CHAPTER 7

INTEGRALS

CASE BASED QUESTIONS

Sl. No.	Read the passage given below and answer the following questions
1.	We know that $\int_a^n f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$ ………+$\int_m^n f(x)dx$ where a<b<c……..<m<n Also |x| =$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$. Now evaluate the following
a)	$\int_0^4 \lvert x-1\rvert$dx = (A) -5 (B) 5 (C) 3 (D) None of these
b)	$\int_{-1}^1 \lvert x\rvert dx$ = (A) -1 (B) 1 (C) 2 (D) -2
c)	$\int_{-\pi/2}^{\pi/2} \lvert sinx\rvert$ dx = (A) 1 (B) 2 (C) -2 (D) -1
d)	$\int_{-2}^1 \lvert x^2-1\rvert$ dx = (A) 3 (B) -3 (C) 2 (D) -2

2.	Understand the following properties and answer the questions $\int_{-a}^{a} f(x)$ dx = 2 $\int_{0}^{a} f(x)$ dx , f(-x) = f(x) even = 0 , f(-x) = -f(x) odd And $\int_{0}^{a} f(x)$dx = $\int_{0}^{a} f(a-x)$dx
a)	$\int_{0}^{\pi/2} \log\ tanx$ dx = (A) 0 (B) 1 (C) 2 (D) 3
b)	$\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)$ dx = (A) 1 (B) 2 (C) 3 (D) 0
c)	$\int_{0}^{\pi/4} log(1+Tanx)$ dx = (A) (π/2) log2 (B) (π/4) log2 (C) (π/8) log2 (D) None of these
d)	$\int_{0}^{1} tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)$ dx = (A) 0 (B) 1 (C) 2 (D) 3
3	The work done by a constant force of magnitude F on a point that moves a displacement d in the direction of the force is the product: W = Fd. Integration approach can be used both

	to calculate work done by a variable force and work done by a constant force. This suggests that integrating the product of force and distance is the general way of determining the work done by a force on a moving body. The work done by a force f(x) which displace a body from a point a to b is define as below. W=$\int_a^b f(x).dx$ Based on the information given above, answer the following questions:
a)	The work done by a variable force f(x) =x^2-4 from x=5 to x=8 is calculated by formula (a) W= (x^2-4) .(8-5) (b) W=$\int_5^8 3(x^2-4)dx$ (c) W=$\int_5^8 (x^2-4)dx$ (d) W=$\int_0^8 (x^2-4)dx$
b)	The work done by a variable force f(x) =x^2-4 from x=5 to x=8 is (a) 118 units (b) 117 units (c) 116 units (d) 115 units
c)	The work done by force f(x)=$\tan^2 x$ from 0 to $\frac{\pi}{4}$. (a) $\frac{4+\pi}{4}$ $units$

	(b) $\frac{4-\pi}{4}$ *units* (c) $\frac{4+\pi}{2}$ *units* (d) $\frac{4-\pi}{2}$ *units*
d)	The work done by force $f(x)=\sin^3 x.\cos^4 x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$. (a) $\frac{\pi}{4}$ *units* (b) $\frac{4\pi}{5}$ *units* (c) 0 units (d) $\frac{19}{20}$ *units*
4	In real life, integrations are used in various fields such as engineering, where engineers use integrals to find the shape of building. In Maths/Physics, used in to find the center of gravity, kinetic energy, mass of a body, volume , surface area, force, work etc. In the field of graphical representation, where three-dimensional models are demonstrated.
a)	Find the area bounded by $y = x^2$,hex − axis and the lines $x = -1$ and $x = 1$. (a) $\frac{4}{3}$ *sq unit* (b) $\frac{3}{2}$ *sq unit* (c) $\frac{2}{3}$ *sq unit* (d)$\frac{1}{3}$ *sq unit*
b)	Find the area bounded by y = x, the x − axis and the lines $x = 0$ and $x = 4$. (a) 4 sq units (b) 16 sq units (c) 32 sq units (d) 8 sq units
c)	Volume of water in 4 minutewhich passes through in a riverwhose width(b) and height(h) are 5m and 4m, and water is flowing at a rate $\frac{dl}{dt}$= $(3t^2 + 2t)$ m/ minute. then formula fot calculating volume of water is

	(a) Volume=4 $\int_0^{20}(3t^2+2t)dt$ (b) Volume=5 $\int_0^{4}(3t^2+2t)dt$ (c) Volume=4$\int_0^{4}(3t^2+2t)dt$ (d) Volume=20 $\int_0^{4}(3t^2+2t)dt$
d)	Volume of water in 4 minute which passes through in a river whose width(b) and height(h) are 5m and 4m, and water is flowing at a rate $\frac{dl}{dt}$= ($3t^2$ + 2t) m/ minute. then volume of water is (a) 128000 liter (b) 1600000 liter (c) 1400000 liter (d) 800000 liter
e)	The value $\int_{-2}^{2}\lvert x\rvert dx$ is (a) 4 (b) 0 (c) 8 (d) 2
5	The given Integral ∫f(x)dx can be transformed into another form by changing the independent variable x to t by substituting x=g(t) Consider I=∫f(x)dx Put x=g(t), so that dx/dt=g'(t) we write dx=g'(t)dt Thus I=∫f(x)dx = ∫f(g(t))g'(t)dt This change of variable formula is one of the important tools available to us in the name of integration by substitution. For example: ∫2xsin(x^2+1)dx Put x^2+1=t 2xdx=dt ∫sin(t)dt= −cos(t)+C = −cos(x^2+1)+C Based on the above information answer the following questions. (i) $\int \frac{\sin(\tan^{-1}x)}{1+x^2}dx$ is equal to:

