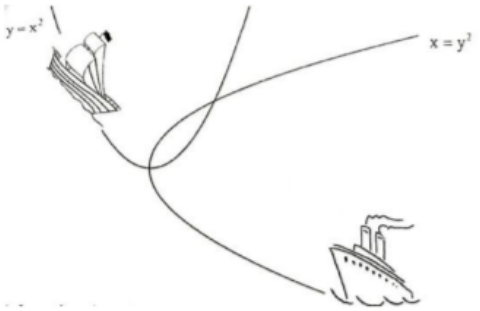
a. Rs.2000	b. Rs.4000
c. Rs.11000	d. Rs.1000

QUE 10

CS 1	A potter made a mud vessel, where the shape of the pot is based on f(x) = |x – 3| + | x-2|, where f(x) represents the height of the pot. 
1	When x > 4 What will be the height in terms of x ? A. x – 2 | B. x – 3 C. 2x – 5 | D. 5 – 2x
2	Will the slope vary with x value? A. Yes | B. No C. Can't Say | D. Data is not sufficient to say
3	What is $\frac{dy}{dx}$ at x = 3 A. 2 | B. – 2 C. Function is not differentiable | D. 1
4	When the x value lies between (2,3) then the function is A. 2x – 5 | B. 5 – 2x C. 1 | D. 5
5	If the potter is trying to make a pot using the function f(x) = [x] ,will he get a pot or not? Why? A. Yes, because it is a continuous function | B. Yes, because it is not continuous C. No, because it is not continuous | D. No, because it is not continuous

QUE 11

Assuming that two ships follow the path of curves C_1: $y = x^2$ and C_2: $x = y^2$ in the sea. There are high chances that these ships may cross the path traced by each other.



1 The points of intersection for the path traced by the ships (intersection of curves) are

A. (0, 0), (1, ±1)	B. (0, 0), (±1, 1)
C. (0, - 1), (1, 0)	D.(1, 0), (0, 1)

2 What are the number of points at which the given two curves intersect?

A. 2	B. 1
C. 3	D. 0

3 The slope of the curve $x = y^2$ at the point of intersection of both the given curves is

A. $\frac{1}{2}, -\frac{1}{2}, \frac{1}{0}(not\ defined)$	B. $\frac{1}{2}, 0$
C. $-\frac{1}{2}, \frac{1}{0}(not\ defined)$	D. $\frac{1}{2}, \frac{1}{0}(not\ defined)$

4 The slope of tangent to the curve $y = x^2$ at the point of intersection of both the given curves, is

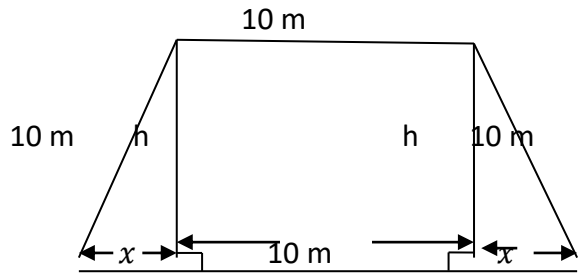
A. 0, 2	B. 2, - 2
C. 0, - 1	D. 2, - 2, 0

5 The angle of intersection of both the curves is

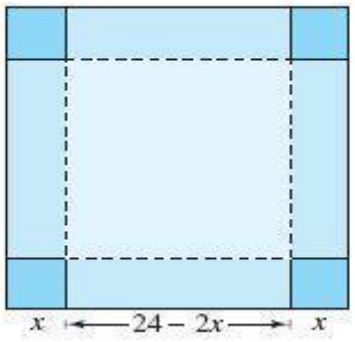
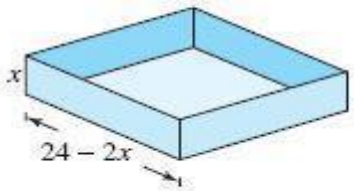
A. $\pi, \tan^{-1}\frac{3}{4}$	B. $\frac{\pi}{2}, \tan^{-1}\frac{4}{3}$
C. $\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$	D. $-\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$

