## **CHAPTER 9**

## DIFFERENTIAL EQUATIONS

## **CASE STUDY BASED QUESTIONS**

QUESTION	Questions
	Read the passage given below and answer the following.
	Radium is a radioactive compound. Radioactivity is a property of some elements in which the
	decomposes continuously in modern time the carbon dating of fossils and old things are used to
	determine their life. Radium decomposes at the rate proportional to the quantity of radium
	present. It is found that in 25 years approximately 1.1% of a certain quantity radium has
	decomposed.
1.	Determine how long it will take for one half of the original amount .
	(a) 1567 yrs
	(b) 1528 yrs
	(c) 50 yrs
	(d) 100 yrs
2	Determine how long it will take for one fourth of the original?
	(d) 100/ yrs
	(b) $3133$ yrs
	(d) 25 vrs

Read the passage given below and answer the following questions:
The rate of change of temperature of a body is proportional to the difference between the
temperature of the body itself and that of the surroundings. This Law is known as Newton's Law
of Cooling.
Let S be the constant temperature of surroundings. Let T be the temperature of the body at any
time t.

3.	Which of the following is correct :
	(A) $T^{-}S^{=}Ce^{kt}$ , where $k^{>0}$ is a constant
	(B) $T^{-}S^{=}Ce^{-kt}$ , where $k^{>0}$ is a constant
	(C) $T^+ S^= Ce^{-kt}$ , where $k^{>0}$ is a constant
	(D) $T^+ S^= Ce^{kt}$ , where $k^{>0}$ is a constant
4	The temperature of a body in a room is 80° F. After five minutes the temperature of the body
	becomes $60^{\circ}$ F. After another 5 minutes the temperature becomes $50^{\circ}$ F. What is the
	temperature of surroundings?
	(A) 60° F
	(B) 20 <sup>o</sup> F
	(C) 80 <sup>o</sup> F
	(D) 40° F
5	Which of the following is a differential equation?
	(A) $x^2 - 3x + 3 = 0$
	(B) sin x + cosx= 0
	(C) $x + y = 7$
	(D) $\frac{dy}{dx} = e^x$
6	Which of the following is correct for this differential equation
	$\frac{dy}{dx} \cos_{x} = 0$
	(A) Only order is defined.
	(B) Only degree is defined.
	(C) Both are defined.
	(D) None of the above.

7	Let us consider the equation $x^{2+}y^{2} = r^{2}$ . It represents
	(A) The family of concentric circles
	(B) The family of straight lines.
	(C) The family of parabola.
	(D) The family of ellipses.
QUESTION	Read the passage given below and answer the following questions:
	We know the study of differential equation began in order to solve the problems that originated
	from different branches of mathematics, physics, biological sciences etc.
	i $i$ $t = 0$ $L$ $t$
	Let us consider RL circuit. This circuit contains resistor ( R ) and Inductor ( L). So it is known RL
	circuit. AT t =0 , the switch is closed and current does not pass through the circuit. When switch is
	on, the current passes through the circuit. As per the the electricity law, when voltage across a
	resistor of resistance R is equal to R <sub>i</sub> the voltage across an inductor is given by $L\frac{di}{dt}$ , where 'i' is
	the current.
8.	The equation of electromotive force (e.m.f) is $E = R_i + \frac{di}{dt}$ , where R is resistance, L is the self
	inductance and " ${ m i}$ " is electric current. Find the equation relating time (t) and electric current ( ${ m i}$ )
	(A) $i = \frac{E}{R} - \frac{\frac{Rt}{L}}{\frac{R}{R}}$ is the required equation.
	(B) $i = \frac{E}{R} + \frac{CeL}{R}$ is the required equation.
	(C) $i = -\frac{E}{R} - \frac{\frac{Rt}{L}}{\frac{Rt}{R}}$ is the required equation.

	(D) None of the above.
9	The general solution of the differential equation
	1+ 2
	$\frac{dy}{dt} = \frac{y}{1-t}$ is:
	$dx  1 + \frac{2}{x}$
	(A) tan y = tan x +C
	(B) $\tan x + \tan y = C$
	(C) $\tan^{-1}y^{=}\tan^{-1}x^{+}C$
	(D) None of the above
10	The integrating factor of the differential equation
	$dy = y^{\pm} \cos x i \sin x$
	$\frac{dx}{dx}$ y cosxis
	(A) e <sup>-x</sup>
	(B) <i>e</i> <sup>x</sup>
	(C) $-e^{-x}$
	(D) $-e^{x}$
11	Degree of the differential equation $\frac{dy}{dx} - 10\cos_x = 0$ is
	(A) 1
	(B) 2
	(C) 3
	(D) 4

12	The number of arbitrary constants in the particular solution of a differential equation of second
	order are:
	(A) 1
	(B) 2
	(C) 3
	(D) 0
QUESTION	Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation. Degree of a differential equation (when it is a polynomial equation in derivatives) is the highest power (positive integral index) of the highest order derivative involved in the given differential equation. Order and degree of a differential equation are always positive integers.
	Based on the above information, answer the following questions.
13	Order and degree of $\left(\frac{ds}{dt}\right)^4 + 3\frac{d^2s}{dt^2} = 0$ is:
	(a) O=2, D=1
	(b) O=1,D=1
	(c) O=3, D=4
	(d) None of these
14	Order and degree of $\left(\frac{d^2 y}{dx^2}\right)^4 + \cos\left(\frac{d y}{dx}\right) = 0$ is:
	(a) O=2, D=2
	(b) O=2, D=not defined
	(c) O=3, D= 1
	(d) O=1, D=1
15	Order and degree of $d^2 y$ = eac2y + ein2y is:
	dx
	(a) O=2, D=0
	(b) O=2, D=2
	(c) O=2, D=1