	$(a) - sin(\tan^{-1} x) + C$ $(b) - \cos(\tan^{-1} x) + C$ $(c)\ tanx + C$ (d)None of these (ii) ∫tanxdx is equal to: (a) secx+C (b) cotx+C (c) log|x|+C (d) None of these (iii) $\int \frac{2x}{1+x^2} dx$ is equal to: (a) $1+x^2+C$ (b) $\log\|1+x^2\|+C$ (c) $\log\|2/(1+x^2)\|+C$ (d) None of these (iv) ∫sin(ax+b)cos(ax+b)dx is equal to: (a) $\cos^2(ax+b)+C$ (b) $\sin^2(ax+b)$ (c) (−1/4a)cos2(ax+b)+C (d) None of these (v) $Integral\ of\ \frac{1}{x+xlogx}$is equal to: (a) |1+logx|+C (b) log|1+logx|+C (c) logx+C (d) None of these
6	The given Integral ∫f(x)dx can be transformed into another form by changing the independent variable x to t by substituting x=g(t) Consider I=∫f(x)dx

	Put x=g(t) so that dx/dt=g'(t) we write dx=g'(t)dt Thus I=∫f(x)dx = ∫f(g(t))g'(t)dtf This change of variable formula is one of the important tools available to us in the name of integration by substitution. Based on the above information, answer the following questions: (i) $\int\frac{e^{\tan^{-1}x}}{1+x^2}dx$ is equal to: (a) $e^{\tan^{-1}x}$+C (b) $\tan^{-1}$x +C (c) e^x+C (d) None of these (ii) $\int\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ is equal to: (a) $(\sin^{-1}x)/2$+C (b) $(\sin^{-1}x)^2/2$+C (c) (sinx)/2+C (d) None of these (iii) $\int\frac{sinx}{(1+cosx)^2}dx$ equal to: (a) 1/(1+cosx)+C (b) sinx+C (c) cosx+C (d) 1/(1−cosx)+C (iv) $\int\frac{x}{e^{x^2}}dx$ equal to: (a) $\frac{1}{2e^{x^2}}$+C (b) $\frac{1}{e^{x^2}}$+C (c) $\frac{-1}{2e^{x^2}}$+C (d) None of these

	(v) $\int \frac{1}{x(\log x)^m} dx$,x>0 equals: (a) logx+C (b) $(\log x)^{1-m}/(1-m)+C$ (c) 1/logx+C (d) None of these
7	The given integral $\int f(x)dx$ can be transformed into another form by changing the independent variable *x* to *t* by substituting $x = g(t)$. Consider $I = \int f(x)dx$ Put $x = g(t)$ so that $\frac{dx}{dt} = g'(t)$ We write $dx = g'(t)dt$ Thus, $I = \int f(x)dx = \int f\{g(t)\}g'(t)\, dt$ This change of variable formula is one of the important tools available to us in the name of integration by substitution. For example $\int 3x^2 \cos(x^3 - 2)dx$ Put $x^3 - 2 = t$ $3x^2dx = dt$ $\therefore \int \cos(t) = \sin(t) + C = \sin(x^3 - 2) + C$ Based on the above information answer the following questions.
a)	$\int \frac{\sin(\tan^{-1} x)}{1 + x^2} dx$ is equal to (A) $-\sin(\tan^{-1} x) + C$ (B) $-\cos(\tan^{-1} x) + C$ (C) $\tan(\sin^{-1} x) + C$ (D) $-\tan(\sin^{-1} x) + C$
b)	$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx$ is equal to (A) $\frac{1}{2}\log\lvert 3\cos x + 2\sin x\rvert + C$ (B) $\frac{1}{2}\log\lvert 3\cos x - 2\sin x\rvert + C$ (C) $\frac{1}{2}\log\lvert 2\cos x + 3\sin x\rvert + C$ (D) $\frac{1}{2}\log\lvert 2\cos x - 3\sin x\rvert + C$

c)	$\int (x^3-1)^{\frac{1}{3}} x^5 dx$ is equal to (A) $\frac{1}{7}(x^3-1)^{\frac{7}{3}} + \frac{1}{4}(x^3-1)^{\frac{4}{3}} + C$ (B) $\frac{1}{7}(x^3-1)^{\frac{7}{3}} - \frac{1}{4}(x^3-1)^{\frac{4}{3}} + C$ (C) $\frac{3}{7}(x^3-1)^{\frac{7}{3}} + \frac{3}{4}(x^3-1)^{\frac{4}{3}} + C$ (D) $\frac{3}{7}(x^3-1)^{\frac{7}{3}} - \frac{3}{4}(x^3-1)^{\frac{4}{3}} + C$
d)	$\int \frac{(x-1)(x-\log x)^3}{x} dx$ is equal to (A) $\frac{1}{3}(x-\log x)^3 + C$ (B) $\frac{1}{3}(x-1)(x-\log x)^3 + C$ (C) $\frac{1}{4}(x-\log x)^4 + C$ (D) $\frac{1}{4}(x-1)(x-\log x)^4 + C$
e)	$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ is equal to (A) $\cos(e^x x) + C$ (B) $-\cos(e^x x) + C$ (C) $\tan(e^x x) + C$ (D) $\tan(x^x e) + C$
8	Consider the integral $I = \int e^x[f(x) + f'(x)]\, dx$ $= \int e^x f(x)\, dx + \int e^x f'(x)\, dx$ $= I_1 + \int e^x f'(x)\, dx, \ldots\ldots\ldots (1)$ where $I_1 = \int e^x f(x) dx$ Taking $f(x)$ as the first function and e^x as second function in I_1andintegrating