QUE 12	The shape of a toy is given as $f(x)=6(2x^4-x^2)$. To make the toy beautiful 2 sticks which are perpendicular to each other we replace data point (2,3),above the toy. 	
1	Which value from the following may be abscissa of critical point?	
	A.$\pm\frac{1}{4}$	B.$\pm\frac{1}{2}$
	C.±1	D.None
2	Find the slope of the normal based on the position of the stick.	
	A.360	B. −360
	C. $\frac{1}{360}$	D.$-\frac{1}{360}$
3	What will be the equation of the tangent at the critical point if it passes through (2,3)?	
	A.x+360y=1082	B.y=360x−717
	C.x =717y +360	D.None
4	Find the second orderderivative of the function atx= 5.	
	A.598	B.1176
	C.3588	D.3312
5	At which of the following intervals will f(x)be increasing?	
	A.$(-\infty,-\frac{1}{2})\cup(\frac{1}{2},\infty)$	B.$(-\frac{1}{2},0)\cup(\frac{1}{2},\infty)$
	C.$(0,\frac{1}{2})\cup(\frac{1}{2},\infty)$	D. $(-\infty,-\frac{1}{2})\cup(0,\frac{1}{2})$

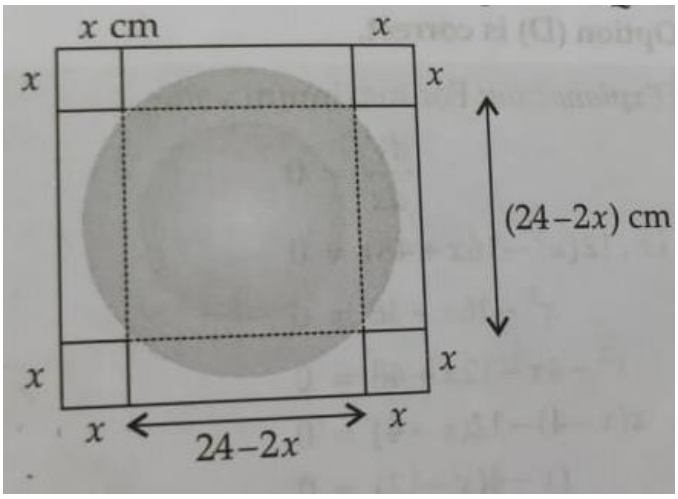
Q.13	You want to make two gardens in the shape of square and circle in front of your house. If you purchase a wire of length 28m to fence these gardens and you have used *x* metres of wire to fence circular garden.  
1.	Radius of the circular garden is? a) $\frac{x}{4}$ b) $\frac{x}{\pi}$ c) $\frac{x}{2\pi}$ d) $\frac{x}{2}$
2.	Side of squared garden is? a) $\frac{x-28}{4}$ b) $\frac{28-x}{4}$

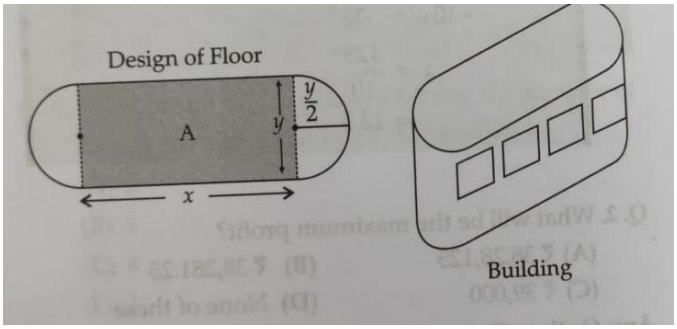
	c) $\frac{28-x}{2\pi}$ d) $\frac{x-28}{2\pi}$
3.	Combined area of circular and squared garden is given by? a) $\pi\left(\frac{x}{2\pi}\right)^2+\left(\frac{x-28}{4}\right)^2$ b) $\pi\left(\frac{28-x}{2\pi}\right)^2+\left(\frac{x}{4}\right)^2$ c) $\pi\left(\frac{x}{2\pi}\right)^2+\left(\frac{28-x}{4}\right)^2$ d) $\pi\left(\frac{x-28}{2\pi}\right)^2+\left(\frac{x}{4}\right)^2$
4.	If you want to minimize the combined area of both gardens without wasting the wire of length 34m. Then How much length of the wire will be needed to fence the circular garden. a) $\frac{112}{\pi+4}m$ b) $\frac{28}{\pi+4}m$ c) $\frac{28\pi}{\pi+4}m$ d) $\frac{112\pi}{\pi+4}m$
5.	And how much length of the wire will be needed to fence the squared garden? a) $\frac{112}{\pi+4}m$ b) $\frac{28}{\pi+4}m$ c) $\frac{28\pi}{\pi+4}m$ d) $\frac{112\pi}{\pi+4}m$
QUE 14	The bridge is in the shape of a trapezium as shown below. Its three sides other than base are of 10 m each. The height of the gate is 'h' meter. 