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	(d) O=2, D=3
16	The degree of the differential equation:
	$\left(\frac{d^2 y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$
	(a) 3
	(b) 2
	(c) 1
	(d) Not defined
17	$xy(\frac{d^2y}{dx^2}) + x(\frac{dy}{dx})^2 - y(\frac{dy}{dx}) = 0$
	(a) O=1, D=1
	(b) O=2, D=1
	(c) O=1, D=1
	(d) O=3, D=2
	Read the passage given below and answer the following questions:
QUESTION	A first order- first degree differential equation is of the form $\frac{dy}{dx}$ =F(x,y)
	If F(x,y) can be expressed as a product of g(x)h(y) where g(x) is a function of x and h(y) is a function of y, then $\frac{dy}{dx} = g(x)h(y)$
	$\int_{-}^{1} dy = \int g(x) dx$ h(y)
	Solution of differential equation by this method is called "variable separable".
	Based on the above information answer the following questions:
18	The general solution of differential equation $\frac{dy}{dx} = \frac{1+2}{1+\frac{y}{x}}$ is:

	(a) $\tan^{-1}y = \tan^{-1}x + c$
	(b) $tan(y) = tan(x)+c$
	(c) $\tan^{-1}(xy) = c$
	(d) None of these
19	Solution of differential equation $\frac{dy}{dx} = -4xy^2$ is:
	(a) $y = \frac{1}{2x^2 - c}$
	(b) $y = 2x^2 + c$
	(c) $y^{2} = x = c$
	(d) None of these
20	Solution of differential equation $\frac{dy_x + 1}{dx 2 - y}$ is:
	(a) $x^2+2x-4y+c=0$
	(b) $x^2+y^2+2x-4y+c=0$
	(c) $y^2 + 2x - 4x^2 + c = 0$
	(d) None of these
21	Solution of differential equation $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ is: $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$
	(a) $y=\tan \frac{x}{2}+c$
	(b) y=2tan $\frac{x}{2}$ -x <sup>2</sup> +c

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	(c) $y=2\tan \frac{x}{2}-x+c$
	(d) None of these
22	Solution of $x^5 \frac{dy}{dx} - y^5$ is:
	(a) $x^{-4} + y^{-4} = c$
	(b) $x^4 + y^4 = c$
	(c) $x^2 + y^2 = c$
	(d) None of these
QUESTION	A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the
	hospital, it was already dead. The pet lover wanted to find its time of death. He took the
	temperature of the cat at 11.30 pm which was 94.6 F. He took the temperature again after one
	hour; the temperature was lower than the first observation. It was 93.4 F. The room in which the
	cat was put is always at 70 F. The normal temperature of the cat is taken as 98.6 F when it was
	alive. The doctor estimated the time of death using Newton law of cooling which is governed by
	the differential equation: $\frac{dT}{dt} \propto$ (T - 70), where 70 F is the room temperature and T is the
	temperature of the object at time t. Substituting the two different observations of T and t made,
	in the solution of the differential equation
	$\frac{dT}{dt}$ = ( <i>T</i> – 70) where k is a constant of proportion, time of death is calculated.
23	Which method of solving a differential equation helped in calculation of the time of death?
	a. Variable separable method
	b. Solving Homogeneous differential equation
	c. Solving Linear differential equation
	d. all of the above

24	The solution of the differential equation
	$\frac{dT}{dt} = k(T - 70)$ is given by,
	a. log   T – 70  = kt + C
	b. log   T - 70  = log  kt  + C
	c. T – 70 = kt + C
	d. T – 70 = kt C
QUESTION	Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is
	directly proportional to the number of children who have not been administered the drops. By
	the end of 2nd week half the children have been given the polio drops. How many will have been
	given the drops by the end of 3rd week can be estimated using the solution to the differential
	equation $\frac{dy}{dx}$ = (50 – y) where x denotes the number of weeks and y the number of children who
	have been given the drops.
25	Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = (50 - y)$ .?
	a. Variable separable method
	b. Solving Homogeneous differential equation
	c. Solving Linear differential equation
	d. all of the above
26	The solution of the differential equation
	$\frac{dy}{dx} = \mathbf{k}(50 - \mathbf{y}) \text{ is given by,}$
	a. $\log   50 - y   = kx + C$
	b log   50 - y  = kx + C
	c. log   50 - y  = log  kx  + C
	d. 50 – y = kx + C

27	The value of c in the particular solution given that
	y(0)=0 and k = 0.049 is.
	a. log 50
	1
	b. log50
	c. 50
	d50

## ANSWER KEY

SR	Answer
1	(a)
2	(b)
3	В
4	D
5	D
6	С
7	А
8	А
9	С
10	А
11	А
12	D
13	(a) O=2, D=1
14	(b) O=2, D=not defined
15	(c) O=2, D=1
16	(d) Not defined
17	(b) O=2, D=1
18	(a) <sup>tan-1</sup> y= <sup>tan-1</sup> x+c
19	

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	1
	(a) $y = \frac{1}{2} a$
	<i>x</i> - <i>c</i>
20	(b) $x^2+y^2+2x-4y+c=0$
21	X
	(c) $y=2\tan 2x+c$
	- 4 4
22	(a) $x^{-4} + y^{-4} = c$
23	(a) Variable separable method
24	(a) $\log   T - 70   = kt + C$
25	(a) Variable separable method
26	(b) - log   50 - y  = kx + C
27	1
	(b) log –
	(~, ~, ~, 50

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