	it by parts, we have $I_1 = f(x)e^x - \int f'(x)\, e^x\, dx + C$ Substituting I_1 in (1), we get $I = e^x f(x) - \int f'(x)\, e^x\, dx + \int e^x f'(x)\, dx + C$ $I = e^x f(x) + C$ Thus, $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$ For example $\int e^x [\sin x + \cos x]\, dx$ Put $f(x) = \sin x$ so that $f'(x) = \cos x$ $\therefore \int e^x [\sin x + \cos x]\, dx = \int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C = e^x \sin x + C$ Based on the above information answer the following questions.
a)	$\int e^x \sec x\, [1 + \tan x] dx$ is equal to (A) $e^x \sec x \tan x + C$ (B) $e^x \sec x + C$ (C) $e^x \tan x + C$ (D) $e^x [1 + \tan x] + C$
b)	$\int e^x \left[\tan^{-1} x + \frac{1}{1+x^2}\right] dx$ is equal to (A) $e^x \tan x + C$ (B) $\frac{e^x}{1+x^2} + C$ (C) $e^x \tan^{-1} x + C$ (D) $e^x \frac{\tan^{-1} x}{1+x^2} + C$
c)	$\int e^x \left[\frac{1+\sin x}{1+\cos x}\right] dx$ is equal to (A) $e^x \tan x + C$ (B) $e^x \tan \frac{x}{2} + C$ (C) $e^x \sec x + C$ (D) $e^x \sec^2 \frac{x}{2} + C$

d)	$\int \frac{x\, e^x}{(1+x)^2}\,dx$ is equal to (A) $\frac{e^x}{1+x}+C$ (B) $\frac{e^x}{1-x}+C$ (C) $\frac{e^x}{(1+x)^2}+C$ (D) $\frac{e^x}{(1+x)^3}+C$
e)	$\int \frac{(x^2+1)e^x}{(x+1)^2}\,dx$ is equal to (A) $e^x\frac{x+1}{x-1}+C$ (B) $e^x\frac{x-1}{x+1}+C$ (C) $e^x\frac{x^2+1}{x+1}+C$ (D) $e^x\frac{x^2+1}{x-1}+C$
9	1) A thermometer reading $80^0F$is taken outside. Five minutes later the thermometer reads 60^0F. After another 5 minutes the thermometer reads 50^0F. At any time t the thermometer reading be $T^0Fandtheoutsidetempraturebe S^0F$.  Based on the above information, answer the following questions: a) Ifλ *is positive constant of proportionality then* $\frac{dT}{dt}$ *is* (i) λ (T-S) (ii) λ (T+S) (iii) - λ (T-S) (iv) λTS b) The value of T(5)

	(i) 30^0F. (ii) 40^0F. (iii) 50^0F. (iv) 60^0F. c) The value of T(10) (i) 50^0F. (ii) 60^0F. (iii) 80^0F. (iv) 90^0F. d) The function T is given by (i) $logT = St + C$ (ii) $\log(T - S) = -\lambda t + C$ (iii) $logS = Tt + C$ (iv) $\log(T + S) = \lambda t + C$ (v) e) The value of the constant of integration C in given situation will be (i) $\log(60 - S)$ (ii) $\log(80 + S)$ (iii) $\log(80 - S)$ (iv) $\log(60 + S)$
10	It is known that if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annumn and the principal. Let P be the principal at any time t and rate of interest be r% per annum.  Based on the above information answer the following: (a) The value of $\frac{dP}{dt} willbeequalto$

	(i) Pr (ii) $\frac{Pr}{1000}$ (iii) $\frac{Pr}{100}$ (iv) $\frac{Pr}{10}$ (b) If $P_0 istheinitialprincipalthenthesolutioninthegivensituationwillbe$ (i) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$ (ii) $\log\left(\frac{P}{P_0}\right) = \frac{rt}{10}$ (iii) $\log\left(\frac{P}{P_0}\right) = rt$ (iv) $\log\left(\frac{P}{P_0}\right) = 100rt$ (c) if the interest is compounded continuously at 5% per annum, in how many years will Rs. 100 double itself (i) 12.728 (ii) 14.789 (iii) 13.862 (iv) 15.872 (d) At what rate interest rate will Rs.100 double itself in 10 years? ($\log_e 2 = 0.6931$) (i) 9.66% (ii) 8.239% (iii) 7.341% (iv) 6.931% (e) How much will Rs. 1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$) (i) Rs.1648 (ii) Rs. 1500 (iii) Rs. 1664 (iv) Rs. 1572.
11	In integral of the type $\int \frac{dx}{a+b\cos x}$ is integrated by substitution method in which converts the integral into the rational fraction . observe the above integral . Answer the following questions
a)	Which one of the following suitable substitution. (a) $\tan\frac{x}{2}$ = t (b)$\sec\frac{x}{2} = t$ (c)Cos x = t (d)sin x =t
b)	Cos x in term of t (a) $1 + t^2$ (b)$\frac{2t}{1+t^2}$ (c) $1 + t^3$ (d) 2t
c)	What is the value of dx in term of t (a) $\frac{2dt}{1+t^2}$ (b)$\frac{2}{1+t^2}$ (c)$1+t^2$ (d) 3t
d)	Sin x in term of t (a) $1 + t^2$ (b)$\frac{2t}{1+t^2}$ (c) $1 + t^3$ (d) 2t
e)	I = $\int \frac{dx}{a+b\cos x}$ in term of t