	
1.	The Area (A) of the gate expressed as a function of x is (a)$(10+x)\sqrt{100+x^2}$ (b) $(10-x)\sqrt{100+x^2}$ (c) $(10+x)\sqrt{100-x^2}$ (d) $(10-x)\sqrt{100-x^2}$
2.	The value of x when A is maximum, is (a) 5 m b) 10 m c) 20 m d) 15 m
3.	The value of 'h' when A is maximum, is a) $5\sqrt{2}$ m b) $5\sqrt{3}$ m c) $10\sqrt{2}$ m d) $10\sqrt{3}$ m
4.	Maximum value of A is a) $\frac{75\sqrt{3}}{2}$ m^2 b) $\frac{75\sqrt{3}}{4}$ m^2 c) $75\sqrt{3}$ m^2 d) 75 m^2
5	The value of $\frac{dA}{dx}$ is a) $\frac{100-10x-2x^2}{\sqrt{100-x^2}}$ b) $\frac{100+10x-2x^2}{\sqrt{100-x^2}}$ c) $\frac{10x-2x^2}{\sqrt{100-x^2}}$ d) $\frac{2x^2-10x}{\sqrt{100-x^2}}$
Q.15	A window is in the form of a rectangle surmounted by a semicircular opening.  Based on the above information answer the following:
1	The perimeter of the windowis:

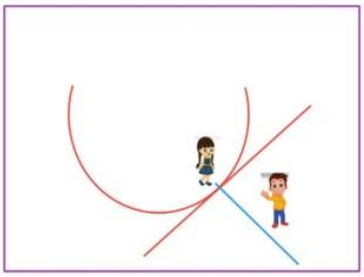
	(a) 2x+4y+πx(b) x + 2y +π (c) 2x +2y +π (d) π
2	The Area as function of x is when perimeter P isgiven: (a) A(x)=Px-2x^2- $\frac{\pi x^2}{2}$ (b) $A(x) = 4x + \pi x^2$ (c) A(x)=Px-4x+ $\frac{\pi x^2}{4}$ (d)$A(x) = 2x + \pi x^2 - Px$
3.	The value of x for which Area is maximum: (a) $2P/(\pi+4)$ (b) P /2(π+4) (c) P /(π+4) (d) π+9
4.	The length of window for which area is maximum: (a) $2P/(\pi+4)$ (b) $P/2(\pi+4)$ (c) $P/(\pi+4)$ (d) π-P
5.	The perimeter of the window when its dimensions are halved is: (a) 2x +4y+π (b) x + 2y+ π (c) x +y +$\frac{\pi x}{2}$ (d) None of these
Q.16	A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box.  
1	Volume of open box formed by folding up the flap: (a) 4(x^3 – 24x^2 + 144x) (b) 4(x^3– 34x^2 + 244x) (c) x^3 – 24x^2 + 144x (d)4x^3 – 24x^2 + 144x
2.	In the first derivative test, if dy/dx changes its sign from positive to negative as x increases through c_1, then function attains a: (a) Local maxima at x = c_1 (b) Local minima at x = c_1 (c)Neither maxima nor minima at x = c_1 (d)None of these
3.	What should be the maximum volume of open box? (a) 1034 cm^3 (b) 1024 cm^3 (c)1204 cm^3 (d)4021 cm^3
4.	What should be the side of the square piece to be cut from each corner of the board to

	behold the maximum volume? (a) 14 cm (b)12 cm (c)4 cm (d)5 cm
5.	What are the dimensions cuboid i.e. l,b,h (a) 16,12,4 (b) 16,16,4 (c)18,16,4 (d)4,4,4

QUE No	Question
17	Read the following test and answer the following questions on the basis of the same. An open box is to be made out of a piece of cardboard measuring (24 cm $\times$ 24 cm) by cutting of equal squares form the corners and turning up the sides.  (i) Find the value of that open box. (A) $4x^3-96x^2+576x$ (B) $4x^3+96x^2-576x$ (C) $2x^3-48x^2+288x$ (D) $2x^3+48x^2-288x$ (ii) Find the value of $\frac{dy}{dx}$. (A) $12\left(x^2+16x-48\right)$ (B) $12\left(x^2-16x+48\right)$ (C) $6\left(x^2+8x-24\right)$ (D) $6\left(x^2-8x+24\right)$ (iii) Find the value of x other than 12. (A) 3 cm (B) 9 cm (C) 1 cm (D) 4 cm (iv) Find the value of $\frac{d^2y}{dx^2}$. (A) $24\left(x+8\right)$