	(a) $2\int \frac{dt}{(a+b)+(a-b)t^2}$ (b) $2\int \frac{dt}{1+(a-b)t^2}$ (c) $2\int \frac{dt}{(a+b)+a\,t^2}$ (d) $2\int \frac{dt}{t^2}$
12	Integration I = $\int \frac{x^4}{x^4-1}$by using partial fraction method . Let$\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$. observe the condition and answer the questions
a)	What is the value of A (a) $\frac{1}{4}$ (b)$\frac{1}{3}$ (c)$\frac{1}{2}$ (d)$\frac{1}{5}$
b)	What is the value of B (b) $\frac{1}{4}$ (b)$\frac{1}{3}$ (c)$\frac{1}{-4}$ (d) $\frac{1}{5}$
c)	What is the value of C (c) $\frac{1}{4}$ (b) 0 (c) $\frac{1}{2}$ (d)$\frac{1}{5}$
d)	What is the value of D (d) $\frac{1}{4}$ (b)$\frac{1}{3}$ (c) -$\frac{1}{2}$ (d) $\frac{1}{5}$
e)	What is the value of I when evaluated (a) I = $x +$ $\frac{1}{4}$log|x-1| -$\frac{1}{4}$ log|x+1| - $\frac{1}{2}tan^{-1}x + C$ (b) I= $1 +$ $\frac{1}{4}$log|x-1| -$\frac{1}{4}$ log|x+1| - $\frac{1}{2}tan^{-1}x + C$ (c) I = $2x +$ $\frac{1}{4}$log|x-1| -$\frac{1}{4}$ log|x+1| - $\frac{1}{2}tan^{-1}x + C$ (d) $I = 3\,x +$ $\frac{1}{4}$log|x-1| -$\frac{1}{4}$ log|x+1| - $\frac{1}{2}tan^{-1}x + C$

13	 A train is moving very fast from one state capital to another.If the acceleration of the moving train is given as (2t + 5) kmh^{-2} where t represents the time taken. On the basis of the above information , choose the correct answer:
a)	What will be the velocity relation? A. $t^3 - t^2 + C$ B. $t^2 - t + C$ C. $t^2 + 2t + C$ D. $t^2 + 5t + C$
b)	At t = 0 , what will be the value of C? A. 0 B. 1 C. 2 D. 3
c)	Write the distance relation at time t? A. $t^3/3 + 5t^2/2$ B. $t^3 - t^2/2$ C. $t^2/3 - 5t/2$ D. $t^2/3 + 5t/2$
d)	Distance(km) covered in 10 hours is A. 3000 B. 3500 C. 4000 D. 1750/3

e)	Speed of the train after 5 hours is A. 35 km/h B. 45 km/h C. 50 km/h D. 60 km/h
14	If g *is a continuous function defined on* $[0,a]$, *then* $\int_0^a g(x)dx = \int_0^a g(a-x)dx.$ On the basis of above information, answer the following questions.
a)	$\int_0^a \frac{g(x)}{g(x)+g(a-x)}dx$= (A)$a$ (B)$\frac{a}{2}$ (C) $-\frac{a}{2}$ (D) $2a$
b)	If $g(x) = \frac{secx-cosecx}{1+secxcosecx}$, then$g\left(\frac{\pi}{2}-x\right)$= A)$2g(x)$ (B)$g(x)$ (C) $-g(x)$ (D) $\frac{1}{g(x)}$
c)	If $g(x) = \frac{\sqrt{cotx}}{\sqrt{cotx}+\sqrt{tanx}}$, *then* $\int_0^{\frac{\pi}{2}} g(x)dx$ = (A)π (B)$\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D)$\frac{\pi}{4}$
d)	If $g(x) = \log(1+tanx)$ *thenthevalueof* $g\left(\frac{\pi}{4}-x\right)+g(x) =$

	(A)$log2$ (B) $2log2$ (C) log3 (D) $2log3$
e)	If $g(x) = \frac{xsinx}{1+cos^2x}, then \int_0^\pi g(x)dx =$ (A)0 (B)$\frac{\pi}{4}$ (C) $\frac{\pi^2}{4}$ (D)$\frac{\pi^2}{2}$
15	For any function f(x), we have $\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \cdots + \int_{c_n}^b f(x)dx$ $, wherea < c_1 < c_2 < c_3 < \cdots < c_n < b.$ On the basis of above information, answer the following questions
a)	$\int_0^3 \|2 - x\|dx$= (A)$\frac{5}{2}$ (B)$\frac{3}{2}$ (C)$-\frac{3}{2}$ (D)$-\frac{5}{2}$
b)	$\int_{-1}^2 x\|x\|dx$= (A)$\frac{5}{3}$ (B)$\frac{3}{5}$

	(C)$\frac{3}{7}$ (D)$\frac{7}{3}$
c)	$\int_0^{\frac{\pi}{2}} \lvert sinx - cosx \rvert dx$= (A)0 (B)$\sqrt{2}-1$ (C)$2(\sqrt{2}-1)$ (D)$2(\sqrt{2}+1)$
d)	$\int_{-2}^{3} \lvert 1-x^2 \rvert dx$= (A)$\frac{1}{3}$ (B)$\frac{7}{3}$ (C)$\frac{14}{3}$ (D)$\frac{28}{3}$
e)	$\int_0^2 x^2[x]dx$= (A)$\frac{5}{3}$ (B)$\frac{1}{3}$ (C)$\frac{7}{3}$ (D)$\frac{8}{3}$
16	A person was walking alon a line (AB) represented by equation x-5y+9 = 0. After some time he started to walk along (BC) represented by equation 2x+3y-21=0 and after reaching at C he turned again and started to walk along (CA) represented by equation 3x-2y+1 = 0 and reached at initial point of starting. (i) What distance he walked along AB? (ii) What distance he walked along BC? (iii) What distance he walked along CA? (iv) If these lines represent three roads then, what is the area enclosed by these

	roads?
17	A motorcyclist was moving along a line (AB) represented by equation x + 2y = 2. After some time he started to walk along (BC) represented by equation x-y = -1 and after reaching at C he turned again and started to walk along (CA) represented by equation 2x + y = 7 and reached at initial point of starting. (i) What distance he walked along AB? (ii) What distance he walked along BC? (iii) What distance he walked along CA? (iv) If these lines represent three roads then, what is the area enclosed by these roads?
18	
	Let [x] denote the greatest integer ≤x and x is a positive integer. (A).[3.2] is equal to (a)2$(b)3$ (c) 1.5 (d) 1 (B) The value of $\int_{-n}^{n}[x]dx$ for n = 1 is equal to (a)2$(b)\frac{1}{2}$ (c) -1 (d) 0 (C) The value of $\int_{-n}^{n}[x]dx$ for n = 2 is equal to (a)-2 $(b)\frac{1}{2}$ (c) -1 (d) 0 (D)The value of $\int_{0}^{n}[x]dx$ for n = 1 is equal to (a)3$(b)2$ (c) -1 (d) 0
19	Read the following text and answer the following questions on the basis of the same: $\int e^x[f(x) + f'(x)]\,dx = \int e^x f(x)dx + \int e^x f'(x)dx$ $= e^x f(x) - \int e^x f'(x)dx + \int e^x f'(x)dx$ $= e^x f(x) + c$ (A) $\int_0^{\pi/2} e^x(\cos x + \sin x)\,dx =$ ______________ (a)0$(b)e^{\pi/2}$ (c) e (d) e-1 (B) $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)$ dx is equal to (a)1$(b)e^{\pi/2}$ (c) e^{π} (d) e-1

	(C) $\int_0^{\pi/2} e^x(x+1)dx$ (a)$\frac{\pi}{2}e^{\pi/2}$ $(b)e^{\pi/2}$ (c) e^{π} (d) e-1 (D)$\int_0^{\pi} e^x(\tan x+\sec^2 x)\ dx$ is equal to (a)1 $(b)0$ (c)-1 (d) e-1
20	Read the following text and answer the following Questions on the basis of the same. Function Odd function Even Function $f(-x)=-f(x) f(-x)=f(x)$ $\int_{-a}^{a} f(x)dx=0 \int_{-a}^{a} f(x)dx=2\int_0^{a} f(x)dx$
	QUESTION
a)	$\int_{-1}^{1} x^{99}dx$ is A. 0 B.1 C.-1 D .2
b)	$\int_{-\pi}^{\pi} x\cos x\, dx$ is A. 0 B.2π C. π D.$\pi/2$
c)	$\int_{-\pi}^{\pi} \cot x\, cosec^2 x\, dx$ is

	A.1 B.-1 C.0 D. 2
d)	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + \tan^5 x + 1)dx$ is A.0 B.2 C.π D.1
21	Read the following text and answer the following Questions on the basis of the same. To evaluate $\int_a^b f(x)\,dx$, by substitution, the steps could be as follows: 1. Consider the integral without limits and substitute, $y = f(x)$ or $x = g(y)$ to reduce the given integral to a known form. 2. Integrate the new integrand with respect to the new variable without mentioning the constant of integration. 3. Resubstitute for the new variable and write the answer in terms of the original variable. 4. Find the values of answers obtained in (3) at the given limits of integral and find the difference of the values at the upper and lower limits.
a)	$\int_2^3 \frac{x\,dx}{x^2+1}$ dx is A.log2 B.log $\sqrt{2}$ C. $\log\frac{3}{2}$ D.$log\frac{2}{3}$
b)	$\int_0^2 x\sqrt{x+2}\ dx$ is A.$\frac{2}{15}(16+8\sqrt{2})$ B. $\frac{1}{15}(16+8\sqrt{2})$ C.$\frac{2}{15}(16-8\sqrt{2})$ D. $\frac{1}{15}(16-8\sqrt{2})$
c)	$\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}\ dx$ is A. e B.$e^{\frac{\pi}{2}} - 1$ C.$e^{\frac{\pi}{4}} - 1$ D.0
d)	$\int_0^1 \frac{x^3 \sin(\tan^{-1}x^4)\ \ dx}{1+x^8}$ is A. $\frac{\sqrt{2}-1}{4}$ B.$\frac{1-\sqrt{2}}{4}$ C.$\frac{\sqrt{2}-1}{8}$ D.$\frac{\sqrt{2}-1}{4\sqrt{2}}$