	(B) $12(x-4)$ (C) $24(x-8)$ (D) $12(x+4)$ (v) Volume is maximum at what height of that open box. (A) 3 cm (B) 9 cm (C) 1 cm (D) 4 cm
QUE 18	Read the following test and answer the following questions on the basis of the same.  The area of the base is 200 sq unit. (i) If x and y represents the length and breadth of the rectangular region then the relation between the variable. (A) $x+\pi y=100$ (B) $2x+\pi y=200$ (C) $\pi x+y=50$ (D) $x+y=100$ (ii) The area of the rectangular region A expressed as a function of x is. (A) $\frac{2}{\pi}(100x-x^2)$ (B) $\frac{1}{\pi}(100x-x^2)$ (C) $\frac{x}{\pi}(100-x)$ (D) $\pi y^2+\frac{2}{\pi}(100x-x^2)$ (iii) The maximum value of area A. (A) $\frac{\pi}{3200}\ \mathrm{m}^2$ (B) $\frac{3200}{\pi}\ \mathrm{m}^2$ (C) $\frac{5000}{\pi}\ \mathrm{m}^2$ (D) $\frac{1000}{\pi}\ \mathrm{m}^2$

	(iv) The CEO of the multinational company is interested in minimizing the area of the whole floor including the semi-circular ends, for this to happen the value of x should be (A) 0 (B) 30 m (C) 50 m (D) 80 m (v) The extra area generated if the area of the whole floor is maximized is. (A) $\frac{3000}{\pi}\ \mathrm{m}^2$ (B) $\frac{5000}{\pi}\ \mathrm{m}^2$ (C) $\frac{7000}{\pi}\ \mathrm{m}^2$ (D) None, both area are equal

19.	A student Alok is running on a playground along the curve given by $y = 2x^2 + 7$. Another student Arunima standing at a point of contact of tangent which is parallel to the line 4x – y +3 = 0. 
(i)	Alok' s position at any value of x will be (a) $(x^2,\ y-7)$ (b) $(x^2,\ y+7)$ (c) $(x\ , 2x^2+\ 7)$ (d) $(x^2, x-7)$
(ii)	Arunima position will be : (a) (2, 10) (b) (2,9) (c) (1 ,10) (d) (1, 9)
(iii)	Distance between alok and arunima will be: (a) $(x-1)\sqrt{1+4\,(x+1)^2}$

	(b)$(x+1)\sqrt{1+4(x-1)^2}$ (c)$4\sqrt{(x-1)^2+\ (x+1)^2}$ (d)$(x-1)\sqrt{4+\ (x+1)^2}$
(iv)	Arunima standing on a tangent line. Then the equation of tangent is : (a)$4x-y+3=0$ (b)$y-x+3=0$ (c) $y+x+9=0$ (d) $4x-y+5=0$
(V)	If Alok standing on a normal line then slope of normal will be : (a)4 (b) -4 (c) ¼ (d) -1/4

ANSWERS

Q.No. 1	(i) C (ii) B (iii) A (iv) C (v) B
Q.No. 2	(i) D (ii) B (iii) D (iv) C (v) A
Q.No. 3	(i) A (ii) B (iii) C (iv) A (v) C
Q NO 4	(i) b (ii) a (iii) a (iv) (b)
Q No 5	(i) b (ii) c (iii) a (iv) c
Q No 6	I (1/4,1/2) ii no iii (0,0) iv (0,1) v x=0
Q No 7	1. X=1,2,8/5 2. Not a turning point 3. $(-\infty, 1) \cup (1, \frac{8}{5}) \cup (2, \infty)$4. $(\frac{8}{5}, 2)$ 5. No
Q No 8	(i) b (ii) a (iii) c (iv) c (v) d
Q No 9	(i) b (ii) c (iii) d (iv) c (v) d
Q No 10	1.(c) 2.(a) 3.(c) 4.(c) 5.(d)
Q No 11	1.(b) 2.(d) 3.(b) 4.(c) 5.(b)
Q No 12	1.B 2.D 3.B 4.C 5.B
Q No 13	1.C 2. B 3. C 4. C 5. A
Q No 14	1. C 2. A 3. B 4. A 5. A
Q No 15	1. A 2. A 3. C 4. A 5.D
Q No 16	1. A 2. A 3. B 4. C 5. B
Q No 17	(i) A (ii) B (iii) D (iv) C (v) D
Q No 18	(i) B (ii) A (iii) C (iv) A (v) D
Q no 19	(i) C (ii)D (iii) A (iv)D (v) D

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