22.	Evaluate : $\int_{-\pi}^{\pi} \frac{2x\,(1+sinx)}{1+cos^2x}$ dx i) By which property of definite integrals the given integration will be solved? a) $\int_0^a f(x)$ dx = $\int_0^a f(a-x)$ dx , b) $\int_{-a}^{a} f(x)$ dx = 2 $\int_0^a f(x)$ dx , f(x) is an even function 0 , f(x) is an odd function c) $\int_0^{2a} f(x)$ dx = 2 $\int_0^a f(x)$ dx , if f(x) = f(2a –x) 0 , if -f(x) = f(2a-x) d) $\int_a^b f(x)$ dx = $\int_a^b f(a+b-x)$ dx ii) For solving $\int_{-\pi}^{\pi} f(x)$ dx , where f(x) = $\frac{2x}{1+cos^2x}$, the property will be used – a) f(x) =f (2a-x) b) f(2a-x)= -f(x) c) f(-x)= f(x) d) f(-x)= -f(x) iii) For solving $\int_{-\pi}^{\pi} g(x)$ dx , where g(x) = $\frac{2xsinx}{1+cos^2x}$, the property will be used – a)f(-x)= f(x) b) f(x) =f (2a-x) c) f(2a-x)= -f(x) d) f(-x)= -f(x) iv) The value of $\int_{-\pi}^{\pi} g(x)$ dx , where g(x) = $\frac{2xsinx}{1+cos^2x}$ is - a) 0 b) $2\pi^2$ c) π^2 d) - π^2

	v) The value of $\int_{-\pi}^{\pi} \frac{2x\,(1+sinx)}{1+cos^2x}$ dx is - a) 0 b) π^2 c) $2\pi^2$ d) $4\pi^2$
23.	Evaluate: I = $\int_0^{\pi} \log\,(1+cosx)$ dx i)By which property of definite integrals the given integration will be solved? a)$\int_a^b f(x)$ dx = $-\int_b^a f(x)$ dx b) $\int_a^b f(x)$ dx = $\int_a^b f(a+b-x)$ dx c) $\int_0^a f(x)$ dx = $\int_0^a f(a-x)$ dx d) $\int_a^b f(x)$ dx = $\int_a^c f(x)$ dx +$\int_c^b f(x)$ dx ii) By applying property of definite integrals the given integration (I) can be reduced as – a) I = $\int_0^{\pi} \log\,(1-cosx)$ dx b) I = $\int_0^{\pi/2} \log\,(1-sinx)$ dx c) I = $\int_0^{\pi} \log\,(1-sinx)$ dx d) I = $\int_0^{\pi/2} \log\,(1-cosx)$ dx iii) By solving the integration can be written as – a) 2I = $\int_0^{\pi} \log\, cos^2x$ dx b) 2I = $\int_0^{\pi} \log\, sin^2x$ dx c) 2I = $\int_0^{\pi/2} \log\, cos^2x$ dx

	d) 2I = $\int_0^{\pi/2} \log tan^2 x$ dx iv) Another property of definite integral can be applied to solve the given integration is – a) $\int_0^a f(x)$ dx = $\int_0^a f(a-x)$ dx b) $\int_0^{2a} f(x)$ dx =2$\int_0^a f(x)$ dx ,if f(x)=f(2a –x) c) $\int_0^{2a} f(x)$ dx = 0 , if -f(x) = f(2a –x) d) $\int_{-a}^a f(x)$ dx = 0, if f(-x) = - f(x) v) The value of the given integration (I) is – a) $\frac{\pi}{2}$ log 2 b) $\frac{\pi}{2}$ log $\frac{1}{2}$ c) πlog $\frac{1}{2}$ d) πlog 2
24.	For a function $f(x)$, if $(i) \quad f(-x) = f(x)$, then $f(x)$ is an even function $(ii) f(-x) = -f(x)$, then $f(x)$ is an odd function. Again we have, $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if\ f\ is\ an\ even\ function \\ 0, \ \ if\ f(x)\ is\ an\ odd\ function. \end{cases}$ On the basis of above information answer the following questions:
a)	$\int_{-\pi}^{\pi} x\ cosx\ \ dx$ is equal to (A) 1 (B) 0

	(C) -1 (D) $\frac{\pi}{2}$
b)	$\int_{-\pi}^{\pi} x\ sinx\ \ dx$ is equal to (A)π (B)2π (C) 3π (D) 4π
c)	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \|sinx\|\ \ dx$ is equal to (A)1 (B)2 (C) 3 (D) 4
d)	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 x\ \ dx$ is equal to (A)0 (B)1 (C) 2 (D) 7
e)	$\int_{-\pi}^{\pi} tanx\ sec^2 x\ \ dx$ is equal to (A)1 (B)-1 (C) 0 (D) 2
25.	Read the passage given below and answer the following If $f(x)$ is a continuous function defined on [a, b], then

	$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$ On the basis of above information answer the following
a)	$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx$ is equal to (A)$\frac{a+b}{2}$ (B)$\frac{a-b}{2}$ (C) $\frac{b-a}{2}$ (D) $b-a$
b)	If $f(x) = log(tanx),$ then $f(\frac{\pi}{2} - x)$ is equal to (A)$f(x)$ (B)$-f(x)$ (C) $\frac{f(x)}{2}$ (D) $2f(x)$
c)	$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \log(tanx)\ dx$ is equal to (A)$\pi/12$ (B)$\pi/6$ (C) $\pi/3$ (D) 0
d)	If $g(x) = \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}$, then $g(a+b-x)$ is equal to (A)$g(x)$ (B)$1-g(x)$ (C) $\frac{g(x)}{2}$ (D) $2g(x)$

e)	$\int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}\ dx$ is equal to (A)0 (B)$\frac{a+b}{2}$ (C) $\frac{b-a}{2}$ (D) $\frac{a-b}{2}$
26.	Let f be a continuous function defined on the closed interval [a,b] and F be an antiderivative of f then $\int_a^b \boldsymbol{f(x)\ dx = [F(x)]_a^b = F(b) - F(a)}$ It is very useful because it gives us a method of calculating the definite integral more easily. There is no need to keep integration constant C because if we consider F(x) + C instead of F(x). we get $\int_a^b \boldsymbol{f(x)dx = [F(x) + C]_a^b}$ $= \boldsymbol{F(b) + C - F(a) - C}$ $= \boldsymbol{F(b) - F(a)}$ Based on the above information, answer the following questions: (i) $\int_2^3 x^2\ dx$ is equal to: (a) $\frac{7}{3}$ (b) 9 (c)$\frac{19}{3}$ (d) $\frac{1}{3}$ (ii)$\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is equal to: (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

	(iii) $\int_{-1}^{1}(x+1)dx$ equals: (a) -1 (b) 2 (c) 1 (d) 3 (iv) $\int_{2}^{3}\frac{1}{x}\,dx$ equals: (a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\log\frac{3}{2}$ (d) $\log 2$ (v) $\int_{4}^{5} e^{x}\ dx$ equals: (a) 1 (b) $e^{5}-1$ (c) e (d) $e^{5}-e^{4}$
27.	Consider the integral $\int_{a}^{b} f\big(g(x)\big)g'(x)\ dx.$ Let g(x) = t, then $g'(x)\ dx$ = dt Also, when x = a, t = g(a) and t = g(b) for x = b. Therefore $\int_{a}^{b} f\big(g(x)\big)g'(x)\ dx = \int_{g(a)}^{g(b)} f(t)dt$ Thus if the variable in a definite integral is changed, then the substitution in terms of new variable is effected in three places: (i)In the integrand (ii) in the differential, say , dx (iii) in the limits Also, limits of the new variable t are simply the values of t corresponding to the values of the original variable x and so they are obtained by putting the values of x in the substitution relation between x and t. Based on the above information, answer the following questions: (i) $\int_{0}^{1}\sin^{-1}x\ \ dx$ is equal to: (a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}-1$ (d) $\frac{\pi}{2}$ (ii) $\int_{0}^{\frac{\pi}{4}}\frac{dx}{x+\sqrt{x}}$ is equal to: (a) $4\ \log 3$ (b) $\log 2$ (c) $\log 3$ (d) $2\log 3$

	(iii) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+cos^2 x} dx$ equals: (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{4}$ (d) 3 (iv) $\int_1^2 \frac{1}{x(1+\log x)^2} dx$ equals: (a) $\frac{3}{2}$ (b) $\frac{\log 2}{1+\log 2}$ (c) $-\frac{\log 2}{1+\log 2}$ (d) $\log 2$ (v) $\int_4^{12} x(x-4)^{\frac{1}{3}} dx$ equals: (a) 1 (b) $\frac{720}{7}$ (c) 720 (d) 0
28.	Read the passage given below and answer the following questions........ If $f(x)$ is a continuous function defined on $[0,2a]$, then $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, f(2a-x) = f(x) \\ 0, if\ f(2a-x) = -f(x) \end{cases}$
a)	The value of $\int_0^{\pi/2} \log\lvert\tan x + \cot x\rvert\, dx$ (A) $\frac{\pi}{2}\log 2$ (B) $-\frac{\pi}{2}\log 2$ (C) $\pi \log 2$ (D) $-\pi \log 2$
b)	The value of $\int_0^{\pi} x\sqrt{2-x} dx$ (A) $\frac{16\sqrt{2}}{15}$ (B) $\frac{14\sqrt{2}}{15}$

	(C) $-\frac{16\sqrt{2}}{15}$ (D) $-\frac{14\sqrt{2}}{15}$
c)	The value of $\int_0^1 x(1-x)^n dx$ (A) $\frac{1}{(n-1)(n+2)}$ (B) $\frac{1}{(n+1)(n+2)}$ (C) $\frac{1}{(n-1)(n-2)}$ (D) $\frac{1}{(n-1)(n-2)}$
d)	$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx =$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) π
e)	$\int_0^1 \log\left(\frac{1}{x}-1\right) dx =$ (A) 0 (B) 1 (C) 2 (D) 3
29.	Three possible prompts in motion problems involving definite integrals

	<table><tr><th>Type</th><th>Common prompt</th><th>Appropriate expression</th></tr><tr><td>**Displacement**</td><td>"What is the particle's displacement between... and..." or "What is the change in the particle's position between... and..."</td><td>$\int_a^b v(t)\,dt$</td></tr><tr><td>**Total distance**</td><td>"What is the total distance the particle has traveled between... and..."</td><td>$\int_a^b \lvert v(t)\rvert\,dt$</td></tr><tr><td>**Actual position**</td><td>"What is the particle's position at..."</td><td>$C + \int_a^b v(t)\,dt$ where C is the initial condition</td></tr></table>
a)	A particle moves in a straight line with velocity $v(t) = 5 - t$ meters per second, where t is time in second. Then the displacement of the particle between $t = 0$ and $t = 10$ seconds is (A) 25 (B) 0 (C) -25 (D) None of these.
b)	In the above question, How much distance will be covered by the particle? (A) 25 (B) 0 (C) -25 (D) None of these.
c)	Rahul received the following problem: A particle moves in a straight line with the velocity $v(t) = -t^2 + 8$ meters per second,

	where t is time in seconds. At $t = 2$ the particle's distance from the starting point was 5 meters. Which expression should Rahul use to find the total distance of the particle has traveled between $t = 2$ and $t = 6$ seconds? (A) $\lvert v(6) - v(2) \rvert$ (B) $\int_2^6 v(t)dt$ (C) $v'(6)$ (D) $\int_2^6 \lvert v(t) \rvert dt$
d)	From the above question, What is the total distance of the particle has traveled between $t = 2$ and $t = 6$ seconds? (A)$\frac{32}{3}(2\sqrt{2} - 1)$ (B)$\frac{16}{3}(2\sqrt{2} - 1)$ (C)$\frac{32}{3}(2\sqrt{2} + 1)$ (D)$\frac{16}{3}(2\sqrt{2} + 1)$
e)	Divya received the following problem: A particle moves in a straight line with velocity $v(t) = \sqrt{3t - 1}$ meters per second, where t is time in seconds. At $t = 2$, the particle's distance from the starting point was 8 meters in the positive direction. What is the particle's position in $t = 7$ seconds? (A) $8 + \frac{9}{2}[22^{3/2} - 5^{3/2}]$ (B) $8 - \frac{9}{2}[22^{3/2} - 5^{3/2}]$ (C) $8 - \frac{2}{9}[22^{3/2} - 5^{3/2}]$ (D) $8 + \frac{2}{9}[22^{3/2} - 5^{3/2}]$

ANSWERS

Q. No.	Ans		Q. No.	Ans		Q. No.	Ans		Q. No.	Ans
1 (a)	B		9 (a)	(iii)		16 (i)	√26		24 (a)	B
1 (b)	A		9 (b)	(iv)		16 (ii)	√13		24 (b)	B
1 (c)	B		9 (c)	(i)		16 (iii)	√13		24 (c)	C
1 (d)	C		9 (d)	(ii)		16 (iv)	13/2		24 (d)	A
2 (a)	A		9 (e)	(iii)		17 (i)	√20		24 (e)	C
2 (b)	D		10 (a)	(ii)		17 (ii)	√8		25 (a)	C
2 (c)	C		10 (b)	(i)		17 (iii)	√20		25 (b)	B
2 (d)	A		10 (c)	(iii)		17 (iv)	6		25 (c)	D
3 (a)	c		10 (d)	(iv)		18 (A)	b		25 (d)	B
3 (b)	b		10 (e)	(i)		18 (B)	c		25 (e)	C
3 (c)	b		11 (a)	(a)		18 (C)	a		26 (i)	c
3 (d)	c		11 (b)	(b)		18 (D)	d		26 (ii)	d
4 (a)	c		11 (c)	(a)		19 (A)	b		26 (iii)	b
4 (b)	d		11 (d)	(b)		19 (B)	b		26 (iv)	c
4 (c)	d		11 (e)	(a)		19 (C)	a		26 (v)	d
4 (d)	b		12 (a)	(a)		19 (D)	b		27 (i)	c
4 (e)	a		12 (b)	(c)		20 (a)	A		27 (ii)	d
5 (i)	b		12 (c)	(b)		20 (b)	B		27 (iii)	a
5 (ii)	c		12 (d)	(c)		20 (c)	C		27 (iv)	b
5 (iii)	b		12 (e)	(a)		20 (d)	C		27 (v)	b
5 (iv)	c		13 (a)	D		21 (a)	A		28 (a)	C
5 (v)	b		13 (b)	A		21 (b)	B		28 (b)	A
6 (i)	a		13 (c)	D		21 (c)	C		28 (c)	B
6 (ii)	b		13 (d)	D		21 (d)	D		28 (d)	C
6 (iii)	a		13 (e)	C		22 (i)	b		28 (e)	A
6 (iv)	c		14 (a)	B		22 (ii)	d		29 (a)	B
6 (v)	b		14 (b)	C		22 (iii)	a		29 (b)	A
7 (a)	B		14 (c)	D		22 (iv)	c		29 (c)	D
7 (b)	A		14 (d)	A		22 (v)	b		29 (d)	C
7 (c)	A		14 (e)	C		23 (i)	c		29 (e)	D
7 (d)	C		15 (a)	A		23 (ii)	a			
7 (e)	C		15 (b)	D		23 (iii)	b			
8 (a)	B		15 (c)	C		23 (iv)	b			
8 (b)	C		15 (d)	D		23 (v)	c			
8 (c)	B		15 (e)	C						
8 (d)	A									
8 (e)	B									

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